Simulations of $\mathcal{N} = 1$ supersymmetric Yang-Mills theory with three colours

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Introduction

Vacua

Tuning SUSY

Mass spectrum

Scale setting

Conclusions

THE END
SUSY on the lattice is important to test non-perturbative aspects of supersymmetric theories

- We look for non-perturbative mechanisms of spontaneous breaking of SUSY

- We study many non-perturbative aspects: confinement/deconfinement, chiral symmetry, topology

- We test effective theories for the low energy spectrum

- We can test the orientifold equivalence:
  \[ N_f = 1 \quad QCD \Leftrightarrow \mathcal{N} = 1 \quad SYM \]
We study $\mathcal{N} = 1$ supersymmetric Yang-Mills theory with gauge group SU(3)

- The Euclidean action in the continuum:
  \[ S(g, m_g) = \int d^4x \left\{ \frac{1}{4} (F_{\mu \nu}^a F_{\mu \nu}^a) + \frac{1}{2} \bar{\lambda}_a (\gamma^\mu D_{\mu}^a + m) \lambda_b - \frac{\Theta}{16\pi} \epsilon_{\mu \nu \rho \sigma} F_{\mu \nu}^a F_{\rho \sigma}^a \right\} \]

- Gauge fields $A_\mu$ (gluons)
- Majorana fermions $\lambda_\alpha$ (gluinons) in the adjoint representation

- SUSY relates boson gauge fields and fermions:
  \[ A_\mu(x) \rightarrow A_\mu(x) - 2i \bar{\lambda}(x) \gamma_\mu \epsilon \]
  \[ \lambda^a(x) \rightarrow \lambda^a(x) - \sigma_{\mu \nu} F_{\mu \nu}^a(x) \epsilon \]
SUSY is broken on the lattice

- SUSY is related to infinitesimal translations \( \{Q_\alpha, Q_\beta\} = (\gamma^\mu C)_{\alpha,\beta} P_\mu \)

- Gluino mass \( m_g \neq 0 \)

- Finite volume
We tune the $m_g = 0$ limit by $a_m \pi$

- The adjoint pion is not a physical particle!
- It is the connected part of the $a_\eta' (\bar{\lambda} \gamma_5 \lambda)$ correlator
- Assumption: $m_{a-\pi}^2 \propto m_{\bar{g}}$

- OZI (Okubo-Zweig-Iizuka) approximation
- Well defined in "Partially Quenched Chiral Perturbation Theory"
  
  G. Münster, H. Stüwe, JHEP1405 (2014) 034
$m^2_\pi$ is linear in $1/\kappa$ ( $\chi^2/dof = 20.8$ )

Vol=$16^3 \times 32$, $\beta = 4.0$
\( \kappa_c \) obtained from \( a - m_\pi \) is compatible (in \( 4.5\sigma \)) with that obtained from SUSY Ward Identities.

Vol=16^3x32, \( \beta = 4.0 \)

- gluino, \( \kappa_c = 0.13858(15) \)
- a-pion, \( \kappa_c = 0.139264(7) \)
\[ \mathcal{N} = 1 \text{ SUSY is characterised by chiral symmetry} \]

\[ \lambda \to \lambda' = \exp(-i\omega \gamma_5) \lambda \]

Classical \( U(1)_A \) axial symmetry

Gluino mass zero!

\[ \partial_\mu J_5^\mu = \partial_\mu (\bar{\lambda} \gamma_5 \gamma^\mu \lambda) = N_c \frac{g^2}{32\pi^2} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \]

This anomalous contribution can be absorbed by the theta-term!

\[ \Theta \to \Theta - 2N_c \omega, \]
\[ \omega = \frac{n\pi}{N_c}, n = 0, \ldots, 2N_c - 1 \]

\[ U(1)_A \to Z_{2N_c} \to Z_2 \]

\[ \langle \lambda^\alpha \lambda^\alpha \rangle = c\Lambda^3 e^{i\frac{2\pi k}{N_c}} \]
For SU(3) we expect 3 vacua (with a first order transition between them)
We know the expected phase structure [this was for SU(2)]:
We see a double peak structure in the scalar condensate but not in the pseudoscalar.
Here the double peak is clearer

\[ 6^4, \beta = 5.6, c_w = 1.587, \kappa_c = 0.166(1) \]

\[ \kappa = 0.1658 \]

\[ \kappa = 0.1665 \]

\[ \kappa = 0.1661 \]
In our last, preliminary, results we see a jump also in the pseudoscalar channel.
We fix the scale using the Sommer Parameter $r_0$

$$V(r) = \sigma r + \frac{c}{r} + b \quad (+APE)$$

$$\hat{r}_0 = \sqrt{\frac{1.65 + c}{\sigma}}$$

$$r_0^2 \left( \frac{\partial V}{\partial r} \right)_{r_0} = 1.65, \quad r_0 = 0.5 \text{ fm}$$
There are many works which describe the two lower supermultiplets

- The gluino mass breaks SUSY softly. One expects:
  - scalar meson: $a-f_0$
  - gluino-glue: $\tilde{g}g$
  - pseudoscalar meson: $a-\eta'$
  - pseudoscalar glueball: $gg$
  - gluino-glue: $\tilde{g}g$
  - scalar glueball: $gg$

The chiral limit for the scalar channels gives compatible results but still large errors.
The pseudoscalar channel has smaller errors. Extrapolations to the chiral limit not compatible: discretisation effects.
The global summary for the chiral limit tells us we are on the right track to see SUSY restoration.
Conclusions

- We started to study SYM with SU(3)
- We can tune the theory using the adjoint-pion (problem WI ?)
- We have started to explore the phase diagram of the theory: clear sign of a first order transition only in the scalar condensate
- We have started to explore the spectrum of the theory: so far mainly one lattice spacing $\beta = 4.0$

Outlook

- Complete measurements at $\beta = 4.30$
- Spectrum in the continuum limit
- A better signal of a first order transition in the pseudoscalar gluino condensate