Quark orbital dynamics in the nucleon –
from Ji to Jaffe-Manohar orbital angular momentum

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Acknowledgments:
M. Burkardt, S. Liuti,
and members of the Lattice TMD Collaboration

Gauge ensembles provided by:
MILC Collaboration
Proton spin decompositions

\[ \frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \sum_q L_q + J_g \] (Ji)

\[ \frac{1}{2} = \frac{1}{2} \sum_q \Delta q + \sum_q \mathcal{L}_q + \Delta g + \mathcal{L}_g \] (Jaffe-Manohar)

... and many more (in fact, we will see a continuous interpolation between the two ...)

There isn’t one unique way of separating quark and gluon orbital angular momentum – the different decompositions have different, legitimate meanings.
Quark orbital angular momentum

Interpreting terms in the energy-momentum tensor:

\[ L_q \sim -i\psi^\dagger(\vec{r} \times \vec{D})_z\psi \]

Can be obtained from \( L_q = J_q - S_q \), where \( S_q \) and \( J_q \) can be related to GPDs (Ji sum rule) – this has been used in Lattice QCD.

\[ L_q \sim -i\psi^\dagger(\vec{r} \times \vec{D})_z\psi \quad \text{in light cone gauge} \]

Hitherto not accessed in Lattice QCD.
Direct evaluation of quark orbital angular momentum

\[ L_3^U = \int dx \int d^2 k_T \int d^2 r_T \ (r_T \times k_T)_3 \ \mathcal{W}^U(x, k_T, r_T) \quad \text{Wigner distribution} \]

\[ \frac{L_3^U}{n} = \frac{\epsilon_{ij} \partial \partial_{z_T,i} \partial_{\Delta T,j}}{\partial T} \langle p', S | \overline{\psi}(-z/2) \gamma^+ U[-z/2, z/2] \psi(z/2) | p, S \rangle |_{z^+ = z^- = 0, \ \Delta_T = 0, \ z_T \to 0} \]

\[ \langle p', S | \overline{\psi}(-z/2) \gamma^+ U[-z/2, z/2] \psi(z/2) | p, S \rangle |_{z^+ = z^- = 0, \ \Delta_T = 0, \ z_T \to 0} \]

\( n \) : Number of valence quarks

\( p' = P + \Delta_T/2, \ p = P - \Delta_T/2, \ P, S \text{ in 3-direction, } P \to \infty \)

This is the same type of operator as used in TMD studies – generalization to off-forward matrix element adds transverse position information
Direct evaluation of quark orbital angular momentum

\[
\frac{L_{ij}^U}{n} = \epsilon_{ij} \frac{\partial}{\partial z_{ij}} \frac{\partial}{\partial \Delta T_{ij}} \langle p', S | \psi(-z/2) \gamma^+ U[-z/2, z/2 | \psi(z/2) | p, S) | z^+ = z^- = 0, \Delta T = 0, z_T \rightarrow 0 \rangle
\]

Role of the gauge link \( U \):

Y. Hatta, M. Burkardt:

- Straight \( U[-z/2, z/2] \) \( \rightarrow \) Ji OAM
- Staple-shaped \( U[-z/2, z/2] \) \( \rightarrow \) Jaffe-Manohar OAM
- Difference is torque accumulated due to final state interaction
Direct evaluation of quark orbital angular momentum

$$L^U_3 = \frac{n}{\varepsilon_i \frac{\partial}{\partial z_{T,i}} \frac{\partial}{\partial \Delta_{T,j}}} \left[ \langle p', S \mid \bar{\psi}(-z/2) \gamma^+ U[-z/2, z/2] \psi(z/2) \mid p', S \rangle \right] \bigg|_{z^+=z^-=0, \Delta T=0, z_T \to 0}$$

Role of the gauge link $U$:

Direction of staple taken off light cone (rapidity divergences)

Characterized by Collins-Soper parameter

$$\hat{\zeta} = \frac{P \cdot v}{|P||v|}$$

Are interested in $\hat{\zeta} \to \infty$; synonymous with $P \to \infty$ in the frame of the lattice calculation ($v = e_3$)
**Ensemble details**

LHPC mixed action scheme: MILC asqtad configurations, domain wall valence quarks

<table>
<thead>
<tr>
<th>$L^3 \times T$</th>
<th>$a$(fm)</th>
<th>$am_{u,d}$</th>
<th>$am_s$</th>
<th>$m_{\pi}^{DWF}$ (MeV)</th>
<th>$m_N^{DWF}$ (GeV)</th>
<th>#conf.</th>
<th>#meas.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20^3 \times 64$</td>
<td>0.11849(14)(99)</td>
<td>0.02</td>
<td>0.05</td>
<td>518.4(07)(49)</td>
<td>1.348(09)(13)</td>
<td>486</td>
<td>3888</td>
</tr>
</tbody>
</table>
Direct evaluation of quark orbital angular momentum

\[
\frac{L^U_3}{n} = \frac{\epsilon_{ij} \frac{\partial}{\partial z_{T,j}} \frac{\partial}{\partial \Delta_{T,i}}}{\epsilon_{ij}} \langle p', S | \bar{\psi}(-z/2)\gamma^+ U[-z/2, z/2] \psi(z/2) \mid p, S \rangle \mid_{z^+ = z^- = 0, \Delta_T=0, \ z_T \to 0} \]

Parameters to consider: $\Delta, \, \hat{\zeta}, \, z, \, \eta$
Direct evaluation of quark orbital angular momentum

\[
\frac{L^U_3}{n} = \frac{\epsilon_{ij}}{\partial \Delta T_i \partial \Delta T_j} \frac{\partial}{\partial z_i} \frac{\partial}{\partial z_j} \langle p', S | \bar{\psi}(-z/2)\gamma^+ U[-z/2, z/2] \psi(z/2) | p, S \rangle |_{z^+ = z^- = 0, \Delta T = 0, z_T \to 0} \\
\langle p', S | \bar{\psi}(-z/2)\gamma^+ U[-z/2, z/2] \psi(z/2) | p, S \rangle |_{z^+ = z^- = 0, \Delta T = 0, z_T \to 0}
\]

Dataset contains only one value of \(|\Delta_T| = 4\pi/aL \approx 1\) GeV

Substantial underestimate of \(\partial f/\partial \Delta_T\) by using

\[
\frac{\partial f}{\partial \Delta T_j} \bigg|_{\Delta T_j = 0} = \frac{1}{2\Delta T_j} (f(\Delta T_j) - f(-\Delta T_j))
\]
Direct evaluation of quark orbital angular momentum

\[ \frac{\partial f}{\partial z_{T,i}} \bigg|_{z_{T,i}=0} = \frac{1}{2da} (f(da_i) - f(-da_i)) \]
Direct evaluation of quark orbital angular momentum

\[
\frac{L^U_{3i}}{n} = \frac{\epsilon_{ij} \partial \partial_{zT,i} \partial \Delta_{T,j}}{n} \langle p', S \mid \bar{\psi}(-z/2)\gamma^+ U[-z/2, z/2] \psi(z/2) \mid p, S \rangle \bigg|_{z^+=z^-=0, \Delta_T=0, z_T \to 0}
\]

Remaining parameters to consider: \( \hat{\zeta}, \eta \)
Ji quark orbital angular momentum: $\eta = 0$

$\frac{L_3(\eta=0)}{\eta(\eta=0)}$

$\xi$

$u$–$d$ quarks

$m_\pi = 518$ MeV

$\rightarrow$ Signature of underestimate of $\partial f/\partial \Delta T$
From Ji to Jaffe-Manohar quark orbital angular momentum

\[ u-d \text{ quarks} \]

\[ m_\pi = 518 \text{ MeV} \]

\[ \hat{\zeta} = 0 \]
From Ji to Jaffe-Manohar quark orbital angular momentum

$u-d$ quarks

$m_{\pi} = 518$ MeV

$\hat{\zeta} = 0.39$
From Ji to Jaffe-Manohar quark orbital angular momentum

$u-d$ quarks

$m_{\pi} = 518$ MeV

$\hat{\zeta} = 0.78$
Burkardt’s torque – extrapolation in $\hat{\zeta}$

\[ \tau_3 = \left( L_3^{(\eta=\infty)} / n^{(\eta=\infty)} \right) - \left( L_3^{(\eta=0)} / n^{(\eta=0)} \right) \]

Integrated torque accumulated by struck quark leaving proton

$u$–$d$ quarks

$m_\pi = 518$ MeV
Flavor separation – from Ji to Jaffe-Manohar quark orbital angular momentum

\[ m_\pi = 518 \text{ MeV} \]
\[ \hat{\zeta} = 0.39 \]

- d quarks
- isoscalar
- 2u quarks
Flavor separation – Burkardt’s torque

\[
\tau_3 = \left( \frac{L_3^{(\eta=\infty)}}{n^{(\eta=\infty)}} \right) - \left( \frac{L_3^{(\eta=0)}}{n^{(\eta=0)}} \right)
\]

Integrated torque accumulated by struck quark leaving proton
Conclusions

• Quark orbital angular momentum can be accessed directly in Lattice QCD, continuously interpolating between the Ji and Jaffe-Manohar definitions.

• In the gathered exploratory dataset, the difference between the Ji and Jaffe-Manohar definitions, i.e., the torque accumulated by the struck quark leaving a proton, is clearly resolvable, sizeable (∼ 50% of the original Ji OAM), and leads to an enhancement of Jaffe-Manohar OAM relative to Ji OAM.

• Principal shortcomings of present dataset are too large momentum transfer, too small proton momentum. These practical issues are planned to be resolved in future work.