Transverse spin densities of octet baryons

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The University of Adelaide

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CSSM/QCDSF/UKQCD Collaborations

- **Jacob Bickerton (Adelaide)**
- R. Horsley (Edinburgh)
- Y. Nakamura (RIKEN, Kobe)
- P. Rakow (Liverpool)
- G. Schierholz (DESY)
- P. Shanahan (MIT)
- R. Young (Adelaide)

Outline

- Motivation: spin densities
- Simulation details
- Flavour-breaking expansion
- Form factors
- Spin densities
- Summary and Future work

Extension of PRL 98 (2007) 222001

Motivation: Electromagnetic Form Factors

$$\langle p', s'|J^\mu(\vec{q})|p, s\rangle = \bar{u}(p', s')\left[\gamma^\mu F_1(q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2m}F_2(q^2)\right]u(p, s)$$

$$F_1(0) = Q$$
$$F_1(0) + F_2(0) = \mu$$

$$Q_p = 1,\ Q^n = 0$$
$$\mu_p = 2.79\mu_N,\ \mu_n = -1.91\mu_N$$

Radii: $$r_i^2 = -6\frac{dF_i(q^2)}{dq^2}\Big|_{q^2=0}$$

$q^2 > 0$: “Look inside” hadron

Motivation: Electromagnetic Form Factors

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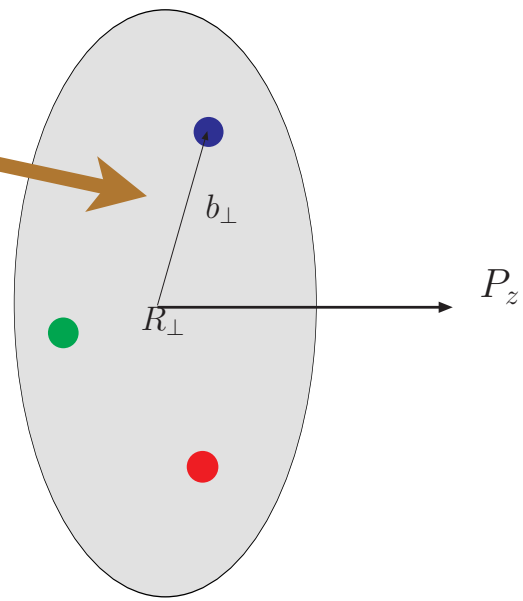
Quark (charge) distribution in transverse plane

$$q(b_\perp^2) = \int d^2q_\perp \, \mathrm{e}^{-i\vec{b}_\perp\cdot q_\perp} F_1(q^2)$$

Distance of (active) quark to the centre of momentum in a fast moving nucleon

Provide information on the size and internal charge densities



Impact Parameter GPDs (M. Burkardt, 2000)

Quark densities in the transverse plane

No momentum transfer in longitunidal direction

- Quark (charge) distribution in the transverse plane

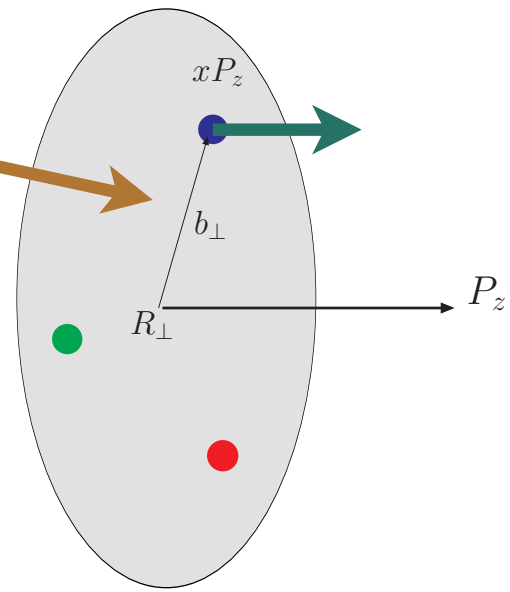
$$q(b_\perp^2) = \int d^2 q_\perp \, \mathrm{e}^{-i\vec{b}_\perp \cdot q_\perp} F_1(q^2)$$

- Probabilistic interpretation of GPDs, e.g. $H(x, \xi, q^2)$ at $\xi = 0$

$$q(x, b_\perp^2) = \int d^2 q_\perp \, \mathrm{e}^{-i\vec{b}_\perp \cdot \vec{q}_\perp} \, H(x, 0, q_\perp^2)$$

Distance of (active) quark to the centre of momentum in a fast moving nucleon

Decompose into contributions from individual quarks with momentum fraction, x



Spin dependence?

Diehl & Hägler, EPJ C44 (2005) 87-101 [hep-ph/0504175]

- **What about nucleon/quark spin?**
  - **How do they affect these quark distributions?**
- **Consider transverse nucleon $\vec{S}_\perp$ and/or quark $\vec{s}_\perp$ polarisations**
  - **Probability density for finding quark at impact parameter $\vec{b}_\perp$ is then**

$$F(b_\perp^2) = \int d^2q_\perp \, \mathrm{e}^{-i\vec{b}_\perp\cdot\vec{q}_\perp} \, F(q_\perp^2)$$

$$\begin{aligned}
\rho^n(b_\perp,\, s_\perp,\, S_\perp) &= \int_{-1}^{1} dx\, x^{n-1}\, \rho(x,\, b_\perp,\, s_\perp,\, S_\perp) \\
&= \frac{1}{2}\Big\{ A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{T\,n0}(b_\perp^2) - \frac{1}{4m^2} \nabla_{b_\perp} \tilde{A}_{T\,n0}(b_\perp^2) \right) \\
&+ \frac{b_\perp^i \epsilon^{ji}}{m} \left(S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{T\,n0}(b_\perp^2) \right) + s_\perp^i \left(2 b_\perp^i b_\perp^j - b_\perp^2 \delta^{ij} \right) S_\perp^j \frac{1}{m^2} \tilde{A}''_{T\,n0}(b_\perp^2) \Big\}
\end{aligned}$$

- **These *A, B, …*, functions define the moments w.r.t *x* of GPDs: "generalised form factors"**
- **This talk: only n=1 (F_1, F_2, g_T, …)**

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$$\rho^n(b_\perp, s_\perp, S_\perp) = \int_{-1}^{1} dx\, x^{n-1}\, \rho(x, b_\perp, s_\perp, S_\perp)$$
$$= \frac{1}{2}\Big\{A_{n0}(b_\perp^2) + s_\perp^i S_\perp^i \left(A_{T\,n0}(b_\perp^2) - \frac{1}{4m^2}\nabla_{b_\perp}\tilde{A}_{T\,n0}(b_\perp^2)\right)$$
$$+ \frac{b_\perp^i \epsilon^{ji}}{m}\left(S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{T\,n0}(b_\perp^2)\right) + s_\perp^i\left(2b_\perp^i b_\perp^j - b_\perp^2\delta^{ij}\right) S_\perp^j \frac{1}{m^2}\tilde{A}''_{T\,n0}(b_\perp^2)\Big\}$$

Unpolarised

$$F(b_\perp^2) = \int d^2q_\perp\, \mathrm{e}^{-i\vec{b}_\perp\cdot\vec{q}_\perp}\, F(q_\perp^2)$$

Spin dependence?

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Polarised Nucleon

$$F(b_\perp^2) = \int d^2q_\perp\, \mathrm{e}^{-i\vec{b}_\perp\cdot\vec{q}_\perp}\, F(q_\perp^2)$$

Spin dependence?

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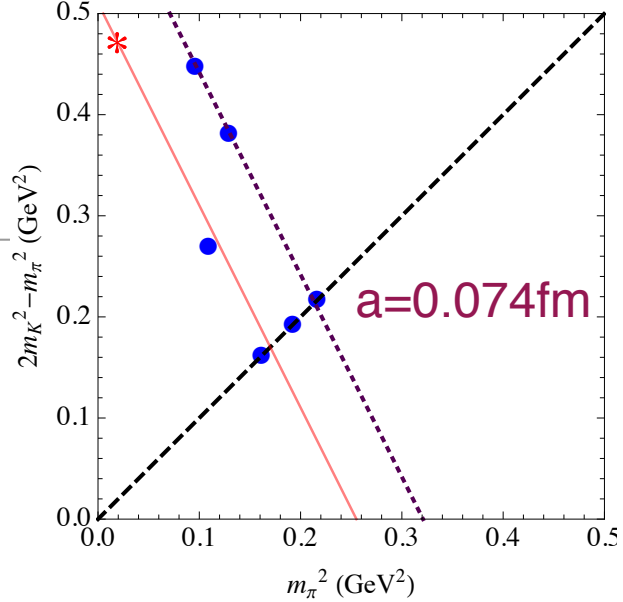
Polarised quark

$$F(b_\perp^2) = \int d^2q_\perp\, \mathrm{e}^{-i\vec{b}_\perp\cdot\vec{q}_\perp}\, F(q_\perp^2)$$

Lattice Set-Up



- N_f =2+1 O(a)-improved Clover fermions ("SLiNC" action)
- Tree-level Symanzik gluon action (plaq + rect)
- Most results from a single lattice spacing (a~0.08fm), but some now available at a~0.06fm
- Novel method for tuning the quark masses [arXiv:1003.1114 (PLB), 1102.5300 (PRD)]

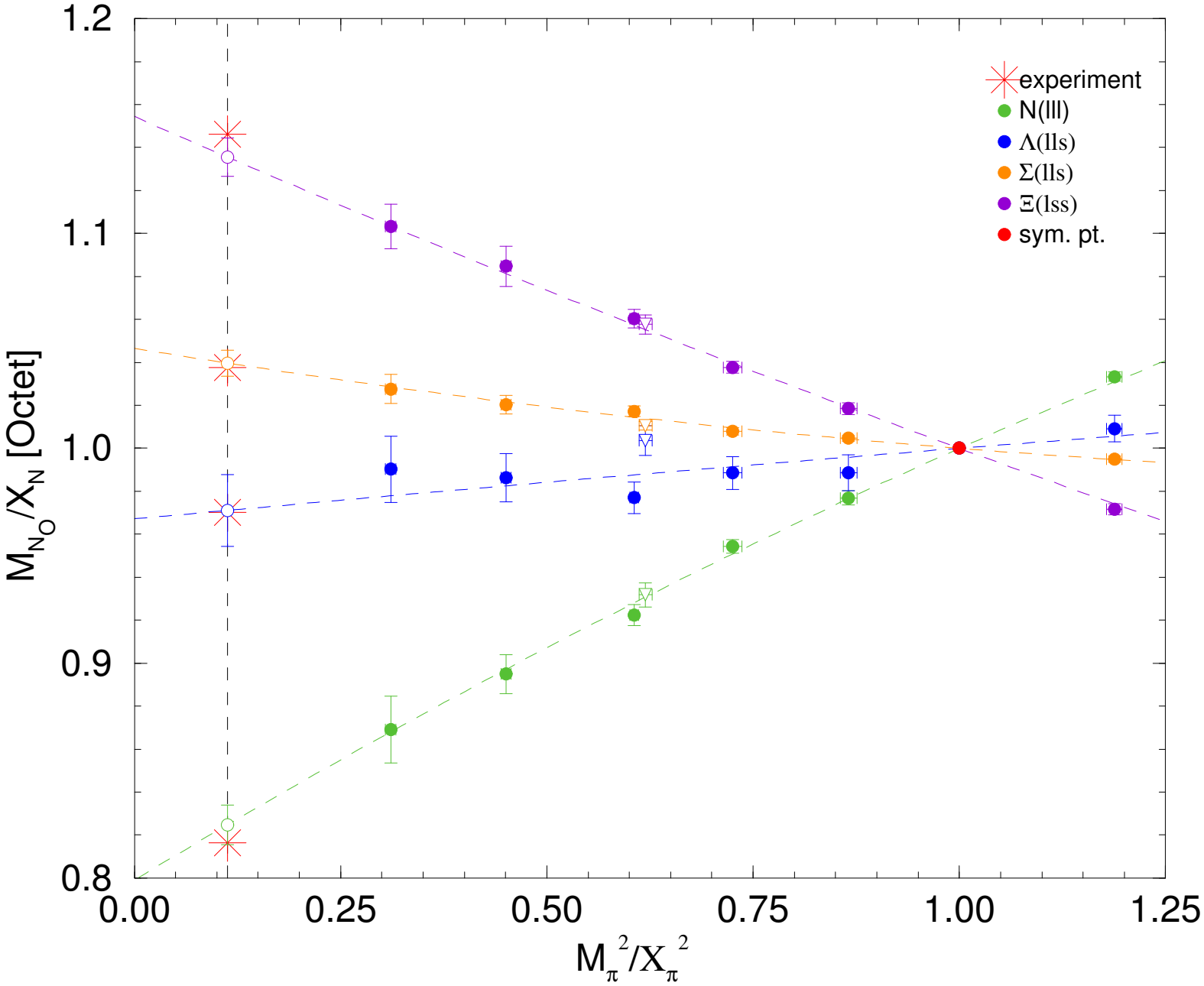
  See also parallel talk by R.Horsley, Thurs 15:00
- Results from O(1500) trajectories for each ensemble
- No study of systematics (disconnected, excited states, ...)

  [J.Dragos et al. arXiv:1606.03195]

  See also parallel talk by A.Chambers Mon 17:30

Baryon Octet ‘fan plot’

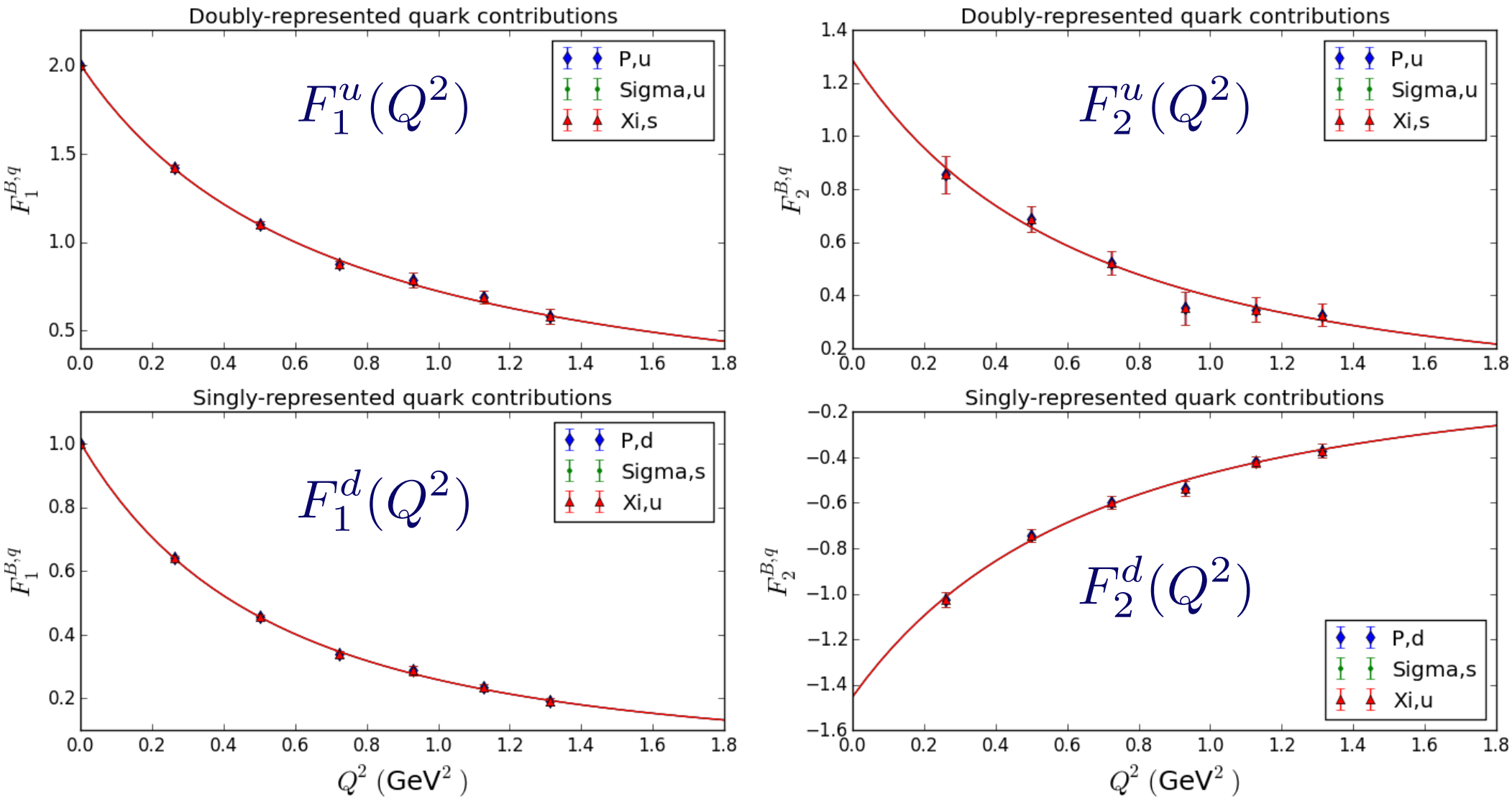
[arXiv:1102.5300 (PRD)]



Vector Form Factors

SU(3) symmetric

$$F_1(Q^2) = \frac{F(0)}{(1+Q^2/M^2)^2}$$

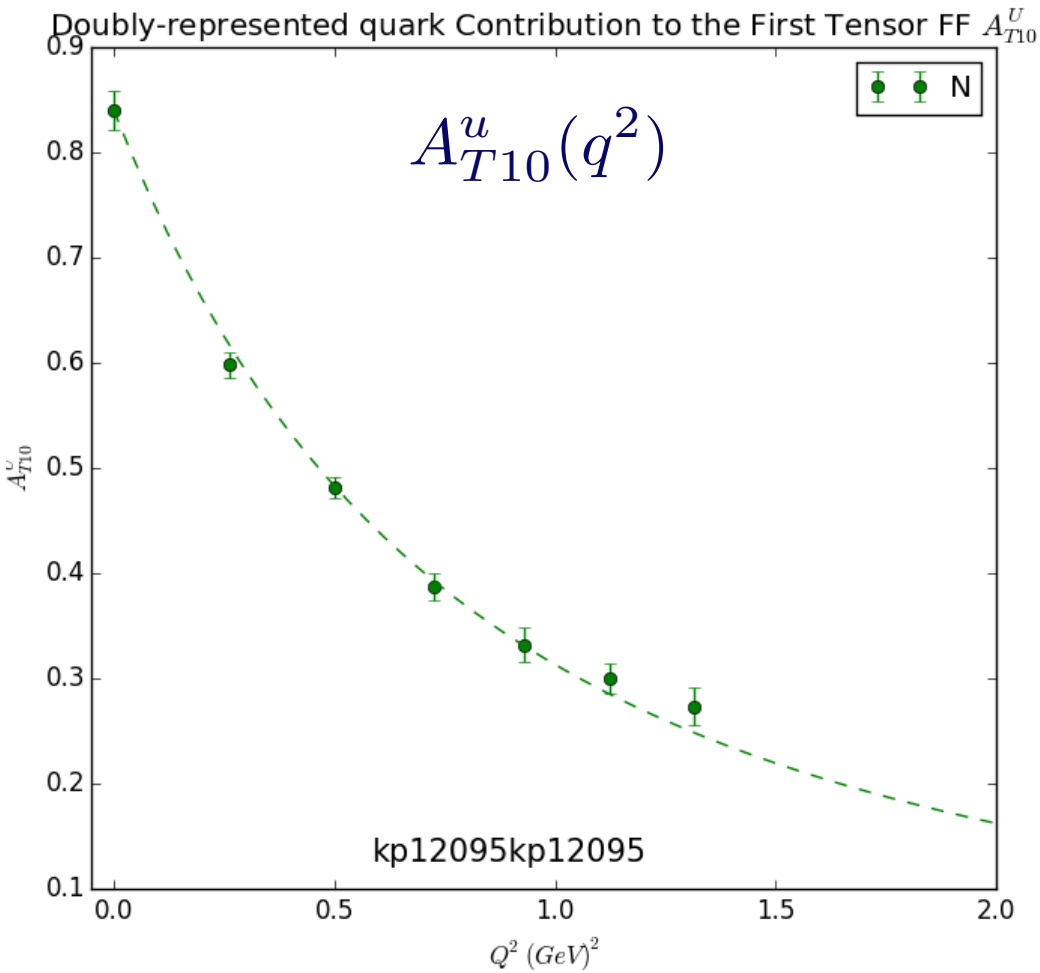


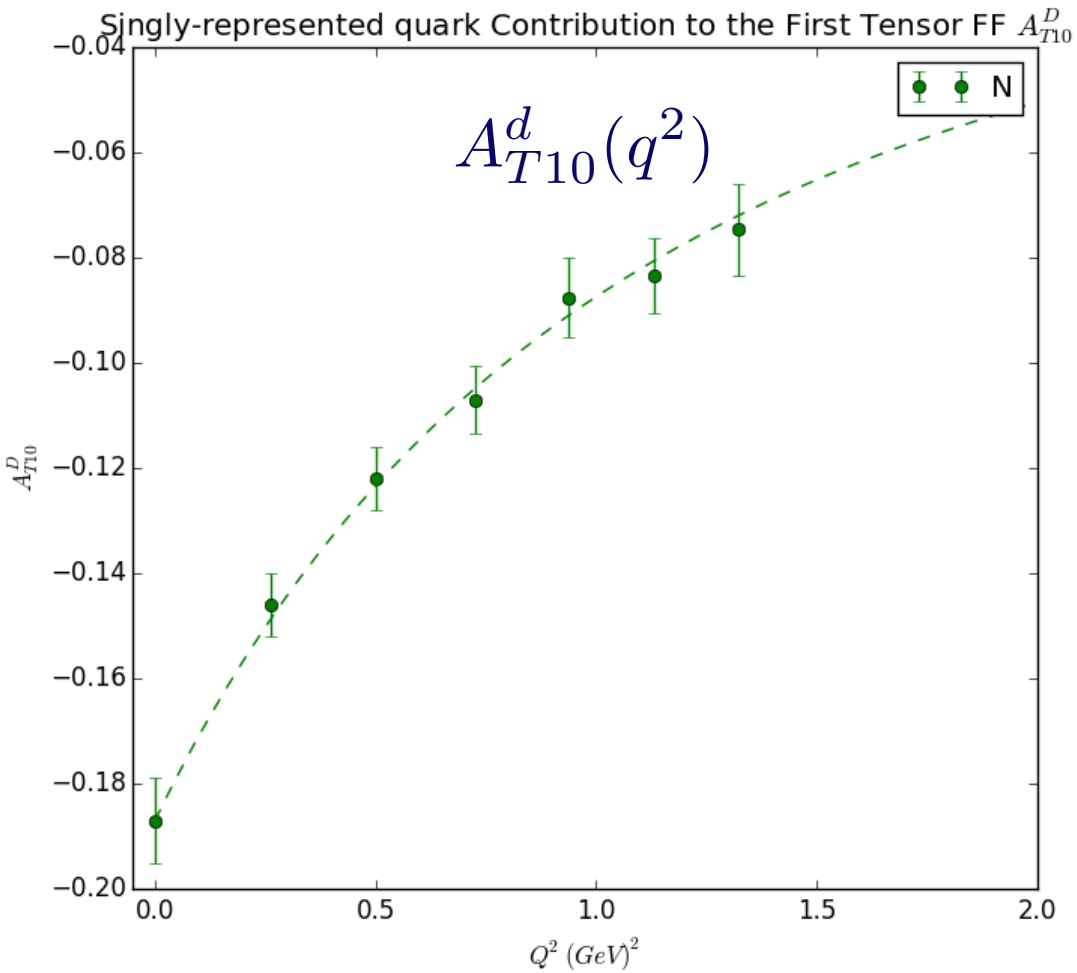
Tensor form factors

$$\bar{P}^\mu = \frac{1}{2}(p' + p)^\mu$$

- Tensor form factors are obtained from the matrix elements

$$\langle p',\, s'|\bar{\psi}(0)i\sigma^{\mu\nu}\psi(0)|p,\, s\rangle = \bar{u}(p',\, s')\{i\sigma^{\mu\nu}A_{T10}(q^2) + \frac{\overline{P}^{[\mu}q^{\nu]}}{m^2}\tilde{A}_{T10}(q^2) + \frac{\gamma^{[\mu}q^{\nu]}}{2m}B_{T10}(q^2)\}u(p,\, s)$$





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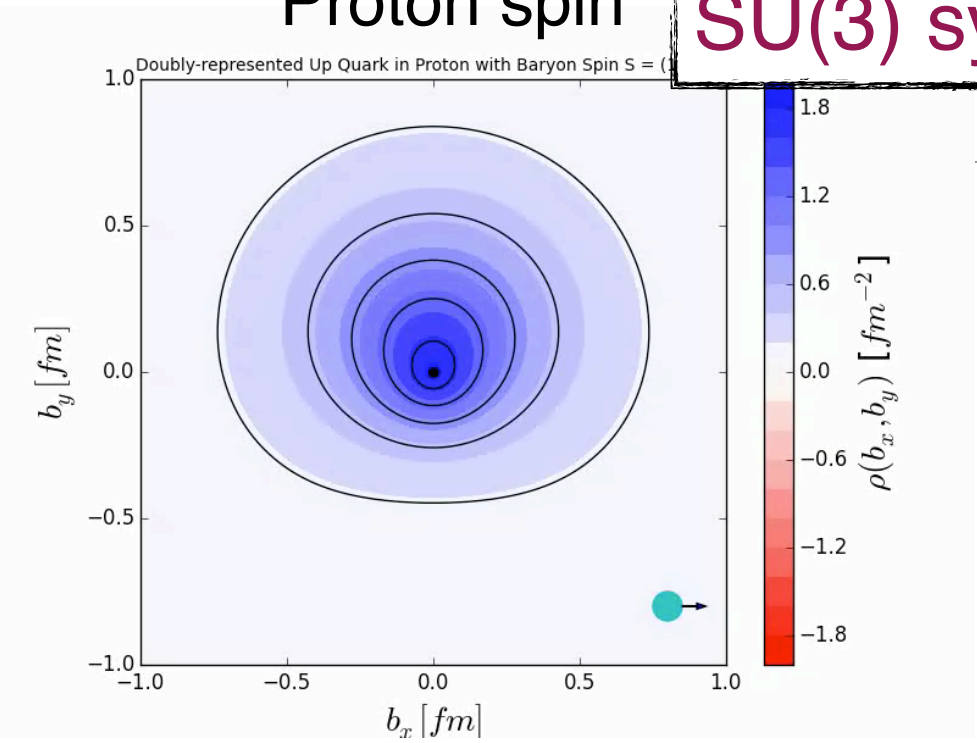
$$\boxed{F(b_\perp^2) = \int d^2q_\perp\, \mathrm{e}^{-i\vec{b}_\perp\cdot\vec{q}_\perp}\, F(q_\perp^2)}$$

Proton spin

SU(3) symmetric

Quark spin



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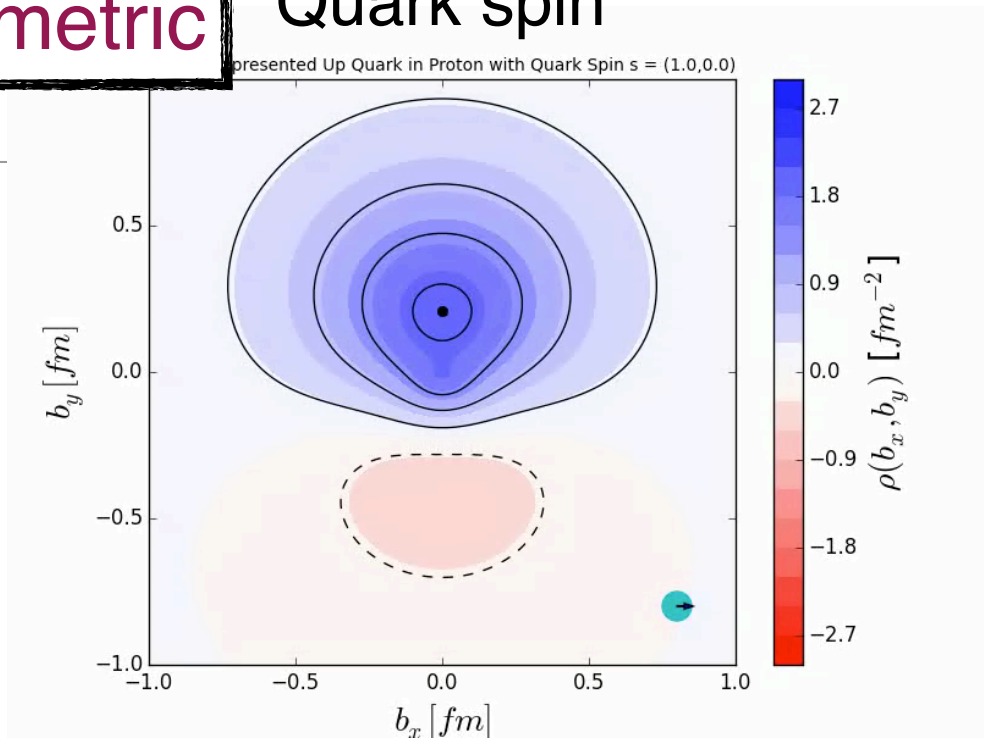

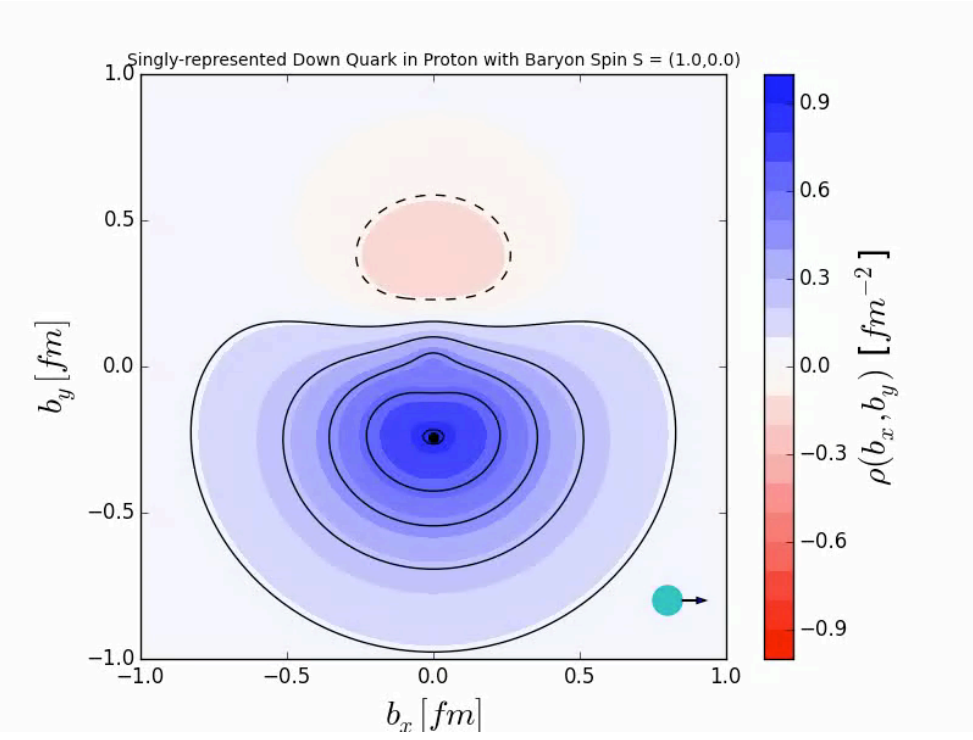

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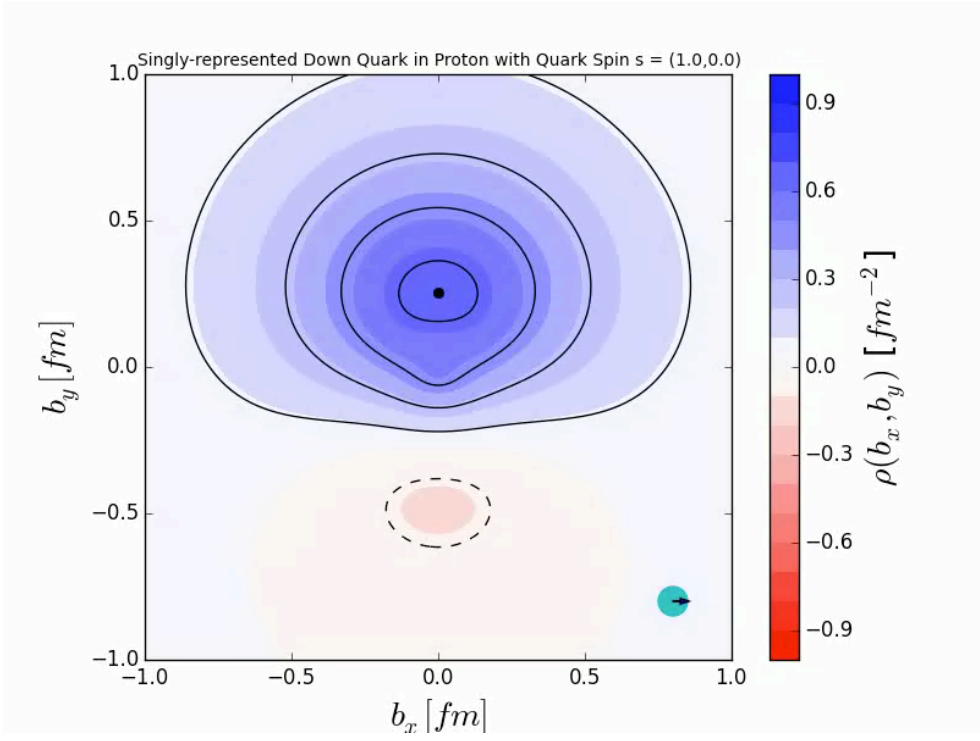


Mass extrapolation

- **Extrapolate to physical masses via flavour breaking expansion**
  - **Similar to masses** PRD 84, 074507 (2011) [arXiv:1106.3580]
  - **And decay constants** [R.Horsley, Thurs 15:00]
- **From SU(3) group theoretical arguments** [P.Rakow, Lattice 2012 (arXiv:1212.2564)]
  - **Construct combinations**

$$\eta = \frac{1}{\sqrt{6}}(\bar{u}\gamma u + \bar{d}\gamma d - 2\bar{s}\gamma s)$$

$$\pi = \frac{1}{\sqrt{2}}(\bar{u}\gamma u - \bar{d}\gamma d)$$

$$F_1 \equiv \frac{1}{\sqrt{3}}\left(A_{\bar{N}\eta N} - A_{\bar{\Xi}\eta\Xi}\right) = 2f - \frac{2}{\sqrt{3}}s_2\delta m_l$$

$$F_2 \equiv \left(A_{\bar{N}\pi N} + A_{\bar{\Xi}\pi\Xi}\right) = 2f + 4s_1\delta m_l$$

$$F_3 \equiv A_{\bar{\Sigma}\pi\Sigma} = 2f + (-2s_1 + \sqrt{3}s_2)\delta m_l$$

$$F_4 \equiv \frac{1}{\sqrt{2}}\mathrm{Re}\left(A_{\bar{\Sigma}K\Xi} - A_{\bar{N}K\Sigma}\right) = 2f - 2s_1\delta m_l$$

$$F_5 \equiv \frac{1}{\sqrt{3}}\mathrm{Re}\left(A_{\bar{\Lambda}K\Xi} - A_{\bar{N}K\Lambda}\right) = 2f + \frac{2}{\sqrt{3}}(\sqrt{3}s_1 - s_2)\delta m_l$$

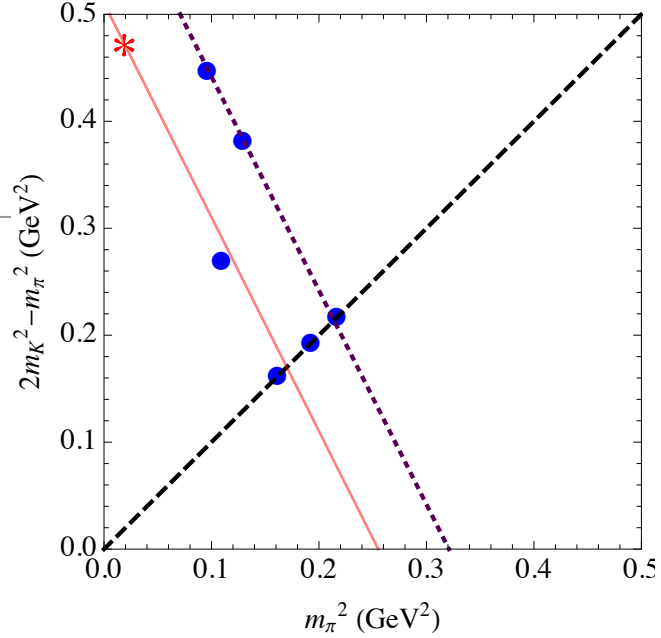
f-fan

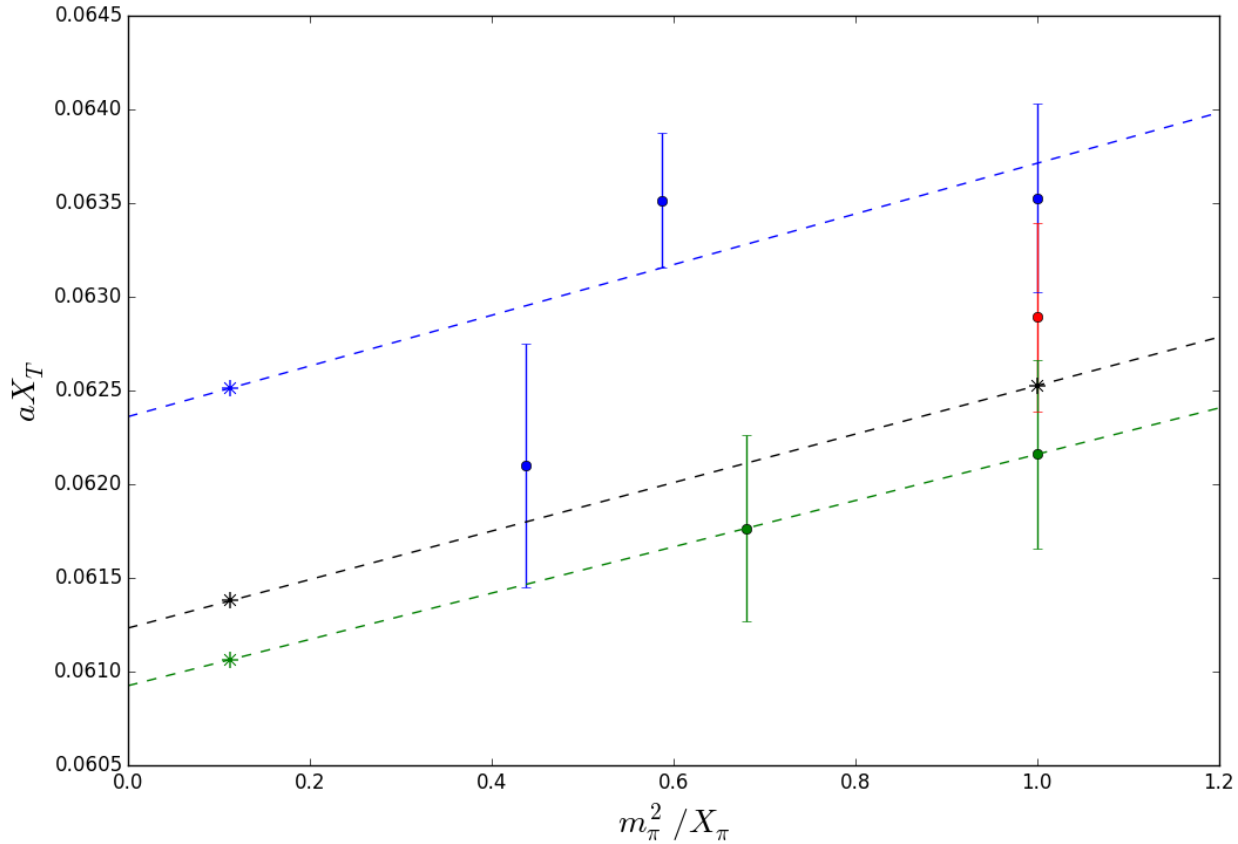
- **Up to quadratic terms,**

$$X_F = \frac{1}{6}(3F_1 + F_2 + 2F_3) = 2f + \mathcal{O}(\delta m_l^2)$$

- **transforms as a SU(3) flavour singlet**

flat along our trajectory $\bar{m} = \text{constant}$





$$Q^2 = 0$$

d-fan

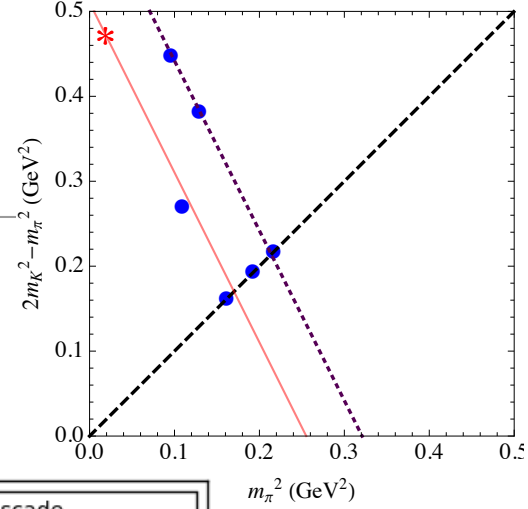
- **Similarly for the "d-fan"**

$$
\begin{aligned}
D_1 \equiv -\left(A_{\bar{N}\eta N} + A_{\bar{\Xi}\eta\Xi}\right) &= 2d - 2r_1\delta m_l \\
D_2 \equiv A_{\bar{\Sigma}\pi\Sigma} &= 2d + (r_1 + 2\sqrt{3}r_3)\delta m_l \\
D_3 \equiv -A_{\bar{\Lambda}\eta\Lambda} &= 2d - (r_1 + 2r_2)\delta m_l \\
D_4 \equiv \frac{1}{\sqrt{3}}\left(A_{\bar{N}\pi N} - A_{\bar{\Xi}\pi\Xi}\right) &= 2d - \frac{4}{\sqrt{3}}r_3\delta m_l \\
D_5 \equiv \mathrm{Re}A_{\bar{\Sigma}K\Lambda} &= 2d + (r_2 - \sqrt{3}r_3)\delta m_l \\
D_6 \equiv \frac{1}{\sqrt{6}}\mathrm{Re}\left(A_{\bar{N}K\Sigma} + A_{\bar{\Sigma}K\Xi}\right) &= 2d + \frac{2}{\sqrt{3}}r_3\delta m_l \\
D_7 \equiv -\mathrm{Re}\left(A_{\bar{N}K\Lambda} + A_{\bar{\Lambda}K\Xi}\right) &= 2d - 2r_2\delta m_l
\end{aligned}
$$

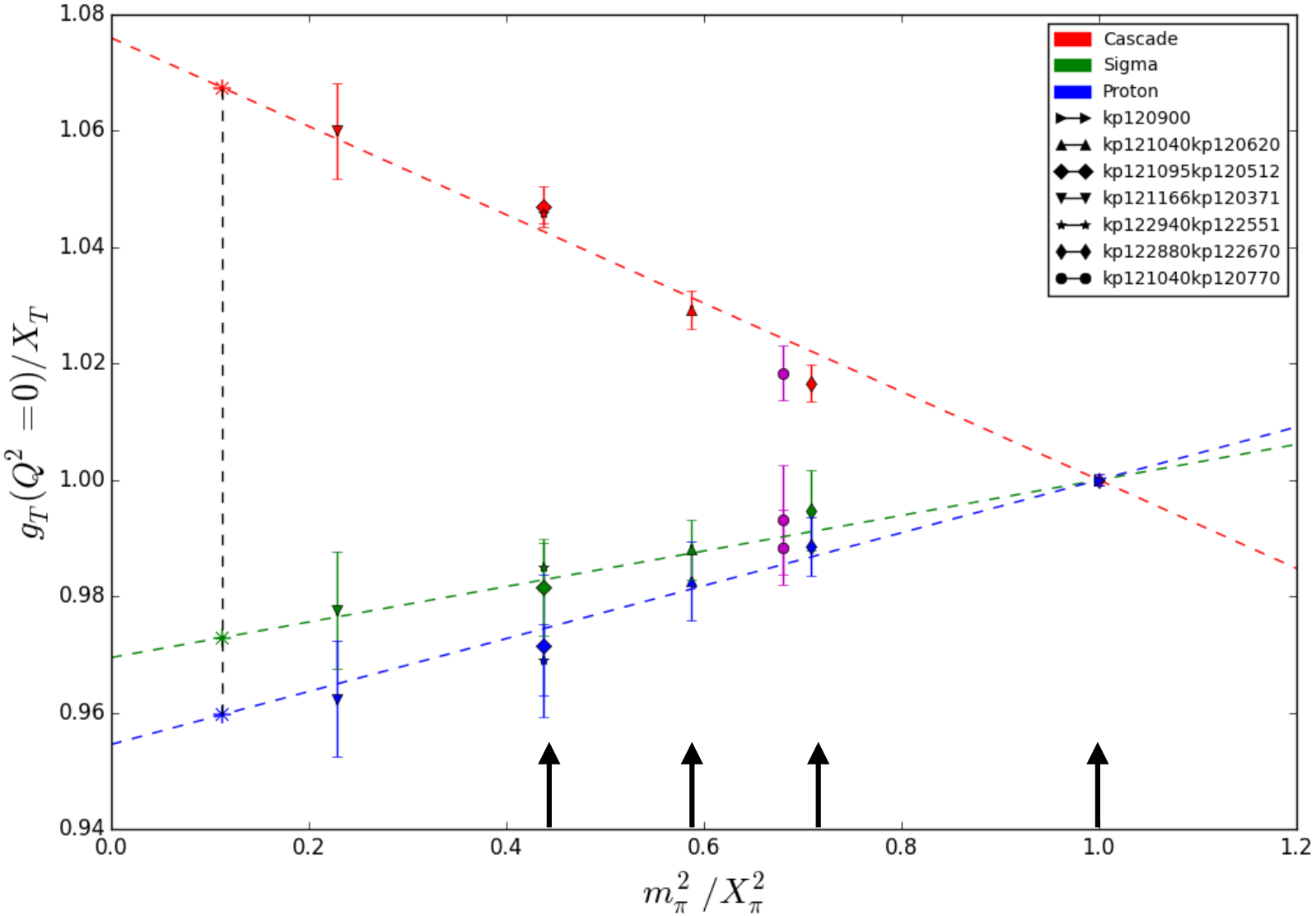
$$
X_D = \frac{1}{4}(D_1 + 2D_2 + D_4) = 2d + \mathcal{O}(\delta m_l^2)
$$

f-fan

$$Q^2 = 0$$



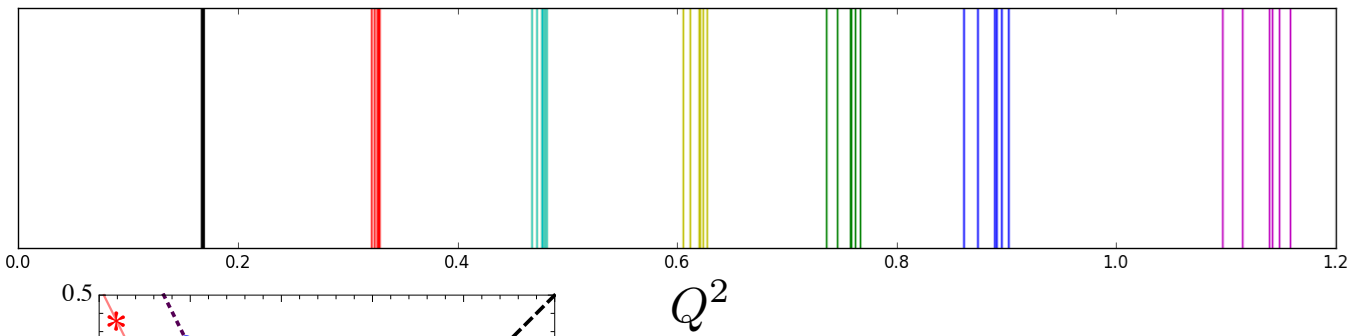
- **Tensor charge (doubly-represented quark contribution)**
- **Ratio "fans out" from SU(3)-symmetric point**

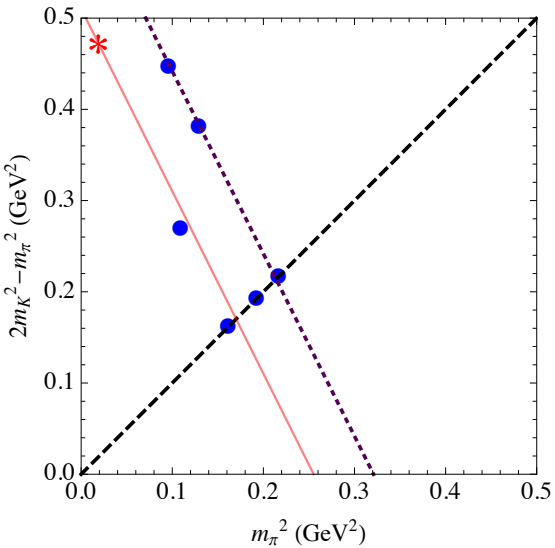


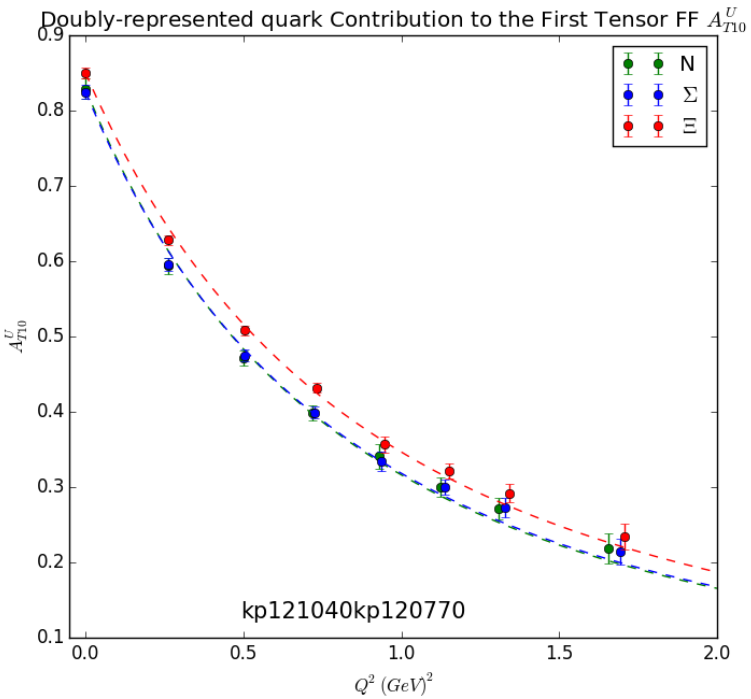
used in fit

Form factors at $Q^2 \neq 0$

- **SU(3) breaking expansion valid at a fixed Q^2**
- **All ensembles have L=32**
- **Small difference in Q^2 comes from difference in baryon masses across ensembles**
- **Bin results in Q^2 and shift form factors to centre of bin using dipole form**

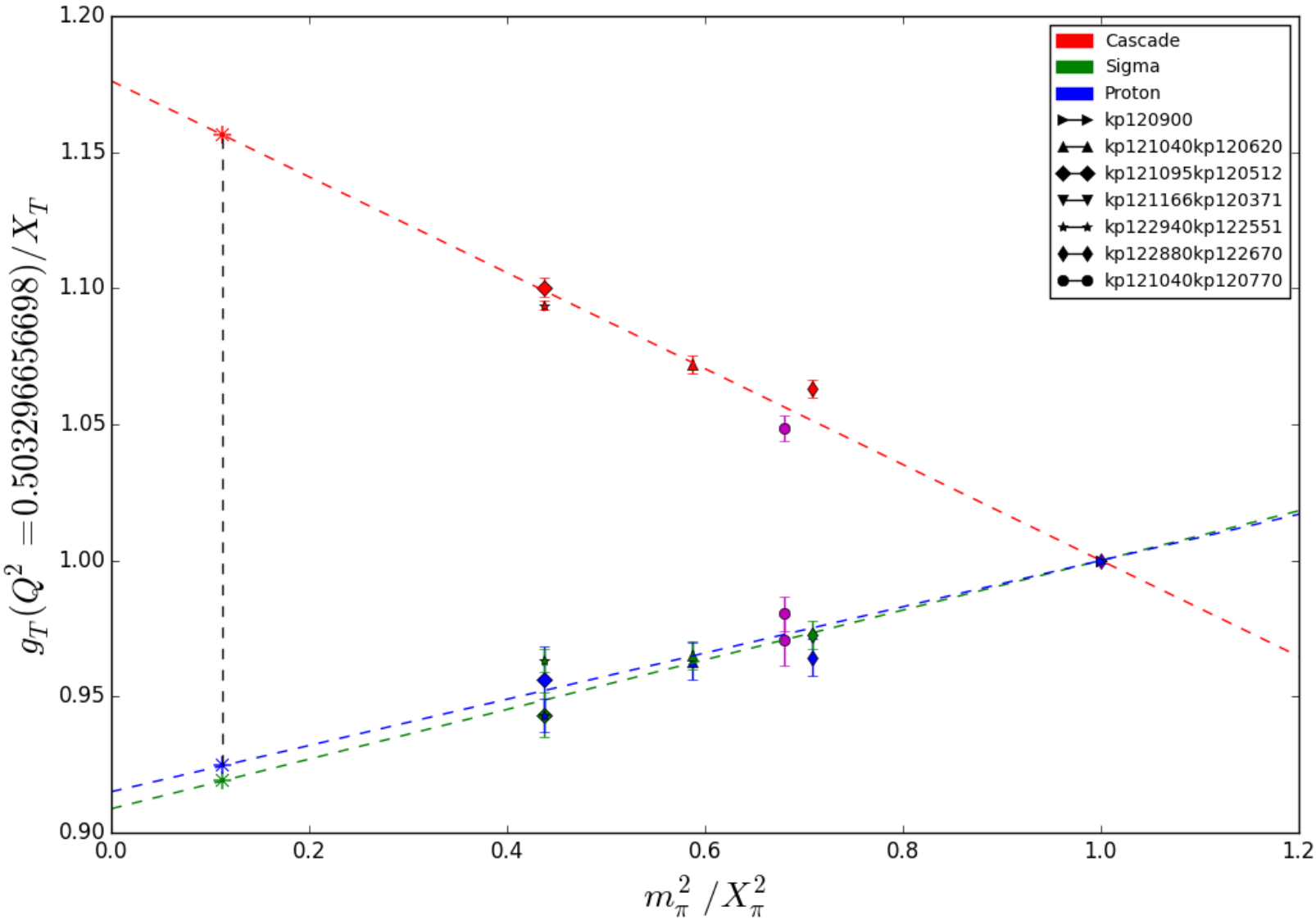






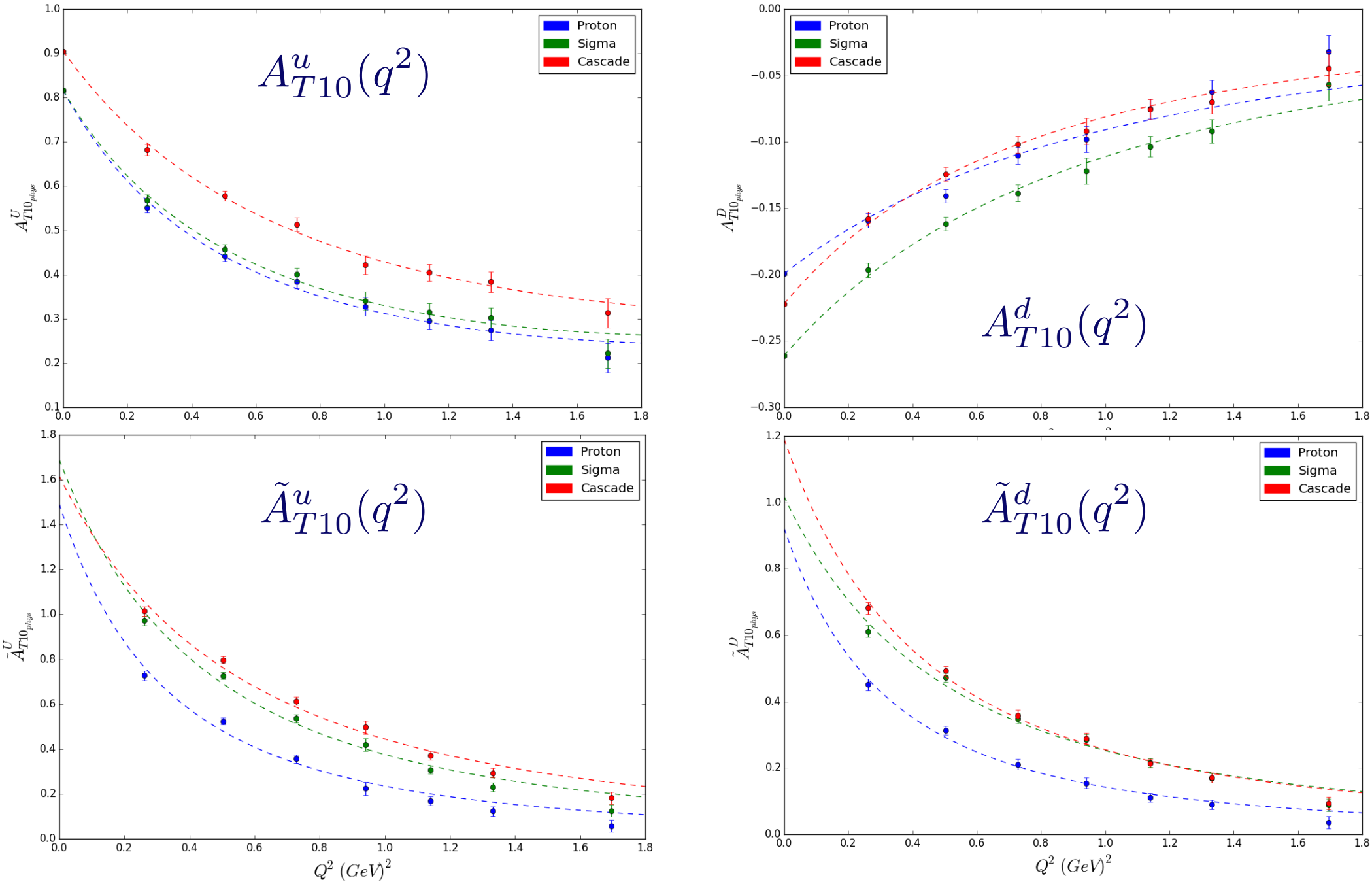
f-fan

$Q^2 \approx 0.5\,\mathrm{GeV}^2$



Physical mass form factors

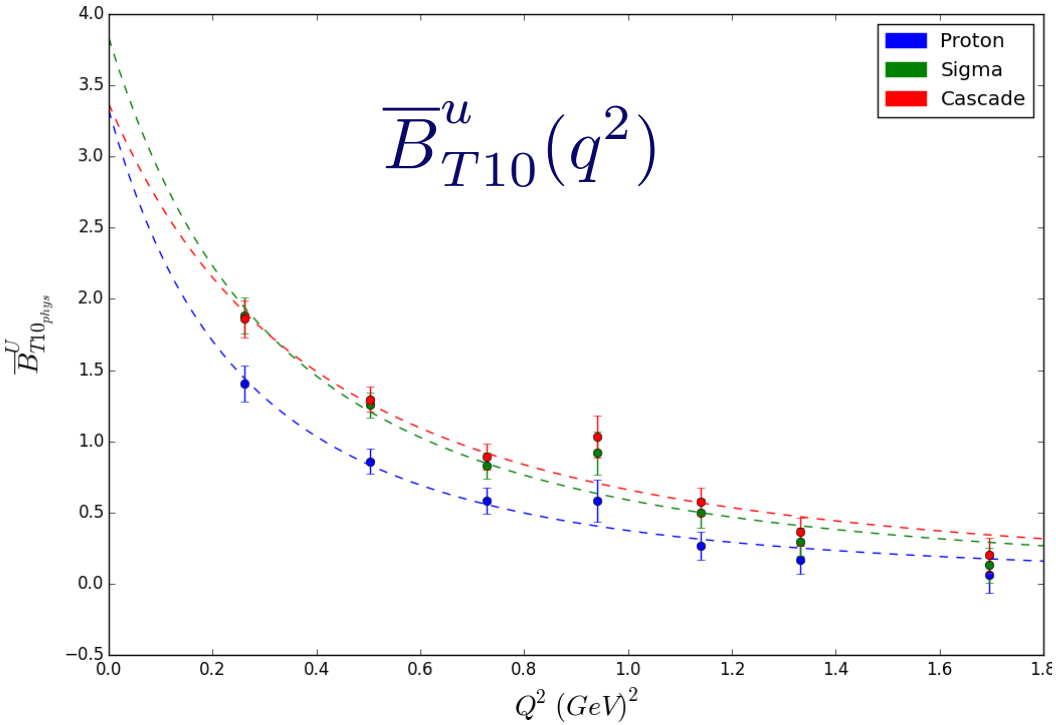
Preliminary

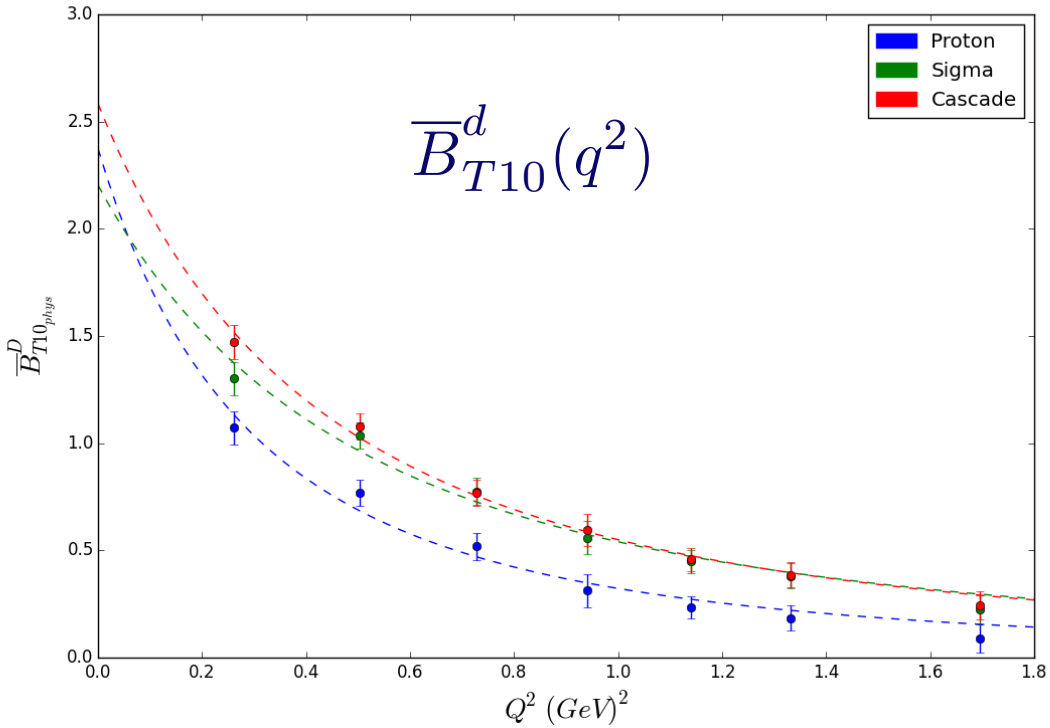


Physical mass form factors

Preliminary

$$\overline{B}_{Tn0}(q^2) = B_{Tn0}(q^2) + 2\tilde{A}_{Tn0}(q^2)$$





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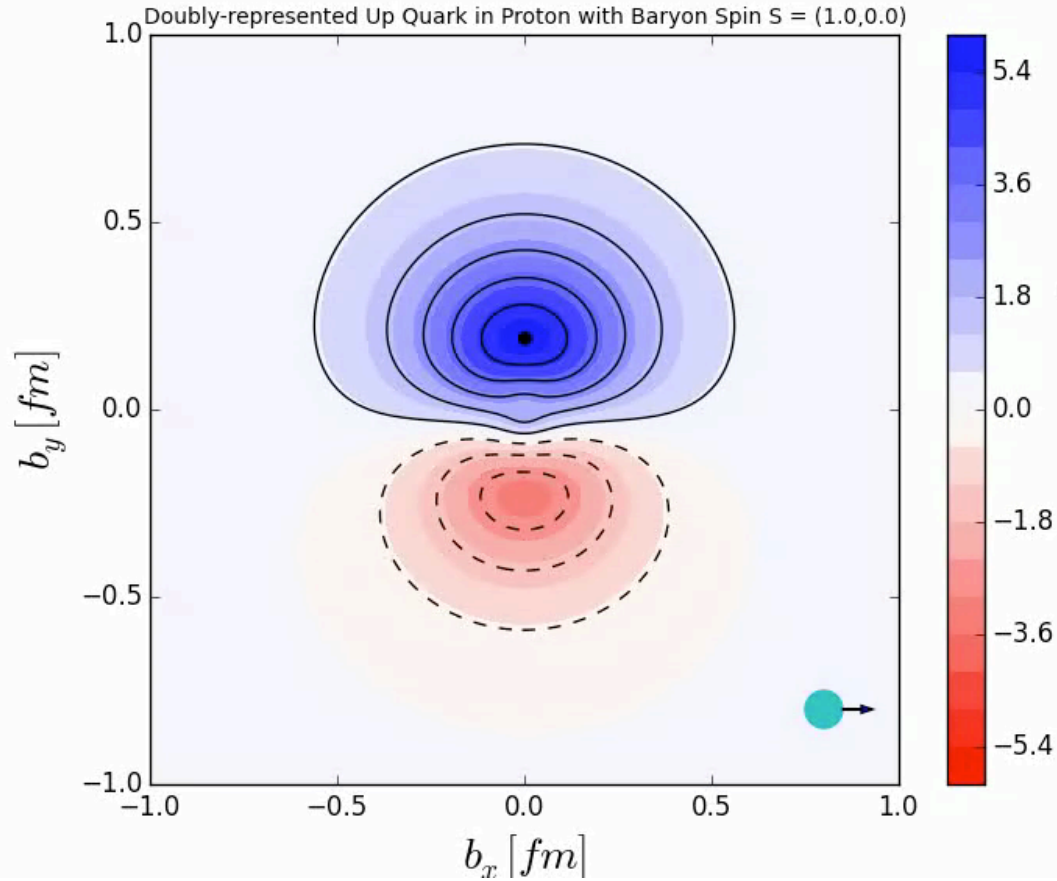
$$+ \frac{b_\perp^i \epsilon^{ji}}{m}\left(S_\perp^i B'_{n0}(b_\perp^2) + s_\perp^i \overline{B}'_{T\,n0}(b_\perp^2)\right) + s_\perp^i\left(2b_\perp^i b_\perp^j - b_\perp^2\delta^{ij}\right)S_\perp^j \frac{1}{m^2}\tilde{A}''_{T\,n0}(b_\perp^2)\Big\}$$

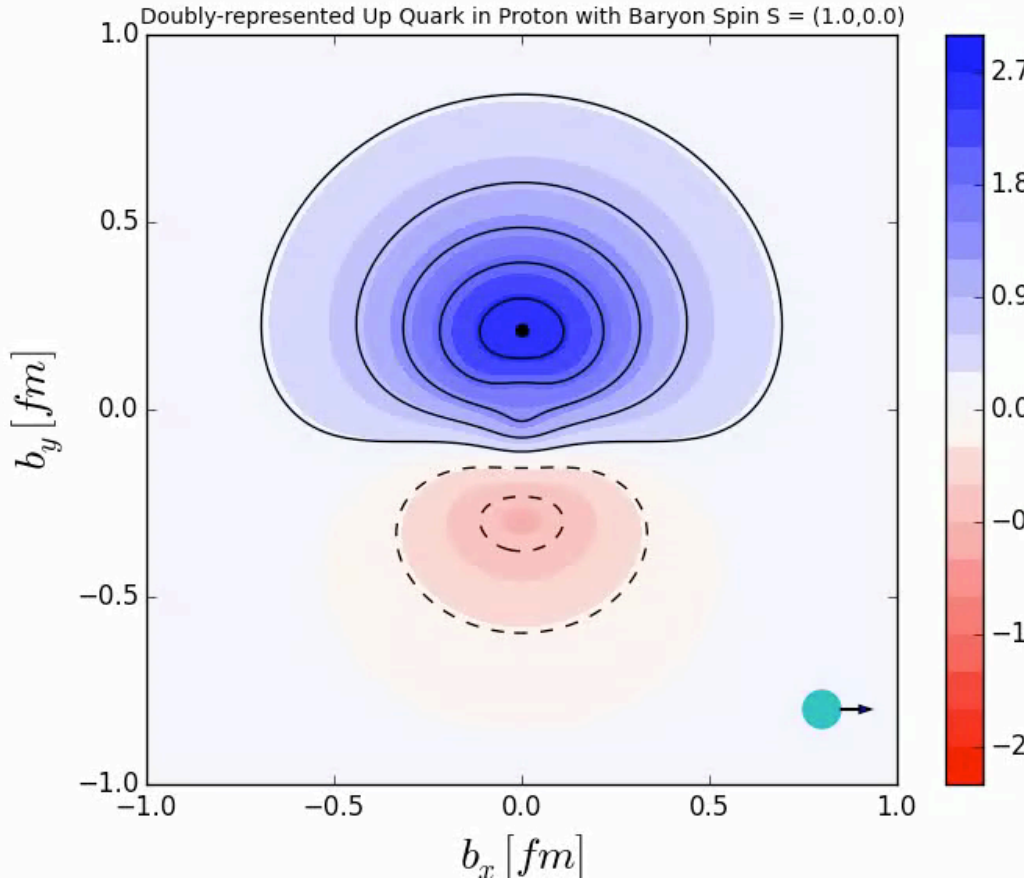
u-quark

Hyperon spin

Proton



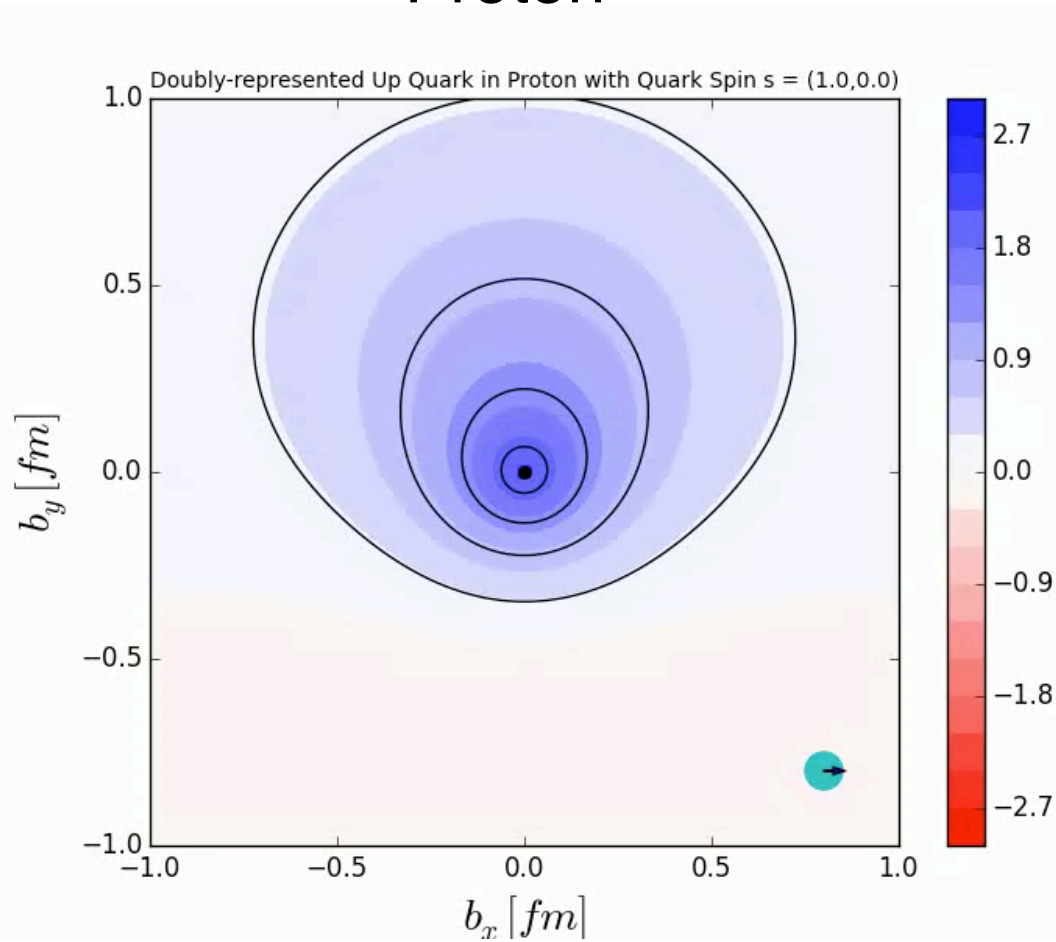
Sigma



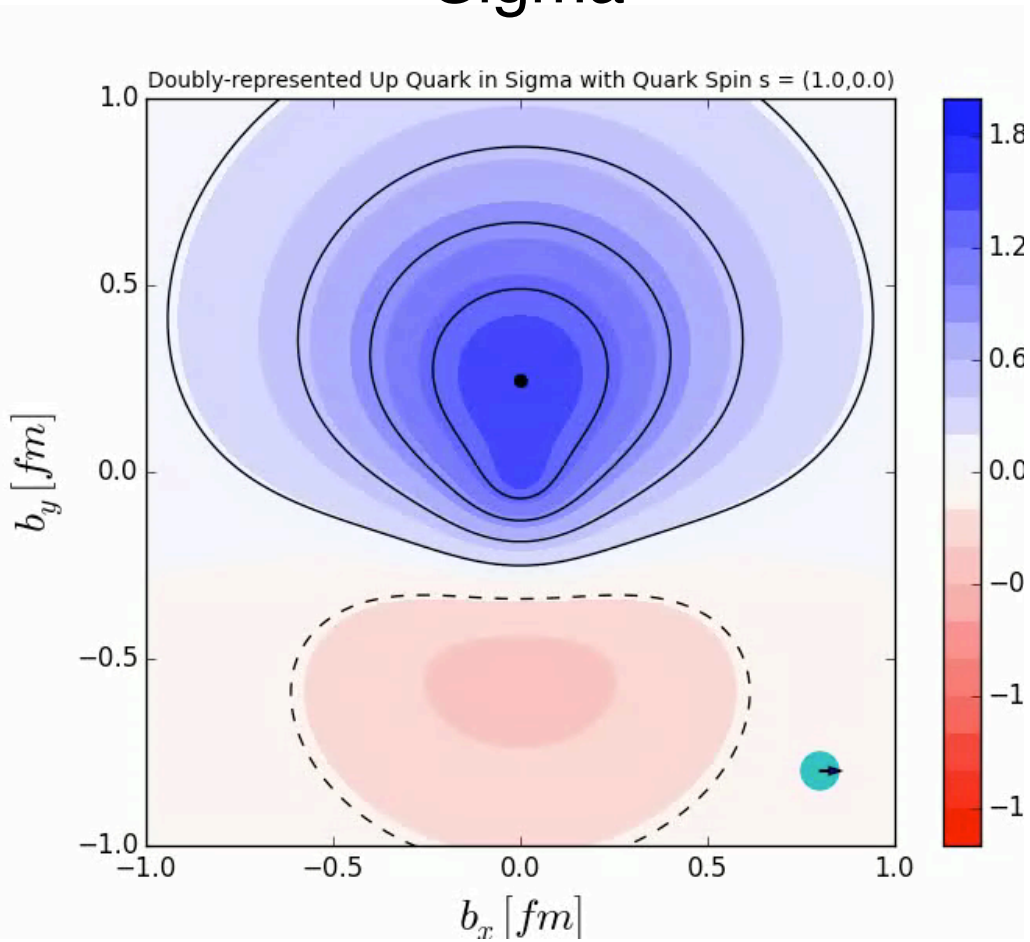
u-quark

Quark spin

Proton

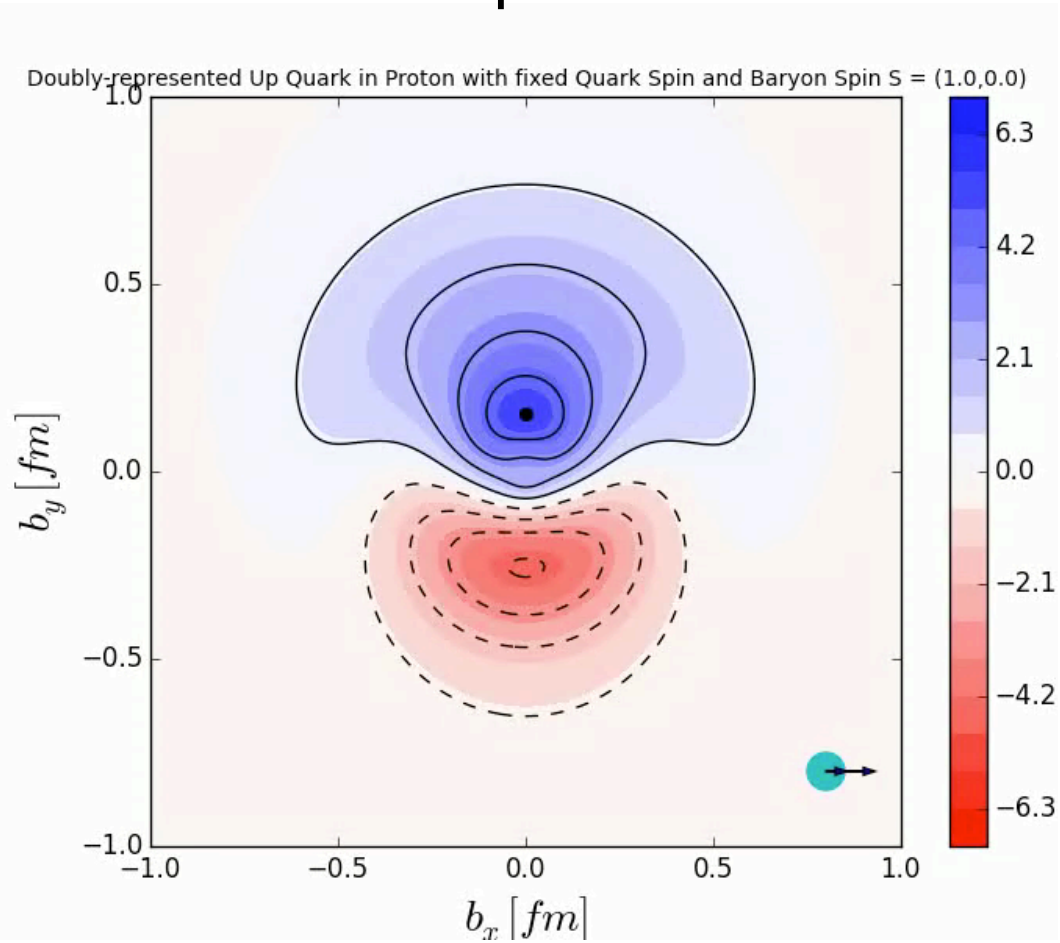


Sigma

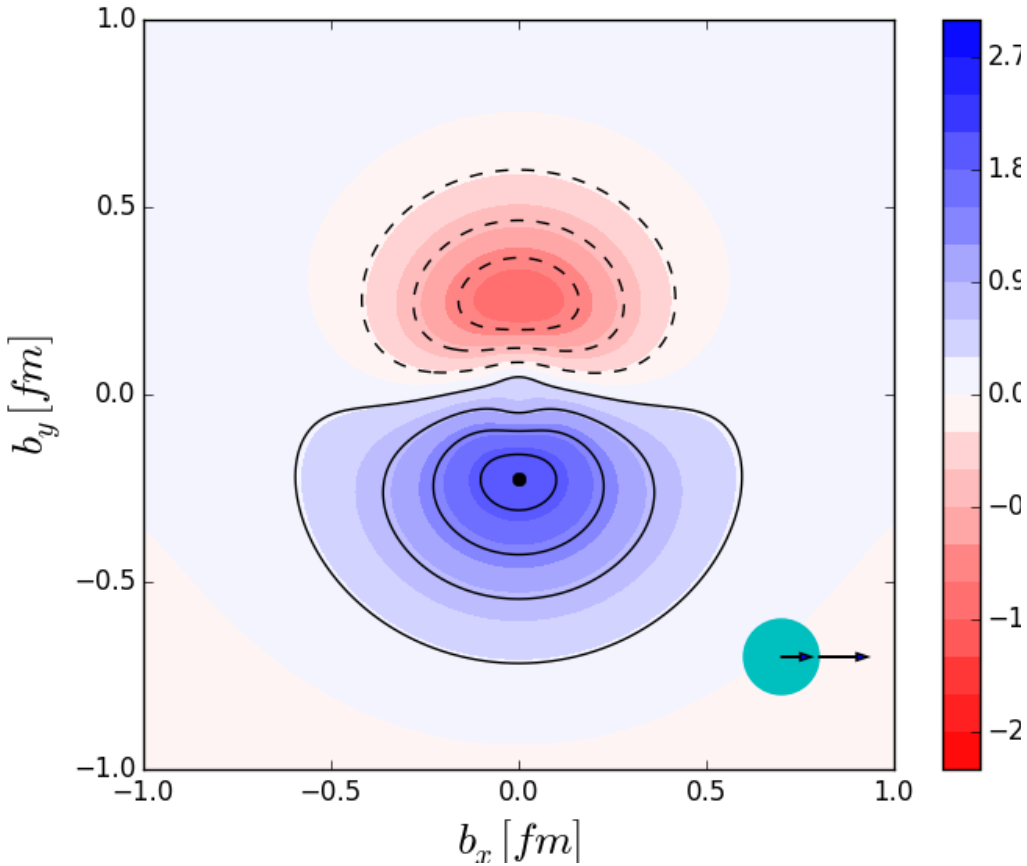


Spin densities - proton

u-quark

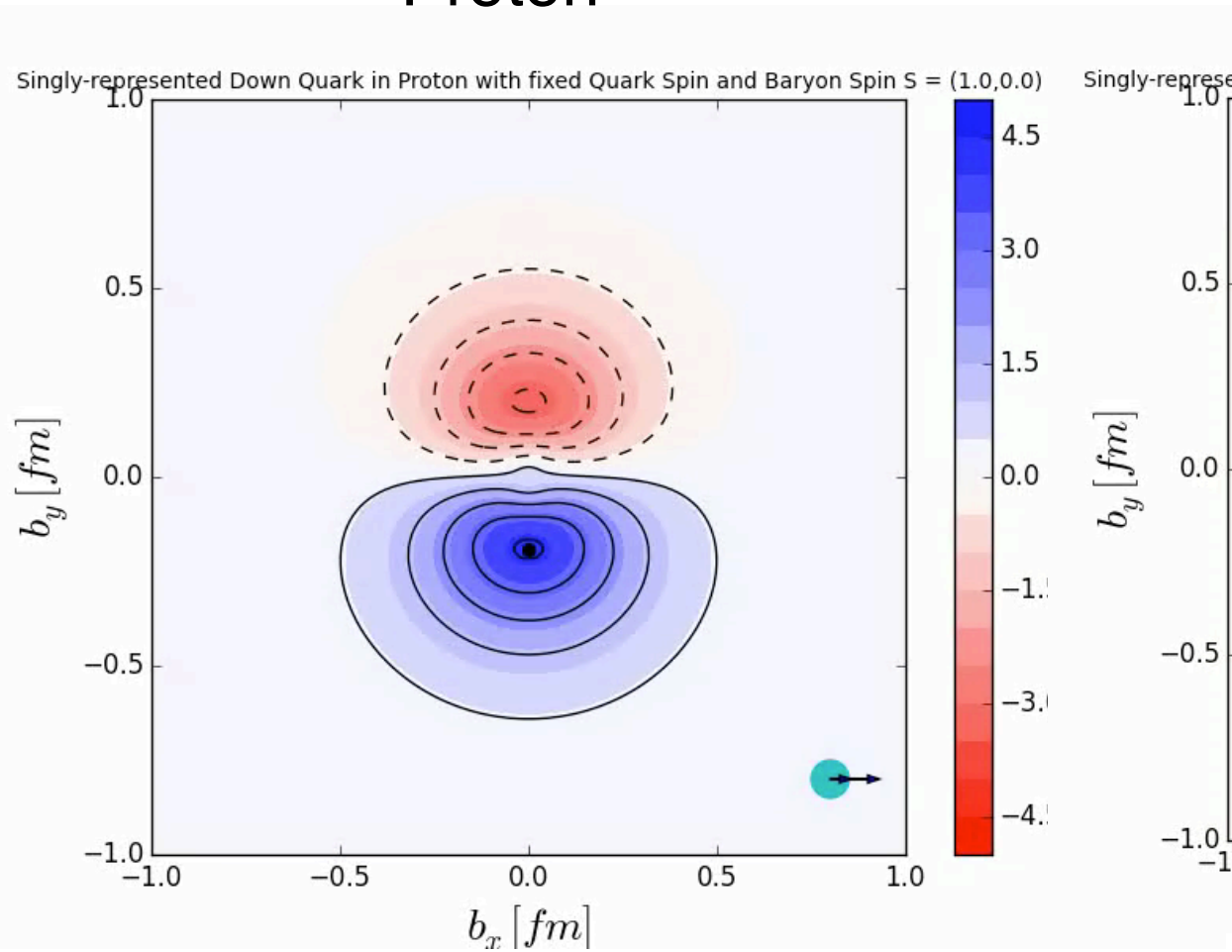


d-quark

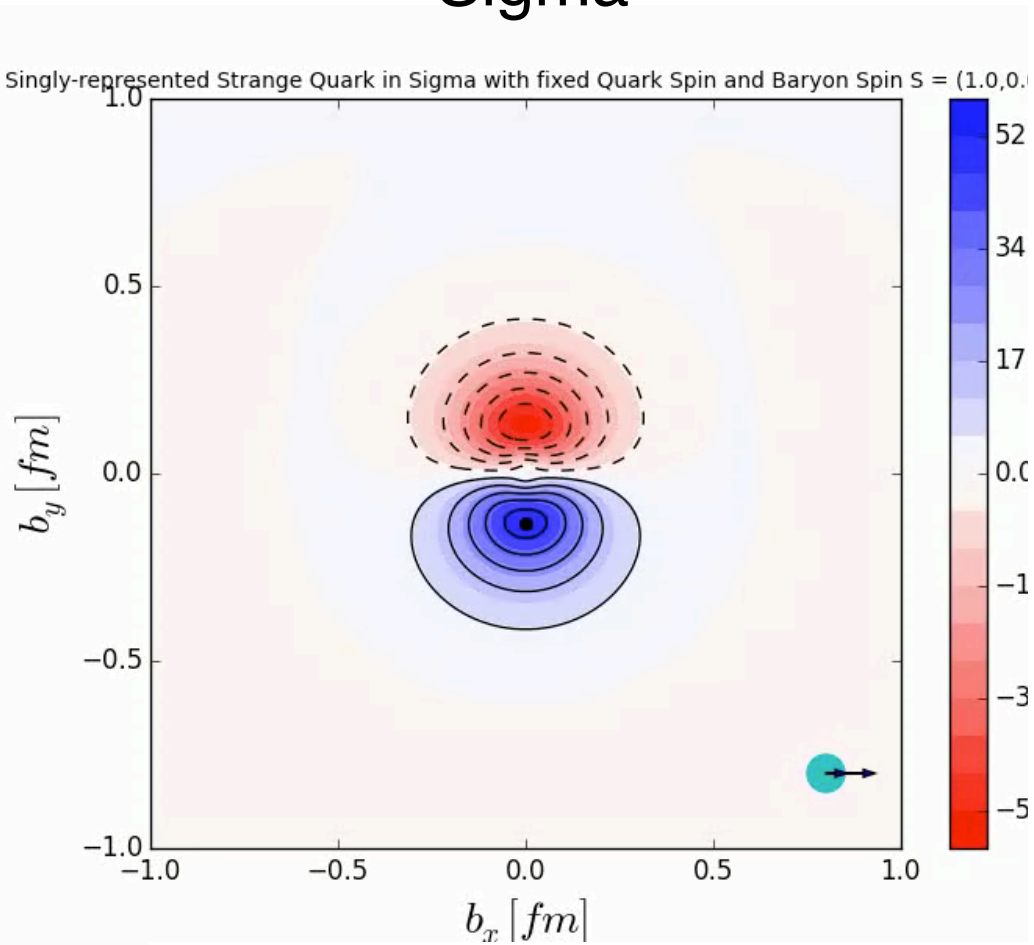


“d”-quark

Proton



Sigma



Summary

- SU(3) flavour breaking expansions
  - Successfully applied to octet baryon form factors (vector & tensor)
- Spin densities
  - Construct combinations of Fourier transformed form factors
  - Reveal non-trivial spin densities in transverse plane
  - Deformation shape similar across baryon octet
  - Small SU(3) flavour breaking effects
- Future work (all form factors)
  - Control excited states through variational approach
  - More ensembles
  - Different Q^2 ansatz (“Kelly”), etc, etc,