Testing the hadro-quarkonium model on the lattice

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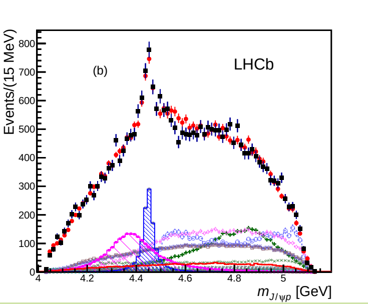
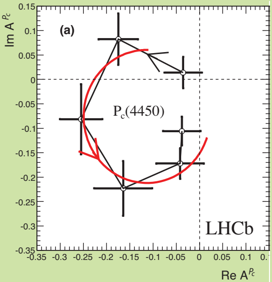
Motivation

LHCb pentaquark candidates





$P_c^+(4380)$ ($J^P = \frac{3}{2}^-$) and $P_c^+(4450)$ ($J^P = \frac{5}{2}^+$) from $\Lambda_b \to J/\psi\, p\, K$ [LHCb: R. Aaij et al, 1507.03414, 1604.05708]

Conjecture of attractive forces between charmonium and pp systems [Brodsky, Schmidt and de Teramond, PRL64 (90) 1011]

Many interpretations

Motivation

Hadro-quarkonia

5 (4 q, 1 $\bar{q}$) quark systems are very difficult to study directly on the lattice
$20\,\mathrm{MeV}$ binding energy for charmonium-nucleon system for $m_\pi \approx 800\,\mathrm{MeV}$ [NPLQCD Collaboration: S. R. Beane et al, 1410.7069]

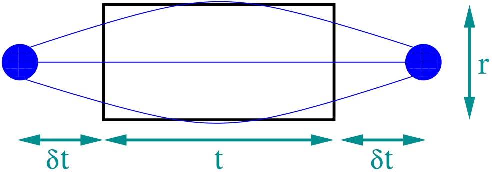
Here we test a particular model instead. Hadro-quarkonia: quarkonia bound "within" ordinary hadrons [S. Dubynskiy and M. Voloshin, 0803.2224]

Examples of close-by charmonium-baryon systems:
$J^P = \frac{3}{2}^- : m(\Delta)+m(J/\psi) \approx 4329\,\mathrm{MeV}$ vs. $4380\,\mathrm{MeV}$ (width $200\,\mathrm{MeV}$)

$J^P = \frac{5}{2}^+ : m(N) + m(\chi_{c2}) \approx 4496\,\mathrm{MeV}$ vs. $4450\,\mathrm{MeV}$ (width $40\,\mathrm{MeV}$)

Correlation function

The hadro-quarkonium model can be tested in the static limit. To leading order in (p)NRQCD, quarkonia can be approximated by the non-relativistic Schrödinger equation with a static potential $V_0(r)$. Does this become more or less attractive, when light hadrons are “added”?



Create a zero-momentum projected hadronic state $|H\rangle$ at the time 0.
Let it propagate to δt, create a quark-antiquark “string”.
Destroy this at $t + \delta t$ and the light hadron at $t + 2\delta t$:

Correlation function

$\overline{Q}Q$ binding energy shift "within" a hadron H

We compute

$$C_H(r,\delta t,t) = \frac{\langle W(r,t) C_{H,2\mathrm{pt}}(t+2\delta t)\rangle}{\langle W(r,t)\rangle\langle C_{H,2\mathrm{pt}}(t+2\delta t)\rangle}$$

where we average over the spatial positions of the Wilson loop $W(r,t)$ and over the hadronic sink positions

The shift of the binding potential is obtained from

$$\Delta V_H(r,\delta t) \equiv V_H(r,\delta t) - V_0(r) = -\lim_{t\to\infty}\frac{\mathrm{d}}{\mathrm{d}t}\ln[C_H(r,\delta t,t)]$$

and extrapolating $\delta t \to \infty$

Lattice details

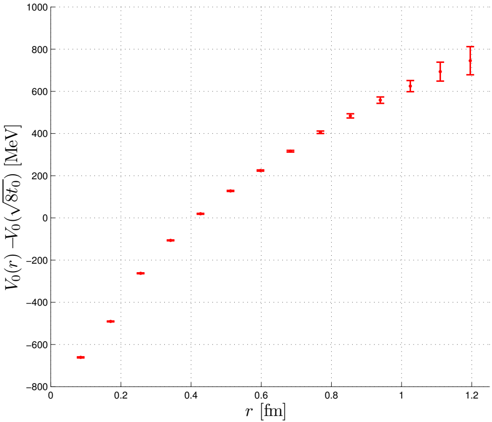
We analyse the $N_f = 2+1$ **CLS** ensemble C101 (96×48^3 sites) [Bruno et al, 1411.3982]:
$m_\pi \approx 223\,\mathrm{MeV}$, $m_\mathrm{K} \approx 476\,\mathrm{MeV}$, $Lm_\pi \approx 4.6$, $L \approx 4.1\,\mathrm{fm}$,
$t_0/a^2 = 2.9085(51)$, $a = 0.0854(15)\,\mathrm{fm}$ from $\sqrt{8t_0}$ extrapolated to physical point [G.S. Bali et al, 1606.09039] and $\sqrt{8t_0} = 0.4144(59)(37)\,\mathrm{fm}$ [Borsanyi et al, 1203.4469]

High statistics: 1552 configs, separated by 4 MDUs, times 12 hadron sources (1 forward, 1 backward, 10 forward and backward propagating $\Rightarrow$ 22 2-point functions). Wilson loops at all positions and in all directions.

Hadronic two-point functions have improved overlap with the ground state. We measure ΔV_H for π, K, ρ, $K^\star$ and ϕ mesons; for N, Σ, Λ and Ξ octet baryons with $J^P = \frac{1}{2}^\pm$; and for Δ, $\Sigma^\star$, $\Xi^\star$ and Ω decuplet baryons with $J^P = \frac{3}{2}^\pm$.

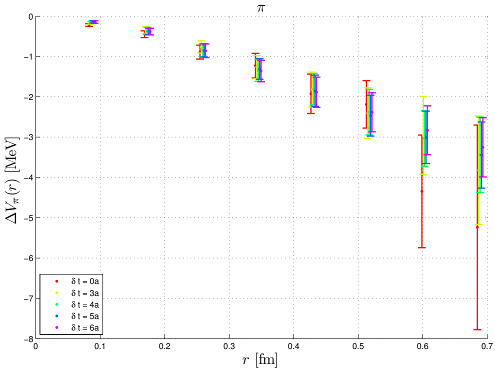
Static potential $V_0(r)$

Using the methods of [Donnellan, FK, Leder and Sommer 1012.3037] variational basis with $0, 5, 7, 12$ spatial HYP levels; open boundary conditions: average between $t = 24$ and $t = 72$; Sommer scale $r_0/a = 5.890(41)$



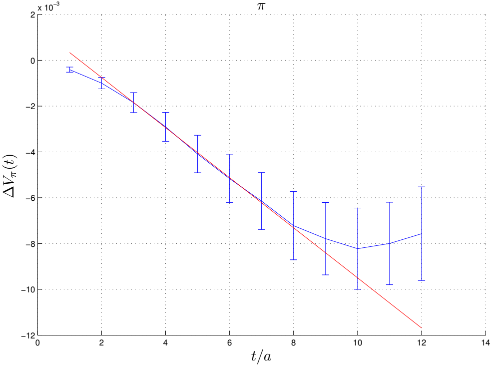
$\overline{Q}Q$ binding energy shift "within" a pion

[M. Alberti et al, in preparation]



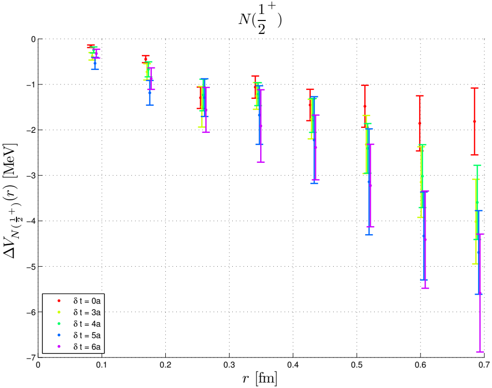
We can resolve small energy differences

Details of the pion analysis



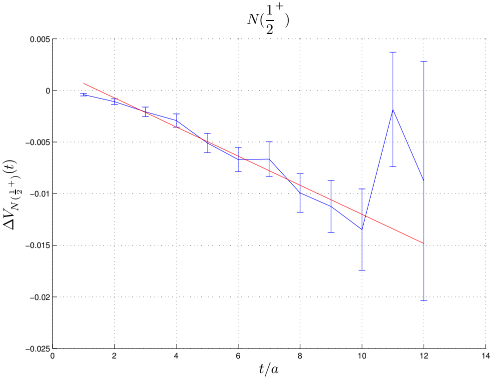
We do a linear fit in t to $\ln[C]$, here for $\delta t = 5a$ and $r = 6a \approx r_0$

$\overline{Q}Q$ binding energy shift "within" a nucleon $N(\frac{1}{2}^+)$



For a baryon of parity P, parity P is taken in the forward propagator and parity $-P$ in the backward propagator

Details of the nucleon analysis



Linear fit in t to $\ln[C(r = 6a \approx r_0, \delta t = 5a, t)]$

The baryon decuplet

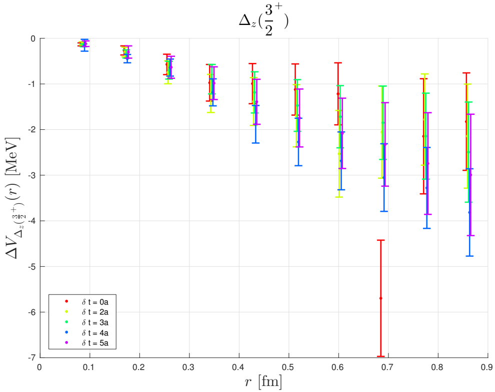
Candidates for pentaquark states

We measure decuplet baryons with **helicity** $\pm\frac{3}{2}$.
Quarkonium in S-wave has $J^P = 0^-$ and 1^-.

Combining 0^- or 1^- quarkonium with a $\frac{3}{2}^+$ decuplet baryon gives a $J^P = \frac{3}{2}^-$ state.

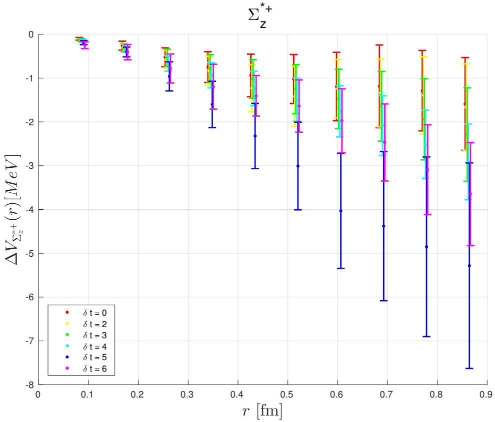
Combining 1^- quarkonium with a $\frac{3}{2}^-$ decuplet baryon gives a $J^P = \frac{5}{2}^+$ state.

$\overline{Q}Q$ binding energy shift "within" a $\Delta(\frac{3}{2}^+)$

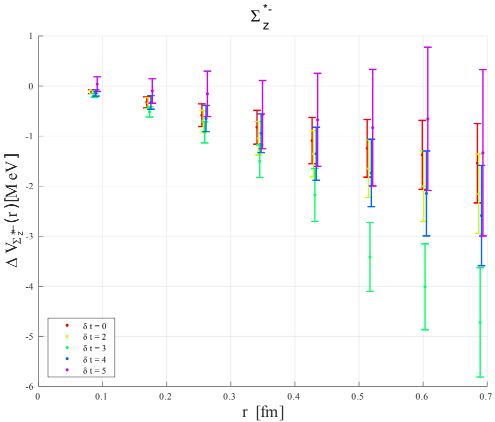


Polarisation: correlation of baryon polarised in z direction with Wilson loops in z direction

$\overline{Q}Q$ binding energy shift "within" a $\Sigma^*(\frac{3}{2}^+)$



$\overline{Q}Q$ binding energy shift "within" a $\Sigma^*(\frac{3}{2}^-)$



The signal for $\frac{3}{2}^-$ decuplet baryons is noisier. Also they do not match the mass of the $\frac{5}{2}^+$ pentaquark.

Summary

Conclusions

- Modifications of the static potential V_0 in the presence of light hadrons are found $\Delta V_H(r) < 0$ and are few MeV at $r = 0.5\,\mathrm{fm}$.
- Solving the Schrödinger equation with V_0 and $V_0 + \Delta_H V$ gives a stronger binding of charmonium $1S$ state by $-1\,\mathrm{MeV}$ to $-2.5\,\mathrm{MeV}$, of $1P$ state by $-2.5\,\mathrm{MeV}$ to $-4.5\,\mathrm{MeV}$ and of $2S$ state by $-2.5\,\mathrm{MeV}$ to $-5.5\,\mathrm{MeV}$. Somewhat inconsistent with the original hadro-charmonium.
- Interestingly, there is a similar attraction in all of the channels investigated so far.

Outlook

study of finite volume effects