Thermodynamics of strongly interacting plasma with high accuracy

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PLAN OF THE TALK

- Introduction
- Equation of State in a moving frame
- Numerical results
- Conclusions

Introduction

- The energy-momentum tensor is a fundamental quantity for a Quantum Field Theory: it contains the currents of Poincare' symmetry and of dilatations.

- For SU(N) Yang-Mills theory in the continuum in D dimensions it is given by

$$T_{\mu\nu}(x) = \frac{1}{g_0^2}\left[F^a_{\mu\rho}(x)F^a_{\nu\rho}(x) - \frac{1}{4}\delta_{\mu\nu}F^a_{\rho\sigma}(x)F^a_{\rho\sigma}(x)\right] = \tau_{\mu\nu} + \delta_{\mu\nu}\tau$$

$$\tau_{\mu\nu} = \frac{1}{g_0^2}\left[F^a_{\mu\alpha}F^a_{\nu\alpha} - \frac{1}{D}\delta_{\mu\nu}F^a_{\alpha\beta}F^a_{\alpha\beta}\right]$$

traceless part: two-index symmetric tensor of SO(D)

$$\tau = \frac{\epsilon}{2Dg_0^2}F^a_{\alpha\beta}F^a_{\alpha\beta}$$

anomalous part: singlet of SO(D)

- The energy-momentum tensor is a physical quantity: directly related to thermodynamic quantities like pressure, entropy and energy density.

- EoS is usually obtained from the direct measurement of the trace anomaly

G. Boyd et al.,
Nucl. Phys. B469 (1996) 419

The energy-momentum tensor on the lattice

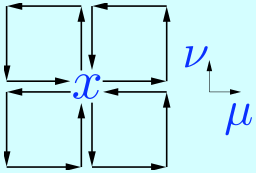
S. Caracciolo et al.,
NPB 375 (1992) 195;
Ann. Phys. 197 (1990) 119

- Lattice: preferred framework for non-perturbative study from first principles

  Explicit breaking of space-time symmetries that must be recovered in the continuum limit; troubles with the energy-momentum tensor

- $T_{\mu\nu}$ on the lattice must generate the correct conserved currents in the cont. limit

$$F_{\mu\nu}(x) = \frac{1}{8}\left[Q_{\mu\nu}(x) - Q_{\nu\mu}(x)\right]_{traceless} \qquad Q_{\mu\nu}(x) = \sum$$



$$T_{\mu\nu} = -\frac{2}{g_0^2}\left[\mathrm{Tr}\left(F_{\mu\alpha}(x)F_{\nu\alpha}(x)\right) - \frac{1}{4}\delta_{\mu\nu}\mathrm{Tr}\left(F_{\alpha\beta}(x)F_{\alpha\beta}(x)\right)\right]$$

- $T_{\mu\nu}$ has dimension 4; on the lattice SO(4) breaks to discrete subgroup

$$T^{[1]}_{\mu\nu} = (1-\delta_{\mu\nu})\frac{1}{g_0^2}\left[F^a_{\mu\alpha}F^a_{\nu\alpha}\right] \qquad T^{[2]}_{\mu\nu} = \delta_{\mu\nu}\frac{1}{4g_0^2}F^a_{\alpha\beta}F^a_{\alpha\beta} \qquad T^{[3]}_{\mu\nu} = \delta_{\mu\nu}\frac{1}{g_0^2}\left[F^a_{\mu\rho}F^a_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}F^a_{\rho\sigma}F^a_{\rho\sigma}\right]$$

- The renormalized energy-momentum tensor correctly generates translations and the trace anomaly. It can be written as

$$T^R_{\mu\nu} = Z_T\left\{T^{[1]}_{\mu\nu} + z_T T^{[3]}_{\mu\nu} + z_S[T^{[2]}_{\mu\nu} - \langle T^{[2]}_{\mu\nu}\rangle_0]\right\}$$

where Z_T, z_T and z_S are renormalizations constants and depend only on g_0^2.

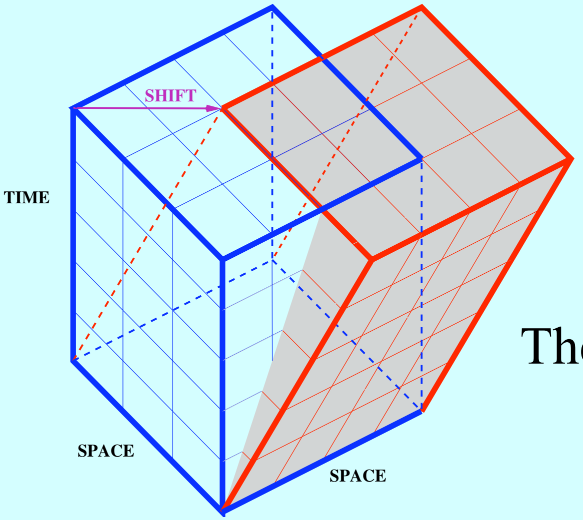
Z_T and z_T have been recently computed non-perturbatively.

L. Giusti and M.P.,
PRD 91 (2015) 11, 114504

A moving frame in Euclidean space: the shift

L. Giusti and H. Meyer,
PRL 2011, JHEP 2011 and 2013

A thermal quantum field theory in a moving reference frame:



$$Z(L_0,\xi) = \mathrm{Tr}\left[e^{-L_0(H-\xi_k T_{0k})}\right]$$

That corresponds to introducing a shift $\vec{\xi}$ when closing the periodic boundary conditions along the temporal direction:

$$A_\mu(L_0,\vec{x}) = A_\mu(0,\vec{x}-L_0\vec{\xi})$$

The Lorentz invariance implies that the free-energy is given by

$$f\left(L_0\sqrt{1+\xi^2}\right) = -\lim_{V\to\infty}\frac{1}{L_0 V}\log Z(L_0,V,\vec{\xi})$$

i.e. the temperature of the system is given by $T = \dfrac{1}{L_0\sqrt{1+\xi^2}}$

Parity symmetry is softly broken: off-diagonal components of $T_{\mu\nu}$ tensor pick up a non-vanishing expectation value

⬇

New equations relating $T_{\mu\nu}$ and the thermodynamics

The EoS in a moving frame

Landau, Lifschiz,
vol 6, "Fluid mechanics"

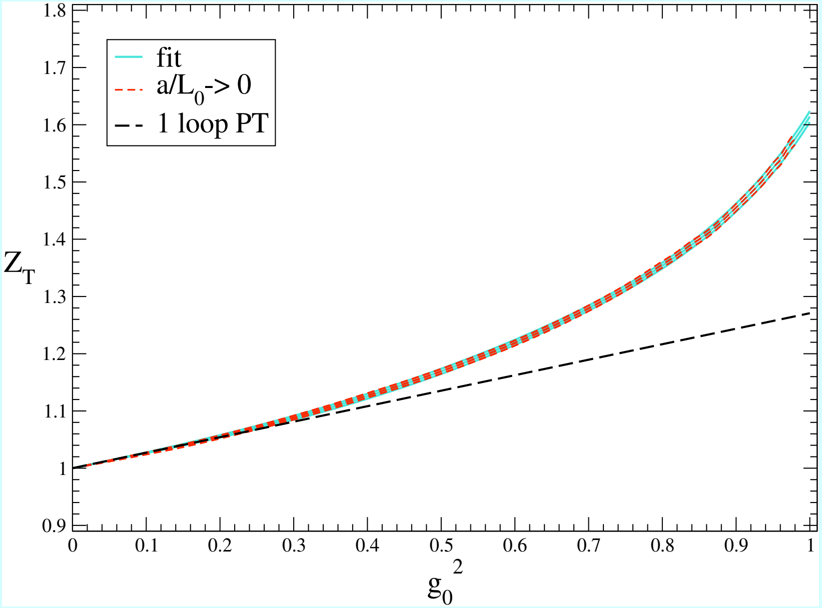
L. Giusti and H. Meyer,
JHEP 2013

$$T_{\mu\nu} = \begin{pmatrix} e & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} \quad \Rightarrow \quad \frac{s}{T^3} = \frac{e+p}{T^4} = -\frac{L_0^4(1+\vec{\xi}^2)^3}{\xi_k} \langle T_{0k} \rangle_\xi Z_T$$

s = entropy density; e = energy density; p = pressure; T = temperature

- s/T^3 is the primary observable that is measured; the other ones follow

$$p(T) - p(T_0) = \int_{T_0}^{T} s(T) dT \quad ; \qquad e(T) = Ts(T) - p(T) \quad \text{or} \quad e(T) - e(T_0) = \int_{s_0}^{s} T(s) ds$$



We only need to compute $\langle T_{0k} \rangle_\xi$ at fixed T

L. Giusti and M.P.,
PRD 91 (2015) 11, 114504

The numerical study

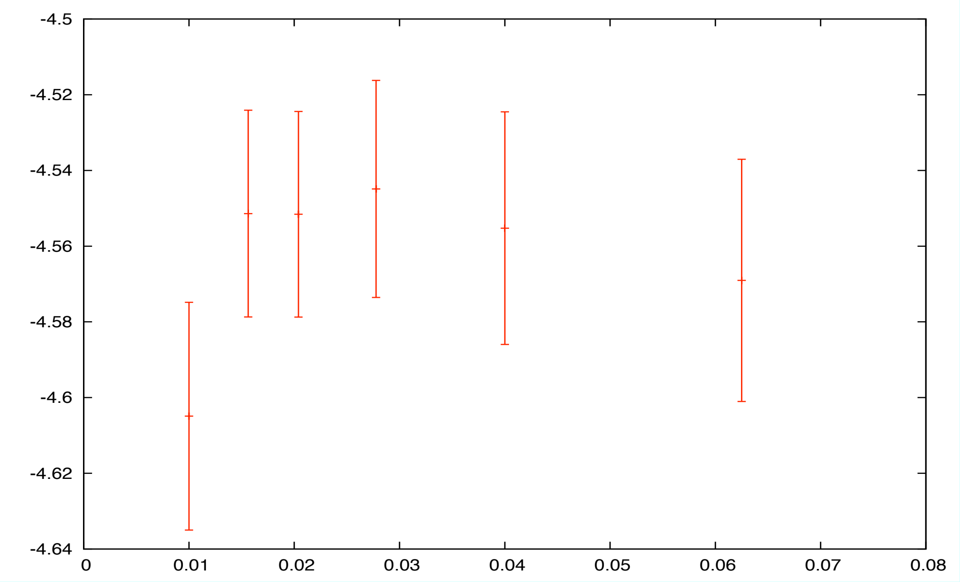
- s/T^3 is extrapolated to the continuum limit in an independent way at every T
- 35 temperatures in the range 0.6 T_c- 230 T_c have been considered
- at every temperature, numerical simulations on lattices with temporal extension $L_0 = 5, 6, 7, 8$ and sometimes $L_0 = 3, 4, 10$; $21 < L/L_0 < 26$
- The gauge couplings have been determined using the Sommer scale and the data of the ALPHA collaboration on the running coupling

Necco and Sommer NPB 622 (2002) 328

Capitani et al. NPB 544 (1999) 669

- the shift $\vec{\xi} = (1,0,0)$ has been considered except for a very few cases with the shifts $\vec{\xi} = (1,1,0)$ and $\vec{\xi} = (1,1,1)$.

$\frac{s}{T^3}$

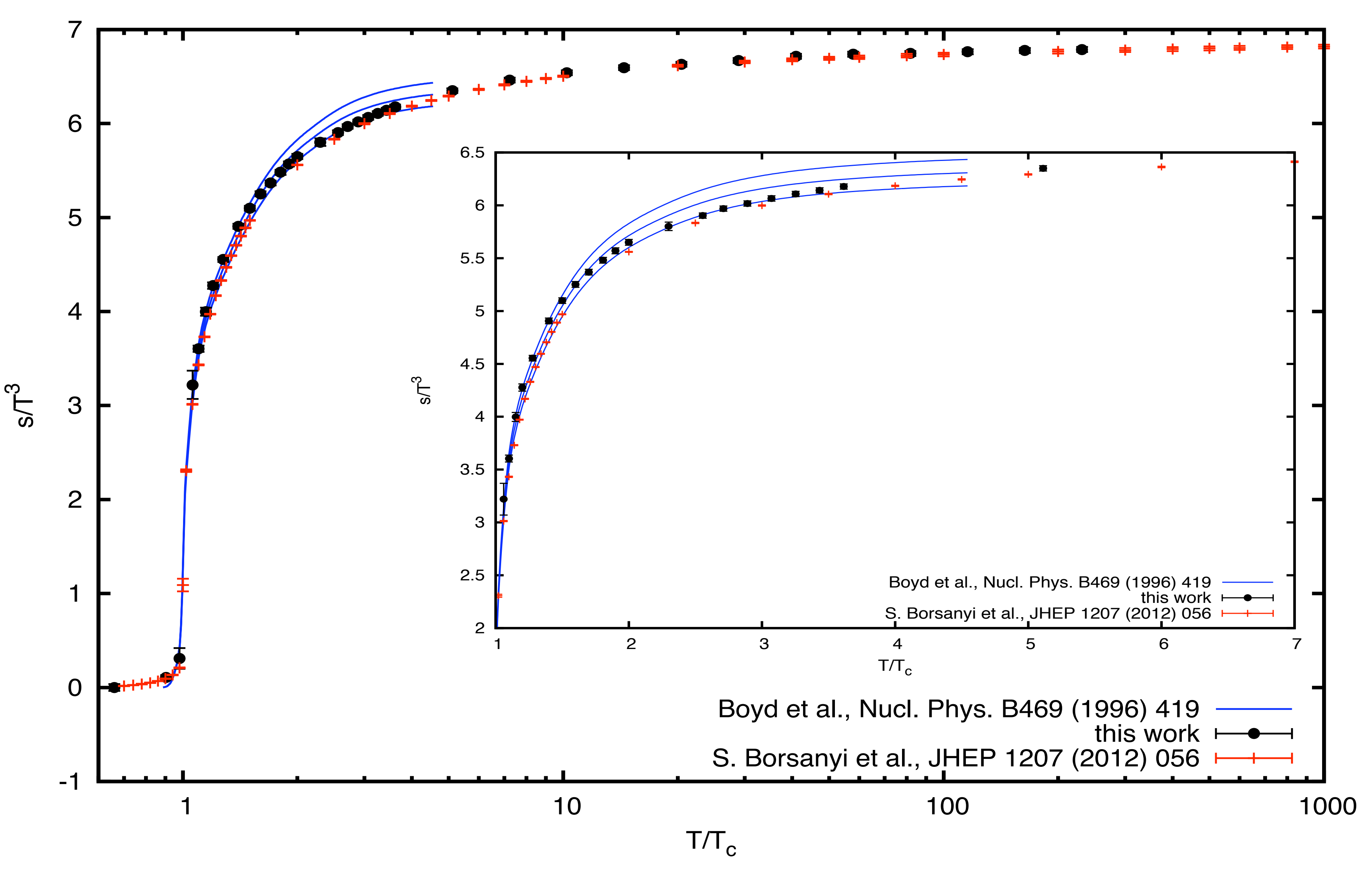


$\left(\frac{a}{L_0}\right)^2$

T=1.277 T_c

very small lattice artifacts

The entropy density

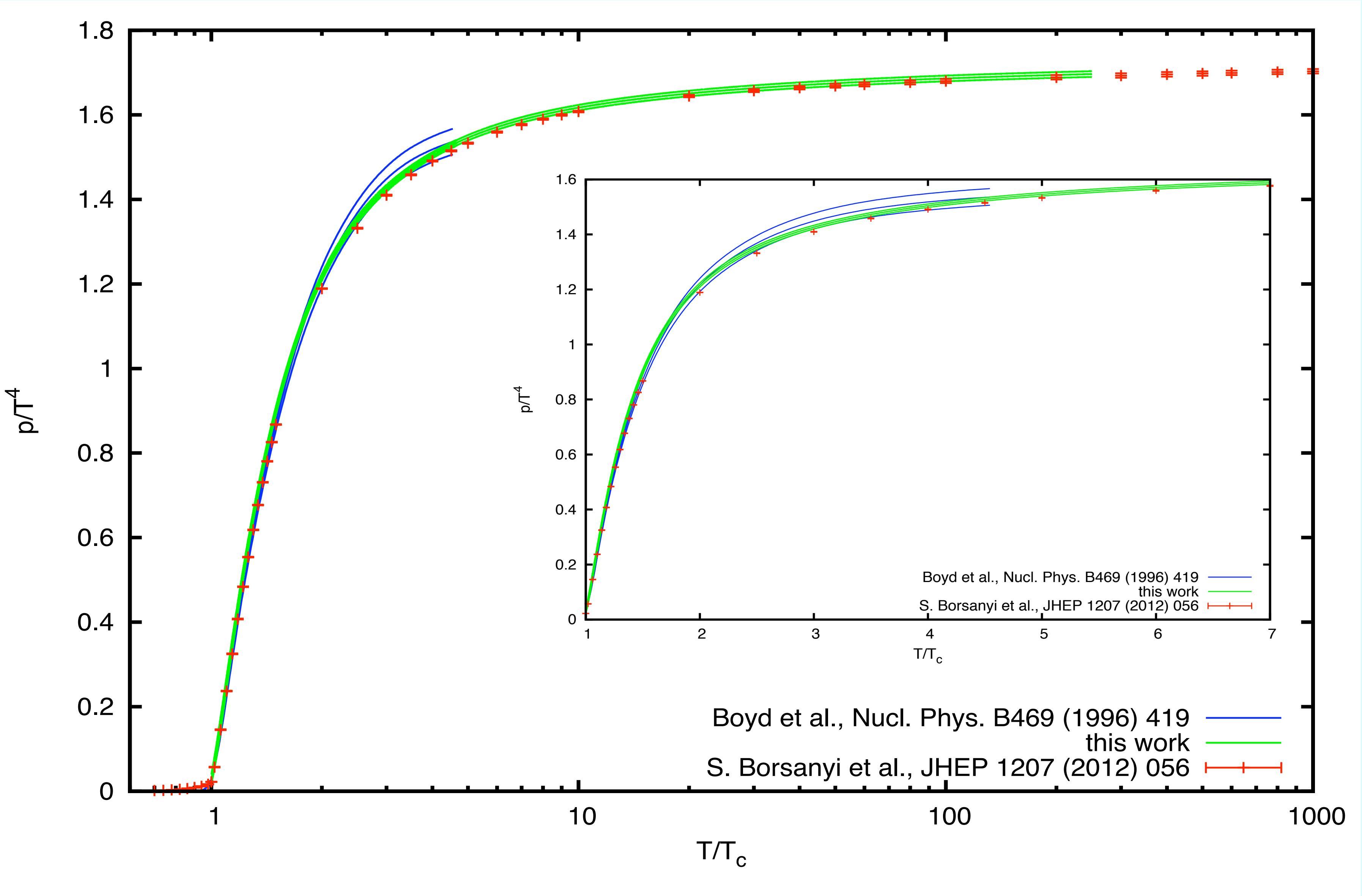


Discrepancy about 5-6 σ

in agreement with

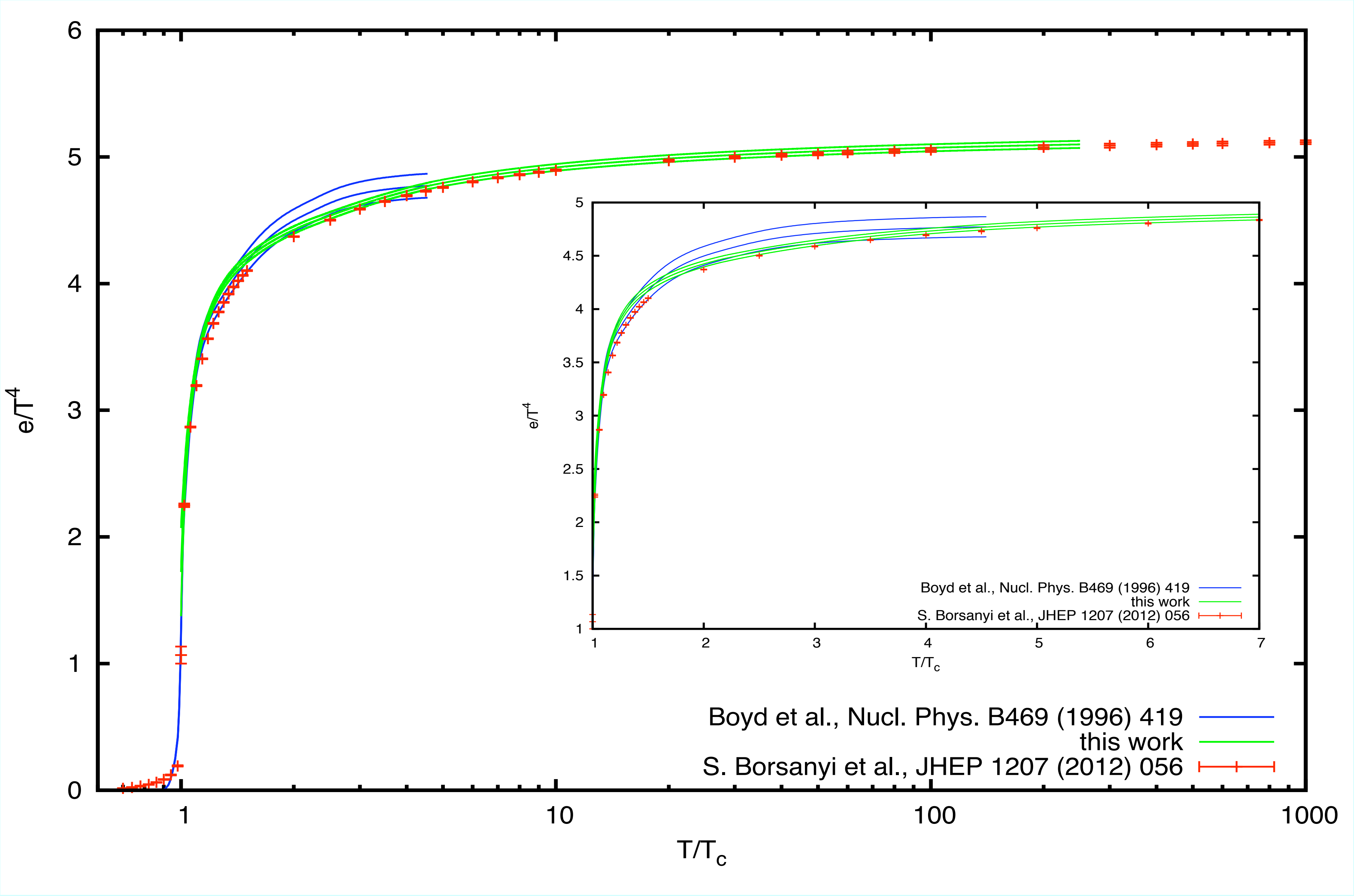
L. Giusti and M.P.
PRL 113 (2014) 031601

The pressure



Discrepancy about 3-4 σ (not fully final)

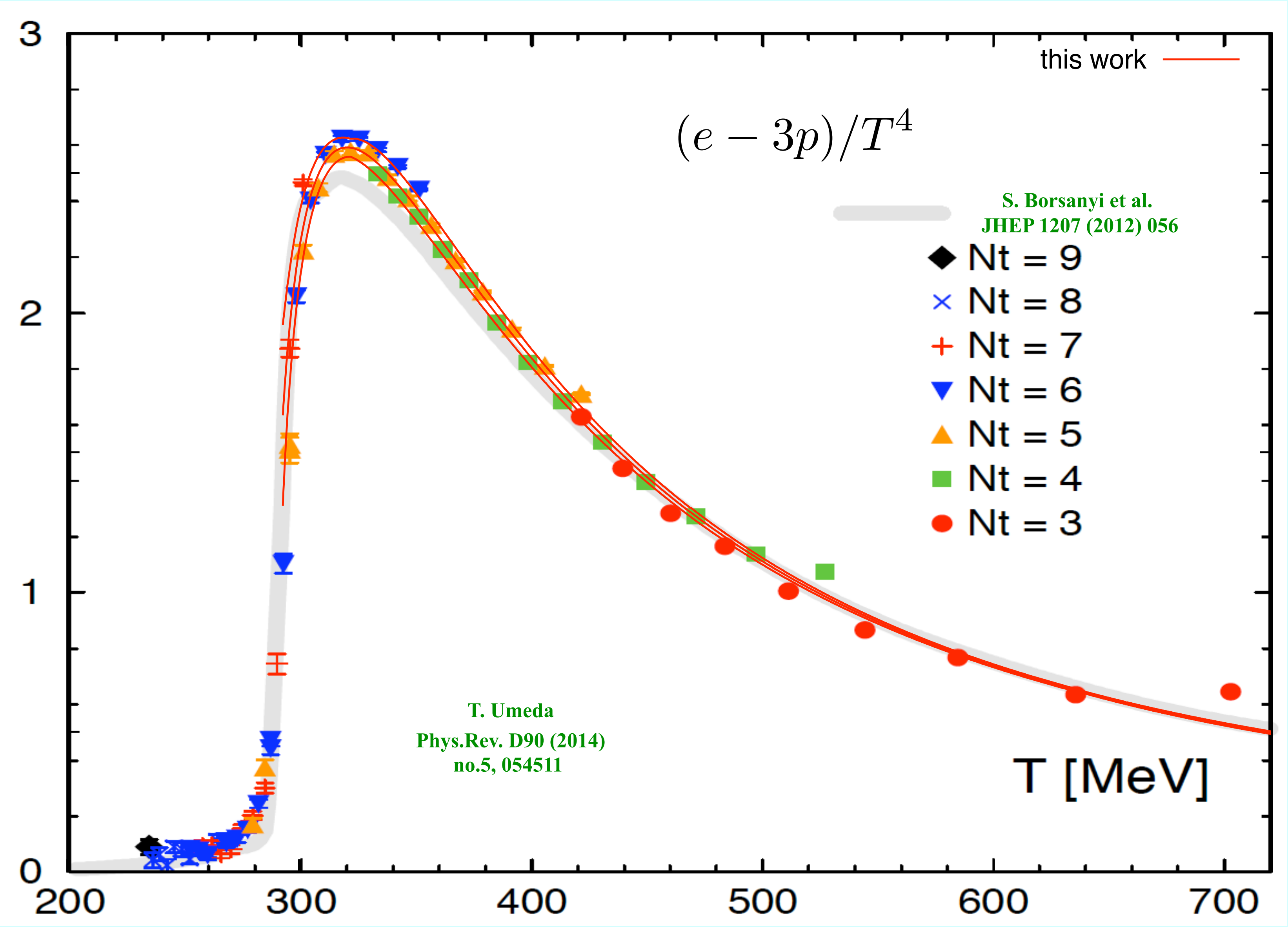
The energy density



Discrepancy about 5 σ (not fully final)

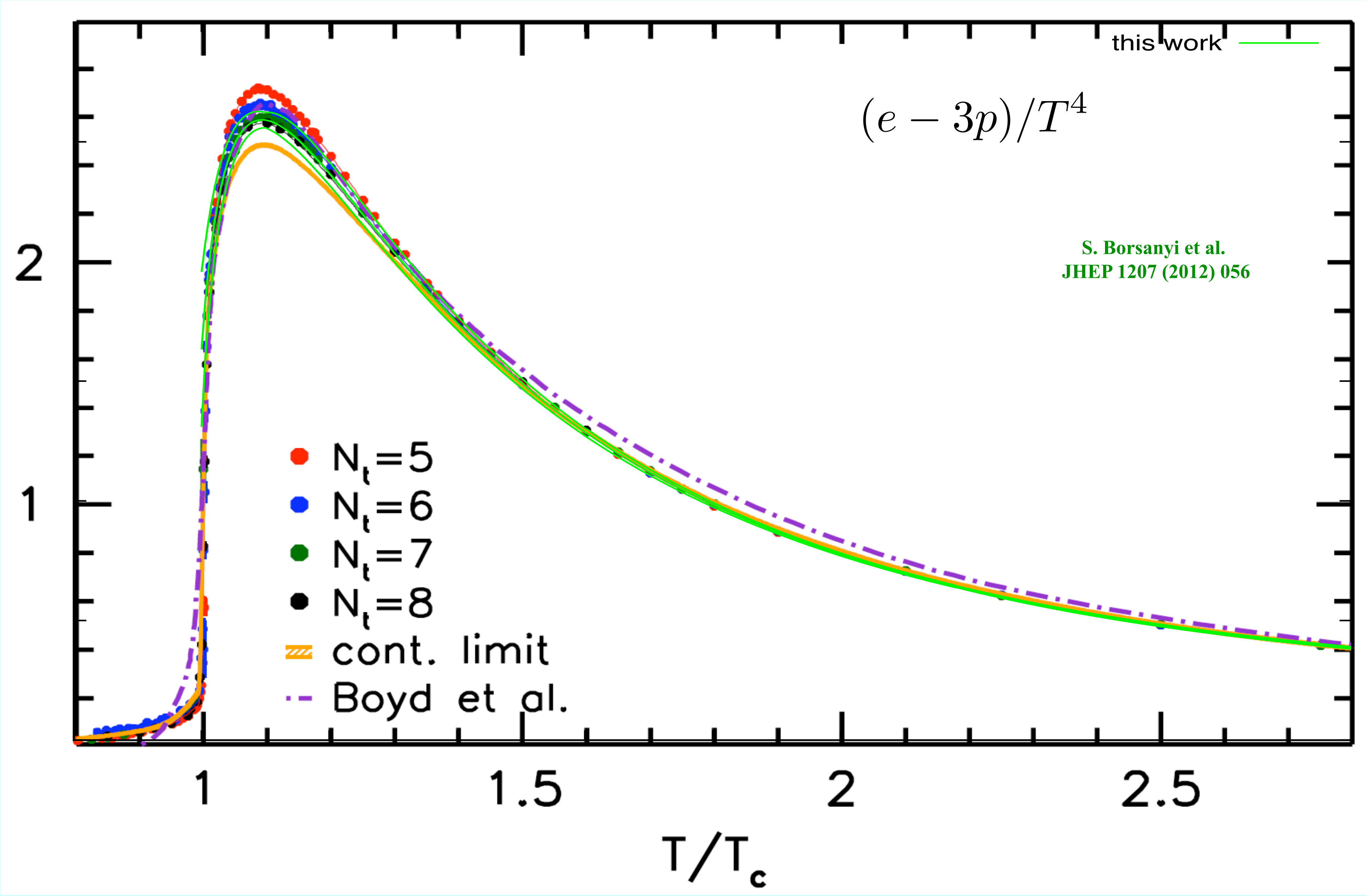
The trace anomaly

(not fully final)



The trace anomaly

(not fully final)



Conclusions

- The framework of shifted boundary conditions provides a simple and robust method to investigate the thermodynamics of a QFT.

- s/T^3 is the primary observable and the continuum limit extrapolation can be carried out in a simple way keeping fixed the physical temperature of the system. The lattice artifacts turn out to be very small: $L_0 = 3, \ldots 10$; $21 < L/L_0 < 26$

- Entropy density, Pressure and Energy density have been accurately measured in the temperature range 0.6 – 250 T_c. Numerically easy to study high temperatures.

- Our results are not in agreement with data in the literature. The discrepancy with the data by Borsanyi et al. just above T_c is significative, especially around 1.1-1.2 T_c where it is about 5σ. Our data are systematically above.

- Work is in progress to consider also dynamical fermions.