$5D$ Maximally Supersymmetric Yang-Mills on the Lattice

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Motivation

A recent claim

M.R. Douglas, [JHEP 02 (2011) 011]
N. Lambert, C. Papageorgakis, M. Schmidt-Sommerfeld, [JHEP 01 (2011) 083]

$$6D\ (2,0)\ \text{superconformal theory} = 5D\ \text{MSYM}$$

D = 5 MSYM captures all the degrees of freedom of the parent 6D theory $\Longrightarrow$ these two theories are the same

Background

$6D$ (2, 0) theory

- Lagrangian not known!
- Existence can be probed through dual gravitational theory on $AdS_7 \times S^4$
- UV finite
- Free energy $\sim N^3$

$5D$ $\mathcal{N} = 2$ theory

- 16 supercharges
- Perturbatively non-renormalizable
- Also takes part in AdS/CFT
- Free energy $\sim N^3$ (at large 't Hooft coupling)

$5D\ \mathcal{N} = 2$ Yang-Mills

Obtained by dimensional reduction of $6D$ (2, 0) theory on a circle

Action

$$S_5 = \frac{1}{g_5^2} \int d^5x \,\mathrm{Tr}\, \Big(\frac{1}{4} F_{mn} F_{mn} + \frac{1}{2} D_m \phi_j D_m \phi_j + \frac{1}{4} [\phi_j, \phi_k]^2 - i \overline{\lambda}^{aX} (\gamma^m)_a{}^b D_m \lambda_{bX} - \overline{\lambda}^{aX} (\gamma^j)_X^{\ Y} [\phi_j, \lambda_{aY}] \Big)$$

Euclidean Lorentz rotation group: $SO_E(5)$

Internal rotation group: $SO_R(5)$

Twist - A Lattice Compatible Form

We are interested in formulating $5D$ theory on the lattice

Our approach: Twist the theory

E. Witten, [Commun. Math. Phys. 117, 353 (1988)]

Define a new rotation group

$$SO'(5) = \text{diag subgroup}\Big(SO_E(5) \times SO_R(5)\Big)$$

On flat space - just a change of variables

(On twisted $4D$ MSYM: See talks by Joel Giedt and David Schaich [this session].)

Twisted $5D$ $\mathcal{N} = 2$ Yang-Mills

Twisted action

$$S_5 = \frac{1}{g_5^2} \int d^5x \; \mathrm{Tr} \Big(\frac{1}{4} \overline{\mathcal{F}}_{mn} \mathcal{F}_{mn} + \frac{1}{2} D_m \phi_m D_n \phi_n - i \overline{\lambda}^{AC} (\gamma^m)_A{}^B D_m \lambda_{BC} - \overline{\lambda}^{CA} (\gamma^m)_A{}^B [\phi_m, \lambda_{CB}] \Big)$$

5 scalars $\rightarrow$ vector field ϕ_m

Can combine with gauge field A_m to form a complexified gauge field $\mathcal{A}_m$

Theory has $\mathcal{A}_m$ and $\overline{\mathcal{A}}_m$

Complexified field strengths: $\mathcal{F}_{mn}$ and $\overline{\mathcal{F}}_{mn}$

Twisted $5D$ $\mathcal{N} = 2$ Yang-Mills

Fermions

$$\lambda_{AB} = \frac{1}{2\sqrt{2}}\Big(\frac{1}{2}(\sigma^{mn})_{AB}\chi_{mn} + (\gamma^m)_{AB}\psi_m - \epsilon_{AB}\eta\Big)$$

η, ψ_m and χ_{mn}: p-forms (Grassmann odd)

Similar decomposition for twisted fields

$\mathcal{Q}, \mathcal{Q}_m, \mathcal{Q}_{mn}$

Important property

$$\mathcal{Q}^2 \ \cdot \ = 0$$

on the lattice

Twisted Action

In terms of p-form fields

$$S_5 = \frac{1}{g_5^2}\int d^5x \ \mathrm{Tr}\left(\frac{1}{4}\overline{\mathcal{F}}_{mn}\mathcal{F}_{mn} - \frac{1}{8}[\mathcal{D}_m, \overline{\mathcal{D}}_m]^2 - i\chi_{mn}\mathcal{D}_m\psi_n - i\psi_m\overline{\mathcal{D}}_m\eta - \frac{i}{8}\epsilon_{mncde}\chi_{de}\overline{\mathcal{D}}_c\chi_{mn}\right)$$

Can be written as a sum of $\mathcal{Q}$-exact and $\mathcal{Q}$-closed terms

$$S_5 = \mathcal{Q}\Lambda - \frac{1}{g_5^2}\int d^5x \ \mathrm{Tr}\ \frac{i}{8}\epsilon_{mncde}\chi_{de}\overline{\mathcal{D}}_c\chi_{mn},$$

$$\Lambda = \frac{1}{g_5^2}\int d^5x \ \mathrm{Tr}\left(\frac{i}{4}\chi_{mn}\mathcal{F}_{mn} - \eta D_m\phi_n - \frac{1}{2}\eta d\right)$$

Twisted Action

$\mathcal{Q}$ transformations on twisted fields

$$
\begin{aligned}
\mathcal{Q}\mathcal{A}_m &= \psi_m \\
\mathcal{Q}\psi_m &= 0 \\
\mathcal{Q}\overline{\mathcal{A}}_m &= 0 \\
\mathcal{Q}\chi_{mn} &= -i\overline{\mathcal{F}}_{mn} \\
\mathcal{Q}\eta &= d \\
\mathcal{Q}d &= 0
\end{aligned}
$$

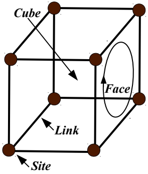
d: auxiliary field

Exactly nilpotent: $\mathcal{Q}^2 \cdot = 0$.

On the Lattice

Straightforward. Use geometric discretization

- Discretize theory on a hypercube.
- Place p-form fields on p-cells of hypercube



$$\begin{aligned}
\widehat{\boldsymbol{\mu}}_1 &= (1,0,0,0,0) \\
\widehat{\boldsymbol{\mu}}_2 &= (0,1,0,0,0) \\
\widehat{\boldsymbol{\mu}}_3 &= (0,0,1,0,0) \\
\widehat{\boldsymbol{\mu}}_4 &= (0,0,0,1,0) \\
\widehat{\boldsymbol{\mu}}_5 &= (0,0,0,0,1)
\end{aligned}$$

Note: We have only 16 fermions not 32, to fill the hypercube!

Lattice covariant difference operators appropriate for p-forms

P. H. Damgaard and S. Matsuura, [JHEP 0707, 051 (2007)]
S. Catterall, [JHEP 0801, 048 (2008)]

$$
\begin{aligned}
\mathcal{D}_m^{(+)} f(\mathbf{n}) &= \mathcal{U}_m(\mathbf{n}) f(\mathbf{n}+\widehat{\boldsymbol{\mu}}_m) - f(\mathbf{n})\mathcal{U}_m(\mathbf{n}) \\
\mathcal{D}_m^{(+)} f_n(\mathbf{n}) &= \mathcal{U}_m(\mathbf{n}) f_n(\mathbf{n}+\widehat{\boldsymbol{\mu}}_m) - f_n(\mathbf{n})\mathcal{U}_m(\mathbf{n}+\widehat{\boldsymbol{\mu}}_n) \\
\overline{\mathcal{D}}_m^{(-)} f_m(\mathbf{n}) &= f_m(\mathbf{n})\overline{\mathcal{U}}_m(\mathbf{n}) - \overline{\mathcal{U}}_m(\mathbf{n}-\widehat{\boldsymbol{\mu}}_m) f_m(\mathbf{n}-\widehat{\boldsymbol{\mu}}_m) \\
\overline{\mathcal{D}}_c^{(+)} f_{mn}(\mathbf{n}) &= f_{mn}(\mathbf{n}+\widehat{\boldsymbol{\mu}}_c)\overline{\mathcal{U}}_c(\mathbf{n}) - \overline{\mathcal{U}}_c(\mathbf{n}+\widehat{\boldsymbol{\mu}}_m+\widehat{\boldsymbol{\mu}}_n) f_{mn}(\mathbf{n})
\end{aligned}
$$

Lattice Theory

$\mathcal{Q}$ transformations on the lattice

$$\begin{aligned}
\mathcal{Q}\mathcal{U}_m(\mathbf{n}) &= \psi_m(\mathbf{n}) \\
\mathcal{Q}\psi_m(\mathbf{n}) &= 0 \\
\mathcal{Q}\overline{\mathcal{U}}_m(\mathbf{n}) &= 0 \\
\mathcal{Q}\chi_{mn}(\mathbf{n}) &= -i\left(\overline{\mathcal{D}}_m^{(+)}\overline{\mathcal{U}}_n(\mathbf{n})\right) = -i\overline{\mathcal{F}}_{mn}^L \\
\mathcal{Q}\eta(\mathbf{n}) &= d(\mathbf{n}) \\
\mathcal{Q}d(\mathbf{n}) &= 0
\end{aligned}$$

Similar to the continuum transformations.

Lattice Theory

$\mathcal{Q}$-exact piece

$$S_{\mathcal{Q}-\text{exact}} = \beta \sum_{\mathbf{n},m,n} \text{Tr} \Big[-\frac{1}{4}\overline{\mathcal{F}}^{L}_{mn}(\mathbf{n})\mathcal{F}_{mn}(\mathbf{n}) - \frac{1}{8}\Big(\overline{\mathcal{D}}^{(-)}_{m}\mathcal{U}_{m}(\mathbf{n})\Big)^{2}$$
$$-i\chi_{mn}(\mathbf{n})\mathcal{D}^{(+)}_{m}\psi_{n}(\mathbf{n}) - i\eta(\mathbf{n})\overline{\mathcal{D}}^{(-)}_{m}\psi_{m}(\mathbf{n})\Big]$$

Gauge-invariant and local

$\mathcal{Q}$-closed piece has to be modified

Open loop on the lattice. Need to close it.

Lattice Theory

Introduce a path ordered link.

Option 1. Use $\overline{\mathcal{U}}$-fields

$$\mathcal{P}_{\text{POL}} \equiv \prod_{l \in C_L} \overline{\mathcal{U}}_l$$

Action is $\mathcal{Q}$-invariant, gauge-invariant but non-local.

Option 2. Use $\mathcal{U}$-fields

$$\mathcal{P}_{\text{POL}} \equiv \prod_{l \in C_L} \mathcal{U}_l$$

Action is $\mathcal{Q}$ non-invariant, gauge-invariant and local.

Need to introduce additional operator

$$\Big(\mathcal{Q}\mathcal{P}_{\text{POL}}\Big) \times (\cdots)$$

to make it $\mathcal{Q}$ invariant

Lattice Theory

Lattice action is: $S = S_{\mathcal{Q}-\text{exact}} + S_{\mathcal{Q}-\text{closed}}$

$$S_{\mathcal{Q}-\text{exact}} = \beta \sum_{\mathbf{n},m,n} \text{Tr} \Big[-\frac{1}{4}\overline{\mathcal{F}}^{L}_{mn}(\mathbf{n})\mathcal{F}_{mn}(\mathbf{n}) - \frac{1}{8}\Big(\overline{\mathcal{D}}^{(-)}_{m}\mathcal{U}_{m}(\mathbf{n})\Big)^2 - i\chi_{mn}(\mathbf{n})\mathcal{D}^{(+)}_{m}\psi_{n}(\mathbf{n}) - i\eta(\mathbf{n})\overline{\mathcal{D}}^{(-)}_{m}\psi_{m}(\mathbf{n})\Big]$$

$$S_{\mathcal{Q}-\text{closed}} = -\frac{i\beta}{8} \sum_{\mathbf{n},m,n,c,d,e} \text{Tr}\; \epsilon_{mncde}\mathcal{P}_{\text{POL}} \times \chi_{de}(\mathbf{n} + \widehat{\boldsymbol{\mu}}_m + \widehat{\boldsymbol{\mu}}_n + \widehat{\boldsymbol{\mu}}_c)\overline{\mathcal{D}}^{(+)}_{c}\chi_{mn}(\mathbf{n})$$

$\mathcal{Q}S = 0$, if we use Option 1. Additional operator needed for Option 2.

Lattice Theory

- A well defined theory at finite lattice spacing
- Can simulate at finite coupling and finite N
- No doublers
- Theory has naive continuum limit
- How about quantum continuum limit?
  - Look for 2nd (or higher) order phase transitions

Future Directions

- Most pressing question:

  Does it have a non-trivial UV fixed point on the lattice?

  If so, it can give UV completion of the $5D$ theory and thus a non-perturbative definition

- One can check the scaling of free-energy on the lattice: N^3? (at all couplings?)

- Validating gauge-gravity duality

Lots of interesting explorations to do...!

THANK YOU!