Effective action for pions and a dilatonic meson — results

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Effective theory for pions and a dilatonic meson: ingredients

Consider $SU(N_c)$ gauge theory with $N_f \equiv n_f N_c$ fundamental fermions close to conformal window with degenerate quark mass $m$

- for fixed $n_f$ theory is function of $g^2 N_c$ and $n_f$ in $N_c \to \infty$ (Veneziano) limit

- $n_f^*$ conformal sill: $n_f < n_f^*$ ChSB ; $n_f > n_f^*$ IRFP

  **assume:**
  $T_{an} \sim n_f - n_f^*$ near ChSB scale  ($T_{an}$ is trace anomaly in chiral limit)

- $m \ll \Lambda \Rightarrow$ approximate chiral symmetry, light pions
  $n_f \lesssim n_f^*$ ⇒ approximate scale symmetry, light dilatonic meson

⇒ power counting in $m, p^2, n_f - n_f^*$ (and $1/N_c \sim 1/N_f$)
  and systematic EFT for pions and light dilatonic meson (Yigal’s talk)
Leading order lagrangian: (Yigal’s talk)

\[
\mathcal{L} = \mathcal{L}_\pi + \mathcal{L}_\tau + \mathcal{L}_m + \mathcal{L}_d
\]

\[
\mathcal{L}_\pi = \left(\frac{f_\pi^2}{4}\right) e^{2\tau} \text{tr} \left( \partial_\mu \Sigma^\dagger \partial_\mu \Sigma \right)
\]

\[
\mathcal{L}_\tau = \left(\frac{f_\tau^2}{2}\right) e^{2\tau} (\partial_\mu \tau)^2
\]

\[
\mathcal{L}_m = -\left( m f_\pi^2 B_\pi / 2 \right) e^{y\tau} \text{tr} \left( \Sigma + \Sigma^\dagger \right)
\]

\[
\mathcal{L}_d = \left[ \tilde{c}_{00} + (n_f - n_f^*) (\tilde{c}_{01} + \tilde{c}_{11}) \right] f_\tau^2 B_\tau e^{4\tau}
\]

- pion field \( \Sigma = \exp(2i\pi / f_\pi) \rightarrow g_L \Sigma g_R^\dagger \) and dilat. meson field \( \tau \rightarrow \tau + \log \lambda \)

- use \( \tau \) shift and redefine LECs to get \( \mathcal{L}_d = \tilde{c}_{11} (n_f - n_f^*) (\tau - 1/4) f_\tau^2 B_\tau e^{4\tau} \)

- \( \chi = \) renormalized source, \( m = \) renormalized mass

\[ y = 3 - \gamma_m^* \] with \( \gamma_m^* \) the IRFP value of the mass anomalous dimension at the sill of the conformal window

- corrections are accounted for by expansion in \( n_f - n_f^* \)
Classical vacuum in the chiral limit

- Dilatonic meson potential: \( V_{\text{cl}}(\tau) \propto V_d(\tau)e^{4\tau} = \tilde{c}_{11}(n_f - n_f^*)(\tau - 1/4) e^{4\tau} \)
- Self-consistency: \( \tilde{c}_{11} < 0 \) (recall \( n_f < n_f^* \)) \( \Rightarrow V_{\text{cl}}(\tau) \) bounded from below
- Effective theory at leading order seems “almost” scale invariant
- But: linear term in \( V_d(\tau) \) is crucial; reflects hard breaking of scale invariance!
- Going to \( n_f > n_f^* \), classical potential becomes unbounded from below
- \( \Rightarrow \) EFT “knows” it cannot be used inside conformal window where no pions exist!
Tree-level masses

- \( m = 0 \): shifted classical vacuum: \( v = \langle \tau \rangle = 0 \)

- dilatonic meson mass: \( m^2_\tau = 4\tilde{c}_{11}(n_f - n^*_f)\hat{B}_\tau \)

- \( m > 0 \): \( V_{cl}(\tau) = V_d(\tau) e^{4\tau} - \frac{m}{\mathcal{M}} e^{y\tau} \Rightarrow v(m) \) increases monotonically with \( m \)

- dilatonic meson: \( m^2_\tau = 4\tilde{c}_{11}(n_f - n^*_f)\hat{B}_\tau e^{2v(m)}(1 + (4 - y)v(m)) \)

- pion: \( m^2_\pi = 2\hat{B}_\pi me^{y-2}v(m) \), increase with \( m \) faster than ordinary ChPT

Varying \( n_f \) towards \( n_f^* \):

- condensate enhancement for \( \tilde{c}_{00} > 0^* \)

\[
\frac{\langle \bar{\psi}\psi \rangle}{f^3_\pi} = -\frac{B_\pi}{f_\pi} \exp \left[ \gamma^*_m \left( \frac{1}{4} + \frac{\tilde{c}_{00}}{\tilde{c}_{11}(n_f - n^*_f)} \right) \right]
\]

* “gauge choice” \( \tilde{c}_{01} = 0 \) \( \Rightarrow V_{cl} = [\tilde{c}_{00} + (n_f - n^*_f)\tilde{c}_{11}\tau] f^2_\tau B_\tau e^{4\tau} \)
Matching the trace anomaly

\[ S_\mu = x_\nu \Theta_{\mu\nu} = x_\nu (T_{\mu\nu} + K_{\mu\nu}/3) \]

\[ \langle 0 | \Theta_{\mu\nu}(x) | \tau \rangle = \frac{f_\tau}{3} (-\delta_{\mu\nu} p^2 + p_\mu p_\nu) e^{ipx} \]

\[ \langle 0 | S_\mu(x) | \tau \rangle = ip_\mu f_\tau e^{ipx} \]

- anomalous divergence shows up at leading order in EFT:

\[ \partial_\mu S_\mu = \tilde{c}_{11} (n_f - n^*_f) f_\tau^2 B_\tau e^{4\tau} + (1 + \gamma^*_m) \frac{f_\pi^2 B_\pi m}{2} e^{y\tau} \text{tr} (\Sigma + \Sigma^\dagger) \]

\[ = -\frac{\beta (g^2)}{4g^2} F^2 (\text{EFT}) - (1 + \gamma^*_m) m \overline{\psi} \psi (\text{EFT}) \]

- GMOR relation when \( m \ll |n_f - n^*_f| \):

\[ -(2m/N_f) \langle \overline{\psi} \psi \rangle = \hat{f}_\pi^2 m_\pi^2 \]

- GMOR-like relation for dilatonic meson:

\[ -(\beta (g^2)/g^2) \langle F^2 \rangle = \hat{f}_\tau^2 m_\tau^2 \]

(worked since \( \Gamma_\tau/m_\tau \sim |n_f - n^*_f| \))
Next-leading order and one loop renormalization

- Examples of various types of NLO operators
  - usual ChPT
    
    \[ e^{4\tau} \text{tr} \left( e^{-\tau} \partial_{\mu} \Sigma \dagger e^{-\tau} \partial_{\mu} \Sigma \right) \text{tr} \left( e^{(y-4)\tau} \chi \dagger \Sigma + \Sigma \dagger e^{(y-4)\tau} \chi \right) \]
  - LO potentials
    
    \[ [\tilde{c}_{02} + \tilde{c}_{12} \tau + \tilde{c}_{22}(\tau^2/2)](n_f - n_f^*)^2 f^2 \pi B \tau e^{4\tau} \]
  - pure dilatonic derivative terms
    
    \[ [(\partial_{\mu} \tau)^2]^2, (\Box \tau)^2, \Box \tau (\partial_{\mu} \tau)^2 \]
  - mixed
    
    \[ e^{(y-2)\tau} (\partial_{\mu} \tau)^2 \text{tr} \left( \chi \dagger \Sigma + \Sigma \dagger \chi \right) \]

- One-loop effective potential from dilatonic meson loop

\[
V_{\text{cl}}(\tau) = f_{\tau}^2 B_{\tau} V_{\text{d}}(\tau) e^{4\tau} - \frac{f_{\pi}^2 B_{\pi}}{2} e^{y\tau} \text{tr} \left( \chi \dagger \Sigma + \Sigma \dagger \chi \right)
\]

\[
V_{\text{eff}, \text{dil.}}^{(1)} = -\frac{1}{64\pi^2} \left( e^{-2\tau V''_{\text{cl}}}(\tau) \right)^2 \times \left( \frac{2}{4 - d} - \gamma + \frac{3}{2} - \log \left( \frac{e^{-2\tau V''_{\text{cl}}}(\tau)}{4\pi \mu^2} \right) + O(d - 4) \right)
\]

\[ \Rightarrow \] divergence expandable in NLO operators
Summary

• Main assumption: $T_{an} \sim (n_f - n_f^*)^\eta$ at the onset of ChSB

• Crude model (2-loop + gap equation): $\beta(g_c^2) \propto n_f - n_f^* = n_f - 4$

• Can be extracted from e.g. $\frac{\langle T_{an}(0) [F^2](x) \rangle_c}{\langle [F^2](0) [F^2](x) \rangle_c}$

• Obtain (by necessity) $n_f^*$ and $\eta$ like other LECs, by fitting data at varying $N_c$ and $N_f$ to EFT. But: predictions for masses at fixed $N_c$ and $N_f$

• For two-index (and higher) irreps, asymptotic freedom forbids $N_f \to \infty$

• Can try the EFT anyway, for fixed model (fixed $N_c$ and fermion content)
  Being lucky: given $V_I = \sum c_n(\tau - \sigma)^n$ if, empirically, $c_0 \gg c_1 \gg c_2 \cdots$
  Can be interpreted as having non-integer $N_f^*$ close to (and above) an integer
Back-up: Yigal’s talk
A light flavor-singlet scalar — the Higgs particle?

- $SU(3), N_f = 8$ fund. [LatKMI, LSD,..]

Consistent low-energy theory must contain both pions and the flavor-singlet scalar

LSD collaboration, PRD 93 (2016) 114514

- $SU(3), N_f = 2$ sextet [Fodor et al.]
Phases of $SU(N_c)$ with $N_f$ fundamental-rep Dirac fermions

- running slows down when $N_f$ is increased
  \[ \frac{\partial g^2}{\partial \log \mu} = -\frac{b_1}{16\pi^2} g^4 - \frac{b_2}{(16\pi^2)^2} g^6 \]

- two-loop IRFP $g^2_*$ develops when $b_1 > 0 > b_2$

- “Walking” gap equation $\Rightarrow$
  ChSB when $g^2(\mu) = g^2_c = \frac{4\pi^2}{3C_2}$

- $SU(3)$, fund. rep: $g^2_c = \pi^2 \simeq 9.87$

- chirally broken if $g_c < g_*(N_f)$

- conformal (IRFP) if $g_c > g_*(N_f)$

- sill of conformal window: $g_*(N_f^*) = g_c$ (note: $N_f^*$ not an integer)
Pseudo Nambu-Goldstone boson of approx dilatation symmetry?

• dilatations: $\Phi_i(x) \rightarrow \lambda^{\Delta_i} \Phi_i(\lambda x)$, $\Delta_i$ scaling dimension of field $\Phi_i(x)$

• dilatation current: $S_\mu = x_\nu T_{\mu\nu}$ classically conserved for $m = 0$

• non-conservation:
  \[
  \partial_\mu S_\mu = T_{\mu\mu} = -T_{cl} - T_{an}
  \]
  \[
  T_{cl} = m[\bar{\psi}\psi]
  \]
  \[
  T_{an} = \frac{\beta(g^2)}{4g^2} [F^2] + \gamma_m m [\bar{\psi}\psi]
  \]

• probe beta fn at the ChSB scale:
  \[
  \frac{\langle T_{an}(0) [F^2](x) \rangle_c}{\langle [F^2](0) [F^2](x) \rangle_c}
  \]

• below conformal sill:
  $\beta(g_c^2) \propto N_f - N_f^*$
  expect: increasing $N_f$ towards $N_f^*$ $\Rightarrow$ smaller $\beta(g_c)$ at ChSB scale
  $\Rightarrow$ better scale invariance $\Rightarrow$ “dilatonic” pNG boson gets lighter

• Q: use $N_f - N_f^*$ as small parameter? (problem: $N_f$ takes discrete values)
Low-energy EFT with dilatonic meson: power counting

- standard ChPT: fermion mass $m$ is a parameter of the microscopic theory
  $m$ can be tuned continuously towards zero
  $\Rightarrow$ **Systematic expansion** in $m$ and $p^2$

- problem: cannot turn off trace anomaly; theory is defined at fixed $N_c, N_f$

- analogy: cannot turn off $U(1)_A$ anomaly;
  but it becomes vanishingly small for $N_c \to \infty$
  $\Rightarrow$ **Systematic expansion** in $m, 1/N_c,$ and $p^2$  
  [Kaiser and Leutwyler, '00]

- Veneziano limit: $N_f, N_c \to \infty$ with $n_f = N_f/N_c$ fixed
  $n_f$ becomes a continuous parameter; theory depends only on $g^2 N_c$ and $n_f$
  $n_f^* = \lim_{N_c \to \infty} N_f^*(N_c)/N_c = \text{sill of conformal window for } N_c \to \infty.$

- assume: $T_{an} \sim (n_f - n_f^*)^\eta$ at the ChSB scale  
  $[\eta = 1$ in this talk$]$

  $\Rightarrow$ **Systematic expansion** in $m, 1/N, n_f - n_f^*, \text{and } p^2$
Constructing an Effective Field Theory

Microscopic theory:

- symmetries
- spurions: external fields transforming under the symmetries
- fixing “VEVs” of spurions $\Rightarrow$ explicit breaking of symmetries

Effective theory:

- same symmetries, same spurion fields, but new dynamical (effective) fields
- explicit breaking of symmetries from same VEVs of spurions
- power counting (previous slide)
- use spurions as probes $\Rightarrow$ fix Low Energy Constants order by order, by matching correlators obtained by differentiation with respect to spurion fields
Spurions in the microscopic theory

- chiral symmetry: \( \mathcal{L}^{\text{MIC}}(\chi) = \frac{1}{4}F^2 + \bar{\psi} \slashed{D} \psi + \bar{\psi}_R \chi \dagger \psi_L + \bar{\psi}_L \chi \psi_R \)

\( \delta \mathcal{L}^{\text{MIC}}(\chi) = 0 \), but: \( \langle \chi \rangle = m \Rightarrow \delta \mathcal{L}^{\text{MIC}}(m) = m \delta(\bar{\psi} \psi) \)

- axial \( U(1)_A \) symmetry: \( \mathcal{L}^{\text{MIC}}(\theta) = \frac{1}{4}F^2 + \bar{\psi} \slashed{D} \psi + \theta icg^2 F\tilde{F} \)

\( \delta \theta = 1 \Rightarrow \delta \mathcal{L}^{\text{MIC}}(\theta) = 0 \) (finite \( U(1)_A \) transf: \( \theta \rightarrow \theta + \alpha \))

but: \( \langle \theta \rangle = \theta_0 \Rightarrow \delta \mathcal{L}^{\text{MIC}}(\theta_0) = -icg^2 F\tilde{F} \)

- dilatations: \( \mathcal{L}^{\text{MIC}}(\sigma, \chi) = \mathcal{L}^{\text{MIC}}(0, \chi) + \sigma Tan(\chi) + \cdots \)

\( \delta \sigma = x_\mu \partial_\mu \sigma + 1 \Rightarrow \delta \mathcal{L}^{\text{MIC}}(\sigma, \chi) = x_\mu \partial_\mu \mathcal{L}^{\text{MIC}}(\sigma, \chi) \)

but: \( \langle \sigma \rangle = 0 \Rightarrow \delta \mathcal{L}^{\text{MIC}}(0, \chi) = x_\mu \partial_\mu \mathcal{L}^{\text{MIC}}(0, \chi) - Tan(\chi) \)
Effective Field Theory with pions and dilatonic meson $\tau(x)$

- dilatation transformation [finite]:

  source fields:  $\sigma(x) \rightarrow \sigma(\lambda x) + \log \lambda$,  $\chi(x) \rightarrow \lambda^{4-y} \chi(\lambda x)$

  effective fields:  $\tau(x) \rightarrow \tau(\lambda x) + \log \lambda$,  $\Sigma(x) \rightarrow \Sigma(\lambda x)$

- invariant low-energy theory:  $\tilde{\mathcal{L}}^{\text{EFT}} = \tilde{\mathcal{L}}_\pi + \tilde{\mathcal{L}}_\tau + \tilde{\mathcal{L}}_m + \tilde{\mathcal{L}}_d$  where

$$
\begin{align*}
\tilde{\mathcal{L}}_\pi &= V_\pi (\tau - \sigma) \left( \frac{f^2_\pi}{4} \right) e^{2\tau} \text{tr} \left( \partial_\mu \Sigma^\dagger \partial_\mu \Sigma \right) \\
\tilde{\mathcal{L}}_\tau &= V_\tau (\tau - \sigma) \left( \frac{f^2_\tau}{2} \right) e^{2\tau} (\partial_\mu \tau)^2 \\
\tilde{\mathcal{L}}_m &= -V_M (\tau - \sigma) \left( \frac{f^2_\pi B_\pi}{2} \right) e^{y\tau} \text{tr} \left( \chi^\dagger \Sigma + \Sigma^\dagger \chi \right) \\
\tilde{\mathcal{L}}_d &= V_d (\tau - \sigma) f^2_\tau B_\tau e^{4\tau}
\end{align*}
$$

with invariant potentials:  $V(\tau(x) - \sigma(x)) \rightarrow V(\tau(\lambda x) - \sigma(\lambda x))$

$\Rightarrow$ No predictability without power counting!
Power counting hierarchy from matching correlation functions

- recall microscopic theory
  \[ \mathcal{L}^{\text{MIC}}(\sigma, \chi) = \mathcal{L}^{\text{MIC}}(0, \chi) + \sigma T_{\text{an}}(\chi) + O(\sigma^2) \]

\[ \frac{\partial}{\partial \sigma(x)} \mathcal{L}^{\text{MIC}} \bigg|_{\sigma=\chi=0} = T_{\text{an}}(x) \bigg|_{\chi=0} = \frac{\beta(g^2)}{4g^2} [F^2(x)] \sim n_f - n_f^* \]

- effective theory

\[ \left( - \frac{\partial}{\partial \sigma(x)} \right)^n \tilde{\mathcal{L}}^{\text{EFT}} \bigg|_{\sigma=\chi=0} = V_d^{(n)}(\tau(x)) f_\tau^2 B_\tau e^{4\tau(x)} + \cdots \]

\[ \Rightarrow \quad V(\tau - \sigma) = \sum_{n=0}^{\infty} c_n (\tau - \sigma)^n \quad \text{where} \quad c_n = O((n_f - n_f^*)^n) \]

\[ \Rightarrow \quad \text{Only a finite number of LECs at each order!} \]
Matching – role of non-coinciding points

- **Magenta**: points at distances $\ll$ meson size collapse to a single point in the EFT
- **Cyan**: points at asympt. large distances

**Upshot:**

$$V(\tau - \sigma) = \sum_{n=0}^{\infty} (\tau - \sigma)^n \sum_{k \geq n} \tilde{c}_{nk} (n_f - n_f^*)^k$$
Leading order lagrangian, finally:

- now set \( \sigma(x) = 0 \), obtaining at order \( m \sim n_f - n_f^* \sim p^2 \):

\[
\mathcal{L} = \mathcal{L}_\pi + \mathcal{L}_\tau + \mathcal{L}_m + \mathcal{L}_d
\]

\[
\mathcal{L}_\pi = (f_\pi^2/4) e^{2\tau} \text{tr} (\partial_\mu \Sigma^\dagger \partial_\mu \Sigma)
\]

\[
\mathcal{L}_\tau = (f_\tau^2/2) e^{2\tau} (\partial_\mu \tau)^2
\]

\[
\mathcal{L}_m = -(m f_\pi^2 B_\pi/2) e^{y\tau} \text{tr} \left( \Sigma + \Sigma^\dagger \right)
\]

\[
\mathcal{L}_d = [\tilde{c}_{00} + (n_f - n_f^*) (\tilde{c}_{01} + \tilde{c}_{11} \tau)] f_\tau^2 B_\tau e^{4\tau}
\]