Effective action for pions and a dilatonic meson — foundations

presenter: Yigal Shamir

Effective action for pions and a dilatonic meson — results

presenter: Maarten Golterman

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A light flavor-singlet scalar — the Higgs particle?

- \( SU(3), N_f = 8 \) fund. [LatKMI, LSD,..]

\[ aM_{\pi}, \rho, a_1, N, 0^+ \]

Consistent low-energy theory must contain both pions and the flavor-singlet scalar

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- \( SU(3), N_f = 2 \) sextet [Fodor et al.]
Phases of $SU(N_c)$ with $N_f$ fundamental-rep Dirac fermions

- Running slows down when $N_f$ is increased

$$\frac{\partial g^2}{\partial \log \mu} = -\frac{b_1}{16\pi^2} g^4 - \frac{b_2}{(16\pi^2)^2} g^6$$

- Two-loop IRFP $g^2_*$ develops when $b_1 > 0 > b_2$

- “Walking” gap equation $\Rightarrow$ ChSB when $g^2(\mu) = g^2_c = \frac{4\pi^2}{3C_2}$

- $SU(3)$, fund. rep: $g^2_c = \pi^2 \simeq 9.87$

- Chirally broken if $g_c < g_*(N_f)$

- Conformal (IRFP) if $g_c > g_*(N_f)$

- Sill of conformal window: $g_*(N_f^*) = g_c$ (note: $N_f^*$ not an integer)
Pseudo Nambu-Goldstone boson of approx dilatation symmetry?

- dilatations: \( \Phi_i(x) \rightarrow \lambda^{\Delta_i} \Phi_i(\lambda x) \), \( \Delta_i \) scaling dimension of field \( \Phi_i(x) \)

- dilatation current: \( S_\mu = x_\nu T_{\mu\nu} \) classically conserved for \( m = 0 \)

- non-conservation:
  \[
  \partial_\mu S_\mu = T_{\mu\mu} = -T_{cl} - T_{an}
  \]
  \[
  T_{cl} = m[\bar{\psi}\psi]
  \]
  \[
  T_{an} = \frac{\beta(g_c^2)}{4g^2} [F^2] + \gamma_m m [\bar{\psi}\psi]
  \]

- probe beta fn at the ChSB scale:
  \[
  \langle T_{an}(0) [F^2](x) \rangle_c / \langle [F^2](0) [F^2](x) \rangle_c
  \]

- below conformal sill:
  \[
  \beta(g_c^2) \propto N_f - N_f^*
  \]
  expect: increasing \( N_f \) towards \( N_f^* \) \( \Rightarrow \) smaller \( \beta(g_c) \) at ChSB scale
  \( \Rightarrow \) better scale invariance \( \Rightarrow \) “dilatonic” pNG boson gets lighter

- Q: use \( N_f - N_f^* \) as small parameter? (problem: \( N_f \) takes discrete values)
Low-energy EFT with dilatonic meson: power counting

- standard ChPT: fermion mass $m$ is a parameter of the microscopic theory
  $m$ can be tuned continuously towards zero
  $\Rightarrow$ Systematic expansion in $m$ and $p^2$

- problem: cannot turn off trace anomaly; theory is defined at fixed $N_c, N_f$
  $\Rightarrow$ Systematic expansion in $m$, $1/N_c$, and $p^2$

- analogy: cannot turn off $U(1)_A$ anomaly;
  but it becomes vanishingly small for $N_c \to \infty$
  $\Rightarrow$ Systematic expansion in $m$, $1/N_c$, and $p^2$ [Kaiser and Leutwyler, ’00]

- Veneziano limit: $N_f, N_c \to \infty$ with $n_f = N_f/N_c$ fixed
  $n_f$ becomes a continuous parameter; theory depends only on $g^2 N_c$ and $n_f$
  $n_f^* = \lim_{N_c \to \infty} N_f^* (N_c)/N_c = \text{sill of conformal window for } N_c \to \infty$.

- assume: $T_{an} \sim (n_f - n_f^*)^\eta$ at the ChSB scale [\(\eta = 1\) in this talk]
  $\Rightarrow$ Systematic expansion in $m$, $1/N$, $n_f - n_f^*$, and $p^2$
Constructing an Effective Field Theory

Microscopic theory:

• symmetries

• spurions: external fields transforming under the symmetries

• fixing “VEVs” of spurions $\Rightarrow$ explicit breaking of symmetries

Effective theory:

• same symmetries, same spurion fields, but new dynamical (effective) fields

• explicit breaking of symmetries from same VEVs of spurions

• power counting (previous slide)

• use spurions as probes $\Rightarrow$ fix Low Energy Constants order by order, by matching correlators obtained by differentiation with respect to spurion fields
Spurions in the microscopic theory

- chiral symmetry: \( \mathcal{L}^{\text{MIC}}(\chi) = \frac{1}{4} F^2 + \bar{\psi} D\psi + \bar{\psi}_R \chi \dagger \psi_L + \bar{\psi}_L \chi \psi_R \)

\( \delta \mathcal{L}^{\text{MIC}}(\chi) = 0 \), but: \( \langle \chi \rangle = m \implies \delta \mathcal{L}^{\text{MIC}}(m) = m \delta(\bar{\psi}\psi) \)

- axial \( U(1)_A \) symmetry: \( \mathcal{L}^{\text{MIC}}(\theta) = \frac{1}{4} F^2 + \bar{\psi} D\psi + \theta icg^2 F \tilde{F} \)

\( \delta \theta = 1 \implies \delta \mathcal{L}^{\text{MIC}}(\theta) = 0 \) \hspace{1cm} (finite \( U(1)_A \) transf: \( \theta \to \theta + \alpha \))

but: \( \langle \theta \rangle = \theta_0 \implies \delta \mathcal{L}^{\text{MIC}}(\theta_0) = -icg^2 F \tilde{F} \)

- dilatations: \( \mathcal{L}^{\text{MIC}}(\sigma, \chi) = \mathcal{L}^{\text{MIC}}(0, \chi) + \sigma \tan(\chi) + \cdots \)

\( \delta \sigma = x_\mu \partial_\mu \sigma + 1 \implies \delta \mathcal{L}^{\text{MIC}}(\sigma, \chi) = x_\mu \partial_\mu \mathcal{L}^{\text{MIC}}(\sigma, \chi) \)

but: \( \langle \sigma \rangle = 0 \implies \delta \mathcal{L}^{\text{MIC}}(0, \chi) = x_\mu \partial_\mu \mathcal{L}^{\text{MIC}}(0, \chi) - \tan(\chi) \)
Effective Field Theory with pions and dilatonic meson $\tau(x)$

- dilatation transformation [finite]:

  source fields: $\sigma(x) \rightarrow \sigma(\lambda x) + \log \lambda, \quad \chi(x) \rightarrow \lambda^{4-y} \chi(\lambda x)$

  effective fields: $\tau(x) \rightarrow \tau(\lambda x) + \log \lambda, \quad \Sigma(x) \rightarrow \Sigma(\lambda x)$

- invariant low-energy theory: $\tilde{\mathcal{L}}^{\text{EFT}} = \tilde{\mathcal{L}}_{\pi} + \tilde{\mathcal{L}}_{\tau} + \tilde{\mathcal{L}}_{m} + \tilde{\mathcal{L}}_{d}$ where

  $\tilde{\mathcal{L}}_{\pi} = V_\pi (\tau - \sigma) \left( \frac{f_\pi^2}{4} \right) e^{2\tau} tr (\partial_\mu \Sigma^\dagger \partial_\mu \Sigma)$

  $\tilde{\mathcal{L}}_{\tau} = V_\tau (\tau - \sigma) \left( \frac{f_\tau^2}{2} \right) e^{2\tau} (\partial_\mu \tau)^2$

  $\tilde{\mathcal{L}}_{m} = -V_M (\tau - \sigma) \left( \frac{f_\pi^2 B_\pi}{2} \right) e^{y\tau} tr \left( \chi^\dagger \Sigma + \Sigma^\dagger \chi \right)$

  $\tilde{\mathcal{L}}_{d} = V_d (\tau - \sigma) \frac{f_\tau^2}{2} B_\tau e^{4\tau}$

  with invariant potentials: $V(\tau(x) - \sigma(x)) \rightarrow V(\tau(\lambda x) - \sigma(\lambda x))$

$\Rightarrow$ No predictability without power counting!
Power counting hierarchy from matching correlation functions

- recall microscopic theory
  \[ \mathcal{L}^{\text{MIC}}(\sigma, \chi) = \mathcal{L}^{\text{MIC}}(0, \chi) + \sigma T_{\text{an}}(\chi) + O(\sigma^2) \]

\[ \frac{\partial}{\partial \sigma(x)} \mathcal{L}^{\text{MIC}} \bigg|_{\sigma=\chi=0} = T_{\text{an}}(x) \bigg|_{\chi=0} = \frac{\beta(g^2)}{4g^2} [F^2(x)] \sim n_f - n_f^* \]

- effective theory

\[ \left( - \frac{\partial}{\partial \sigma(x)} \right)^n \tilde{\mathcal{L}}^{\text{EFT}} \bigg|_{\sigma=\chi=0} = V_d^{(n)}(\tau(x)) f_\tau^2 B_\tau e^{4\tau(x)} + \cdots \]

\[ \Rightarrow \quad V(\tau - \sigma) = \sum_{n=0}^{\infty} c_n (\tau - \sigma)^n \quad \text{where} \quad c_n = O((n_f - n_f^*)^n) \]

\[ \Rightarrow \quad \text{Only a finite number of LECs at each order!} \]
Matching – role of non-coinciding points

- **Magenta**: points at distances $\ll$ meson size collapse to a single point in the EFT
- **Cyan**: points at asympt. large distances

- **Upshot**:

$$V(\tau - \sigma) = \sum_{n=0}^{\infty} (\tau - \sigma)^n \sum_{k \geq n} \tilde{c}_{nk} (n_f - n_f^*)^k$$
Leading order lagrangian, finally:

- now set \( \sigma(x) = 0 \), obtaining at order \( m \sim n_f - n_f^* \sim p^2 \):

\[
\begin{align*}
\mathcal{L} &= \mathcal{L}_\pi + \mathcal{L}_\tau + \mathcal{L}_m + \mathcal{L}_d \\
\mathcal{L}_\pi &= \left( f_\pi^2 / 4 \right) e^{2\tau} \text{tr} \left( \partial_\mu \Sigma^\dagger \partial_\mu \Sigma \right) \\
\mathcal{L}_\tau &= \left( f_\tau^2 / 2 \right) e^{2\tau} \left( \partial_\mu \tau \right)^2 \\
\mathcal{L}_m &= -\left( m f_\pi^2 B_\pi / 2 \right) e^{y\tau} \text{tr} \left( \Sigma + \Sigma^\dagger \right) \\
\mathcal{L}_d &= \left[ \tilde{c}_{00} + (n_f - n_f^*) \left( \tilde{c}_{01} + \tilde{c}_{11} \tau \right) \right] f_\tau^2 B_\tau e^{4\tau}
\end{align*}
\]

- \( \chi = \text{renormalized source}, \quad m = \text{renormalized mass} \)

\[ y = 3 - \gamma_m^* \text{, with } \gamma_m^* \text{ the IRFP value of the mass anomalous dimension at the sill of the conformal window} \]

- corrections are accounted for by expansion in \( n_f - n_f^* \)