Latest results from lattice $\mathcal{N} = 4$ super Yang–Mills

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Lattice 2016, 26 July

arXiv:1505.03135   arXiv:1508.00884   arXiv:1512.01137
& more to come with Simon Catterall, Poul Damgaard and Joel Giedt

Brief review of motivations for lattice supersymmetry

- Much interesting physics in 4D supersymmetric gauge theories:
  dualities, holography, confinement, conformality, BSM, ...
- Lattice promises non-perturbative insights from first principles

Problem: Discrete spacetime breaks supersymmetry algebra

$$\left\{Q_{\alpha}^{\mathrm{I}}, \overline{Q}_{\dot{\alpha}}^{\mathrm{J}}\right\} = 2\delta^{\mathrm{IJ}}\sigma_{\alpha\dot{\alpha}}^{\mu} P_{\mu} \text{ where } \mathrm{I}, \mathrm{J} = 1, \cdots, \mathcal{N}$$

$\Longrightarrow$ Impractical fine-tuning generally required to restore susy, especially for scalar fields from matter multiplets or $\mathcal{N} > 1$

Solution: Preserve (some subset of) the susy algebra on the lattice
Possible for $\mathcal{N} = 4$ supersymmetric Yang–Mills (SYM)

Brief review of $\mathcal{N} = 4$ SYM

$\mathcal{N} = 4$ SYM is a particularly interesting theory

—AdS/CFT correspondence

—Testing ground for reformulations of scattering amplitudes

—Arguably simplest non-trivial field theory in four dimensions

Basic features:

- SU(*N*) gauge theory with four Majorana Ψ^{I} and six scalars Φ^{IJ}, all massless and in adjoint rep.
- Action consists of kinetic, Yukawa and four-scalar terms with coefficients related by symmetries
- Supersymmetric: 16 supercharges Q_α^{I} and $\overline{Q}_{\dot{\alpha}}^{\mathrm{I}}$ with $\mathrm{I} = 1, \cdots, 4$
  Fields and *Q*'s transform under global SU(4) $\simeq$ SO(6) R symmetry
- Conformal: β function is zero for any 't Hooft coupling λ

Exact supersymmetry on the lattice

Equivalent constructions from orbifolding and "topological" twisting:

The 16 spinor supercharges $Q^{\rm I}_{\alpha}$ and $\overline{Q}^{\rm I}_{\dot\alpha}$ fill a Kähler–Dirac multiplet:

$$\begin{pmatrix} Q^1_\alpha & Q^2_\alpha & Q^3_\alpha & Q^4_\alpha \\ \overline{Q}^1_{\dot\alpha} & \overline{Q}^2_{\dot\alpha} & \overline{Q}^3_{\dot\alpha} & \overline{Q}^4_{\dot\alpha} \end{pmatrix} = \mathcal{Q} + \mathcal{Q}_\mu\gamma_\mu + \mathcal{Q}_{\mu\nu}\gamma_\mu\gamma_\nu + \overline{\mathcal{Q}}_\mu\gamma_\mu\gamma_5 + \overline{\mathcal{Q}}\gamma_5$$

$$\longrightarrow \mathcal{Q} + \mathcal{Q}_a\gamma_a + \mathcal{Q}_{ab}\gamma_a\gamma_b$$

with $a, b = 1, \cdots, 5$

$\mathcal{Q}$'s transform with **integer spin** under "twisted rotation group"

$$\text{SO}(4)_{tw} \equiv \text{diag}\Big[\text{SO}(4)_{\rm euc} \otimes \text{SO}(4)_R\Big] \qquad\qquad \text{SO}(4)_R \subset \text{SO}(6)_R$$

This change of variables gives a susy subalgebra $\{\mathcal{Q}, \mathcal{Q}\} = 2\mathcal{Q}^2 = 0$

This subalgebra can be exactly preserved on the lattice

Pertinent features of the lattice theory

All fields transform with **integer spin** under $SO(4)_{tw}$ — **no spinors**

$$Q^{\mathrm{I}}_{\alpha} \text{ and } \overline{Q}^{\mathrm{I}}_{\dot{\alpha}} \longrightarrow \mathcal{Q},\ \mathcal{Q}_a \text{ and } \mathcal{Q}_{ab} \qquad (a, b = 1, \cdots, 5)$$

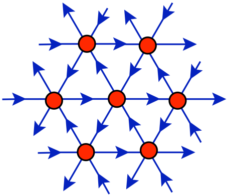
$$\Psi^{\mathrm{I}} \longrightarrow \eta,\ \psi_a \text{ and } \chi_{ab} \qquad \text{(site, link, plaq.)}$$

$$U_\mu \text{ and } \Phi^{\mathrm{IJ}} \longrightarrow \mathcal{U}_a = (U_\mu, \phi) + i(B_\mu, \overline{\phi}) \text{ and } \overline{\mathcal{U}}_a$$

Supersymmetry transformations include $\mathcal{Q}\,\mathcal{U}_a = \psi_a$

$\Longrightarrow$ Links must be in algebra, with continuum limit $\mathcal{U}_a = \mathbb{I}_N + \mathcal{A}_a$

$\Longrightarrow$ $U(N) = SU(N) \otimes U(1)$ gauge invariance



Five links symmetrically span four dimensions
$\longrightarrow$ A_4^* lattice (4D analog of triangular lattice)

Basis vectors are linearly dependent
and non-orthogonal $\longrightarrow \lambda = \lambda_{\mathrm{lat}}/\sqrt{5}$

Improvement 1: Lattice action

arXiv:1505.03135

Exact zero modes and flat directions must be regulated

in both the SU(N) and U(1) sectors

—Soft $\mathcal{Q}$ breaking scalar potential $\propto \mu^2 \sum_a \left(\text{Tr}\left[\mathcal{U}_a \overline{\mathcal{U}}_a\right] - N\right)^2$

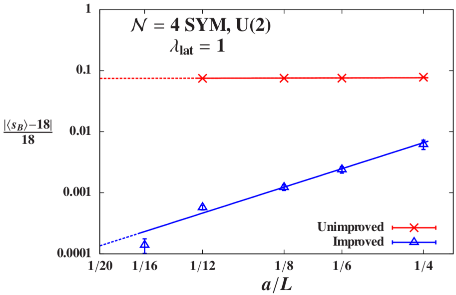
lifts SU(N) flat directions

—Constraint on plaquette det. lifts U(1) zero mode & flat directions

Improved lattice action introduces
$\mathcal{Q}$-exact plaquette det. deformation

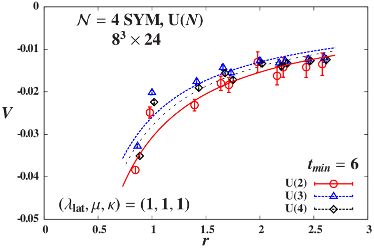
Ward identity violations
decrease $\sim 500\times$ for $L = 16$,
vanish $\langle \mathcal{Q}\mathcal{O} \rangle \propto (a/L)^2$

($\mathcal{Q}$ forbids all dim-5 operators)



Improvement 2: Lattice perturbation theory

Previous results for static potential $V(r)$ showed discretization artifacts



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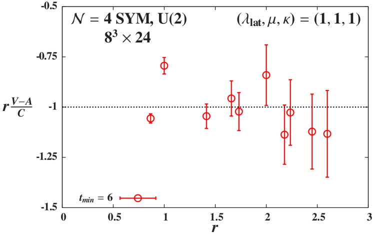


Improve by applying tree-level lattice perturbation theory for the $\mathcal{N} = 4$ SYM bosonic propagator on the A_4^* lattice:

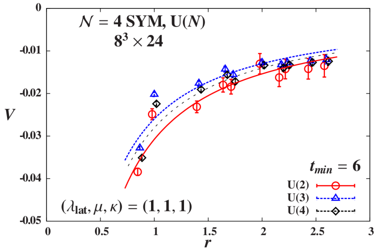
$$V(r) \longrightarrow V_{\rm tree}(r_I) \quad \text{where} \quad \frac{1}{r_I^2} \equiv 4\pi^2 \int \frac{d^4k}{(2\pi)^4} \frac{\exp\left[ir\cdot k\right]}{\sum_{\mu=1}^{4} \sin^2\left(k\cdot\widehat{e}_\mu \;/\; 2\right)}$$

$\widehat{e}_\mu$ are A_4^* lattice basis vectors (arXiv:1102.1725)

Momenta $k = \frac{2\pi}{L}\sum_{\mu=1}^{4} n_\mu \widehat{g}_\mu$ depend on dual basis vectors

Improvement 2: Lattice perturbation theory

Previous results for static potential $V(r)$ showed discretization artifacts



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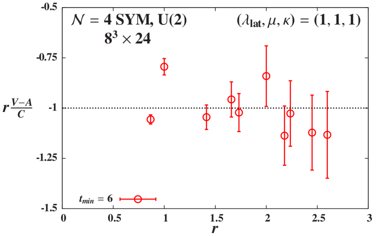
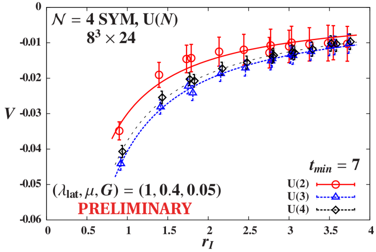


Tree-level improvement significantly reduces discretization artifacts



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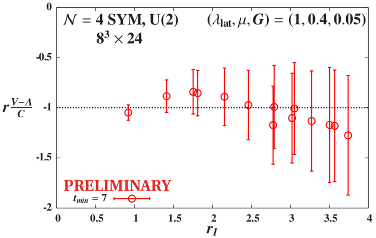


Coupling dependence of Coulomb coefficient

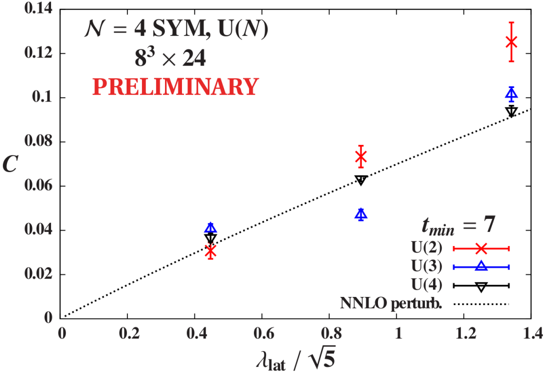
Fit $V(r)$ to Coulombic
or confining form

$$V(r) = A - C/r$$

$$V(r) = A - C/r + \sigma r$$

C is Coulomb coefficient

σ is string tension



$V(r)$ is Coulombic at all λ:
fits to confining form produce vanishing string tension

C for U(4) in good agreement with perturbation theory for $\lambda \lesssim 3/\sqrt{5}$

U(2) and U(3) results less stable — working on further improvements

Anomalous dimensions

$\mathcal{N} = 4$ SYM is conformal at all $\lambda \longrightarrow$ spectrum of scaling dimensions that govern power-law decay of correlation functions

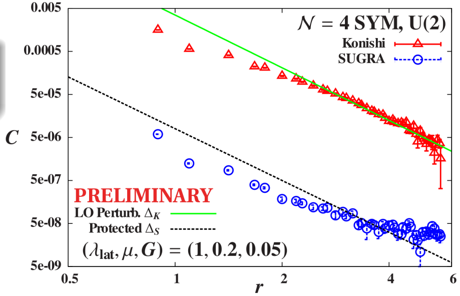
The Konishi operator is the simplest conformal primary operator

$$\mathcal{O}_K(x) = \sum_{\rm I} \text{Tr}\left[\Phi^{\rm I}(x)\Phi^{\rm I}(x)\right], \qquad C_K(r) \equiv \mathcal{O}_K(x+r)\mathcal{O}_K(x) \propto r^{-2\Delta_K(\lambda)}$$

On lattice, extract scalar fields from polar decomposition

$$\mathcal{U}_a(n) \longrightarrow e^{\varphi_a(n)} U_a(n)$$

$$\mathcal{O}_K^{\rm lat}(n) = \sum_a \text{Tr}\left[\varphi_a(n)\varphi_a(n)\right] - \text{vev}$$



Improvement 3: Lattice Konishi operator mixing

$$\mathcal{O}_K(x) = \sum_{\mathrm{I}} \mathrm{Tr}\left[\Phi^{\mathrm{I}}(x)\Phi^{\mathrm{I}}(x)\right] \longrightarrow \mathcal{O}_K^{\mathrm{lat}}(n) = \sum_a \mathrm{Tr}\left[\varphi_a(n)\varphi_a(n)\right] - \mathrm{vev}$$

Recall twisted $SO(4)_{tw}$ involves only $SO(4)_R \subset SO(6)_R$

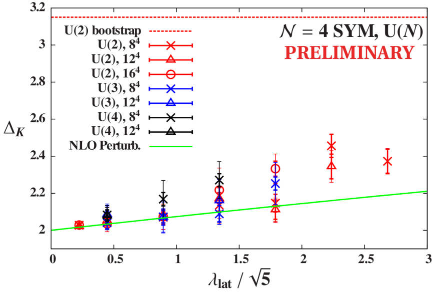
$\Longrightarrow$ The lattice Konishi operator mixes with the $SO(4)_R$-singlet part of an $SO(6)_R$-nonsinglet operator $\mathcal{O}_S$ (the "SUGRA" or 20′)

Need joint analyses including both operators

Konishi scaling dimension from MCRG stability matrix including both $\mathcal{O}_K^{\mathrm{lat}}$ and $\mathcal{O}_S^{\mathrm{lat}}$

Impose protected $\Delta_S = 2$

Systematic uncertainties from different amounts of smearing



Recapitulation

- Continuing progress in lattice $\mathcal{N} = 4$ SYM
- Improved action dramatically reduces Ward identity violations
- Tree-level improved static potential reduces discretization artifacts
- Promising initial results for Konishi anomalous dimension
- Many more directions are being — or can be — pursued
  - Understanding the (absence of a) sign problem
  - Exploring the Coulomb branch (Higgs mechanism)
  - Reducing to lower dimensions, possibly with less supersymmetry
  - Adding matter fields for spontaneous supersymmetry breaking

Advertisement: Public code for lattice $\mathcal{N} = 4$ SYM

so that the full improved action becomes

$$S_{\text{imp}} = S'_{\text{exact}} + S_{\text{closed}} + S'_{\text{soft}} \tag{3.10}$$

$$S'_{\text{exact}} = \frac{N}{2\lambda_{\text{lat}}} \sum_n \text{Tr}\Bigg[-\overline{\mathcal{F}}_{ab}(n)\mathcal{F}_{ab}(n) - \chi_{ab}(n)\mathcal{D}^{(+)}_{[a}\psi_{b]}(n) - \eta(n)\overline{\mathcal{D}}^{(-)}_a\psi_a(n)$$

$$+ \frac{1}{2}\left(\overline{\mathcal{D}}^{(-)}_a\mathcal{U}_a(n) + G\sum_{a\neq b}\left(\det\mathcal{P}_{ab}(n) - 1\right)\mathbb{I}_N\right)^2\Bigg] - S_{\text{det}}$$

$$S_{\text{det}} = \frac{N}{2\lambda_{\text{lat}}} G \sum_n \text{Tr}\left[\eta(n)\right] \sum_{a\neq b}\left[\det\mathcal{P}_{ab}(n)\right] \text{Tr}\left[\mathcal{U}_b^{-1}(n)\psi_b(n) + \mathcal{U}_a^{-1}(n+\widehat{\mu}_b)\psi_a(n+\widehat{\mu}_b)\right]$$

$$S_{\text{closed}} = -\frac{N}{8\lambda_{\text{lat}}} \sum_n \text{Tr}\left[\epsilon_{abcde}\ \chi_{de}(n+\widehat{\mu}_a+\widehat{\mu}_b+\widehat{\mu}_c)\overline{\mathcal{D}}^{(-)}_c\chi_{ab}(n)\right],$$

$$S'_{\text{soft}} = \frac{N}{2\lambda_{\text{lat}}}\mu^2 \sum_n \sum_a \left(\frac{1}{N}\text{Tr}\left[\mathcal{U}_a(n)\overline{\mathcal{U}}_a(n)\right] - 1\right)^2$$

The lattice action is obviously very complicated

(the fermion operator involves $\gtrsim$100 gathers)

To reduce barriers to entry our parallel code is publicly developed at

github.com/daschaich/susy

Evolved from MILC code, presented in arXiv:1410.6971

Thank you!

Thank you!

Collaborators

Simon Catterall, Poul Damgaard and Joel Giedt

Funding and computing resources

Supplement: The sign problem

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int [d\mathcal{U}][d\overline{\mathcal{U}}] \;\; \mathcal{O} \;\; e^{-S_B[\mathcal{U},\overline{\mathcal{U}}]} \;\; \text{pf}\, \mathcal{D}[\mathcal{U},\overline{\mathcal{U}}]$$

Pfaffian can be complex for lattice $\mathcal{N} = 4$ SYM, $\text{pf}\,\mathcal{D} = |\text{pf}\,\mathcal{D}| e^{i\alpha}$

Complicates interpretation of $\left\{e^{-S_B}\, \text{pf}\,\mathcal{D}\right\}$ as Boltzmann weight

We carry out phase-quenched calculations with $\text{pf}\,\mathcal{D} \longrightarrow |\text{pf}\,\mathcal{D}|$

In principle need to reweight phase-quenched (pq) observables:

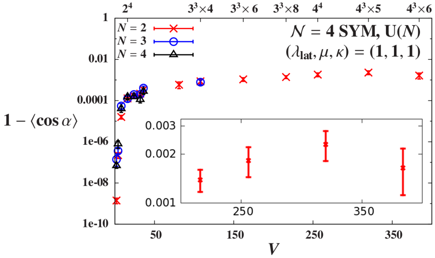
$$\langle \mathcal{O} \rangle = \frac{\left\langle \mathcal{O} e^{i\alpha} \right\rangle_{pq}}{\left\langle e^{i\alpha} \right\rangle_{pq}} \qquad \text{with } \left\langle \mathcal{O} e^{i\alpha} \right\rangle_{pq} = \frac{1}{\mathcal{Z}_{pq}} \int [d\mathcal{U}][d\overline{\mathcal{U}}]\, \mathcal{O} e^{i\alpha}\, e^{-S_B}\, |\text{pf}\,\mathcal{D}|$$

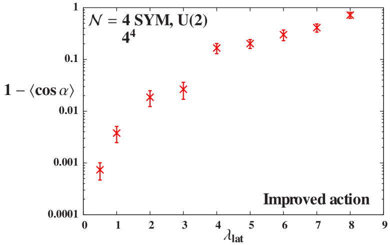
$\Longrightarrow$ Monitor $\left\langle e^{i\alpha} \right\rangle_{pq}$ as function of volume, coupling, N

Pfaffian phase dependence on volume and coupling

Left: $1 - \langle\cos(\alpha)\rangle_{pq} \ll 1$ independent of volume and N at $\lambda_{\text{lat}} = 1$

Right: Newer 4^4 results at $4 \leq \lambda_{\text{lat}} \leq 8$ show much larger fluctuations





May be interesting to check more volumes and N for improved action

Extremely expensive computation despite parallelization:

$\mathcal{O}(n^3)$ scaling $\longrightarrow$ ~50 hours for single U(2) 4^4 measurement

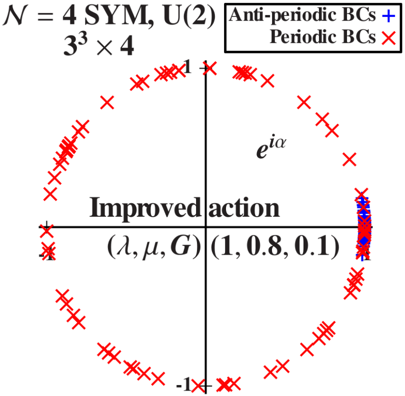
Two puzzles posed by the sign problem

- With periodic temporal boundary conditions for the fermions
  we have an obvious sign problem, $\left\langle e^{i\alpha} \right\rangle_{pq}$ consistent with zero
- With anti-periodic BCs and all else the same $e^{i\alpha} \approx 1$,
  phase reweighting has negligible effect

Why such sensitivity to the BCs?

Also, other pq observables
are nearly identical
for these two ensembles

Why doesn't the sign problem
affect other observables?



Backup: Failure of Leibnitz rule in discrete space-time

Given that $\left\{Q_\alpha, \overline{Q}_{\dot\alpha}\right\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu = 2i\sigma^\mu_{\alpha\dot\alpha}\partial_\mu$ is problematic,

why not try $\left\{Q_\alpha, \overline{Q}_{\dot\alpha}\right\} = 2i\sigma^\mu_{\alpha\dot\alpha}\nabla_\mu$ for a discrete translation?

Here $\nabla_\mu\phi(x) = \frac{1}{a}\left[\phi(x + a\widehat{\mu}) - \phi(x)\right] = \partial_\mu\phi(x) + \frac{a}{2}\partial_\mu^2\phi(x) + \mathcal{O}(a^2)$

Essential difference between ∂_μ and ∇_μ on the lattice, $a > 0$

$$\begin{aligned}\nabla_\mu\left[\phi(x)\chi(x)\right] &= a^{-1}\left[\phi(x + a\widehat{\mu})\chi(x + a\widehat{\mu}) - \phi(x)\chi(x)\right] \\ &= \left[\nabla_\mu\phi(x)\right]\chi(x) + \phi(x)\nabla_\mu\chi(x) + a\left[\nabla_\mu\phi(x)\right]\nabla_\mu\chi(x)\end{aligned}$$

We only recover the Leibnitz rule $\partial_\mu(fg) = (\partial_\mu f)g + f\partial_\mu g$ when $a \to 0$

$\Longrightarrow$ "Discrete supersymmetry" breaks down on the lattice

(Dondi & Nicolai, "Lattice Supersymmetry", 1977)

Backup: Twisting $\longleftrightarrow$ Kähler–Dirac fermions

The Kähler–Dirac representation is related to the spinor $Q^{\mathrm{I}}_{\alpha}, \overline{Q}^{\mathrm{I}}_{\dot{\alpha}}$ by

$$\begin{pmatrix} Q^1_\alpha & Q^2_\alpha & Q^3_\alpha & Q^4_\alpha \\ \overline{Q}^1_{\dot{\alpha}} & \overline{Q}^2_{\dot{\alpha}} & \overline{Q}^3_{\dot{\alpha}} & \overline{Q}^4_{\dot{\alpha}} \end{pmatrix} = \mathcal{Q} + \mathcal{Q}_\mu \gamma_\mu + \mathcal{Q}_{\mu\nu}\gamma_\mu\gamma_\nu + \overline{\mathcal{Q}}_\mu \gamma_\mu \gamma_5 + \overline{\mathcal{Q}}\gamma_5$$
$$\longrightarrow \mathcal{Q} + \mathcal{Q}_a \gamma_a + \mathcal{Q}_{ab}\gamma_a\gamma_b \qquad \text{with } a, b = 1, \cdots, 5$$

The 4×4 matrix involves R symmetry transformations along each row and (euclidean) Lorentz transformations along each column

$\Longrightarrow$ Kähler–Dirac components transform under "twisted rotation group"

$$\mathrm{SO}(4)_{tw} \equiv \mathrm{diag}\left[\mathrm{SO}(4)_{\mathrm{euc}} \otimes \mathrm{SO}(4)_R\right]$$

$\uparrow$ only $\mathrm{SO}(4)_R \subset \mathrm{SO}(6)_R$

Backup: Twisted $\mathcal{N} = 4$ SYM fields and $\mathcal{Q}$

Everything transforms with **integer spin** under $\text{SO}(4)_{tw}$ — **no spinors**

$$Q^{\text{I}}_{\alpha} \text{ and } \overline{Q}^{\text{I}}_{\dot{\alpha}} \longrightarrow \mathcal{Q},\ \mathcal{Q}_a \text{ and } \mathcal{Q}_{ab}$$

$$\Psi^{\text{I}} \longrightarrow \eta,\ \psi_a \text{ and } \chi_{ab}$$

$$A_{\mu} \text{ and } \Phi^{\text{IJ}} \longrightarrow \mathcal{A}_a = (A_{\mu}, \phi) + i(B_{\mu}, \overline{\phi}) \text{ and } \overline{\mathcal{A}}_a$$

The twisted-scalar supersymmetry $\mathcal{Q}$ acts as

$$\mathcal{Q}\ \mathcal{A}_a = \psi_a \qquad \mathcal{Q}\ \psi_a = 0$$

$$\mathcal{Q}\ \chi_{ab} = -\overline{\mathcal{F}}_{ab} \qquad \mathcal{Q}\ \overline{\mathcal{A}}_a = 0$$

$$\mathcal{Q}\ \eta = d \qquad \mathcal{Q}\ d = 0$$

↖ bosonic auxiliary field with e.o.m. $d = \overline{\mathcal{D}}_a \mathcal{A}_a$

1. $\mathcal{Q}$ directly interchanges bosonic $\longleftrightarrow$ fermionic d.o.f.
2. The susy subalgebra $\mathcal{Q}^2 \cdot = 0$ is manifest

Backup: Lattice $\mathcal{N} = 4$ SYM

The lattice theory is nearly a direct transcription,
despite breaking the 15 $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$

- Covariant derivatives $\longrightarrow$ finite difference operators
- Complexified gauge fields $\mathcal{A}_a \longrightarrow$ gauge links $\mathcal{U}_a \in \mathfrak{gl}(N, \mathbb{C})$

$$\mathcal{Q}\,\mathcal{A}_a \longrightarrow \mathcal{Q}\,\mathcal{U}_a = \psi_a \qquad\qquad \mathcal{Q}\,\psi_a = 0$$
$$\mathcal{Q}\,\chi_{ab} = -\overline{\mathcal{F}}_{ab} \qquad\qquad \mathcal{Q}\,\overline{\mathcal{A}}_a \longrightarrow \mathcal{Q}\,\overline{\mathcal{U}}_a = 0$$
$$\mathcal{Q}\,\eta = d \qquad\qquad \mathcal{Q}\,d = 0$$

Geometry manifest: η and d on sites, $\mathcal{U}_a$ and ψ_a on links, etc.

- Supersymmetric lattice action ($\mathcal{Q}S = 0$)
  follows from $\mathcal{Q}^2 \cdot = 0$ and Bianchi identity

$$S = \frac{N}{2\lambda_{\text{lat}}}\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \eta\overline{\mathcal{D}}_a\mathcal{U}_a - \frac{1}{2}\eta d\right) - \frac{N}{8\lambda_{\text{lat}}}\epsilon_{abcde}\,\chi_{ab}\overline{\mathcal{D}}_c\,\chi_{de}$$

Backup: A_4^* lattice with five links in four dimensions

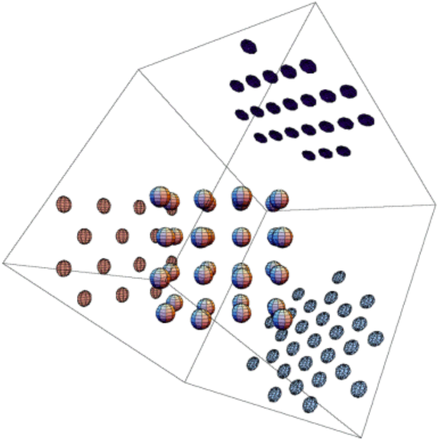
$A_a = (A_\mu, \phi)$ may remind you of dimensional reduction

On the lattice we want to treat all five $\mathcal{U}_a$ symmetrically
to obtain $S_5 \longrightarrow \text{SO}(4)_{tw}$ symmetry

—Start with hypercubic lattice
in 5d momentum space

—**Symmetric** constraint $\sum_a \partial_a = 0$
projects to 4D momentum space

—Result is A_4 lattice
$\longrightarrow$ dual A_4^* lattice in real space

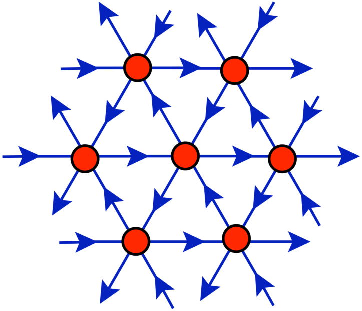


Backup: Twisted SO(4) symmetry on the A_4^* lattice

—Can picture A_4^* lattice
as 4D analog of 2D triangular lattice

—Basis vectors are linearly dependent
and non-orthogonal $\longrightarrow \lambda = \lambda_{\text{lat}}/\sqrt{5}$

—Preserves S_5 point group symmetry



S_5 irreps precisely match onto irreps of twisted $\text{SO}(4)_{tw}$

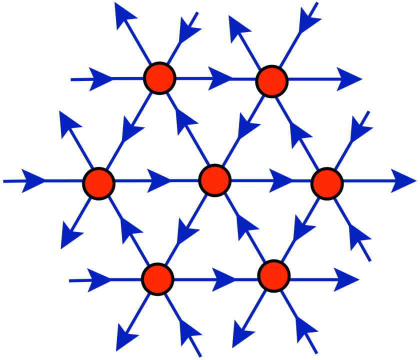
$$\mathbf{5} = \mathbf{4} \oplus \mathbf{1} : \quad \psi_a \longrightarrow \psi_\mu, \;\; \overline{\eta}$$

$$\mathbf{10} = \mathbf{6} \oplus \mathbf{4} : \quad \chi_{ab} \longrightarrow \chi_{\mu\nu}, \;\; \overline{\psi}_\mu$$

$S_5 \longrightarrow \text{SO}(4)_{tw}$ in continuum limit restores the rest of $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$

Backup: Hypercubic representation of A_4^* lattice

In the code it is very convenient to represent the A_4^* lattice as a hypercube with a backwards diagonal



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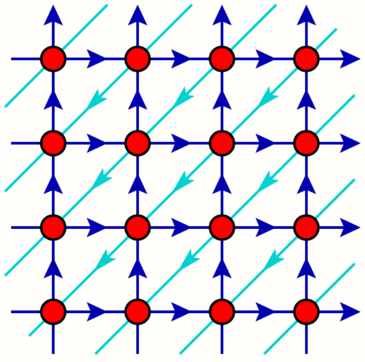


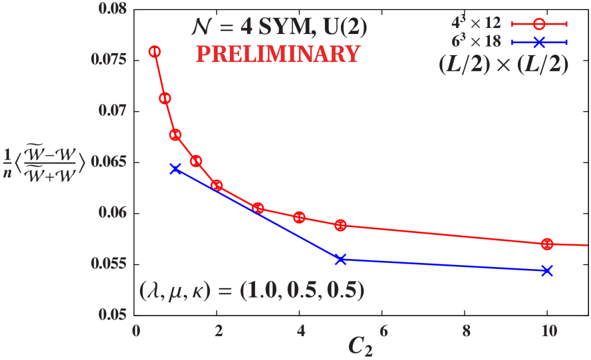
Backup: Restoration of $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$ supersymmetries

Results from arXiv:1411.0166 to be revisited with improved action

$\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$ from restoration of R symmetry (motivation for A_4^* lattice)

Modified Wilson loops test R symmetries at non-zero lattice spacing

Parameter c_2 may need log. tuning in continuum limit



Backup: More on flat directions

Supersymmetry transformations include $\mathcal{Q}\,\mathcal{U}_a = \psi_a$

$\Longrightarrow$ Links must be in algebra, with continuum limit $\mathcal{U}_a = \mathbb{I}_N + \mathcal{A}_a$

$\Longrightarrow$ U(N) = SU(N) $\otimes$ U(1) gauge invariance

Flat directions in SU(N) sector are physical,

those in U(1) sector decouple only in continuum limit

Both must be regulated in calculations $\longrightarrow$ two deformations needed:

SU(N) scalar potential $\propto \mu^2 \sum_a \left(\text{Tr}\left[\mathcal{U}_a\overline{\mathcal{U}}_a\right] - N\right)^2$

U(1) plaquette determinant $\sim G \sum_{a \neq b} \left(\det \mathcal{P}_{ab} - 1\right)$

Scalar potential **softly** breaks $\mathcal{Q}$ supersymmetry

$\nwarrow$ susy-violating operators vanish as $\mu^2 \to 0$

Plaquette determinant can be made $\mathcal{Q}$-invariant $\longrightarrow$ improved action

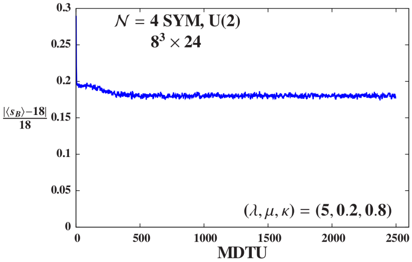
Backup: One problem with flat directions

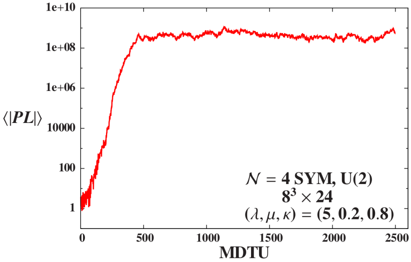
Gauge fields $\mathcal{U}_a$ can move far away from continuum form $\mathbb{I}_N + \mathcal{A}_a$ if $N\mu^2/(2\lambda_{\text{lat}})$ becomes too small

Example for two-color $(\lambda_{\text{lat}}, \mu, \kappa) = (5, 0.2, 0.8)$ on $8^3 \times 24$ volume

Left: Bosonic action is stable $\sim$18% off its supersymmetric value

Right: Polyakov loop wanders off to $\sim 10^9$

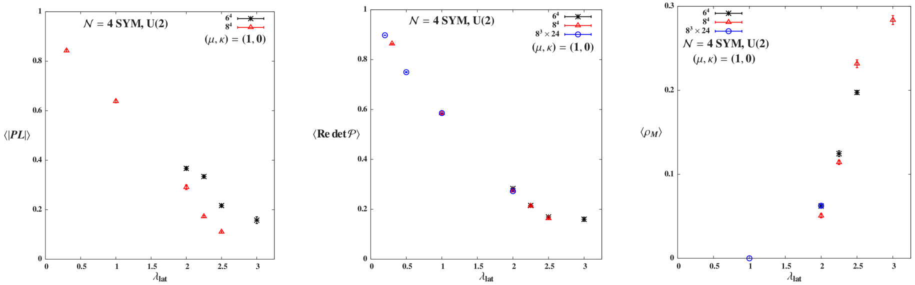




Backup: Another problem with U(1) flat directions

Flat directions in U(1) sector can induce transition to confined phase

This lattice artifact is not present in continuum $\mathcal{N} = 4$ SYM



Around the same $\lambda_{\text{lat}} \approx 2\ldots$

Left: Polyakov loop falls towards zero

Center: Plaquette determinant falls towards zero

Right: Density of U(1) monopole world lines becomes non-zero

Backup: More on soft susy breaking

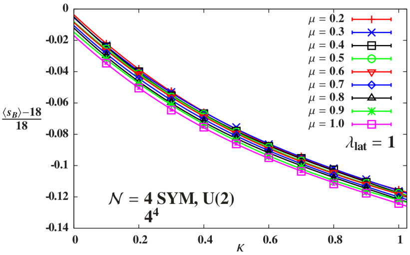
Until 2015 we used a more naive constraint on plaquette det.:

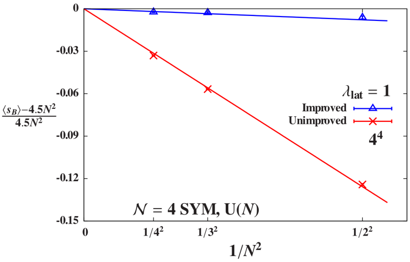
$$S_{soft} = \frac{N}{2\lambda_{\text{lat}}}\mu^2\left(\frac{1}{N}\text{Tr}\left[\mathcal{U}_a\overline{\mathcal{U}}_a\right] - 1\right)^2 + \kappa\left|\det\mathcal{P}_{ab} - 1\right|^2$$

Both terms explicitly break $\mathcal{Q}$ but $\det\mathcal{P}_{ab}$ effects dominate

Left: The breaking is **soft** — guaranteed to vanish as $\mu, \kappa \longrightarrow 0$

Right: Soft $\mathcal{Q}$ breaking also suppressed $\propto 1/N^2$





Backup: More on supersymmetric constraints

Improved action from arXiv:1505.03135
imposes $\mathcal{Q}$-invariant plaquette determinant constraint

$$S = \frac{N}{2\lambda_{\text{lat}}}\mathcal{Q}\left(\chi_{ab}\mathcal{F}_{ab} + \quad \downarrow \quad -\frac{1}{2}\eta d\right) - \frac{N}{8\lambda_{\text{lat}}}\epsilon_{abcde}\,\chi_{ab}\overline{\mathcal{D}}_c\,\chi_{de} + \mu^2 V$$

$$\eta\left(\overline{\mathcal{D}}_a\mathcal{U}_a + G\sum_{a\neq b}\left[\det\mathcal{P}_{ab} - 1\right]\mathbb{I}_N\right)$$

Basic idea: Modify the equations of motion $\longrightarrow$ moduli space

$$d(n) = \overline{\mathcal{D}}_a\mathcal{U}_a(n) \longrightarrow \overline{\mathcal{D}}_a\mathcal{U}_a(n) + G\sum_{a\neq b}\left[\det\mathcal{P}_{ab}(n) - 1\right]\mathbb{I}_N$$

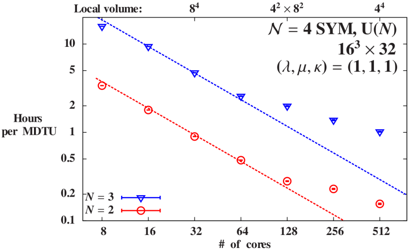
Produces much smaller $\mathcal{Q}$ Ward identity violations
that vanish $\propto (a/L)^2$ in the continuum limit

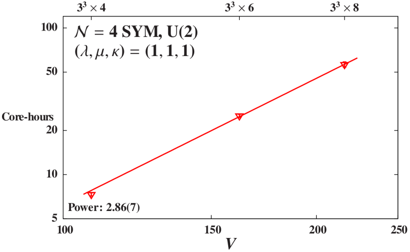
Backup: Code performance—weak and strong scaling

Results from arXiv:1410.6971 to be revisited with improved action

Left: Strong scaling for U(2) and U(3) $16^3 \times 32$ RHMC

Right: Weak scaling for $\mathcal{O}(n^3)$ pfaffian calculation (fixed local volume)
$n \equiv 16N^2L^3N_T$ is number of fermion degrees of freedom





Dashed lines are optimal scaling

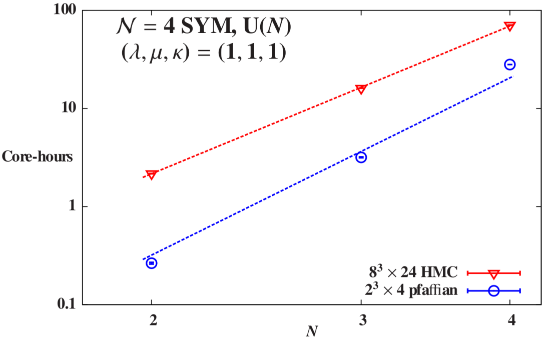
Solid line is power-law fit

Backup: Numerical costs for 2, 3 and 4 colors

Results from arXiv:1410.6971 to be revisited with improved action

Red: RHMC cost scaling $\sim N^5$ should now be better
thanks to recent optimizations (specific to adjoint fermions)

Blue: Pfaffian cost scaling consistent with expected N^6

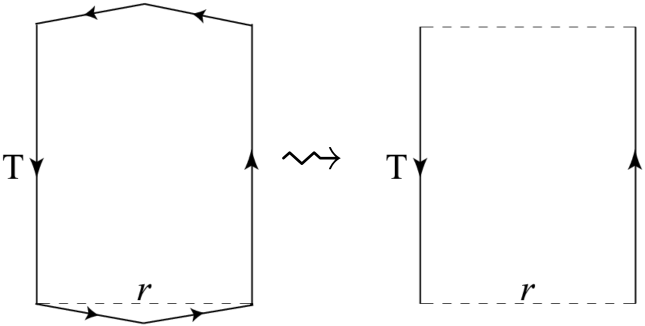


Backup: $\mathcal{N} = 4$ SYM static potential from Wilson loops

Extract static potential $V(r)$ from $r \times T$ Wilson loops

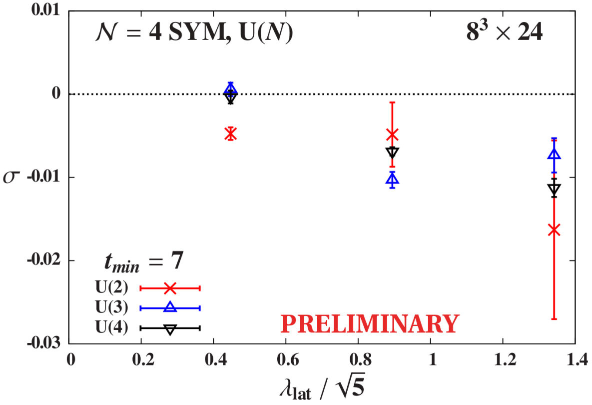
$$W(r,T) \propto e^{-V(r)\,T} \qquad\qquad V(r) = A - C/r + \sigma r$$

Coulomb gauge trick from lattice QCD provides off-axis loops



Backup: Static potential is Coulombic at all λ

String tension σ from fits to confining form $V(r) = A - C/r + \sigma r$



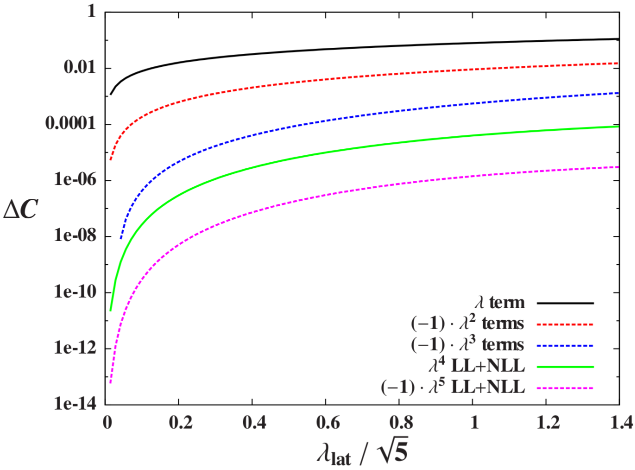
Slightly negative values make $V(r_I)$ flat for $3 \lesssim r_I \lesssim 4$

$\sigma \to 0$ as accessible range of r_I increases on larger volumes

Backup: Perturbation theory for Coulomb coefficient

For range of couplings currently being studied

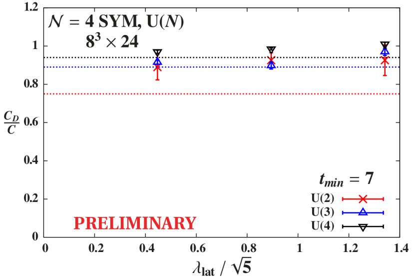
(continuum) perturbation theory for $C(\lambda)$ is well behaved

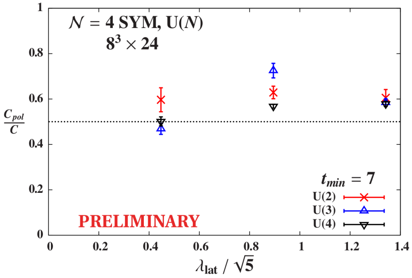


Backup: More tests of the static potential

Left: Projecting Wilson loops from U(N) $\longrightarrow$ SU(N) $\Longrightarrow$ factor of $\frac{N^2-1}{N^2}$

Right: Unitarizing links removes scalars $\Longrightarrow$ factor of $1/2$





Some results slightly above expected factors

May be related to fixed $L = 8$ or non-zero auxiliary couplings (μ, G)

Backup: Real-space RG for lattice $\mathcal{N} = 4$ SYM

Lattice RG blocking transformation must preserve symmetries
$\mathcal{Q}$ and $S_5 \longleftrightarrow$ geometric structure of the system

Simple scheme constructed in arXiv:1408.7067

$$\mathcal{U}_c'(x') = \xi\, \mathcal{U}_c(x)\mathcal{U}_c(x + \widehat{\mu}_c) \qquad \eta'(x') = \eta(x)$$
$$\psi_c'(x') = \xi\left[\psi_c(x)\mathcal{U}_c(x + \widehat{\mu}_c) + \mathcal{U}_c(x)\psi_c(x + \widehat{\mu}_c)\right] \qquad \text{etc.}$$

Doubles lattice spacing $a \longrightarrow a' = 2a$, with ξ a tunable rescaling factor

Scalar fields from polar decomposition $\mathcal{U}_c(n) = e^{\varphi_c(n)} U_c(n)$
are shifted $\varphi_c \longrightarrow \varphi_c + \log \xi$, since blocked U_c must remain unitary

$\mathcal{Q}$-preserving RG blocking is necessary ingredient to derive that
at most one log. tuning needed to recover $\mathcal{Q}_a$ and $\mathcal{Q}_{ab}$ in the continuum

Backup: Scaling dimensions from Monte Carlo RG

Write system as (infinite) sum of operators, $H = \sum_i c_i \, \mathcal{O}_i$
with couplings c_i that flow under RG blocking transformation R_b

n-times-blocked system is $H^{(n)} = R_b H^{(n-1)} = \sum_i c_i^{(n)} \mathcal{O}_i^{(n)}$

Fixed point defined by $H^\star = R_b H^\star$ with couplings $c_i^\star$

Linear expansion around fixed point defines stability matrix $T_{ik}^\star$

$$c_i^{(n)} - c_i^\star = \sum_k \left. \frac{\partial c_i^{(n)}}{\partial c_k^{(n-1)}} \right|_{H^\star} \left(c_k^{(n-1)} - c_k^\star \right) \equiv \sum_k T_{ik}^\star \left(c_k^{(n-1)} - c_k^\star \right)$$

Correlators of $\mathcal{O}_i$, $\mathcal{O}_k$ $\longrightarrow$ elements of stability matrix (Swendsen, 1979)

Eigenvalues of $T_{ik}^\star$ $\longrightarrow$ scaling dimensions of corresponding operators

Backup: Smearing for Konishi analyses

As in glueball analyses, use smearing to enlarge operator basis

Using APE-like smearing: $(1-\alpha)\,—\quad + \quad \frac{\alpha}{8}\sum \sqcap$,

with staples built from unitary parts of links but no final unitarization

(unitarized smearing — e.g. stout — doesn't affect scalar fields)

Average plaquette is stable upon smearing (**right**)

while minimum plaquette steadily increases (**left**)