Lattice QCD Searches for Tetraquarks containing Charm Quarks

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Outline

Introduction

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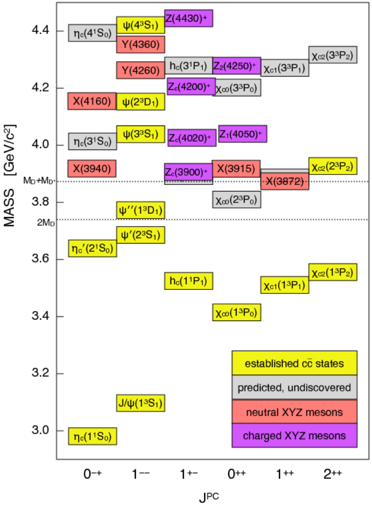
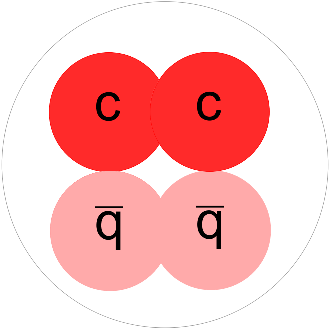
- Tetraquarks may be able to explain some of the X, Y, Z states.
- Exciting era of scattering on the lattice.
- Inclusion of diquark-antidiquark operators important in first principles lattice calcuations.

Alexandrou et al. arXiv:1212.1418
Prelovsek et al. arXiv:1405.7623
Guerrieri et al. arXiv:1411.2247
Padmanath et al. arXiv:1503.03257
Francis et al. arXiv:1607.05214

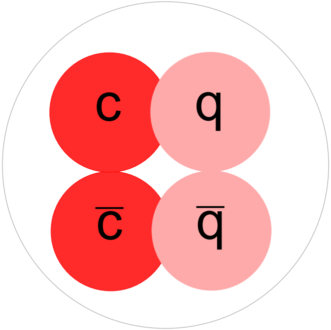


S. Olsen, arxiv:1511.01589

Double and Hidden-Charm Tetraquarks



No experimental candidate but have been hypothesised.



Especially relevant for the charged Z_c states. Isospin-1 more approachable when there is no charm quark annihilation.

Methodology

Calculation Method

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- Anisotropic lattice with $N_f = 2 + 1$ dynamical flavours. Wilson clover actions for the fermions. Symanzik-improved for the gauge action.

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$16^3 \times 128$	≈ 0.12	≈ 3.5	391	478

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- Quark fields smeared using the distillation framework.
- Spectra determined by solving GEVP $C_{ij}(t) = \langle \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)\rangle$ with large bases of operators consisting of:
  - Meson-Meson, $N_{\text{dist}} = 64$
  - Diquark-Antidiquark, $N_{\text{dist}} = 24$

Meson-Meson Operator Construction

- $q\bar{q}$ operator, $\mathcal{O}_{\Lambda,\lambda}(\vec{p},t) \sim \bar{q}\Gamma \overleftrightarrow{D} \cdots \overleftrightarrow{D} q$ subduced into lattice irrep with quantised momenta $\vec{p}$.

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- Two $q\bar{q}$ operators coupled using generalised Clebsch-Gordan coefficients,

$$\mathcal{O}^{MM}_{\Lambda,\lambda}(\vec{p},t) = \sum_{\substack{\vec{p}_1+\vec{p}_2=\vec{p} \\ \lambda_1,\lambda_2}} C^{\vec{p}}_{\Lambda,\lambda} \begin{pmatrix} \vec{p}_1 & \Lambda_1 & \lambda_1 \\ \vec{p}_2 & \Lambda_2 & \lambda_2 \end{pmatrix} \mathcal{O}_{\Lambda_1,\lambda_1}(\vec{p}_1,t)\mathcal{O}'_{\Lambda_2,\lambda_2}(\vec{p}_2,t).$$

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- Advantageous to use variationally optimised $q\bar{q}$ operators.

Non-zero Momentum - An Example

- A pseudoscalar (P) and vector (V) meson both with one unit of momentum coupled to a meson-meson operator in $T_1^+(1^+)$.

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$A_2 \otimes A_1$	T_1^+, A_1^-, E^-
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- Two operators from PV and we expect two energy levels.

Diquark-Antidiquark Operator Construction

- Construct the continuum diquark operator

$$\delta_{\Gamma}^{c} = c_{\alpha ij} q_i^T C \Gamma q_j$$

where C is the charge-conjugation matrix and $c_{\alpha ij}$ are the colour coupling coefficients that couple the two quarks to either the colour $\underline{\overline{3}}$ or $\underline{6}$ irrep.

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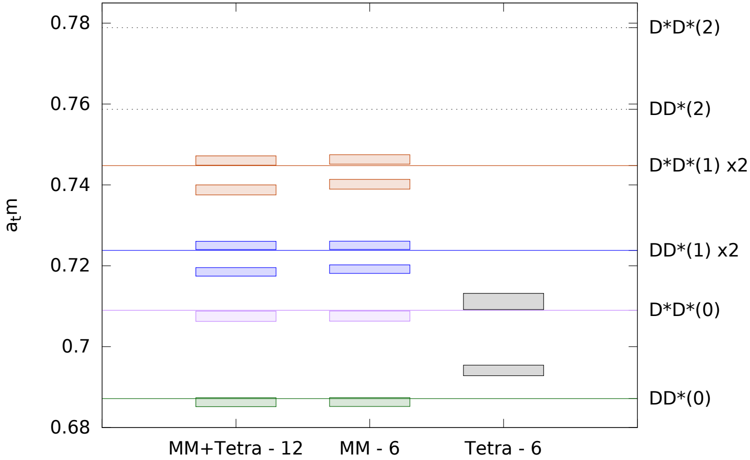
- Tetraquark operator by coupling the diquark and anti-diquark to a colour singlet,

$$T = \mathbb{C}_{c\bar{c}} \delta^c_\Gamma \bar{\delta}^{\bar{c}}_{\Gamma'}$$

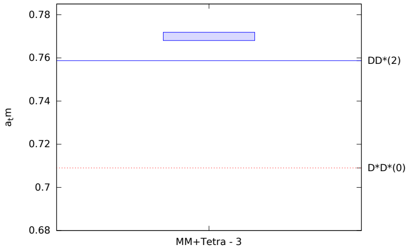
where $\mathbb{C}_{c\bar{c}}$ are the coefficients for either $\underline{\bar{3}} \otimes \underline{3}$ or $\underline{6} \otimes \underline{\bar{6}}$ to $\underline{1}$.

Results

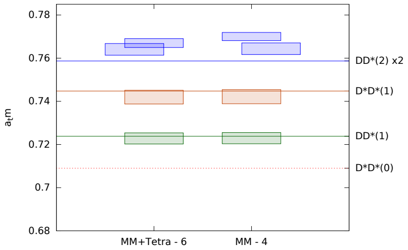
$cc\bar{q}\bar{q}$, $T_1^+(1^+)$ isospin-0



$cc\bar{q}\bar{q}, 2^+$, isospin-0

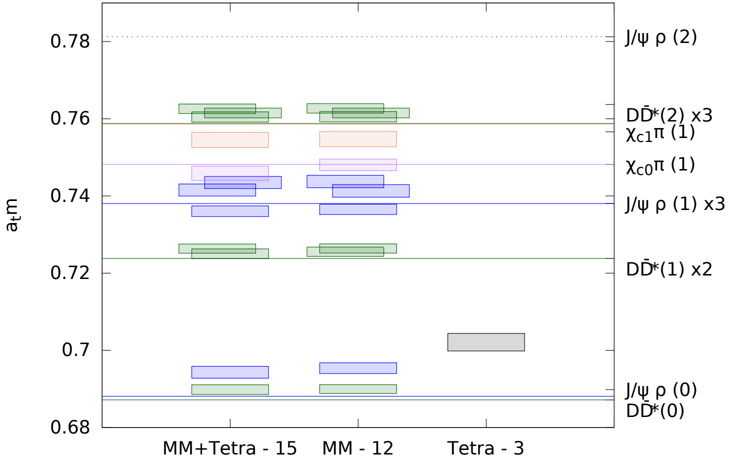


E^+

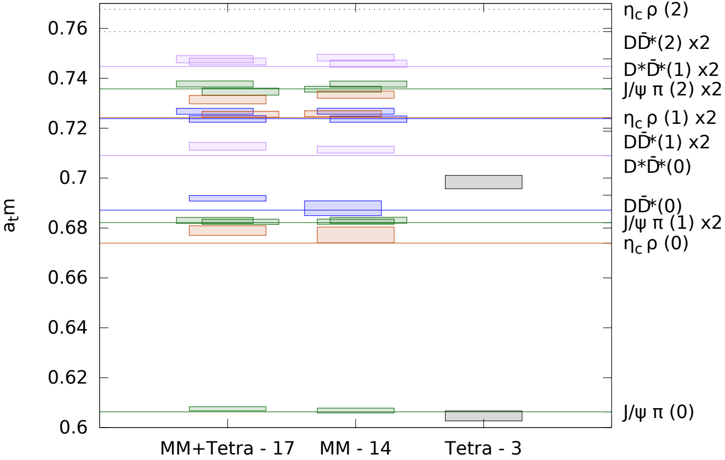


T_2^+

$c\bar{c}q\bar{q}$, $T_1^{++}(1^{++})$ isospin-1



$c\bar{c}q\bar{q}$, $T_1^{+-}(1^{+-})$ isospin-1



Conclusions

- We do not find a hint of a narrow resonance with the inclusion of tetraquark operators.
- Further investigations to be done on bigger lattices and lighter quark masses.
- Prospective studies in isospin-0 $c\bar{c}q\bar{q}$.

Backup - Variational Method

In order to determine the spectrum, we compute a matrix of correlators,

$$C_{ij}(t) = \sum_n \langle 0|\mathcal{O}_i|n\rangle\langle n|\mathcal{O}_j^\dagger|0\rangle e^{-E_n t}.$$

E_n can be extracted by solving the generalised eigenvalue problem $C(t)v^n = \lambda^n(t, t_0)C(t_0)v^n$ and we perform the fit

$$\lambda^n(t, t_0) = (1 - A_n)e^{-E_n(t-t_0)} + A_n e^{-E_n'(t-t_0)}.$$

Backup - Single Mesons

1. Construct a fermion bilinear with a gamma matrix Γ and a number of gauge covariant derivatives D.

$$\mathcal{O}(\vec{p}, t) = \int d^3x \; e^{i\vec{p}\cdot\vec{x}} \; \bar{q}\Gamma D \ldots q.$$

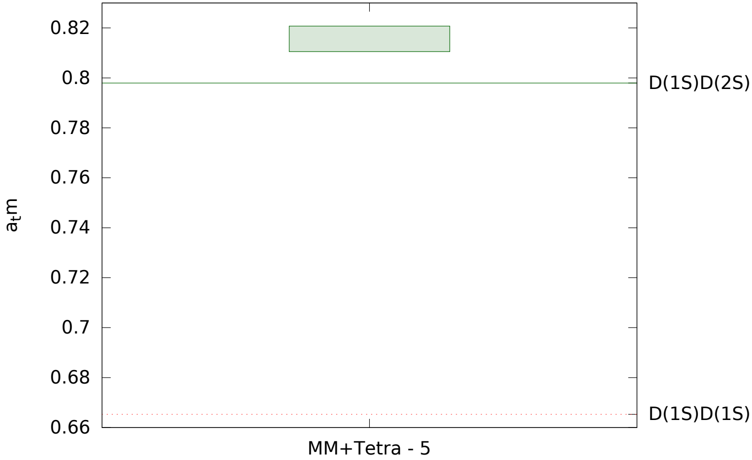
2. Couple to a continuum angular momentum irrep using Clebsch-Gordan coefficients. For example, for one gamma matrix and one derivative at rest,

$$\mathcal{O}^{J,M}(t) = \sum_{m_1, m_2} \langle J_1, m_1; J_2, m_2 | J, M \rangle \bar{q}\Gamma_{m_1} D_{m_2} q.$$

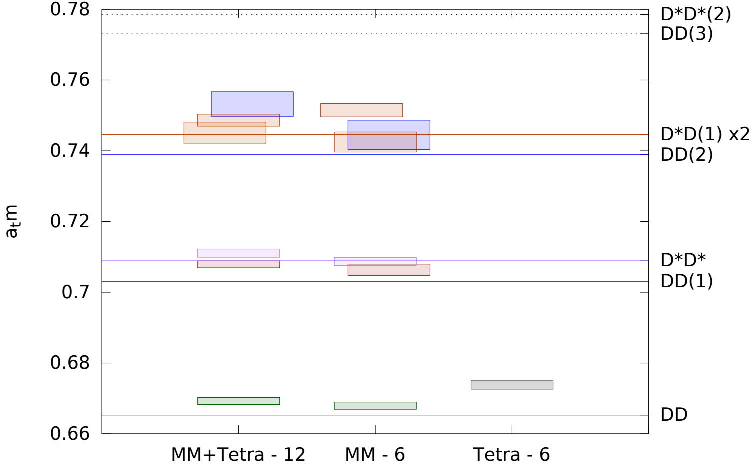
3. Project onto a lattice irrep using 'subduction' coefficients.

$$\mathcal{O}_{\Lambda,\lambda}(\vec{p}, t) = \sum_M S^{J,M}_{\Lambda,\lambda} \mathcal{O}^{J,M}(\vec{p}, t).$$

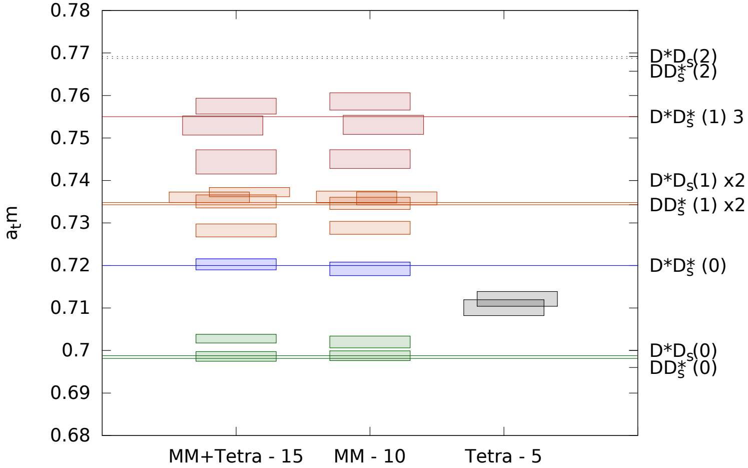
$cc\bar{q}\bar{q}, A_1^+, \mathcal{I} = 0$



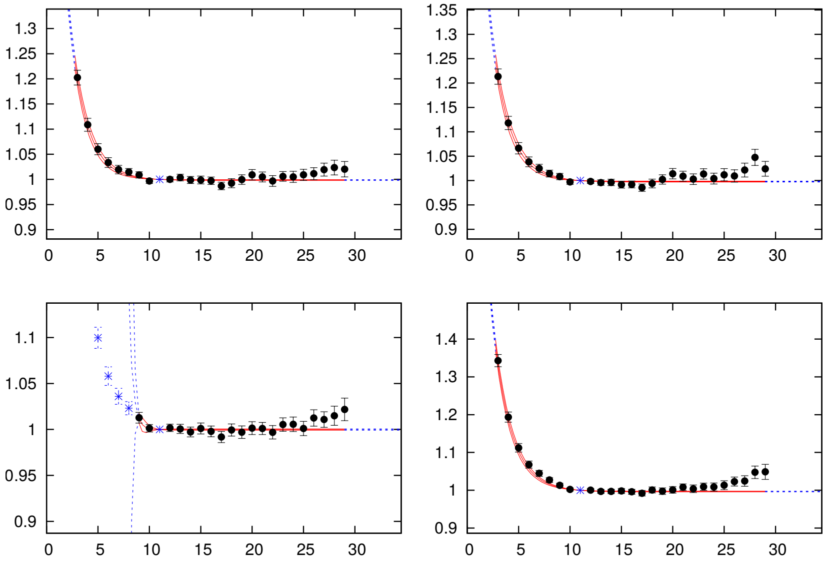
$cc\bar{q}\bar{q}, A_1^+, \mathcal{I} = 1$



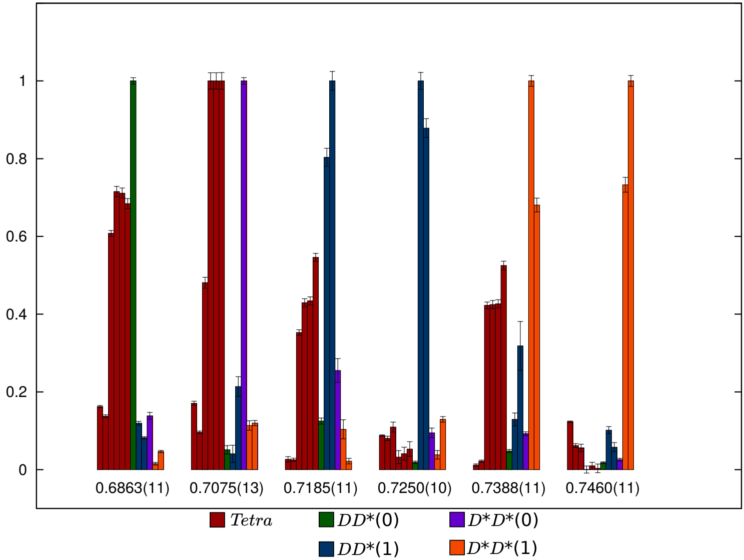
$cc\bar{q}\bar{s}$, $T_1^+(1^+)$ isospin-$\frac{1}{2}$



$cc\bar{q}\bar{q}$, $T_1^+(1^+)$ isospin-0 $\lambda^n e^{E_n(t-t_0)}$



$cc\bar{q}\bar{q}$, $T_1^+(1^+)$ isospin-0 Overlaps $Z_i = \langle 0|\mathcal{O}_i|n\rangle$



T_1^+ Varying N_{dist}