New Noise Subtraction Methods in Lattice QCD

SUMAN BARAL*, WALTER WILCOX*, RONALD MORGAN**
*Physics Department,Baylor University,Waco,TX76798 **Mathematics Department, Baylor University,Waco,TX 76798

Background

Lattice QCD is a set of numerical techniques which use a finite space-time lattice to simulate the interactions between quarks and gluons. But, Lattice QCD amplitudes are affected by the background of quark-antiquark loops in particles such as a proton. The evaluation of loop effects on a given lattice, such as is shown in Figure 1, is extremely computer time intensive and approximation techniques must be introduced. In this context, we are attempting to employ matrix deflation algorithms to reduce statistical uncertainty in these time-consuming lattice calculations.

In addition, we are developing noise suppression algorithms using polynomial subtraction techniques, as well as combining deflation and polynomial methods in an original way.

The overall goal is to improve computer algorithms leading to the solution of massive sets of equations, especially those for Quantum Chromodynamics(QCD).

We achieved same results for a small lattice of size 8^4both in MatLab and Fortran and the results presented are of a large lattice of size $24^3 \times 32$ calculated in Fortran.

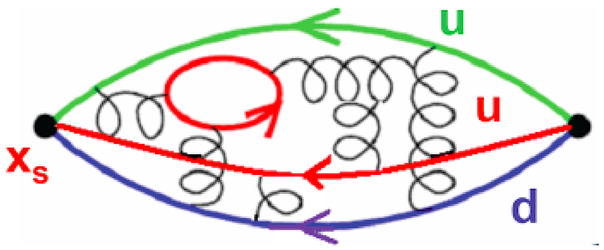

Figure 1: Disconnected quark loops in a proton

https://encryptedtbn1.gstatic.com/images?q=tbn:ANd9GcQt5G21Xs0RtqZs3bN_O55LTgm92sRQf9Z_HQGvPWNXZ-IGCSZumA

Methods

Non Subtracted Methods(NS)

Solution vectors are computed directly without using any of noise subtraction techniques

Eigenvalue Subtraction (ES)

Performance of iterative solvers is often limited by low eigenvalue spectrum of associated matrices. Deflation attempts to remove the effect of such eigenvalues. Deflating out eigenvalues with linear equation solver GMRES-DR can mimic the structure for off diagonal quark matrix $\widetilde{M'}_{eig}^{-1} \equiv \widetilde{V'}_R \widetilde{\Lambda'}^{-1} \widetilde{V'}_L^*$ Where $\widetilde{V'}_R$ and $\widetilde{V'}_L^*$ are the right and left Eigen vectors and $\widetilde{\Lambda'}^{-1}$ is the inverse of eigenvalues.

Hermitian Forced Eigenvalue Subtraction(HFES)

Convergence rates are harder to analyze for non-symmetric matrices. So, if one naively subtracts on a non-Hermitian matrix, one often increases the size of error bars. This can happen if the right handed eigenvectors of a non-Hermitian system are all pointing in the same direction. To combat this problem we have forced our problem to be formulated in a Hermitian manner.This is similar to Eigenvalue Subtraction. Only difference is Matrix is made Hermitian multiplying by γ_5

Perturbative Subtraction(PS)

This is the tried and true method of subtraction used in many lattice calculations. Our goal is to find more efficient methods than this .

$$\widetilde{M}_{pert}^{-1} = 1 + kP + (kP)^2 + (kP)^3 + (kP)^4 + (kP)^5 + (kP)^6$$

Where P is the quark matrix and k = 0.1550

Methods

Polynomial Subtraction (POLY)

This method is similar to that one of Perturbative Subtraction. The only difference is coefficients are now different from one. These coefficients are calculated using (Minimal Residual) Min-Res Projection.

$$\widetilde{M}_{poly}^{-1} = a_1 + a_2 kP + a_3 (kP)^2 + a_4 (kP)^3 + a_5 (kP)^4 + a_6 (kP)^5 + a_7 (kP)^6$$

Where $a_i's$ are the coefficients obtained from Min-Res Projection

Also, this polynomial can be used in a technique called polynomial preconditioning which efficiently evaluates the linear equations for lattice QCD. Convergence of linear solvers is improved by partially projecting the low-lying eigenvalue spectrum of an arbitrary matrix to the unit matrix. One replaces the original problem $Ax^i = b^i$ by $\mathrm{p}(A)Ax^i = p(A)b^i$

Where b^i is the i^{th} right hand side noise vector and x^i is the corresponding solution vector

HFPS combo = Hermitian Forced Eigenvalue Subtraction + Perturbative Subtraction

HFPOLY combo = Hermitian Forced Eigenvalue Subtraction +Polynomial Subtraction

ESPS combo = Eigenvalue Subtraction +Perturbative Subtraction

There are special considerations which have to be satisfied in these combinations ,since deflation methods can be rendered ineffective by methods which add back the removed eigenvalue information.

Size of the lattice is $24^3 \times 32$, the number of noise used is 200,kappa value is 0.155 and the performance is carried out in Fortran.

Linear equations are solved using GMRES-DR (Generalized Minimum RESidual algorithm-Deflated and Restarted) for the first noise, and GMRES-Proj (similar algorithm projected over eigenvectors) for remaining noises.

Results

The error bars for scalar and vector lattice operators are shown in figs.2 and 3

NS method is not efficient as compared to other methods.

ES method seems to increase the error bars as the number of deflated eigenvectors is increased.

HFES method in both cases reduces the error bars. However in neither cases it is better than the standard perturbative subtraction (PS) method although after 140 deflated eigenvectors it is close for scalar operator .

PS method seems to be effective than **HFES** and close to **POLY** method.

POLY method is better than perturbative in both cases, especially so for scalar operator.

ESPS combo method is inefficient.

HFPS combo method is the second best method**.**

HFPOLY combo is the best method.

Relative Efficiency, RE, of the two methods

$$RE \equiv \left(\frac{1}{(relative\ error\ bar)^2} - 1 \right) \times 100$$

Since esgPOLY combo relative error bar is ~0.77 of the perturbative subtraction error in first figure and ~ 0.75 in the second figure ,this means this method is approximately ***68% more efficient*** in the first case and ***77% more effective*** in the second case, compared to perturbative subtraction. These are significant improvements because these calculations are carried out at high kappa value which are very small quark masses. This is where most analysis algorithms breakdown.

Results

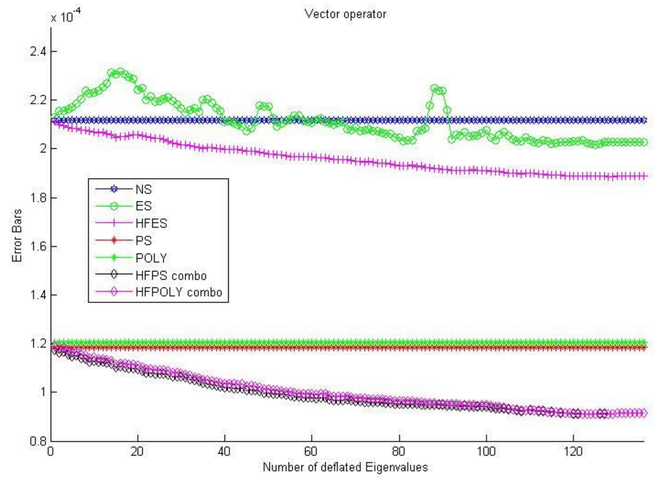

Figure 2: Error bars for a vector quark operator as a function of deflated eigenvalues

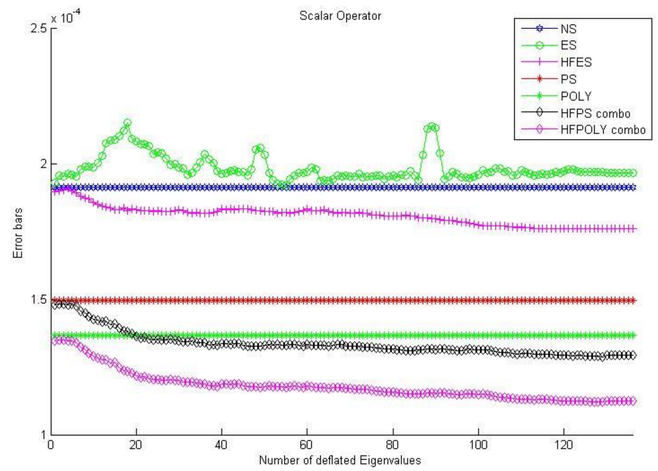

Figure 3: Error bars for a scalar quark operator as a function of deflated eigenvalues

Conclusions

- At a relatively small quark mass,(kappa=0.155) where most analysis algorithms begin to break down, our methods are still very effective.
- Our polynomial/deflation combination methods produce an amazingly good signal. Many lattice QCD groups will be very interested in applying our algorithm to extract signals faster and more efficiently.