Disconnected contributions to nucleon observables with $N_f = 2$ twisted-mass clover fermions at the physical light quark mass

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Outline

- Methods
  - Stochastic methods and variance reduction
  - Exact deflation, low-mode reconstruction
  - Analysis of stochastic errors
  - Hierarchical probing
- Results
  - Removing the excited states
  - σ-terms and g_A
  - g_T, $\langle x \rangle$ and the helicity
- Conclusions

Methods

- Stochastic estimation — Bitar *et al* 1989; Dong, Liu 1994

  Fill N vectors $|\eta_j\rangle$ with Z_N noise and compute $M|s_j\rangle = |\eta_j\rangle$

  $$M_E^{-1} := \frac{1}{N}\sum_{j=1}^{N} |s_j\rangle\langle\eta_j| \approx M^{-1}$$

  Poor performance, error decreases as $1/\sqrt{N}$

- Truncated Solver Method — Bali, Collins, Schäffer 2007

  Increases N cheaply with low-precision (LP) estimation, correct afterwards

  $$M_E^{-1} := \frac{1}{N_{HP}}\sum_{j=1}^{N_{HP}} \left(|s_j\rangle\langle\eta_j|_{HP} - |s_j\rangle\langle\eta_j|_{LP}\right) + \frac{1}{N_{LP}}\sum_{j=N_{HP}+1}^{N_{HP}+N_{LP}} |s_j\rangle\langle\eta_j|_{LP}$$

  Fails for light masses due to loss of correlation between HP and LP

- One-End Trick — Foster, Michael 1998; McNeile, Michael 2006

  Twisted-mass exclusive, based on identities. Example:

  $$\mathrm{Tr}\left[X\left(M_d^{-1} - M_u^{-1}\right)\right] = 2i\mu\mathrm{Tr}\left[\left(M_u^{\dagger}\right)^{-1} X\gamma_5 M_u^{-1}\right] \implies 2i\mu\sum_r \left\langle s^{\dagger} X\gamma_5 s\right\rangle_r$$

- Exact deflation

Exact reconstruction, inversion acceleration

- Exact deflation of the noise vector $|\eta_D\rangle = |\eta\rangle - \langle v_j|\eta\rangle |v_j\rangle$, $|s_D\rangle = M^{-1}|\eta_D\rangle$

Exact (Full op!)

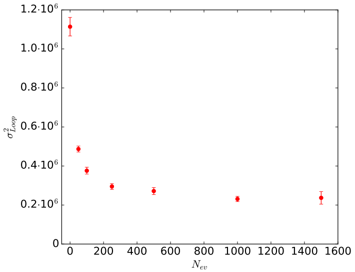
$$M_{low}^{-1} = \sum_{j=1}^{N} \frac{1}{\lambda_j} |v_j\rangle \langle v_j|$$

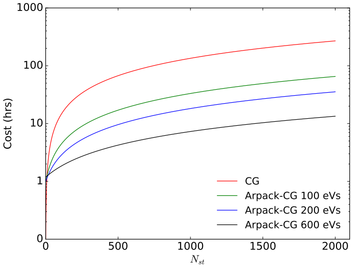
Stochastic

$$|s\rangle = |s_D\rangle + \sum_{j=1}^{N} \frac{1}{\lambda_j} \langle v_j|\eta\rangle$$

- How many low-modes do I need?

Gambhir, Stathopoulos, Orginos 2016





- From $N_{ev} \approx 250$ on there is little to gain in the exact reconstruction
- Inversions are still accelerated as N_{ev} increases

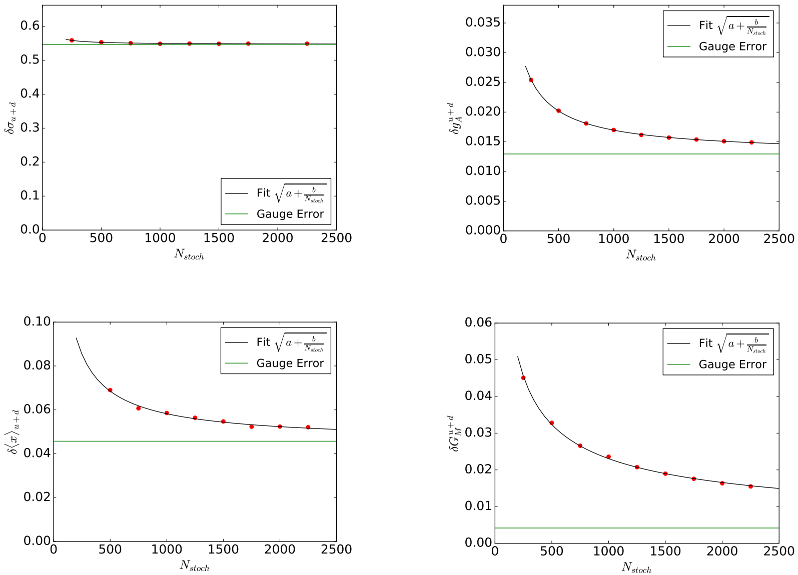
Exact reconstruction

- Idea: Solve with EO, calculate exact part with full operator
- Requires to compute eigenvectors *twice*, for the full and the EO operators

Method	Setup	N_{st}	Cost
Deflated EO 500eV	1.00	2250	1.00
Deflated FullOp 100eV + LM	0.70	750	1.54
Deflated FullOp 250eV + LM	1.40	600	0.97
Deflated EO 500eV + 100eV LM	1.52	750	0.61
Deflated EO 500eV + 250eV LM	2.18	600	0.77

- Exact reconstruction reduces stochastic errors
- In our runs we reduced the cost with respect to EO deflation only by 40%

Are stochastic errors under control?



Hierarchical probing for the EM

- We tested our methods against hierarchical probing

  Stathopoulos, Laechli, Orginos 2013

  - Removes exactly contributions to the trace up to distance 2^p
  - Use Hadamard vectors as basis
    - Vectors for p coloring can be reuse for $p+1$ coloring
    - Allows for a continuous increase in the number of vectors
- In our test we used a particular version of hierarchical probing
  - We probe in 4D up to distance $2^2 = 4$ (no time-dilution)
  - We tested the effects of spin-color dilution
- Rationale behind dropping time-dilution
  - Combine with the one-end trick
  - Use analysis methods that require all the insertion times
- Compare against Truncated Solver Method

Hierarchical probing

- 230 configurations in a $N_F = 2 + 1 + 1$ ensemble with $a \approx 0.086$ fm, $m_\pi \approx 373$ MeV
- Figure of merit: $E = \sigma^2_{stch} \times$ *Cost* (the lower the better)

Method	E_{g_A}	E_{G_M}
Simple stochastic	$3.073 \ \pm 0.012$	8.5 ± 0.5
Hierarchical probing	2.06 ± 0.04	102 ± 0.18
Hierarchical probing + dilution	1.30 ± 0.06	2.3 ± 0.6
TSM all operators	0.65 ± 0.07	2.30 ± 0.05

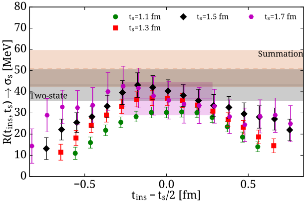
- Our version of hierarchical probing per se is an improvement over simple stochastic sources
- For g_A the TSM bests our hierarchical probing by roughly a factor of 2
- For G_M dilution plays a very important role
- The performance of our hierarchical probing without spin-color dilution is not so impressive for G_M

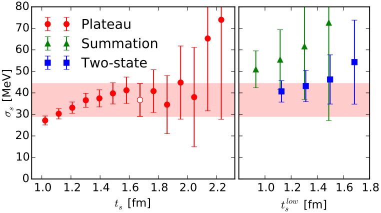
Ensemble and observables

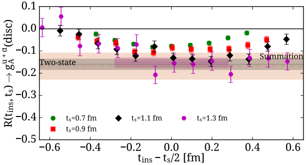
- $V = 48^3 \times 96$, $a \approx 0.09$ fm, $N_F = 2$ with $m_\pi \approx 130\text{MeV}$,
- Stats 2150 configurations $\times$ 400 nucleon 2pt functions per configuration
- Light ultralocal only 2250 noise vectors, deflated with EO, 2136 configurations
- Strange ultralocal TSM with 63HP / 1024LP vectors, 2153 configurations
- Charm ultralocal TSM with 5HP / 1250LP vectors 2153 configurations
- Light one-derivative 1000 noise vectors with exact low-mode reconstruction, deflated with EO, 715 configurations
- Strange one-derivative TSM with 30HP / 960LP vectors, 2153 configurations

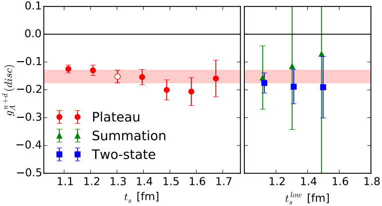
- Three different analysis methods to remove the excited states

Analysis: Removing the excited states



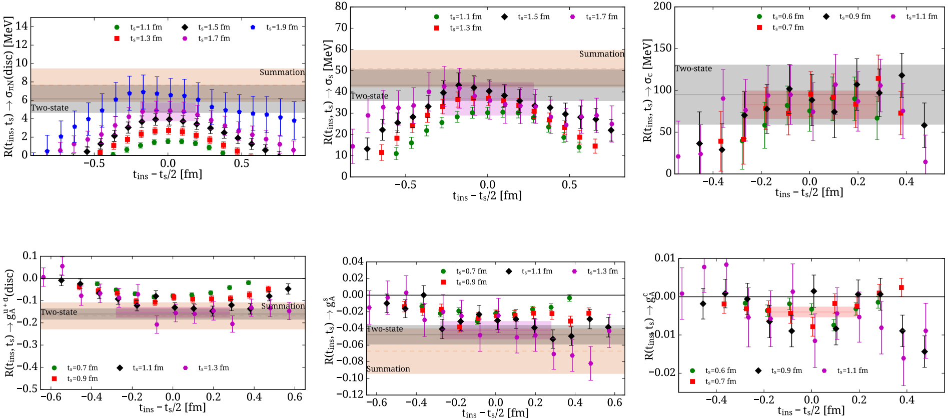






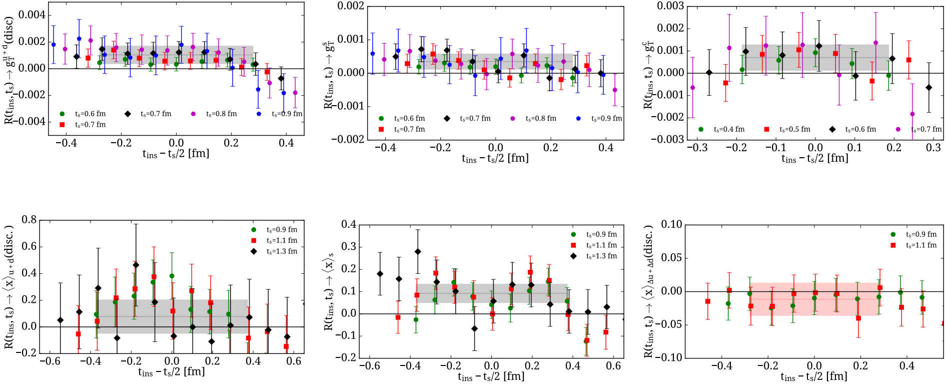
- Must find agreement between at least two of the methods
- Must see convergence as the sink increases

Results: Sigma terms and axial charges



- $\sigma_{u+d,s}$ present large contamination from excited states ($t_s \approx 1.8$ fm)
- σ_c seems to be free from contamination
- g_A shows little contamination compared to the σ-terms

Results: Tensor charges, $\langle x \rangle$ and helicity



- Noisy results, limited mostly by the size of the gauge ensemble

Summary, future work

- Towards high precision computation of disconnected diagrams
  - Stochastic noise fully under control for most observables
  - Must focus on reducing the gauge noise
- Deflation must be used for light masses
  - Strange and charm computations can be done efficiently with the TSM and w/o deflation
  - Deflation might introduce a penalty in the charm computation
- The EM shows an impressive reduction of errors with spin-color dilution
- Much work to be done
  - Keep improving our code with new ideas
  - Aim at high precision, high quality disconnected calculations