Effective action for pions and a dilatonic meson — results

Maarten Golterman

San Francisco State University

MG, Yigal Shamir, arXiv:1603.04575

Lattice 2016, Southampton, July 25, 2016

Effective theory for pions and a dilatonic meson: ingredients

Consider $SU(N_c)$ gauge theory with $N_f \equiv n_f N_c$ fundamental fermions close to conformal window with degenerate quark mass m

- for fixed n_f theory is function of $g^2 N_c$ and n_f in $N_c \to \infty$ (Veneziano) limit

- n_f^* conformal sill: $n_f < n_f^*$ ChSB ; $n_f > n_f^*$ IRFP
  assume:
  $T_{an} \sim n_f - n_f^*$ near ChSB scale (T_{an} is trace anomaly in chiral limit)

- $m \ll \Lambda \Rightarrow$ approximate chiral symmetry, light pions
  $n_f \lesssim n_f^* \Rightarrow$ approximate scale symmetry, light dilatonic meson

$\Rightarrow$ power counting in m, p^2, $n_f - n_f^*$ (and $1/N_c \sim 1/N_f$) and systematic EFT for pions and light dilatonic meson (Yigal's talk)

Leading order lagrangian: (Yigal's talk)

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_\pi + \mathcal{L}_\tau + \mathcal{L}_m + \mathcal{L}_d \\
\mathcal{L}_\pi &= (f_\pi^2/4)\, e^{2\tau} \mathrm{tr}\,(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma) \\
\mathcal{L}_\tau &= (f_\tau^2/2)\, e^{2\tau} (\partial_\mu \tau)^2 \\
\mathcal{L}_m &= -(m f_\pi^2 B_\pi/2)\, e^{y\tau} \mathrm{tr}\left(\Sigma + \Sigma^\dagger\right) \\
\mathcal{L}_d &= [\tilde{c}_{00} + (n_f - n_f^*)(\tilde{c}_{01} + \tilde{c}_{11}\tau)]\, f_\tau^2 B_\tau\, e^{4\tau}
\end{aligned}
$$

- pion field $\Sigma = \exp(2i\pi/f_\pi) \to g_L \Sigma g_R^\dagger$ and dilat. meson field $\tau \to \tau + \log\lambda$
- use τ shift and redefine LECs to get $\mathcal{L}_d = \tilde{c}_{11}(n_f - n_f^*)(\tau - 1/4)\, \hat{f}_\tau^2 \hat{B}_\tau\, e^{4\tau}$
- χ = renormalized source, m = renormalized mass

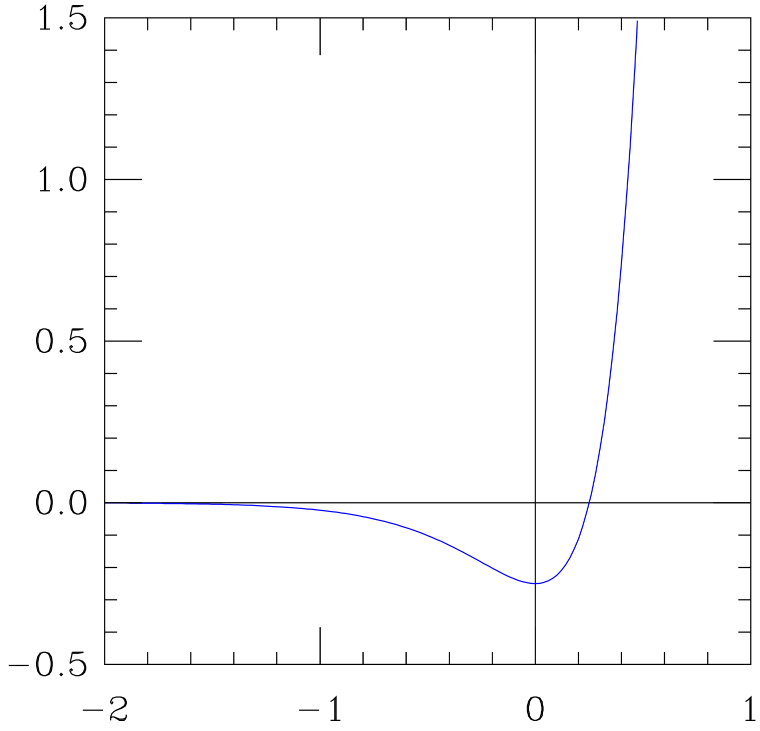
$\Rightarrow$ $y = 3 - \gamma_m^*$ with γ_m^* the IRFP value of the mass anomalous dimension at the sill of the conformal window

- corrections are accounted for by expansion in $n_f - n_f^*$

Classical vacuum in the chiral limit

- Dilatonic meson potential: $V_{\rm cl}(\tau) \propto V_d(\tau)e^{4\tau} = \tilde{c}_{11}(n_f - n_f^*)(\tau - 1/4)\,e^{4\tau}$
- Self-consistency: $\tilde{c}_{11} < 0$ (recall $n_f < n_f^*$) $\Rightarrow$ $V_{\rm cl}(\tau)$ bounded from below
- Effective theory at leading order seems "almost" scale invariant
- But: linear term in $V_d(\tau)$ is crucial; reflects hard breaking of scale invariance!
- Going to $n_f > n_f^*$, classical potential becomes unbounded from below

$\Rightarrow$ EFT "knows" it cannot be used inside conformal window where no pions exist!



Tree-level masses

- $m = 0$: shifted classical vacuum: $v = \langle\tau\rangle = 0$
- dilatonic meson mass: $m_\tau^2 = 4\tilde{c}_{11}(n_f - n_f^*)\hat{B}_\tau$ $\qquad (\hat{B}_\tau = e^{2v[\text{pre-shift}]}B_\tau)$
- $m > 0$: $V_{\rm cl}(\tau) = V_d(\tau)\, e^{4\tau} - \frac{m}{\mathcal{M}}\, e^{y\tau} \Rightarrow v(m)$ increases monotonically with m
- dilatonic meson: $m_\tau^2 = 4\tilde{c}_{11}(n_f - n_f^*)\hat{B}_\tau e^{2v(m)}\big(1 + (4-y)v(m)\big)$
- pion: $m_\pi^2 = 2\hat{B}_\pi m e^{(y-2)v(m)}$, increase with m faster than ordinary ChPT

Varying n_f towards n_f^*:

- condensate enhancement for $\tilde{c}_{00} > 0^\star$

$$\frac{\langle\overline{\psi}\psi\rangle}{\hat{f}_\pi^3} = -\frac{B_\pi}{f_\pi}\exp\left[\gamma_m^*\left(\frac{1}{4} + \frac{\tilde{c}_{00}}{\tilde{c}_{11}(n_f - n_f^*)}\right)\right]$$

⋆ "gauge choice" $\tilde{c}_{01} = 0 \Rightarrow V_{\rm cl} = [\tilde{c}_{00} + (n_f - n_f^*)\tilde{c}_{11}\tau]\, f_\tau^2 B_\tau\, e^{4\tau}$

Matching the trace anomaly

- dilatation current: $S_\mu = x_\nu \Theta_{\mu\nu} = x_\nu (T_{\mu\nu} + K_{\mu\nu}/3)$

$$\langle 0 | \, \Theta_{\mu\nu}(x) \, | \tau \rangle = \frac{\hat{f}_\tau}{3} (-\delta_{\mu\nu} p^2 + p_\mu p_\nu) \, e^{ipx}$$

$$\langle 0 | \, S_\mu(x) \, | \tau \rangle = i p_\mu \hat{f}_\tau \, e^{ipx}$$

- anomalous divergence shows up at leading order in EFT:

$$\partial_\mu S_\mu = \tilde{c}_{11} (n_f - n_f^*) f_\tau^2 B_\tau e^{4\tau} + (1 + \gamma_m^*) \frac{f_\pi^2 B_\pi m}{2} e^{y\tau} \mathrm{tr} \, (\Sigma + \Sigma^\dagger)$$

$$= -\frac{\beta(g^2)}{4g^2} F^2(\mathrm{EFT}) - (1 + \gamma_m^*) m \, \overline{\psi} \psi(\mathrm{EFT})$$

- GMOR relation when $m \ll |n_f - n_f^*|$: $\quad -(2m/N_f) \, \langle \overline{\psi} \psi \rangle = \hat{f}_\pi^2 m_\pi^2$

- GMOR-like relation for dilatonic meson: $\quad -(\beta(g^2)/g^2) \, \langle F^2 \rangle = \hat{f}_\tau^2 m_\tau^2$
  (works since $\Gamma_\tau / m_\tau \sim |n_f - n_f^*|$)

Next-leading order and one loop renormalization

- Examples of various types of NLO operators
  - usual ChPT $\quad e^{4\tau}\,\mathrm{tr}\left(e^{-\tau}\partial_\mu\Sigma^\dagger\, e^{-\tau}\partial_\mu\Sigma\right)\,\mathrm{tr}\left(e^{(y-4)\tau}\chi^\dagger\,\Sigma+\Sigma^\dagger e^{(y-4)\tau}\chi\right)$
  - LO potentials $\quad [\tilde{c}_{02}+\tilde{c}_{12}\tau+\tilde{c}_{22}(\tau^2/2)](n_f-n_f^*)^2 f_\tau^2 B_\tau\, e^{4\tau}$
  - pure dilatonic derivative terms $\quad [(\partial_\mu\tau)^2]^2,\ (\Box\tau)^2,\ \Box\tau(\partial_\mu\tau)^2$
  - mixed $\quad e^{(y-2)\tau}\,(\partial_\mu\tau)^2\,\mathrm{tr}\left(\chi^\dagger\Sigma+\Sigma^\dagger\chi\right)$

- One-loop effective potential from dilatonic meson loop

$$
\begin{aligned}
V_{\mathrm{cl}}(\tau) &= f_\tau^2 B_\tau V_d(\tau) e^{4\tau} - \frac{f_\pi^2 B_\pi}{2}\, e^{y\tau}\,\mathrm{tr}\left(\chi^\dagger\Sigma+\Sigma^\dagger\chi\right) \\
V^{(1)}_{\mathrm{eff,\ dil.}} &= -\frac{1}{64\pi^2}\left(e^{-2\tau}V''_{\mathrm{cl}}(\tau)\right)^2 \\
&\quad \times\left(\frac{2}{4-d}-\gamma+\frac{3}{2}-\log\left(\frac{e^{-2\tau}V''_{\mathrm{cl}}(\tau)}{4\pi\mu^2}\right)+O(d-4)\right)
\end{aligned}
$$

$\Rightarrow$ divergence expandable in NLO operators

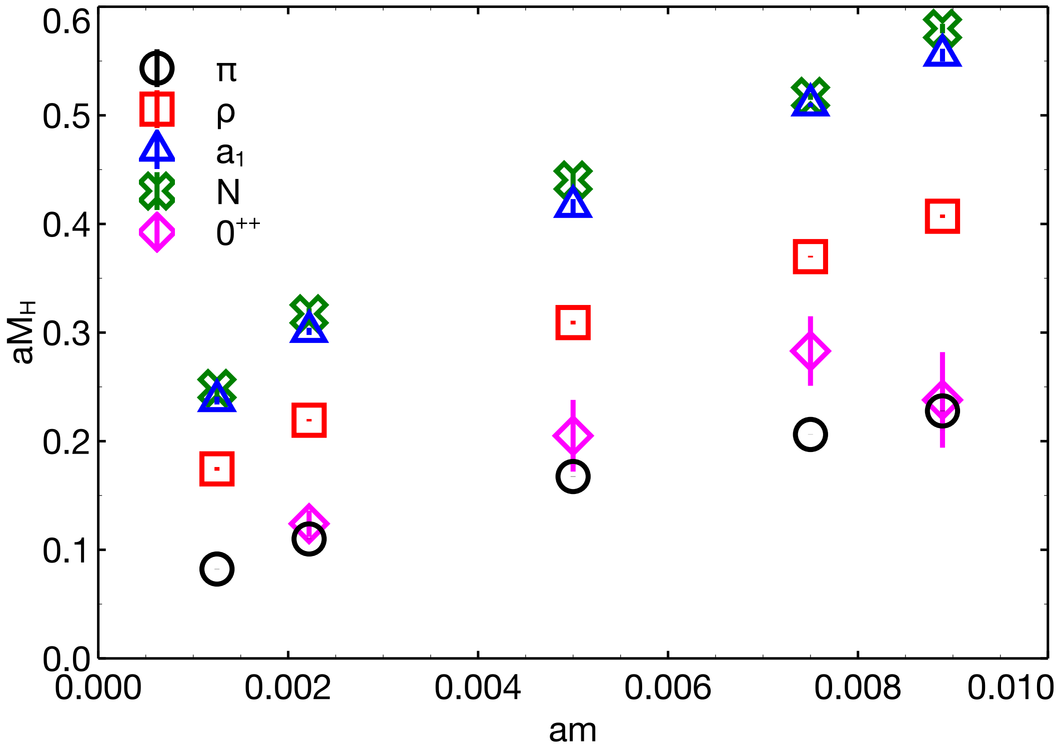
Summary

- Main assumption: $T_{an} \sim (n_f - n_f^*)^\eta$ at the onset of ChSB
- Crude model (2-loop + gap equation): $\beta(g_c^2) \propto n_f - n_f^* = n_f - 4$
- Can be extracted from e.g. $\dfrac{\langle T_{an}(0)\,[F^2](x)\rangle_c}{\langle [F^2](0)\,[F^2](x)\rangle_c}$
- Obtain (by necessity) n_f^* and η like other LECs, by fitting data at varying N_c and N_f to EFT. But: predictions for masses at fixed N_c and N_f
- For two-index (and higher) irreps, asymptotic freedom forbids $N_f \to \infty$
- Can try the EFT anyway, for fixed model (fixed N_c and fermion content)
  Being lucky: given $V_I = \sum c_n(\tau - \sigma)^n$ if, empirically, $c_0 \gg c_1 \gg c_2 \cdots$
  Can be interpreted as having *non-integer* N_f^* close to (and above) an integer

Back-up: Yigal's talk

A light flavor-singlet scalar — the Higgs particle?

- $SU(3), N_f = 8$ fund. [LatKMI, LSD,..]



LSD collaboration, PRD 93 (2016) 114514

Consistent low-energy theory must contain both pions and the flavor-singlet scalar

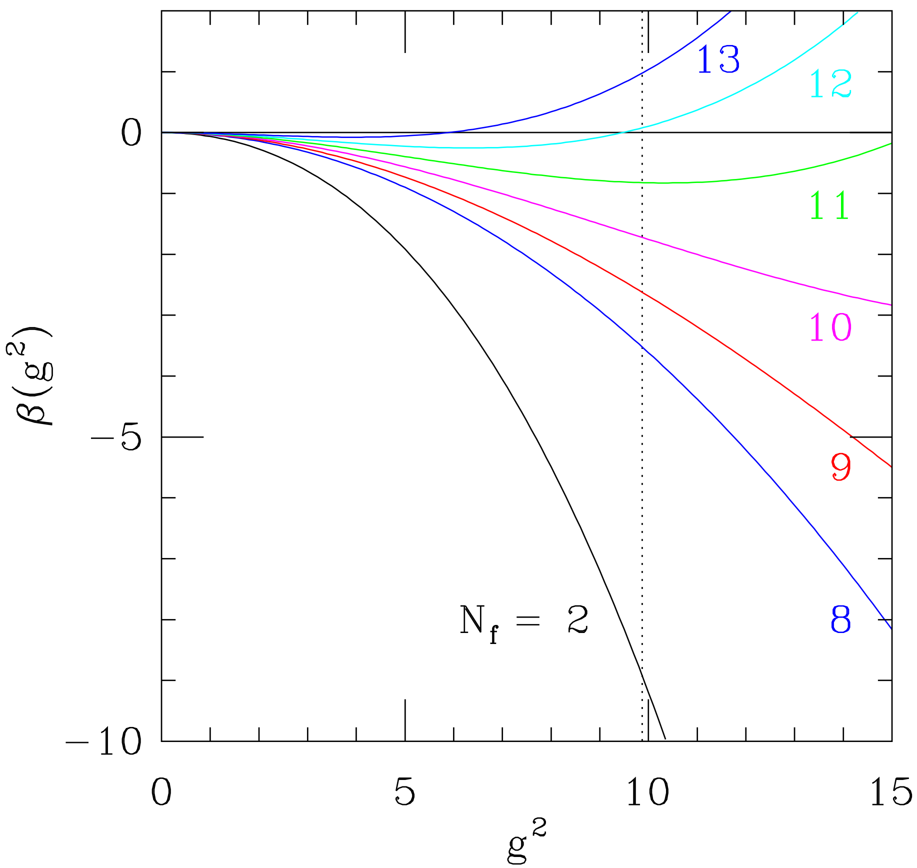
- $SU(3), N_f = 2$ sextet [Fodor et al.]

Phases of $SU(N_c)$ with N_f fundamental-rep Dirac fermions

- running slows down when N_f is increased

$$\frac{\partial g^2}{\partial \log \mu} = -\frac{b_1}{16\pi^2} g^4 - \frac{b_2}{(16\pi^2)^2} g^6$$

- two-loop IRFP g_*^2 develops when $b_1 > 0 > b_2$



- "Walking" gap equation ⇒ ChSB when $g^2(\mu) = g_c^2 = \dfrac{4\pi^2}{3C_2}$

- $SU(3)$, fund. rep: $g_c^2 = \pi^2 \simeq 9.87$

- chirally broken if $g_c < g_*(N_f)$

- conformal (IRFP) if $g_c > g_*(N_f)$

- sill of conformal window: $g_*(N_f^*) = g_c$ (note: N_f^* not an integer)

Pseudo Nambu-Goldstone boson of approx dilatation symmetry?

- dilatations: $\Phi_i(x) \to \lambda^{\Delta_i}\,\Phi_i(\lambda x)$, Δ_i scaling dimension of field $\Phi_i(x)$
- dilatation current: $S_\mu = x_\nu T_{\mu\nu}$ classically conserved for $m = 0$
- non-conservation:

$$\begin{aligned} \partial_\mu S_\mu &= T_{\mu\mu} = -T_{cl} - T_{an} \\ T_{cl} &= m[\overline{\psi}\psi] \\ T_{an} &= \frac{\beta(g^2)}{4g^2}\,[F^2] + \gamma_m\, m\,[\overline{\psi}\psi] \end{aligned}$$

- probe beta fn at the ChSB scale: $\left\langle T_{an}(0)\,[F^2](x)\right\rangle_c \,/\, \left\langle [F^2](0)\,[F^2](x)\right\rangle_c$
- below conformal sill: $\beta(g_c^2) \propto N_f - N_f^*$
  expect: increasing N_f towards N_f^* $\Rightarrow$ smaller $\beta(g_c)$ at ChSB scale
  $\Rightarrow$ better scale invariance $\Rightarrow$ "dilatonic" pNG boson gets lighter
- Q: use $N_f - N_f^*$ as small parameter? (problem: N_f takes discrete values)

Low-energy EFT with dilatonic meson: power counting

- standard ChPT: fermion mass m is a parameter of the microscopic theory
  m can be tuned continuously towards zero

$\Rightarrow$ Systematic expansion in m and p^2

- problem: cannot turn off trace anomaly; theory is defined at fixed N_c, N_f

- analogy: cannot turn off $U(1)_A$ anomaly;
  but it becomes vanishingly small for $N_c \to \infty$

$\Rightarrow$ Systematic expansion in m, $1/N_c$, and p^2 [Kaiser and Leutwyler, '00]

- Veneziano limit: $N_f, N_c \to \infty$ with $n_f = N_f/N_c$ fixed
  n_f becomes a continuous parameter; theory depends only on $g^2 N_c$ and n_f
  $n_f^* \;=\; \lim_{N_c\to\infty} N_f^*(N_c)/N_c \;=\;$ sill of conformal window for $N_c \to \infty$.

- assume: $T_{an} \sim (n_f - n_f^*)^\eta$ at the ChSB scale [$\eta = 1$ in this talk]

$\Rightarrow$ Systematic expansion in m, $1/N$, $n_f - n_f^*$, and p^2

Constructing an Effective Field Theory

Microscopic theory:

- symmetries
- spurions: external fields transforming under the symmetries
- fixing "VEVs" of spurions $\Rightarrow$ explicit breaking of symmetries

Effective theory:

- same symmetries, same spurion fields, but new dynamical (effective) fields
- explicit breaking of symmetries from same VEVs of spurions
- power counting (previous slide)
- use spurions as probes $\Rightarrow$ fix Low Energy Constants order by order, by matching correlators obtained by differentiation with respect to spurion fields

Spurions in the microscopic theory

- chiral symmetry: $\mathcal{L}^{\mathrm{MIC}}(\chi) = \frac{1}{4}F^2 + \overline{\psi}\not{D}\psi + \overline{\psi}_R\chi^\dagger\psi_L + \overline{\psi}_L\chi\psi_R$

$\delta\mathcal{L}^{\mathrm{MIC}}(\chi) = 0\,,$ but: $\langle\chi\rangle = m \quad \Rightarrow \quad \delta\mathcal{L}^{\mathrm{MIC}}(m) = m\delta(\overline{\psi}\psi)$

- axial $U(1)_A$ symmetry: $\mathcal{L}^{\mathrm{MIC}}(\theta) = \frac{1}{4}F^2 + \overline{\psi}\not{D}\psi + \theta icg^2 F\tilde{F}$

$\delta\theta = 1 \quad \Rightarrow \quad \delta\mathcal{L}^{\mathrm{MIC}}(\theta) = 0$ (finite $U(1)_A$ transf: $\theta \to \theta + \alpha$)

but: $\langle\theta\rangle = \theta_0 \quad \Rightarrow \quad \delta\mathcal{L}^{\mathrm{MIC}}(\theta_0) = -icg^2 F\tilde{F}$

- dilatations: $\mathcal{L}^{\mathrm{MIC}}(\sigma,\chi) = \mathcal{L}^{\mathrm{MIC}}(0,\chi) + \sigma T_{an}(\chi) + \cdots$

$\delta\sigma = x_\mu\partial_\mu\,\sigma + 1 \quad \Rightarrow \quad \delta\mathcal{L}^{\mathrm{MIC}}(\sigma,\chi) = x_\mu\partial_\mu\,\mathcal{L}^{\mathrm{MIC}}(\sigma,\chi)$

but: $\langle\sigma\rangle = 0 \quad \Rightarrow \quad \delta\mathcal{L}^{\mathrm{MIC}}(0,\chi) = x_\mu\partial_\mu\,\mathcal{L}^{\mathrm{MIC}}(0,\chi) - T_{an}(\chi)$

Effective Field Theory with pions and dilatonic meson $\tau(x)$

- dilatation transformation [finite]:

source fields: $\sigma(x) \to \sigma(\lambda x) + \log\lambda$, $\chi(x) \to \lambda^{4-y}\chi(\lambda x)$

effective fields: $\tau(x) \to \tau(\lambda x) + \log\lambda$, $\Sigma(x) \to \Sigma(\lambda x)$

- invariant low-energy theory: $\tilde{\mathcal{L}}^{\rm EFT} = \tilde{\mathcal{L}}_\pi + \tilde{\mathcal{L}}_\tau + \tilde{\mathcal{L}}_m + \tilde{\mathcal{L}}_d$ where

$$\begin{aligned}
\tilde{\mathcal{L}}_\pi &= V_\pi(\tau-\sigma)\,(f_\pi^2/4)\,e^{2\tau}{\rm tr}\,(\partial_\mu\Sigma^\dagger\partial_\mu\Sigma) \\
\tilde{\mathcal{L}}_\tau &= V_\tau(\tau-\sigma)\,(f_\tau^2/2)\,e^{2\tau}(\partial_\mu\tau)^2 \\
\tilde{\mathcal{L}}_m &= -V_M(\tau-\sigma)\,(f_\pi^2 B_\pi/2)\,e^{y\tau}{\rm tr}\left(\chi^\dagger\Sigma + \Sigma^\dagger\chi\right) \\
\tilde{\mathcal{L}}_d &= V_d(\tau-\sigma)\,f_\tau^2 B_\tau\,e^{4\tau}
\end{aligned}$$

with invariant potentials: $V(\tau(x)-\sigma(x)) \;\to\; V(\tau(\lambda x)-\sigma(\lambda x))$

$\Rightarrow$ No predictability without power counting!

Power counting hierarchy from matching correlation functions

- recall microscopic theory $\mathcal{L}^{\mathrm{MIC}}(\sigma,\chi) = \mathcal{L}^{\mathrm{MIC}}(0,\chi) + \sigma T_{an}(\chi) + O(\sigma^2)$

$$\frac{\partial}{\partial\sigma(x)}\mathcal{L}^{\mathrm{MIC}}\Big|_{\sigma=\chi=0} = T_{an}(x)\Big|_{\chi=0} = \frac{\beta(g^2)}{4g^2}[F^2(x)] \sim n_f - n_f^*$$

- effective theory

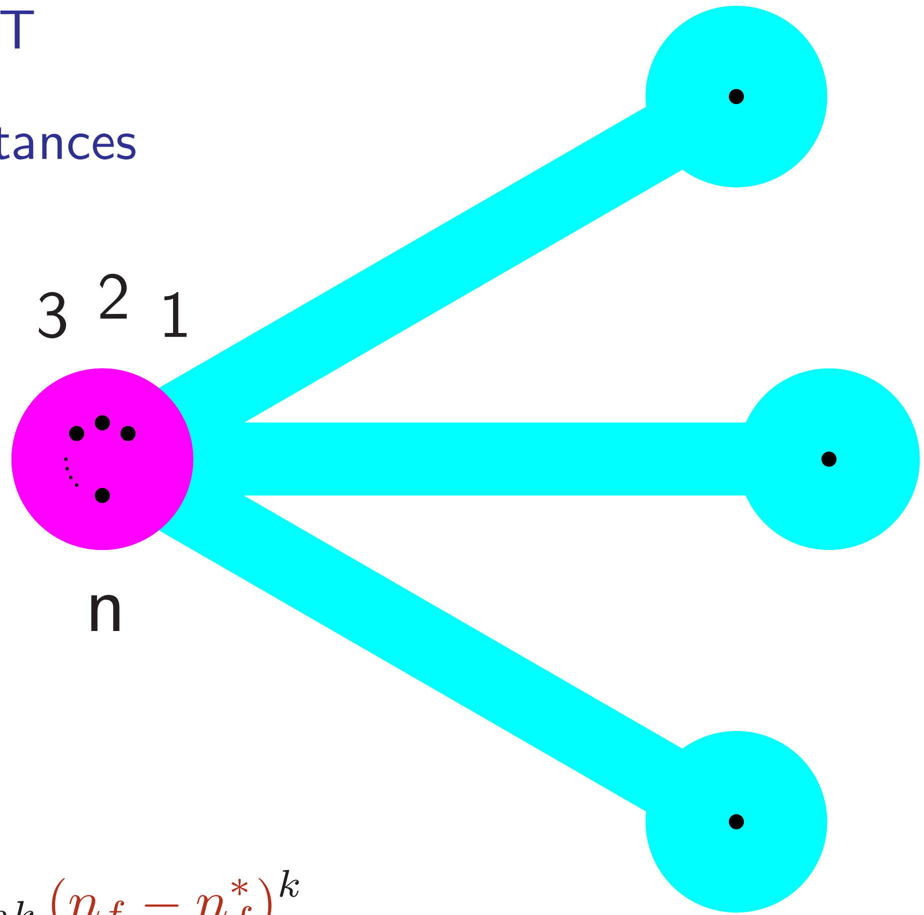
$$\left(-\frac{\partial}{\partial\sigma(x)}\right)^n \tilde{\mathcal{L}}^{\mathrm{EFT}}\Big|_{\sigma=\chi=0} = V_d^{(n)}(\tau(x))\, f_\tau^2 B_\tau\, e^{4\tau(x)} + \cdots$$

$\Rightarrow$ $V(\tau-\sigma) = \sum_{n=0}^{\infty} c_n(\tau-\sigma)^n$ where $c_n = O((n_f - n_f^*)^n)$

$\Rightarrow$ Only a finite number of LECs at each order!

Matching – role of non-coinciding points

- Magenta: points at distances $\ll$ meson size collapse to a single point in the EFT
- Cyan: points at asympt. large distances



- Upshot:

$$V(\tau - \sigma) = \sum_{n=0}^{\infty} (\tau - \sigma)^n \sum_{k \geq n} \tilde{c}_{nk} \, (n_f - n_f^*)^k$$

Leading order lagrangian, finally:

- now set $\sigma(x) = 0$, obtaining at order $m \sim n_f - n_f^* \sim p^2$:

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_\pi + \mathcal{L}_\tau + \mathcal{L}_m + \mathcal{L}_d \\
\mathcal{L}_\pi &= (f_\pi^2/4)\, e^{2\tau} \mathrm{tr}\,(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma) \\
\mathcal{L}_\tau &= (f_\tau^2/2)\, e^{2\tau} (\partial_\mu \tau)^2 \\
\mathcal{L}_m &= -(m f_\pi^2 B_\pi/2)\, e^{y\tau} \mathrm{tr}\left(\Sigma + \Sigma^\dagger\right) \\
\mathcal{L}_d &= [\tilde{c}_{00} + (n_f - n_f^*)(\tilde{c}_{01} + \tilde{c}_{11}\tau)]\, f_\tau^2 B_\tau\, e^{4\tau}
\end{aligned}
$$