Up and down quark masses and corrections to Dashen's theorem from lattice QCD and quenched QED

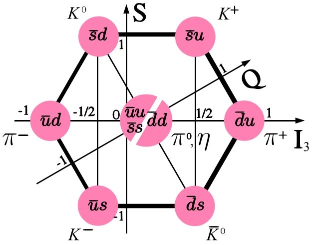
arXiv:1604.07112[hep-lat], to appear in PRL

Lukas Varnhorst
for the BMW collaboration

Bergische Universitt Wuppertal
FB C - Department of Physics

Introduction

The main contribution to hadron masses comes from the energy associated with the non-perturbative QCD interactions inside the particles. It is common to perform lattice computations at a common light quark mass $m_{ud} = \frac{1}{2}(m_u + m_d)$ and a strange quark mass m_s. Such calculations are commonly referred to as $N_f = 2 + 1$.

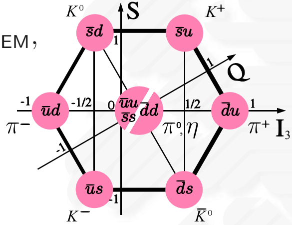


We have to consider $m_u \neq m_d$ and include QED in order to properly account for isospin breaking.

Introduction

Dashens' theorem [1] states that in the $SU(3)$ flavor symmetric limit the electromagnetic mass splittings of the pseudoscalar mesons behave as

$$(M^2_{\pi^\pm} - M^2_{\pi^0})_{\text{EM}} = (M^2_{K^\pm} - M^2_{K^0})_{\text{EM}},$$
$$(M_{\pi^0})_{\text{EM}} = (M_{K^0})_{\text{EM}},$$



Violations to Dashens' theorem can be parametrized by

$$\varepsilon = \frac{\Delta_{\text{QED}} M^2_K - \Delta_{\text{QED}} M^2_\pi}{\Delta M^2_\pi}$$

The K mass splitting can be at leading $\mathcal{O}(\alpha, \delta m)$ order expanded:

$$\Delta M^2_K = C_K \alpha + D'_K \delta m$$

where $\delta m = m_u - m_d$. Knowing $\Delta_{\text{QED}} M^2_K$ and the PQχPT parameter B_2 we could determine δm.

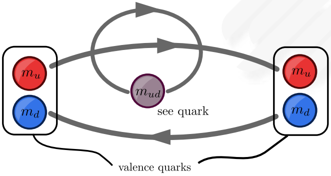
[1] R. F. Dashen, Phys. Rev. **183** (1969) 1245. doi:10.1103/PhysRev.183.1245

QCD action

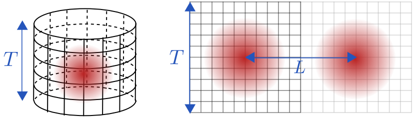
Technical details of the lattice caclulation:

- Tree-Level improved Symmanzik gauge action.
- Tree-Level Clover-improved Wilson fermion action with two levels of gauge smearing.
- Coulomb gauge for the non-compact $U(1)$ gauge fields.
- Zero mode substraction for the $U(1)$ field according to $a^4 \sum_x A_\mu(x) = 0$

In this work: Two approximations:



Finite-Volume-Corrections



QED in a box with periodic boundary condition → mirror particle. Full calculation for e.g. charged scalar particles yields ($\kappa = 2.837\ldots$) [1,2]

$$\frac{M^L}{M^\infty} = -\frac{\kappa}{M^\infty L}\left[1 + \frac{2}{M^\infty L}\left(1 - \frac{\pi}{2\kappa}\frac{T}{L}\right)\right] + c\frac{1}{L^3}$$

QCD has a mass gap → Finite volume corrections to masses scale exponential with the decay length determined by the lightest propagating degree of freedom. E.g. for Pion

$$\frac{M_\pi^L}{M_\pi^\infty} = 1 + c\sqrt{\frac{M_\pi}{L^3}}\exp(-M_\pi L)$$

[1] Z. Davoudi and M. J. Savage, Phys. Rev. D **90** (2014) no.5, 054503 doi:10.1103/PhysRevD.90.054503 [arXiv:1402.6741 [hep-lat]].

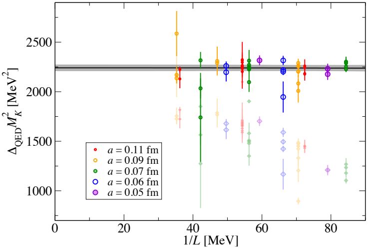
[2] S. Borsanyi *et al.*, Science **347** (2015) 1452 doi:10.1126/science.1257050 [arXiv:1406.4088 [hep-lat]].

Finite-Volume-Corrections

$$\frac{M^L}{M^\infty} = -\frac{\kappa M^\infty}{L}\left[1 + \frac{2}{M^\infty L}\left(1 - \frac{\pi}{2\kappa}\frac{T}{L}\right)\right] + c\frac{1}{L^3}$$

First two orders are universal, third order is structure dependent.

The behavior depends on the dimensionless parameter T/L. This is related to the zero mode subtraction scheme chosen.



Faded points: No finite volume correction, Solid points: First two orders corrected.

The fit function

If one defines

$$\Delta M = M^2_{\bar{u}u} - M^2_{\bar{d}d}$$
$$\Delta M^2_K = M^2_{K^+} - M^2_{K^0}$$

one can write

$$\Delta M^2_K = C_K \alpha + D_K \Delta M^2$$

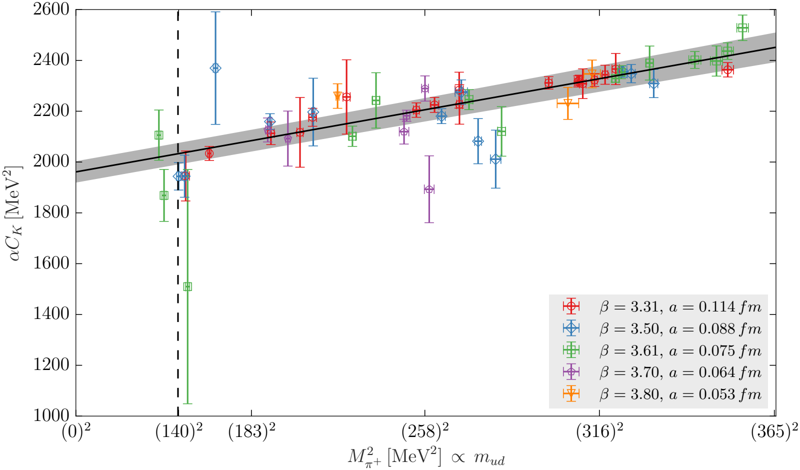
Here C_K parametrizes the QED contributions to the Kaon isospin splitting and D_K parametrizes the QCD contributions.

$$C_K = c_0 + c_1 \underbrace{M^2_\pi}_{\propto m_{ud}} + c_2 \underbrace{(M^2_K - 0.5 M^2_\pi)}_{\propto m_s} + c_3 a + c_4 \frac{1}{L^3}$$
$$D_K = d_0 + d_1 \underbrace{M^2_\pi}_{\propto m_{ud}} + d_2 \underbrace{(M^2_K - 0.5 M^2_\pi)}_{\propto m_s} + d_3 f(a)$$

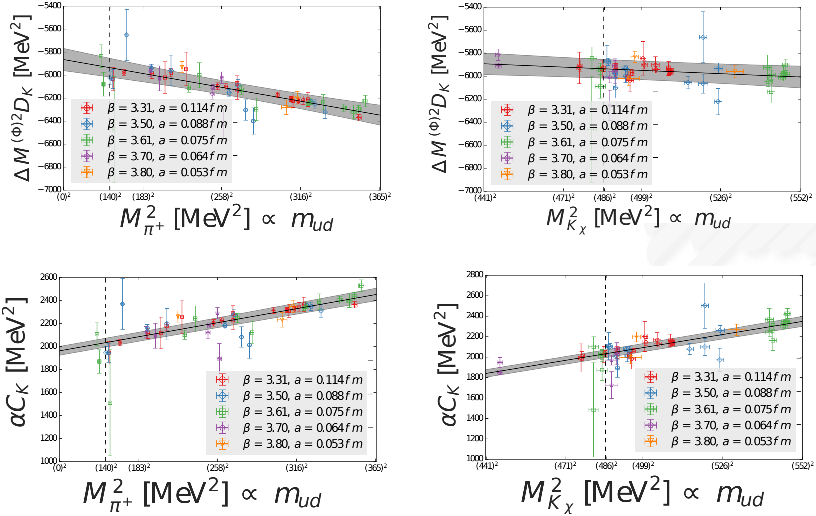
with $f(a)$ being either a^2 or αa.

The fit function

A fully correlated fit is performed:



The fit function



Error estimation

To estimate systematic errors all higher order terms that can not be properly dealt with are treated in two different ways:

- Two different plateau ranges for mass extractions.
- Scale setting either with Ω or Ξ^-.
- Different cuts on M_π^2:
  - 400 MeV or 450 MeV for the scale setting.
  - 350 MeV or 400 MeV for ΔM_K^2
- Replacing Taylor expansions in C_K and D_K by Padé approximations.
- Using αa or a^2 continuum extrapolation for the QCD part.

Resulting in 128 different full analysis. Systematic error is the spread of these analysis. Each analysis is weighted with its Q value.

The statistical error is estimated with a 2000 samples bootstrap procedure.

Results

$$\varepsilon = \frac{\Delta_{\text{QED}} M_K^2 - \Delta_{\text{QED}} M_\pi^2}{\Delta M_\pi^2} = \frac{\Delta_{\text{QED}} M_K^2 - \Delta M_\pi^2}{\Delta M_\pi^2}$$

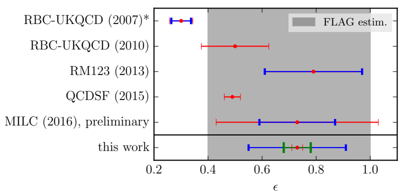
at leading order in δm one can show that $\Delta_{\text{QED}} M_\pi^2 = \Delta M_\pi^2$. $\Delta_{\text{QED}} M_\pi^2$ is expensive to calculate but ΔM_π^2 is known experimentally.

We can use $\Delta_{\text{QED}} M_K^2 = \alpha^{(\phi)} C_K$ to calculate ε:

Violation of Dashen's theorem

$$\varepsilon = 0.73(2)(5)(17)$$

where the errors are statistical, lattice systematics and quenching uncertainty (10% in $\Delta_{\text{QED}} M_K^2$)



red: statistical error
blue: systematic error
green: systematic error without quenching

*: Systematic error: Difference of two results extracted from paper.

Results

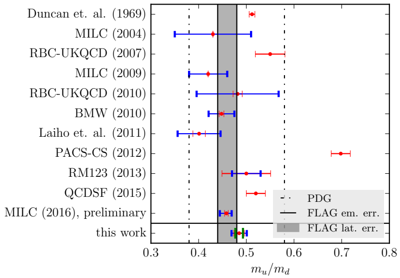
We use $\Delta M^2 = 2B_2\delta m + \mathcal{O}(m_{ud}\alpha, m_{ud}\delta m, \alpha^2, \alpha\delta m, \delta m^2)$ with B_2 from [1] to get

$$\delta m = \frac{\Delta M}{2B_2} = -2.41(6)(4)(9)\,\text{MeV}$$

Using m_{ud} from [2] we arrive at

Light quark mass ratio

$$\frac{m_u}{m_d} = 0.485(11)(8)(14)$$



red: statistical error
blue: systematic error
green: systematic error without quenching

[1] S. Durr, Z. Fodor, C. Hoelbling, S. Krieg, T. Kurth, L. Lellouch, T. Lippert, R. Malak, T. Metivet, A. Portelli, A. Sastre, and K. K. Szabo (BMW Collaboration), Phys. Rev. **D90**, 114504 (2014), arXiv:1310.3626 [hep-lat].

[2] S. Durr, Z. Fodor, C. Hoelbling, S. Katz, S. Krieg, et al. (BMW Collaboration), Phys.Lett. **B701**, 265 (2011), arXiv:1011.2403 [hep-lat].

Results

Using $m_s/m_{ud} = 27.53(20)(8)$ [1] we can calculate the flavor braking ratios R and Q:

Flavor breaking ratios

$$R = \frac{m_s - m_{ud}}{m_d - m_s} = 38.2(1.1)(0.8)(1.4)$$

$$Q = \sqrt{\frac{m_s^2 - m_{ud}^2}{m_d^2 - m_u^2}} = 23.4(0.4)(0.3)(0.4)$$

[1] S. Durr, Z. Fodor, C. Hoelbling, S. Katz, S. Krieg, et al. (BMW Collaboration), Phys.Lett. **B701**, 265 (2011), arXiv:1011.2403 [hep-lat].

Summary

- We calculated the violation to Dashens' theorem

$$\varepsilon = 0.73(2)(5)(17)$$

  and the light quark mass ratio

$$\frac{m_u}{m_d} = 0.485(11)(8)(14).$$

- All systematics from the lattice calculation have been systematically estimated. The quenching uncertainty is taken into account.
- $m_u = 0$ can be excluded with 24σ by these results.

More details: `arXiv:1604.07112[hep-lat]`, to appear in PRL