Computing the static potential using non-string-like trial states

Lattice 2016 - Southhampton

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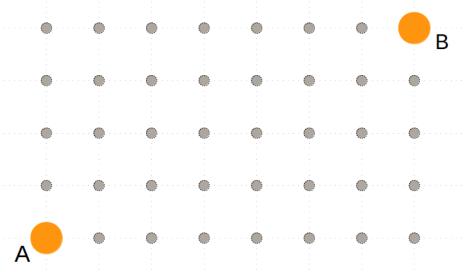
Motivation

- For calculating the static potential with a high resolution we have to work with off axis separated quarks.

- e.g. matching the lattice QCD potential with the perturbative potential to determine $\Lambda_{\overline{MS}}$ in Fourier space. [F. Karbstein, A. Peters and M. Wagner, JHEP **1409**, 114 (2014) [arXiv:1407.7503 [hep-ph]]



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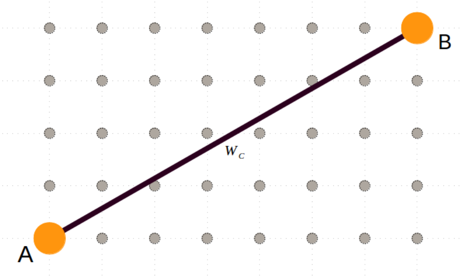
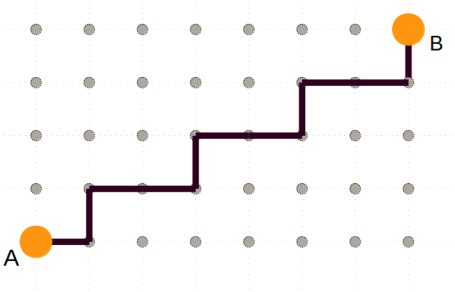


- The quantity of interest is the Wilson loop, which connects the two quarks like a string.

- To compute the spatial part of the Wilson loop one has to go over stair-like paths through the lattice.



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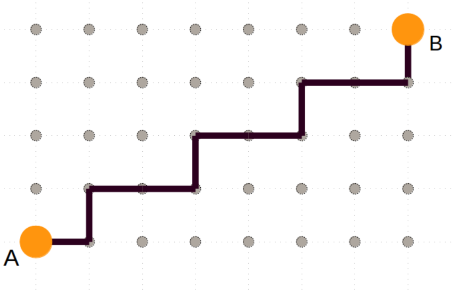

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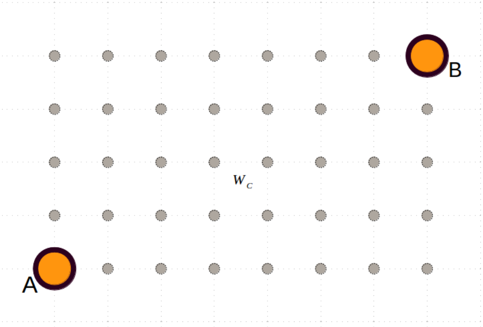

- These stair-like paths are causing a big computational effort for a large number of lattice points.

- Idea: Substitute the spatial part of the Wilson loop by an other object to avoid the calculation of stair-like paths.

The Technical Part

- The spatial Wilson line is needed to ensure gauge invariance of the $q\bar{q}$ trial state.

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- Transformation behavior required: $U'(x,y) = G(x)U(x,y)G^{\dagger}(y)$

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- We explore an idea, which has been used in the context of Polyakov loops and the static potential at finite temperature.

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- Consider the covariant lattice Laplace operator:

$$\begin{aligned}\Delta f =& \frac{1}{a^2}\Big(U_1^\dagger(x-a,y,z)f(x-a,y,z) - 2f(\mathbf{x}) + U_1(x,y,z)f(x+a,y,z)\Big)\\ &+\frac{1}{a^2}\Big(U_2^\dagger(x,y-a,z)f(x,y-a,z) - 2f(\mathbf{x}) + U_2(x,y,z)f(x,y+a,z)\Big)\\ &+\frac{1}{a^2}\Big(U_3^\dagger(x,y,z-a)f(x,y,z-a) - 2f(\mathbf{x}) + U_3(x,y,z)f(x,y,z+a)\Big)\end{aligned}$$

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- Apply an gauge transformation on the eigenvector-equation.

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- Apply an gauge transformation on the eigenvector-equation.
- We see: $G^\dagger(x)f'(x)$ is again eigenvector to the covariant Laplace operator.

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- In SU(3) the eigenvalues are in general nondegenerate. This means:

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- In SU(2) however, the eigenvalues are always two fold degenrate. This means:

$$\alpha f_1(x) + \beta f_2(x) = G^\dagger(x)f'(x)$$

- Where f_1 and f_2 are an orthonormal basis of the corresponding eigenspace.

Transformation law for SU(3):

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Transformation law for SU(2):

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Wilson Line:

$$U'(x,y) = G(x)U(x,y)G^\dagger(y)$$

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SU(3) - Case: $$f'(x)f'^\dagger(y) = G(x)f(x)f^\dagger(y)G^\dagger(y)$$

SU(2) - Case: $$\sum_{i=1}^{2} f_i'(x)f_i'^\dagger(y) = G(x)\Big(\sum_{i=1}^{2} f_i(x)f_i^\dagger(y)\Big)G^\dagger(y)$$

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- Where f_1 and f_2 are an orthonormal basis of the corresponding eigenspace.

- We found objects with the required transformation behavior given by

$$f(x)f^{\dagger}(y) \quad \text{for SU(3) and} \quad \sum_{i=1}^{2} f_i(x)f_i^{\dagger}(y) \quad \text{for SU(2).}$$

- We found objects with the required transformation behavior given by $f(x)f^\dagger(y)$ for SU(3) and $\sum_{i=1}^{2} f_i(x)f_i^\dagger(y)$ for SU(2).

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The computation of stair-like paths is not longer needed.

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Price to pay:

One has to compute the eigenvectors of Δ first.

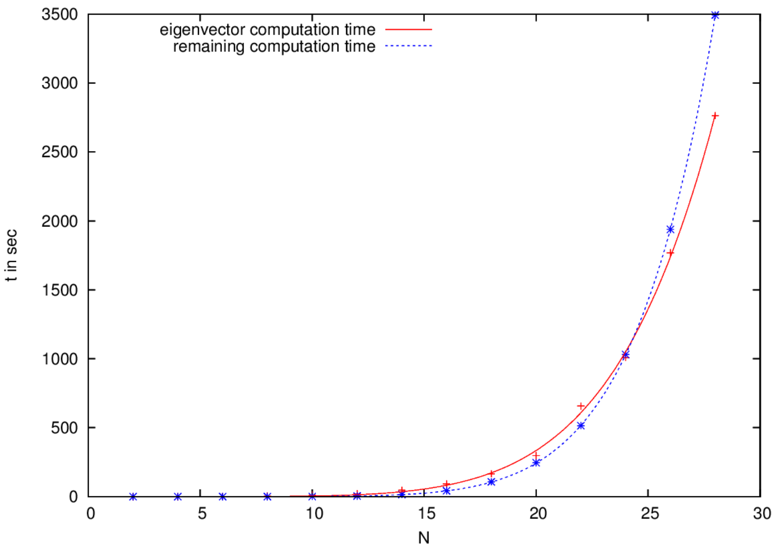


Figure : Runtime of the eigenvector calculation and the remaining computations using the new method

Results

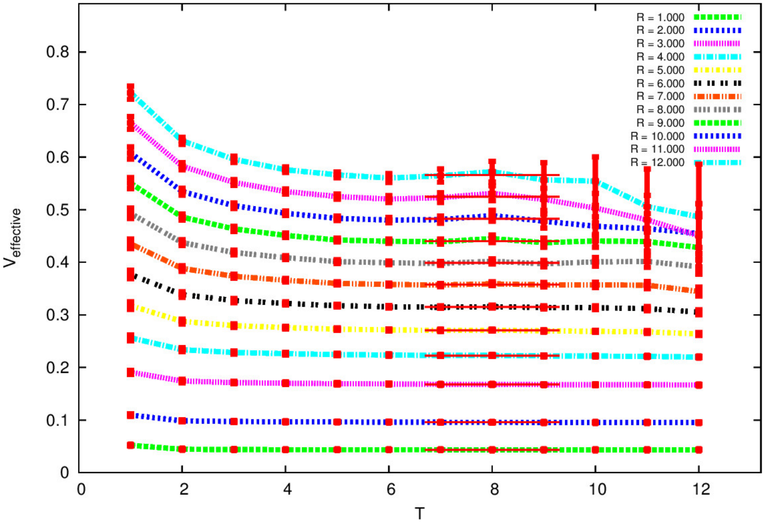


Figure : Effective mass in units of the lattice spacing for the ordinary Wilson loop - using 100 basically independent SU(2) gaugelink configurations with $\beta = 2.5$ ($\approx 0.089 fm$) on a 24x24 Lattice

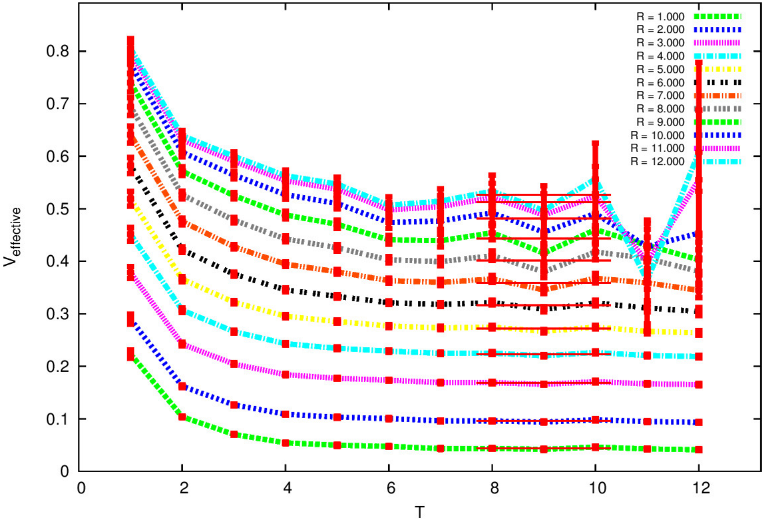


Figure : Effective mass in units of the lattice spacing for the new method - using 100 basically independent SU(2) gaugelink configurations with $\beta = 2.5$ ($\approx 0.089 fm$) on a 24x24 Lattice

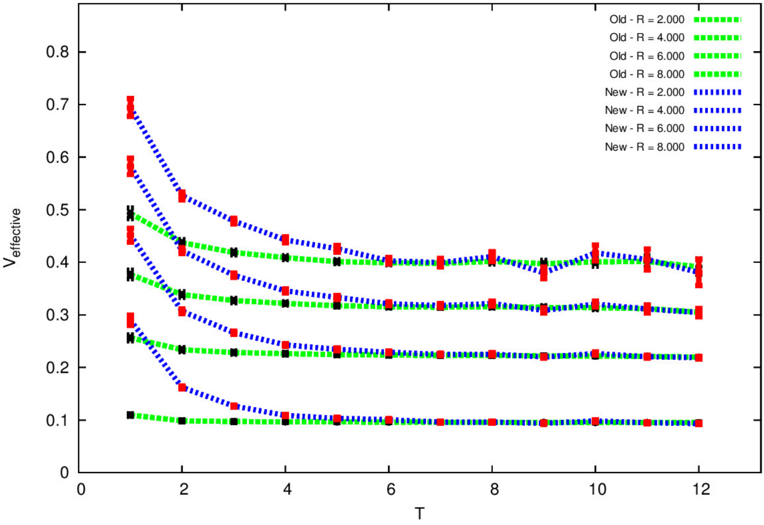


Figure : Comparison of the effective masses from the ordinary Wilson loop and the new method - using 100 basically independent SU(2) gaugelink configurations with $\beta = 2.5$ ($\approx 0.089 fm$) on a 24x24 Lattice

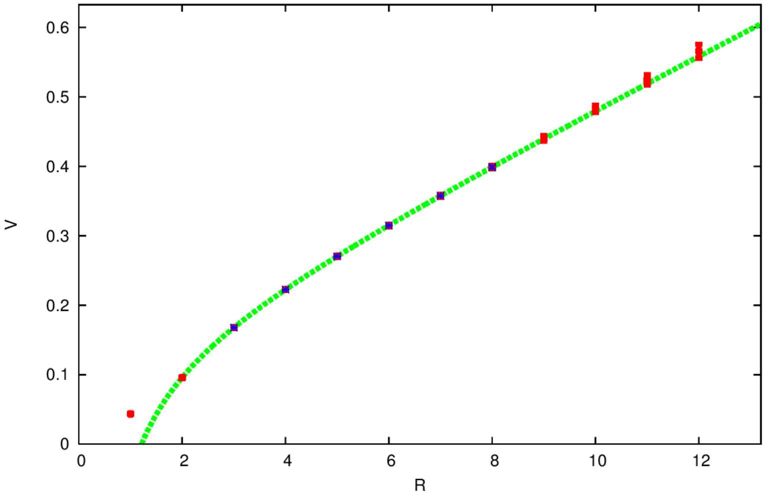
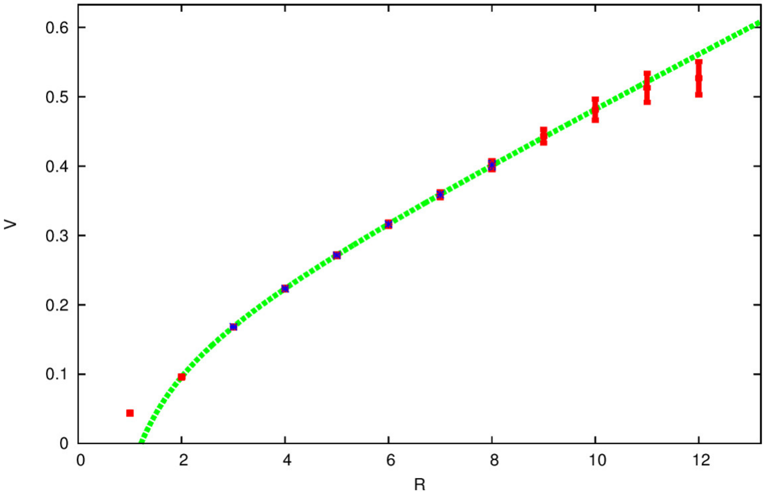


Figure : Potential for the static $q\bar{q}$ pair in units of the lattice spacing - using the ordinary Wilson loop on 100 basically independent SU(2) gaugelink configurations with $\beta = 2.5$ ($\approx 0.089 fm$) on a 24x24 Lattice



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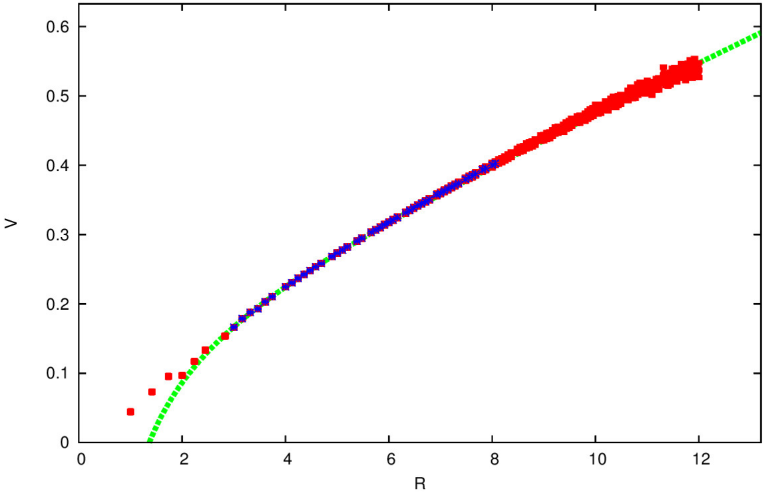

Figure : Off-axis potential for the static $q\bar{q}$ pair in units of the lattice spacing - using the new method on 100 basically independent SU(2) gaugelink configurations with $\beta = 2.5$ ($\approx 0.089 fm$) on a 24x24 Lattice

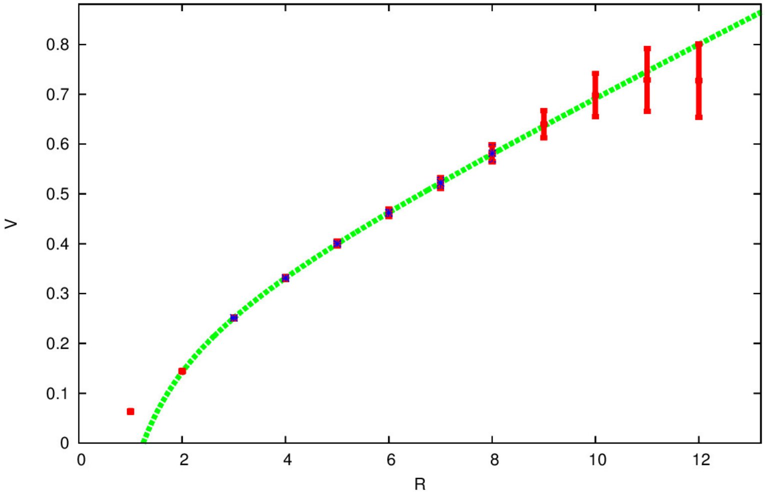

Figure : On-axis potential for the static $q\bar{q}$ pair in units of the lattice spacing - using the new method on 60 basically independent SU(3) gaugelink configurations with $\beta = 3.9$ on a 48x24 Lattice

Summary

- In the ordinary approach the calculation of the static $q\bar{q}$-potential, for off-axis separations, requires the computation of time consuming stair-like paths.

- These stair-like paths come from the Wilson loop, an object that ensures gauge invariance of the used $q\bar{q}$ trial state.

- By using the eigenvectors of the covariant Laplace operator we were able to substitute the spatial part of the Wilson loop by a new object.

- Advantages: Fast computation times for off axis calculations and nearly similar quality of the results (error bars).

- Possible application: The potential with fine resolution can be used for better modeling and comparison with perturbative theories ($\Lambda_{\overline{MS}}$-determination, $b\bar{b}$-spectrum).