Disconnected contributions to nucleon observables with $N_f = 2$ twisted-mass clover fermions at the physical light quark mass

Alejandro Vaquero

INFN Sezione Milano Bicocca

July 28th, 2016

On behalf of the TMC collaboration, with:

- Abdou Abdel-Rehim, CaSToRC at the Cyprus Institute
- Constantia Alexandrou, CaSToRC and University of Cyprus
- Jacob Finkenrath, CaSToRC at The Cyprus Institute
- Christos Kallidonis, CaSToRC at The Cyprus Institute
- Giannis Koutsou, CaSToRC at The Cyprus Institute
- Martha Constantinou, CaSToRC at The Cyprus Institute
- Kyriakos Hadjiyiannakou, The George Washington University

- Karl Jansen, DESY, NIC
- Julia Volmer, DESY, NIC
Outline

- Methods
  - Stochastic methods and variance reduction
  - Exact deflation, low-mode reconstruction
  - Analysis of stochastic errors
  - Hierarchical probing
- Results
  - Removing the excited states
  - $\sigma$-terms and $g_A$
  - $g_T$, $\langle x \rangle$ and the helicity
- Conclusions
Methods

- **Stochastic estimation**
  Fill $N$ vectors $|\eta_j\rangle$ with $Z_N$ noise and compute $M |s_j\rangle = |\eta_j\rangle$
  \[
  M_E^{-1} := \frac{1}{N} \sum_{j=1}^{N} |s_j\rangle \langle \eta_j| \approx M^{-1}
  \]
  Poor performance, error decreases as $1/\sqrt{N}$

- **Truncated Solver Method**
  Increases $N$ cheaply with low-precision (LP) estimation, correct afterwards
  \[
  M_E^{-1} := \frac{1}{N_{HP}} \sum_{j=1}^{N_{HP}} (|s_j\rangle \langle \eta_j|_{HP} - |s_j\rangle \langle \eta_j|_{LP}) + \frac{1}{N_{LP}} \sum_{j=N_{HP}+1}^{N_{HP}+N_{LP}} |s_j\rangle \langle \eta_j|_{LP}
  \]
  Fails for light masses due to loss of correlation between HP and LP

- **One-End Trick**
  Twisted-mass exclusive, based on identities. Example:
  \[
  \text{Tr} \left[ X (M_d^{-1} - M_u^{-1}) \right] = 2i\mu \text{Tr} \left[ (M_u^\dagger)^{-1} X \gamma_5 M_u^{-1} \right] \implies 2i\mu \sum_r \langle s^\dagger X \gamma_5 s \rangle_r
  \]
  Exact deflation
Exact reconstruction, inversion acceleration

- Exact deflation of the noise vector $|\eta_D\rangle = |\eta\rangle - \langle v_j | \eta \rangle \langle v_j |$, $|s_D\rangle = M^{-1} |\eta_D\rangle$

Exact (Full op!)

\[
M_{\text{low}}^{-1} = \sum_{j=1}^{N} \frac{1}{\lambda_j} |v_j\rangle \langle v_j|
\]

Stochastic

\[
|s\rangle = |s_D\rangle + \sum_{j=1}^{N} \frac{1}{\lambda_j} \langle v_j | \eta \rangle
\]

- How many low-modes do I need?

\[
\sigma_{\text{Loop}}^2 \approx 250 \text{ on there is little to gain in the exact reconstruction}
\]

- Inversions are still accelerated as $N_{ev}$ increases

Gambhir, Stathopoulos, Orginos 2016
Exact reconstruction

- Idea: Solve with EO, calculate exact part with full operator
- Requires to compute eigenvectors twice, for the full and the EO operators

<table>
<thead>
<tr>
<th>Method</th>
<th>Setup</th>
<th>$N_{st}$</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deflated EO 500eV</td>
<td>1.00</td>
<td>2250</td>
<td>1.00</td>
</tr>
<tr>
<td>Deflated FullOp 100eV + LM</td>
<td>0.70</td>
<td>750</td>
<td>1.54</td>
</tr>
<tr>
<td>Deflated FullOp 250eV + LM</td>
<td>1.40</td>
<td>600</td>
<td>0.97</td>
</tr>
<tr>
<td>Deflated EO 500eV + 100eV LM</td>
<td>1.52</td>
<td>750</td>
<td>0.61</td>
</tr>
<tr>
<td>Deflated EO 500eV + 250eV LM</td>
<td>2.18</td>
<td>600</td>
<td>0.77</td>
</tr>
</tbody>
</table>

- Exact reconstruction reduces stochastic errors
- In our runs we reduced the cost with respect to EO deflation only by 40%
Are stochastic errors under control?

\[ \delta \langle x \rangle_{u+d} \]
\[ \delta G_{M} \]
\[ \delta \sigma_{u+d} \]
Hierarchical probing for the EM

- We tested our methods against hierarchical probing

  - Removes exactly contributions to the trace up to distance $2^p$
  - Use Hadamard vectors as basis
    - Vectors for $p$ coloring can be reuse for $p + 1$ coloring
    - Allows for a continuous increase in the number of vectors

- In our test we used a particular version of hierarchical probing
  - We probe in 4D up to distance $2^2 = 4$ (no time-dilution)
  - We tested the effects of spin-color dilution

- Rationale behind dropping time-dilution
  - Combine with the one-end trick
  - Use analysis methods that require all the insertion times

- Compare against Truncated Solver Method
Hierarchical probing

- 230 configurations in a $N_F = 2 + 1 + 1$ ensemble with $a \approx 0.086$ fm, $m_\pi \approx 373$ MeV
- Figure of merit: $E = \sigma^2_{stch} \times \text{Cost}$ (the lower the better)

<table>
<thead>
<tr>
<th>Method</th>
<th>$E_{g_A}$</th>
<th>$E_{G_M}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple stochastic</td>
<td>3.073 ± 0.012</td>
<td>8.5 ± 0.5</td>
</tr>
<tr>
<td>Hierarchical probing</td>
<td>2.06 ± 0.04</td>
<td>102 ± 0.18</td>
</tr>
<tr>
<td>Hierarchical probing + dilution</td>
<td>1.30 ± 0.06</td>
<td>2.3 ± 0.6</td>
</tr>
<tr>
<td>TSM all operators</td>
<td>0.65 ± 0.07</td>
<td>2.30 ± 0.05</td>
</tr>
</tbody>
</table>

- Our version of hierarchical probing per se is an improvement over simple stochastic sources
- For $g_A$ the TSM bests our hierarchical probing by roughly a factor of 2
- For $G_M$ dilution plays a very important role
- The performance of our hierarchical probing without spin-color dilution is not so impressive for $G_M$
Ensemble and observables

- $V = 48^3 \times 96$, $a \approx 0.09$ fm, $N_F = 2$ with $m_\pi \approx 130$MeV,
- Stats 2150 configurations $\times$ 400 nucleon 2pt functions per configuration
- Light ultralocal only 2250 noise vectors, deflated with EO, 2136 configurations
- Strange ultralocal TSM with 63HP / 1024LP vectors, 2153 configurations
- Charm ultralocal TSM with 5HP / 1250LP vectors 2153 configurations
- Light one-derivative 1000 noise vectors with exact low-mode reconstruction, deflated with EO, 715 configurations
- Strange one-derivative TSM with 30HP / 960LP vectors, 2153 configurations
- Three different analysis methods to remove the excited states
Analysis: Removing the excited states

- Must find agreement between at least two of the methods
- Must see convergence as the sink increases
\( \sigma_{u+d,s} \) present large contamination from excited states \((t_s \approx 1.8 \text{ fm})\)

\( \sigma_c \) seems to be free from contamination

\( g_A \) shows little contamination compared to the \( \sigma \)-terms
Results: Tensor charges, $\langle x \rangle$ and helicity

- Noisy results, limited mostly by the size of the gauge ensemble
Towards high precision computation of disconnected diagrams
- Stochastic noise fully under control for most observables
- Must focus on reducing the gauge noise

Deflation must be used for light masses
- Strange and charm computations can be done efficiently with the TSM and w/o deflation
- Deflation might introduce a penalty in the charm computation

The EM shows an impressive reduction of errors with spin-color dilution

Much work to be done
- Keep improving our code with new ideas
- Aim at high precision, high quality disconnected calculations