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Charmed meson physics from three-flavour lattice QCD

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LATTICE 2016

Graduiertenkolleg 2149 Research Training Group

Motivation for distance preconditioning method

In order to extract charmed observables such as meson masses and the leptonic decay constants f_{D} and $f_{\mathrm{D_s}}$ given by the non-perturbative QCD matrix elements $\langle 0|\overline{q}\gamma_\mu\gamma_5 c|\mathrm{D}_q(p)\rangle = \mathrm{i} f_{\mathrm{D}_q} p_\mu$, $q = \mathrm{d},\mathrm{s}$ it is imperative to efficiently compute the propagator of the heavy charm-quark with sufficient accuracy. Numerically one checks if the condition

$$\left|\sum_y (D[U]+m_0)_{x,y} S^n(y) - \eta_t(x)\right| < r_{gl}$$

is satisfied, where $D[U]$ is the discretized lattice Dirac operator, m_0 is the quark mass in lattice units, $S^n(y)$ is the approximate solution at the n-th iteration of the solver procedure, $\eta_t(x)$ is a stochastic noise source located on a single time-slice t and r_{gl} is the **global** numerical accuracy one likes to achieve.

- Problem: time-slices y_0 far away from source at x_0 exponentially suppressed by factor $\propto \exp(-m y_0)$
- Contributions to norm negligible for heavy quarks
- Solutions for large time extents $|x_0 - y_0|$ increasingly inaccurate

▶ Proposed improvement: implement Distance Preconditioning [1] via diagonal preconditioning matrix P:

$$P = \begin{pmatrix} p_1 & 0 & \cdots & \cdots & 0 \\ 0 & p_2 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & \cdots & 0 & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & p_T \end{pmatrix} \quad \text{with} \quad p_i = \exp\left(\alpha_0 \cdot |x_0 - y_{0_i}|\right)$$

- Instead of original system consider preconditioned system:

$$AS = \eta \ \text{ with } \ A = (D[U]+m_0) \longrightarrow A'S' = \eta' \Leftrightarrow \left(PAP^{-1}\right)(PS) = (P\eta)$$

$\Rightarrow$ solve for PS and scale with P^{-1} to obtain original solution S

Computational details & techniques

First numerical tests were performed on several Coordinated Lattice Simulations ensembles (https://twiki.cern.ch/twiki/bin/view/CLS/WebHome) with tree level improved Lüscher-Weisz gauge action [2] & Sea of $N_\mathrm{f} = 2+1$ (2 light mass degenerate + strange) non-perturbatively $\mathrm{O}(a)$ improved Wilson quarks on three representative ensembles:

id	H105r002	U101r001	H200r001
$T \times L^3$	96×32^3	128×24^3	96×32^3
β	3.4	3.4	3.55
$a\,[\mathrm{fm}]$	0.086	0.086	0.064
κ_l	0.136970	0.136970	0.137
κ_s	0.13634079	0.13634079	0.137
$m_\pi\,[\mathrm{MeV}]$	280	280	420
$m_\mathrm{K}\,[\mathrm{MeV}]$	460	460	420

- Simulations performed using openQCD code [3], with overall computational setup described in detail in [4]
- Two-point correlation functions of the **pseudoscalar** density $P^{rs} = \overline{\psi}_r\gamma_5\psi_s$ and the time component of the **axial** vector current $A_0^{rs} = \overline{\psi}_r\gamma_0\gamma_5\psi_s$ are constructed from two mass non-degenerate valence quarks r and s as

$$f_{\mathrm{PP}}^{rs}(x_0) = -a^3 \sum_{\vec{x}} \langle P^{rs}(x) P^{sr}(0)\rangle \ , \ f_{\mathrm{AP}}^{rs}(x_0) = -a^3 \sum_{\vec{x}} \langle A_0^{rs}(x) P^{sr}(0)\rangle$$

- Using 16 $U(1)$ noise sources $\eta_t(x) = \delta_{t,x_0}\exp(i\phi(\vec{x}))$ located on randomly chosen time slices t [5] so that solving the Dirac equation once for each noise vector $\zeta_t^r = Q^{-1}(m_{0,r})\eta_t = a^{-1}(D+m_{0,r})^{-1}\gamma_5\eta_t$ suffices to estimate the two-point functions projected onto zero momentum

$$a^3 f_{\mathrm{XP}}^{rs}(x_0) = \sum_{\vec{x}} \langle[\zeta_t^r(x_0+t,\vec{x})]^\dagger \Gamma \zeta_t^s(x_0+t,\vec{x})\rangle \ , \ \Gamma = \mathbf{1}/\gamma_0 \text{ for } \mathrm{X} = \mathrm{P}/\mathrm{A}$$

- Unmodified solver setup: locally deflated Schwarz preconditioned general conjugate residual solver (DFL_SAP_GCR) for light and strange quarks, conjugate gradient on the normal equations solver (CGNE) and DFL_SAP_GCR solver for heavy charm quarks
- Modified solver setup: DFL_SAP_GCR solver for l,s, distance preconditioned CGNE solver (CGNE_DP) & distance preconditioned SAP_GCR solver (SAP_GCR_DP) for h

References

[1] G.M. de Divitiis, R. Petronzio, N. Tantalo, Phys.Lett. B 692 (2010) 157-160, arXiv:1006.4028.

[2] M. Lüscher and P. Weisz, Commun.Math.Phys. 97 (1985) 59, doi:10.1007/BF01206178.

[3] M. Lüscher and S. Schaefer, JHEP 1107 (2011) 036, arXiv:1105.4749.

[4] M.Bruno et al., JHEP 1502 (2015) 043, arXiv:1411.3982.

[5] R. Sommer, Nucl. Phys. Proc. Suppl. 42 (1995) 186, hep-lat/9411024; M. Foster and C. Michael, Phys. Rev. D 59 (1999) 074503, hep-lat/9810021.

Observables & strategies

- Average $\mathrm{O}(a)$ improved bare PCAC quark masses of flavours r and s :

$$\tfrac{1}{2}(m_{rr}+m_{ss})(x_0) = m_{rs}(x_0) = \frac{\tfrac{1}{2}(\partial_0+\partial_0^*) f_{\mathrm{AP}}(x_0) + c_\mathrm{A}\, a\, \partial_0^*\partial_0 f_{\mathrm{PP}}(x_0)}{2 f_{\mathrm{PP}}(x_0)}$$

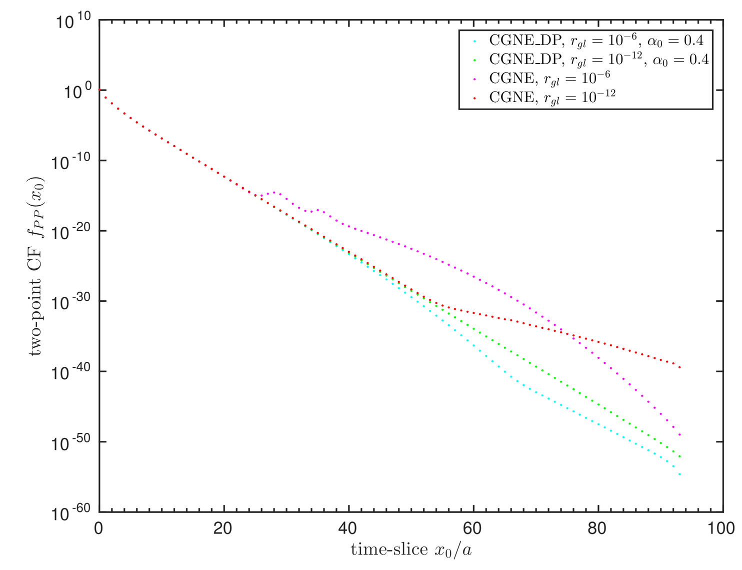
- Pseudoscalar (PS) meson mass derived from spectral decomposition for infinite T:

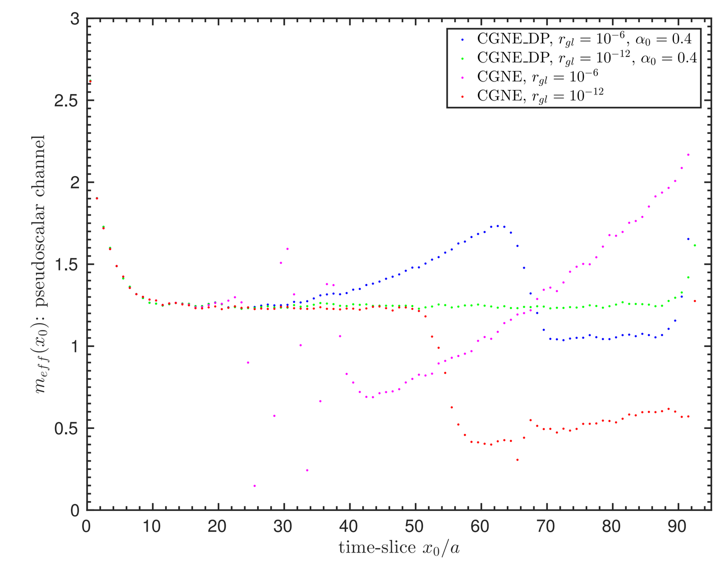
$$f_{\mathrm{PP}}(x_0) = \sum_{i=1}^{\infty} c_i \exp(-E_i x_0) \ \text{ with } \ E_1 = m_{\mathrm{PS}} \ , \ E_{i\geq 2} : \text{excited states}$$

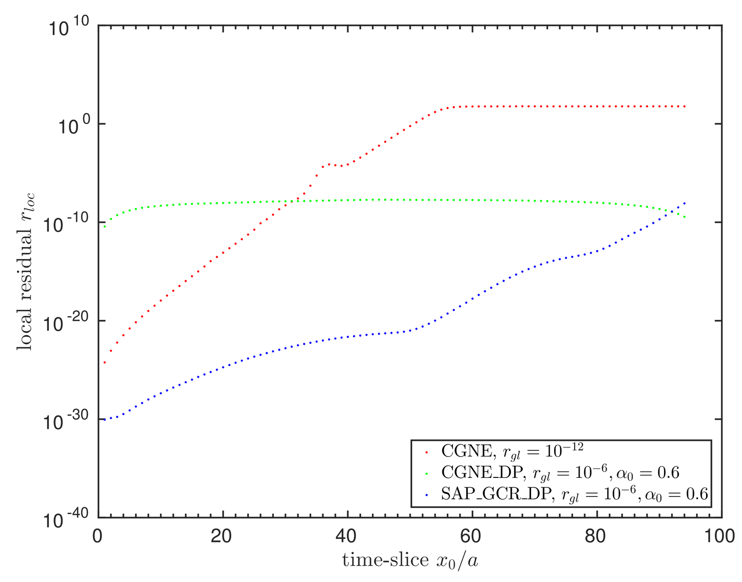
- Chosen method for extraction of bare pseudoscalar decay constant:

$$f_{\mathrm{PS}}^{bare} = 2\sqrt{2c_1} m_{rs} m_{\mathrm{PS}}^{-\frac{3}{2}} \ \text{ and } \ f_{\mathrm{PS}} = f_{\mathrm{PS}}^{bare} \times \text{renormalization constant}$$

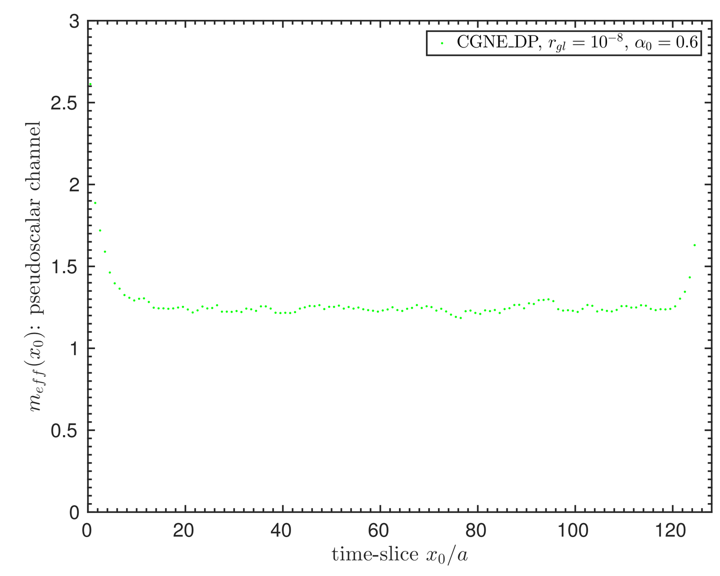
Numerical tests of distance preconditioning

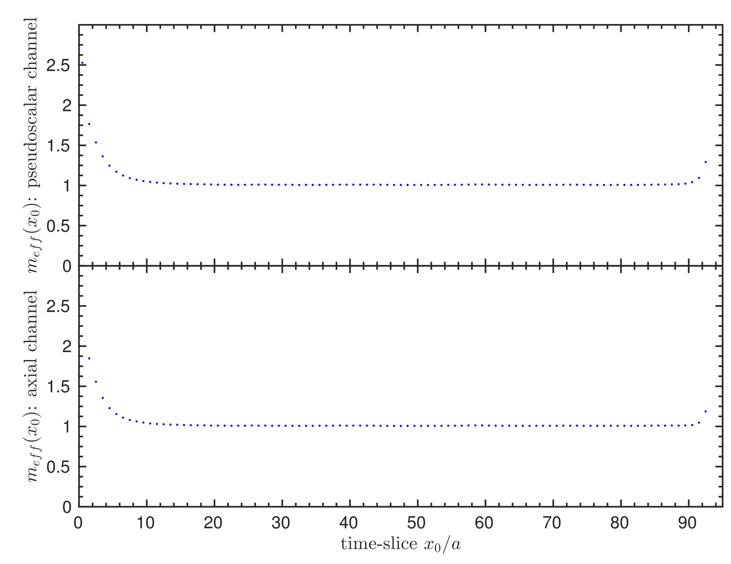




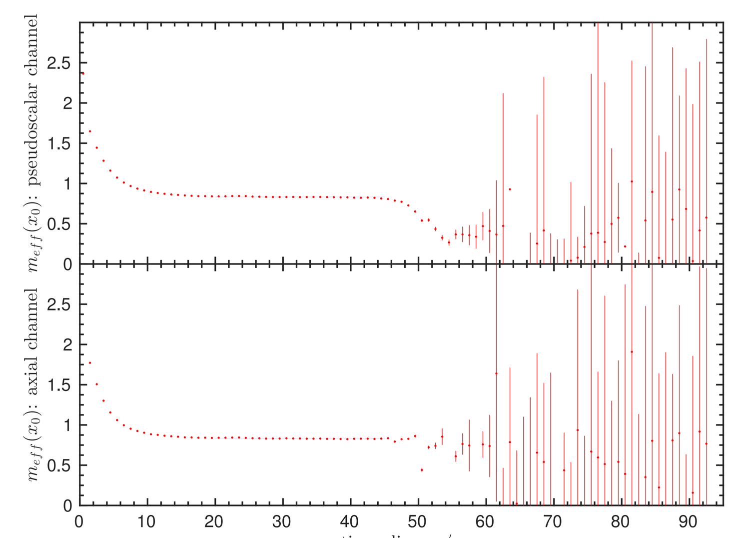


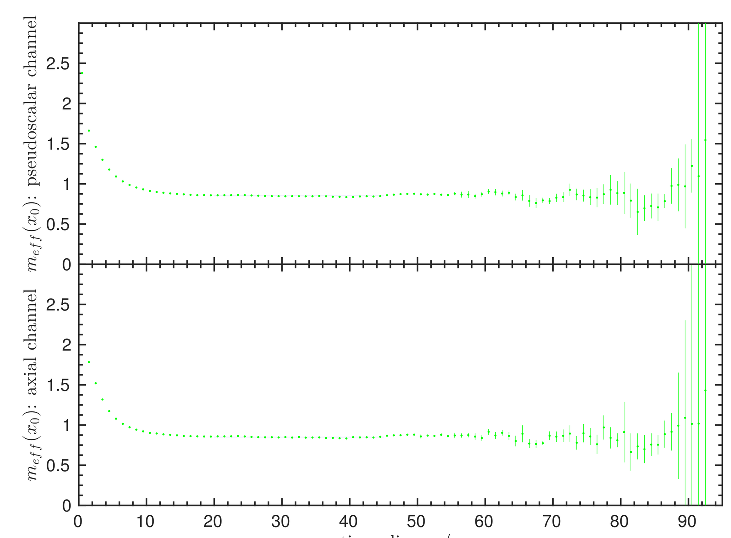
First numerical checks with modified CGNE_DP solver show increased accuracy for solution of heavy-heavy **correlator** on sample of H105r001 configurations (top left) and derived **effective** PS meson mass $m_{eff} = \ln\left(\frac{f_{PP}(x_0)}{f_{PP}(x_0+1)}\right)$ (top right). The behaviour of the **local** residual $r_{loc} = \frac{|AS-\eta_t|(x_0)}{|S(x_0)|}$ was tested for unmodified CGNE, modified CGNE_DP and modified SAP_GCR_DP solver (right)



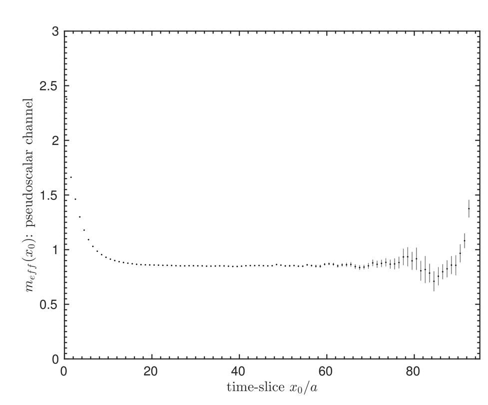


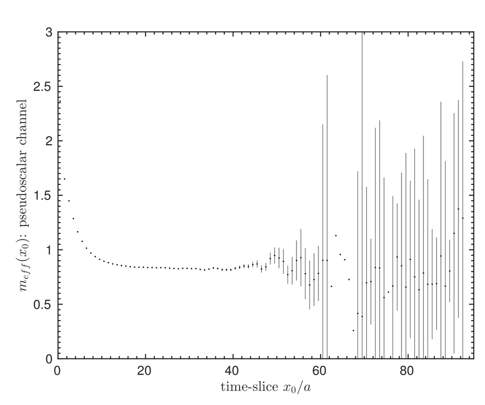
Method also applicable for larger time-extents (10 configurations of U101, top left) and different choice of parameters (50 configurations of H200r001, top right).

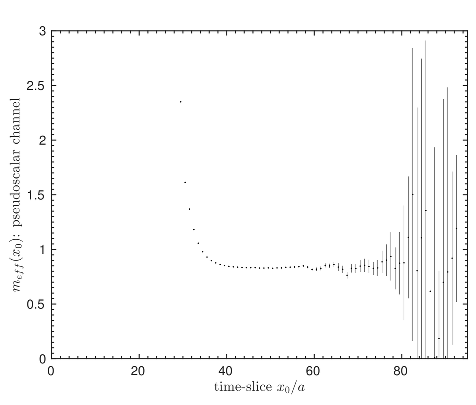




Considerable accuracy gain from unmodified solver setup (50 configurations of H105r002, top left) to modified setup for heavy-strange (with $r_{gl} = 10^{-4}$, $\alpha_0 = 0.7$, $r_{loc} < 10^{-10}$, top right). Sample run with 200 configurations of H105r002 shows increased accuracy for $m_{eff}(x_0)$ in the PS channel for heavy-strange (bottom-left) and heavy-light with source at $x_0 = 1$ (bottom-middle) and at $x_0 = 30$ (bottom-right).







Outlook: increase of statistics and ensembles, streamlining of fitting procedure, smearing techniques to allow for an additional method for extraction of $f_{D_{(s)}}$