Effective action for pions and a dilatonic meson — foundations

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Effective action for pions and a dilatonic meson — results

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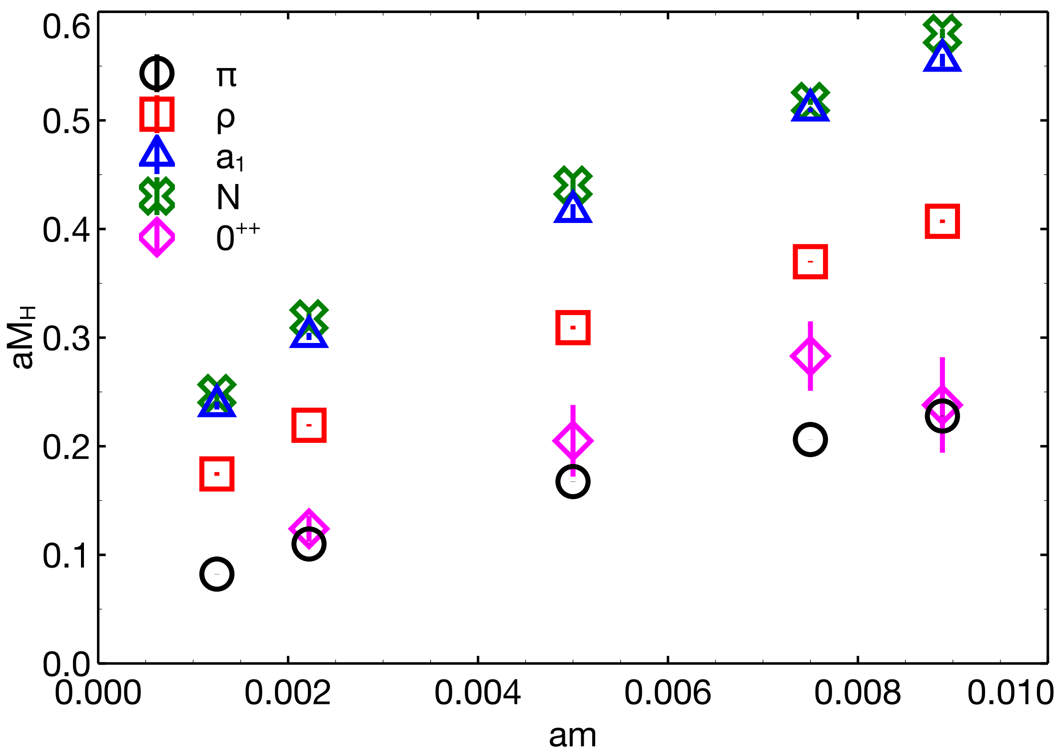
arXiv:1603.04575

A light flavor-singlet scalar — the Higgs particle?

- $SU(3), N_f = 8$ fund. [LatKMI, LSD,..]



LSD collaboration, PRD 93 (2016) 114514

Consistent low-energy theory must contain both pions and the flavor-singlet scalar

- $SU(3), N_f = 2$ sextet [Fodor et al.]

Phases of $SU(N_c)$ with N_f fundamental-rep Dirac fermions

- running slows down when N_f is increased

$$\frac{\partial g^2}{\partial \log \mu} = -\frac{b_1}{16\pi^2} g^4 - \frac{b_2}{(16\pi^2)^2} g^6$$

- two-loop IRFP g_*^2 develops when $b_1 > 0 > b_2$

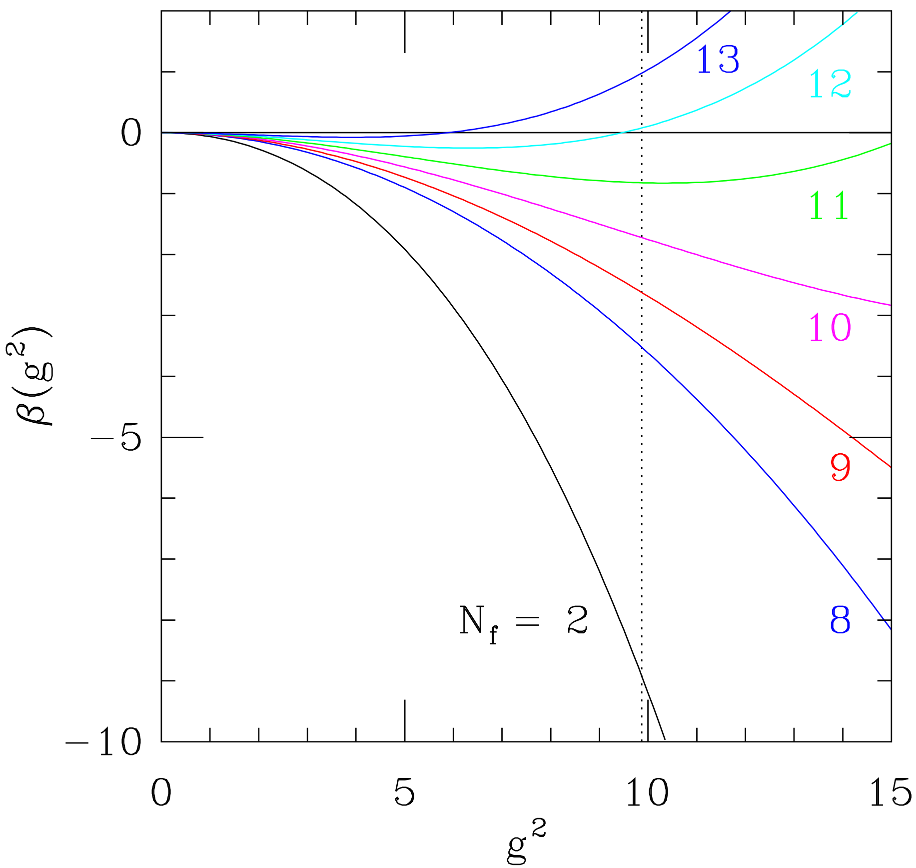
- "Walking" gap equation $\Rightarrow$

  ChSB when $g^2(\mu) = g_c^2 = \frac{4\pi^2}{3C_2}$

- $SU(3)$, fund. rep: $g_c^2 = \pi^2 \simeq 9.87$

- chirally broken if $g_c < g_*(N_f)$

- conformal (IRFP) if $g_c > g_*(N_f)$



- sill of conformal window: $g_*(N_f^*) = g_c$ (note: N_f^* not an integer)

Pseudo Nambu-Goldstone boson of approx dilatation symmetry?

- dilatations: $\Phi_i(x) \to \lambda^{\Delta_i}\,\Phi_i(\lambda x)$, Δ_i scaling dimension of field $\Phi_i(x)$
- dilatation current: $S_\mu = x_\nu T_{\mu\nu}$ classically conserved for $m = 0$
- non-conservation:

$$
\begin{aligned}
\partial_\mu S_\mu &= T_{\mu\mu} = -T_{cl} - T_{an} \\
T_{cl} &= m[\overline{\psi}\psi] \\
T_{an} &= \frac{\beta(g^2)}{4g^2}\,[F^2] + \gamma_m\, m\,[\overline{\psi}\psi]
\end{aligned}
$$

- probe beta fn at the ChSB scale: $\big\langle T_{an}(0)\,[F^2](x)\big\rangle_c \;/\; \big\langle [F^2](0)\,[F^2](x)\big\rangle_c$
- below conformal sill: $\beta(g_c^2) \propto N_f - N_f^*$
  expect: increasing N_f towards N_f^* $\Rightarrow$ smaller $\beta(g_c)$ at ChSB scale
  $\Rightarrow$ better scale invariance $\Rightarrow$ "dilatonic" pNG boson gets lighter
- Q: use $N_f - N_f^*$ as small parameter? (problem: N_f takes discrete values)

Low-energy EFT with dilatonic meson: power counting

- standard ChPT: fermion mass m is a parameter of the microscopic theory
  m can be tuned continuously towards zero

$\Rightarrow$ Systematic expansion in m and p^2

- problem: cannot turn off trace anomaly; theory is defined at fixed N_c, N_f

- analogy: cannot turn off $U(1)_A$ anomaly;
  but it becomes vanishingly small for $N_c \to \infty$

$\Rightarrow$ Systematic expansion in m, $1/N_c$, and p^2 [Kaiser and Leutwyler, '00]

- Veneziano limit: $N_f, N_c \to \infty$ with $n_f = N_f/N_c$ fixed
  n_f becomes a continuous parameter; theory depends only on $g^2 N_c$ and n_f
  $n_f^* = \lim_{N_c \to \infty} N_f^*(N_c)/N_c$ = sill of conformal window for $N_c \to \infty$.

- assume: $T_{an} \sim (n_f - n_f^*)^\eta$ at the ChSB scale [$\eta = 1$ in this talk]

$\Rightarrow$ Systematic expansion in m, $1/N$, $n_f - n_f^*$, and p^2

Constructing an Effective Field Theory

Microscopic theory:

- symmetries
- spurions: external fields transforming under the symmetries
- fixing "VEVs" of spurions $\Rightarrow$ explicit breaking of symmetries

Effective theory:

- same symmetries, same spurion fields, but new dynamical (effective) fields
- explicit breaking of symmetries from same VEVs of spurions
- power counting (previous slide)
- use spurions as probes $\Rightarrow$ fix Low Energy Constants order by order, by matching correlators obtained by differentiation with respect to spurion fields

Spurions in the microscopic theory

• chiral symmetry: $\mathcal{L}^{\rm MIC}(\chi) = \frac{1}{4}F^2 + \overline{\psi}\not{D}\psi + \overline{\psi}_R\chi^\dagger\psi_L + \overline{\psi}_L\chi\psi_R$

$\delta\mathcal{L}^{\rm MIC}(\chi) = 0\,,$ but: $\langle\chi\rangle = m \quad\Rightarrow\quad \delta\mathcal{L}^{\rm MIC}(m) = m\delta(\overline{\psi}\psi)$

• axial $U(1)_A$ symmetry: $\mathcal{L}^{\rm MIC}(\theta) = \frac{1}{4}F^2 + \overline{\psi}\not{D}\psi + \theta icg^2F\tilde{F}$

$\delta\theta = 1 \quad\Rightarrow\quad \delta\mathcal{L}^{\rm MIC}(\theta) = 0$ (finite $U(1)_A$ transf: $\theta \to \theta + \alpha$)

but: $\langle\theta\rangle = \theta_0 \quad\Rightarrow\quad \delta\mathcal{L}^{\rm MIC}(\theta_0) = -icg^2F\tilde{F}$

• dilatations: $\mathcal{L}^{\rm MIC}(\sigma,\chi) = \mathcal{L}^{\rm MIC}(0,\chi) + \sigma T_{an}(\chi) + \cdots$

$\delta\sigma = x_\mu\partial_\mu\,\sigma + 1 \quad\Rightarrow\quad \delta\mathcal{L}^{\rm MIC}(\sigma,\chi) = x_\mu\partial_\mu\,\mathcal{L}^{\rm MIC}(\sigma,\chi)$

but: $\langle\sigma\rangle = 0 \quad\Rightarrow\quad \delta\mathcal{L}^{\rm MIC}(0,\chi) = x_\mu\partial_\mu\,\mathcal{L}^{\rm MIC}(0,\chi) - T_{an}(\chi)$

Effective Field Theory with pions and dilatonic meson $\tau(x)$

- dilatation transformation [finite]:

source fields: $\sigma(x) \to \sigma(\lambda x) + \log\lambda, \quad \chi(x) \to \lambda^{4-y}\chi(\lambda x)$

effective fields: $\tau(x) \to \tau(\lambda x) + \log\lambda, \quad \Sigma(x) \to \Sigma(\lambda x)$

- invariant low-energy theory: $\tilde{\mathcal{L}}^{\mathrm{EFT}} = \tilde{\mathcal{L}}_\pi + \tilde{\mathcal{L}}_\tau + \tilde{\mathcal{L}}_m + \tilde{\mathcal{L}}_d$ where

$$\begin{aligned}
\tilde{\mathcal{L}}_\pi &= V_\pi(\tau-\sigma)\,(f_\pi^2/4)\,e^{2\tau}\mathrm{tr}\,(\partial_\mu\Sigma^\dagger\partial_\mu\Sigma) \\
\tilde{\mathcal{L}}_\tau &= V_\tau(\tau-\sigma)\,(f_\tau^2/2)\,e^{2\tau}(\partial_\mu\tau)^2 \\
\tilde{\mathcal{L}}_m &= -V_M(\tau-\sigma)\,(f_\pi^2 B_\pi/2)\,e^{y\tau}\mathrm{tr}\left(\chi^\dagger\Sigma+\Sigma^\dagger\chi\right) \\
\tilde{\mathcal{L}}_d &= V_d(\tau-\sigma)\,f_\tau^2 B_\tau\,e^{4\tau}
\end{aligned}$$

with invariant potentials: $V(\tau(x)-\sigma(x)) \;\to\; V(\tau(\lambda x)-\sigma(\lambda x))$

$\Rightarrow$ No predictability without power counting!

Power counting hierarchy from matching correlation functions

- recall microscopic theory $\mathcal{L}^{\rm MIC}(\sigma,\chi) = \mathcal{L}^{\rm MIC}(0,\chi) + \sigma T_{an}(\chi) + O(\sigma^2)$

$$\frac{\partial}{\partial\sigma(x)}\mathcal{L}^{\rm MIC}\Big|_{\sigma=\chi=0} = T_{an}(x)\Big|_{\chi=0} = \frac{\beta(g^2)}{4g^2}\,[F^2(x)] \sim n_f - n_f^*$$

- effective theory

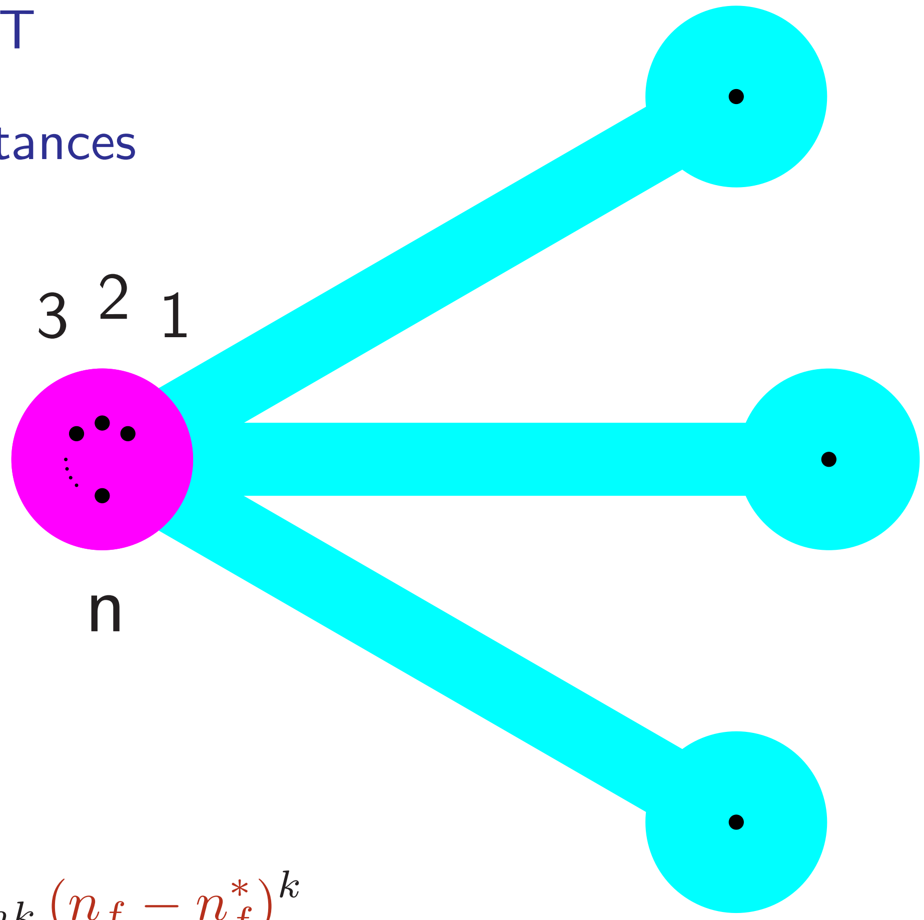
$$\left(-\frac{\partial}{\partial\sigma(x)}\right)^n \tilde{\mathcal{L}}^{\rm EFT}\Big|_{\sigma=\chi=0} = V_d^{(n)}(\tau(x))\, f_\tau^2 B_\tau\, e^{4\tau(x)} + \cdots$$

$\Rightarrow$ $V(\tau-\sigma) = \sum_{n=0}^{\infty} c_n(\tau-\sigma)^n$ where $c_n = O((n_f - n_f^*)^n)$

$\Rightarrow$ Only a finite number of LECs at each order!

Matching – role of non-coinciding points

- Magenta: points at distances $\ll$ meson size collapse to a single point in the EFT
- Cyan: points at asympt. large distances



- Upshot:

$$V(\tau - \sigma) \;=\; \sum_{n=0}^{\infty} (\tau - \sigma)^n \sum_{k \geq n} \tilde{c}_{nk} \, (n_f - n_f^*)^k$$

Leading order lagrangian, finally:

- now set $\sigma(x) = 0$, obtaining at order $m \sim n_f - n_f^* \sim p^2$:

$$
\begin{aligned}
\mathcal{L} &= \mathcal{L}_\pi + \mathcal{L}_\tau + \mathcal{L}_m + \mathcal{L}_d \\
\mathcal{L}_\pi &= (f_\pi^2/4)\, e^{2\tau} \mathrm{tr}\,(\partial_\mu \Sigma^\dagger \partial_\mu \Sigma) \\
\mathcal{L}_\tau &= (f_\tau^2/2)\, e^{2\tau} (\partial_\mu \tau)^2 \\
\mathcal{L}_m &= -(m f_\pi^2 B_\pi/2)\, e^{y\tau} \mathrm{tr}\left(\Sigma + \Sigma^\dagger\right) \\
\mathcal{L}_d &= [\tilde{c}_{00} + (n_f - n_f^*)(\tilde{c}_{01} + \tilde{c}_{11}\tau)]\, f_\tau^2 B_\tau\, e^{4\tau}
\end{aligned}
$$

- $\chi =$ renormalized source, $m =$ renormalized mass

$\Rightarrow$ $y = 3 - \gamma_m^*$, with γ_m^* the IRFP value of the mass anomalous dimension at the sill of the conformal window

- corrections are accounted for by expansion in $n_f - n_f^*$