# Semi-leptonic $B$ and $B_{s}$-decays with charming hadronic final state 

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## THE UNIVERSITY of EDINBURGH

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## RBC- and UKQCD collaborations

| BNL/RBRC | Columbia U | U Edinburgh | U Southampton |
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| Mattia Bruno | Ziyuan Bai | Peter Boyle | Jonathan Flynn |
| Tomomi Ishikawa | Norman Christ | Guido Cossu | Vera Gülpers |
| Taku Izubuchi | Luchang Jin | Luigi Del Debbio | James Harrison |
| Chulwoo Jung | Christopher Kelly | Richard Kenway | Andreas Jüttner |
| Christoph Lehner | Bob Mawhinney | Julia Kettle | Andrew Lawson |
| Meifeng Lin | Greg McGlynn | Ava Khamseh | Edwin Lizarazo |
| Hiroshi Ohki | David Murphy | Antonin Portelli | Chris Sachrajda |
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| CERN | U Connecticut | FZ Jülich | KEK |
| Marina Marinkovic | Tom Blum | Taichi Kawanai | Julien Frison |
| Peking U | U Plymouth | York U (Toronto) |  |
| Xu Feng | Nicolas Garron | Renwick Hudspith |  |

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## Motivation

- Form factors for $B \rightarrow D^{(*)} \ell \nu$
$\rightarrow$ Allow to determine the CKM matrix-element $\left|V_{c b}\right|$
$\rightarrow\left|V_{c b}\right|$ enters as normalization in the unitary triangle fit
$\rightarrow 2-3 \sigma$ discrepancy between $\left|V_{c b}\right|^{\text {incl }}$ and $\left|V_{c b}\right|^{\text {excl }}$
$\rightarrow$ Atlas, CMS, LHCb and Belle II will improve experimental results
- $2-3 \sigma$ tension in $R_{\left.D^{*}\right)}$ ratio - independent of $\left|V_{c b}\right|$
[Fajfer et al. PRD 85 (2012) 094025],[J. Bailey et al. PRL 109 (2012) 071802], [BaBar PRL 109 (2012) 101802]

$$
R_{D^{(*)}}=\mathcal{B}\left(B \rightarrow D^{(*)} \tau \nu_{\tau}\right) / \mathcal{B}\left(B \rightarrow D^{(*)} \ell \nu_{\ell}\right) \text {, with } \ell=e, \mu
$$

$\rightarrow$ Due to its mass $\tau$ is sensitive to both form factors $f_{+}\left(q^{2}\right)$ and $f_{0}\left(q^{2}\right)$, $\ell=e, \mu$ are dominated by $f_{+}\left(q^{2}\right)$
$\rightarrow$ Anomaly in $R_{D^{*}}$ is seen by BaBar, LHCb, and Belle
$\rightarrow$ New physics?

## Motivation: CKM unitarity triangle fit


$\left|V_{c b}\right|$ enters crucially as normalization of the unitarity triangle

$$
\varepsilon_{K} \propto\left|V_{c b}\right|^{4}
$$

## Motivation: $R_{D^{(*)}}$

$$
B \rightarrow D^{(*)} T V
$$

Very preliminary \& unofficial average including new LHCb \& Belle results

- Not using latest lattice results: $\bar{B} \rightarrow D^{*} \ell \nu:$
Fermilab/MILC [PRD 79 (2014) 014506] [PRD 89 (2014) 114504] $B \rightarrow D \ell \nu:$
Fermilab/MILC [PRD 92 (2015) 034506] HPQCD
[PRD 92 (2015) 054510] Atoui et al.
[EPJ. C74 (2014) 2861]

Figure: [Talk by T. Gershon at MIAPP June 2015]

## Our RHQ Project

- Use domain-wall light quarks and nonperturbatively tuned relativistic
$b$-quarks to compute at few-percent precision
- Nonperturbative tuning of RHQ parameters [PRD 86 (2012) 116003]
- Decay constants $f_{B}$ and $f_{B_{s}}$ [PRD 91 (2015) 054502]
$-B \rightarrow \pi \ell \nu$ and $B_{s} \rightarrow K \ell \nu$ form factors [PRD 91 (2015) 074510]
- $g_{B^{*} B \pi}$ coupling constant [PRD 93 (2016) 014510]
- $B^{0}-\overline{B^{0}}$ mixing
- Rare $B$ decays [arXiv:1511.06622] Talk by E. Lizarazo, Friday, July 29, 17:10
- $f_{B}, f_{B_{s}}$, and semi-leptonic form factors
- $O$ (a) improvement at 1 -loop and mostly nonperturbative renormalization
- Correction factors and coefficients computed at 1-loop
- $B$ mixing
- Tree-level $O(a)$ improvement
- Perturbative or mostly nonperturbative renormalization


## $B \rightarrow D \ell \nu$ and $B_{s} \rightarrow D_{s} \ell \nu$ charged current decays


$-\left\langle D\left(p_{D}\right)\right| \mathcal{V}^{\mu}\left|B\left(p_{B}\right)\right\rangle=f_{+}\left(q^{2}\right)\left[\left(p_{B}+p_{D}\right)^{\mu}-\frac{M_{B}^{2}-M_{D}^{2}}{q^{2}} q^{\mu}\right]+f_{0}\left(q^{2}\right) \frac{M_{B}^{2}-M_{D}^{2}}{q^{2}} q^{\mu}$

## $B_{(s)} \rightarrow D_{(s)}^{(*)}$ form factors



- Re-use DWF point-source light and strange quark propagators
- Generate Gaussian smeared MDWF charm quark propagators (on the fly)
- Create Gaussian smeared-source sequential heavy quark propagators
- Compute all possible contractions for pseudoscalar or vector final states
- General building blocks code incl. terms for 1-loop $O\left(\alpha_{s} a\right)$ improvement
- Coefficients to be computed in lattice perturbation theory


## 2+1 Flavor Domain-Wall Iwasaki ensembles

| L | $a^{-1}(\mathrm{GeV})$ | $a m_{l}$ | $a m_{s}$ | $M_{\pi}(\mathrm{MeV})$ | \# configs. | \#sources |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 24 | 1.784 | 0.005 | 0.040 | 338 | 1636 | 1 | [PRD 78 (2008) 114509] |
| 24 | 1.784 | 0.010 | 0.040 | 434 | 1419 | 1 | [PRD 78 (2008) 114509] |
| 32 | 2.383 | 0.004 | 0.030 | 301 | 628 | 2 | [PRD 83 (2011) 074508] |
| 32 | 2.383 | 0.006 | 0.030 | 362 | 889 | 2 | [PRD 83 (2011) 074508] |
| 32 | 2.383 | 0.008 | 0.030 | 411 | 544 | 2 | [PRD 83 (2011) 074508] |
| 48 | 1.730 | 0.00078 | 0.0362 | 139 | 40 | $81 / 1^{\star}$ | [PRD 93 (2016) 074505] |
| 64 | 2.359 | 0.000678 | 0.02661 | 139 | - | - | [PRD 93 (2016) 074505] |
| 48 | $\sim 2.7$ | 0.002144 | 0.02144 | $\sim 250$ | $>50$ | 24 | [in progress] |

* All mode averaging: 81 "sloppy" and 1 "exact" solve [Blum et al. PRD 88 (2012) 094503]
- Lattice spacing determined from combined analysis [Blum et al. PRD 93 (2016) 074505]
- $a: \sim 0.11 \mathrm{fm}, \sim 0.08 \mathrm{fm}, \sim 0.07 \mathrm{fm}$


## Up, down, and strange quarks

- Domain-wall fermions with same parameters as in the sea-sector (domain-wall hight $M_{5}$, extension of $5^{\text {th }}$ dimension $L_{s}$ )
- Unitary and partially quenched quark masses
- Strange quarks at/near physical the physical value


## Charm quarks

- Möbius DWF optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]
- $M_{5}=1.6, L_{s}=12$
- Discretization errors well under control for $a m_{c}<0.45$
$\rightarrow$ On coarse ( $a^{-1}=1.784 \mathrm{GeV}$ ) ensembles we simulate just below $m_{c}^{\text {phys }}$
$\rightarrow$ Simulate 3 or 2 charm-like masses and then extrapolate/interpolate
$\rightarrow$ Linear extrapolation is small and benign; interpolation is safe


## Charm extrapolation Talk by T. Tsang, Friday, July 29, 13:00



Figure by T . Tsang

- Small extrapolation for $a^{-1}=1.784 \mathrm{GeV}$ ensembles
- Interpolation for $a^{-1} \geq 2.383 \mathrm{GeV}$ ensembles


## MDWF charm quarks

Advantages

- Very similar setup for computing $B_{s} \rightarrow D_{s}$ as for $B_{s} \rightarrow K$
$\rightarrow$ Only minor modifications for the perturbative calculations
- No nonperturbative tuning of the RHQ action for charm quarks
- Allows to explore new concept of heavy DWF for semileptonic decays
$\rightarrow$ Fully nonperturbative renormalization of $f_{D}$ in progress Talk by A. Khamseh, Wednesday, July 27, 10:20

Disadvantages

- Larger numerical costs than RHQ charm
- On coarse ensembles small extrapolation needed


## Bottom quarks

- Relativistic Heavy Quark action developed by Christ, Li, and Lin [Christ et al. PRD 76 (2007) 074505], [Lin and Christ PRD 76 (2007) 074506]
- Allows to tune the three parameters ( $m_{0} a, c_{P}, \zeta$ ) nonperturbatively [PRD 86 (2012) 116003]
- Builds upon Fermilab approach [El-Khadra et al. PRD 55 (1997) 3933] by tuning all parameters of the clover action non-perturbatively; close relation to the Tsukuba formulation [S. Aoki et al. PTP 109 (2003) 383]
- Heavy quark mass is treated to all orders in $\left(m_{b} a\right)^{n}$
- Expand in powers of the spatial momentum through $O(\vec{p} a)$
- Resulting errors will be of $O\left(\vec{p}^{2} a^{2}\right)$
- Allows computation of heavy-light quantities with discretization errors of the same size as in light-light quantities
- Applies for all values of the quark mass
- Has a smooth continuum limit
- Recently re-tuned to account for updated values of $a^{-1}$


## First results

- Define (single) ratios for $B_{s} \rightarrow D_{s} \ell \nu$, with $B_{s}$ meson at rest

$$
R_{3, \mu}\left(t, t_{\text {snk }}, \vec{p}_{D_{s}}\right)=\frac{C_{3, \mu}\left(t, t_{\text {ssk }}, \vec{p}_{D_{s}}\right)}{\left.\sqrt{C_{2}^{D_{s}}\left(t, \vec{p}_{D_{s}}\right.}\right)_{C_{2}^{s}}^{B_{s}\left(t_{\text {sskk }}-t\right)}} \frac{\sqrt{2 E_{D_{s}}}}{\exp \left(-E_{D_{s}} t\right) \exp \left(-M_{B_{s}}\left(t_{\text {snkk }}-t\right)\right)}
$$

$-24^{3} \times 64$ ensemble with $a^{-1}=1.784 \mathrm{GeV}$ and $a m_{l}=0.005\left(M_{\pi} \approx 338 \mathrm{MeV}\right)$



## $q^{2}$ dependence



- Data on further ensembles exists
- Have to obtain renormalization factors for meaningful combination


## Alternative determination via double ratios

- Introduced by Hashimoto et al. for $B \rightarrow D \ell \nu$ at zero recoil [PRD 66 (2002) 014503]
- Extended to nonzero recoil by Fermilab/MILC [PRD92 (2015) 034506]
- Get form factors from double ratio at zero and single ratios at nonzero recoil

$$
R_{+}=\frac{\langle D(\overrightarrow{0})| V_{c b}^{4}|B(\overrightarrow{0})\rangle\langle B(\overrightarrow{0})| V_{c b}^{4}|D(\overrightarrow{0})\rangle}{\langle D(\overrightarrow{0})| V_{c c}^{4}|D(\overrightarrow{0})\rangle\langle B(\overrightarrow{0})| V_{b b}^{4}|B(\overrightarrow{0})\rangle}
$$

$Q_{+}(\vec{p}) \equiv \frac{\langle D(\vec{p})| V^{4}|B(\overrightarrow{0})\rangle}{\langle D(\overrightarrow{0})| V^{4}|B(\overrightarrow{0})\rangle} \quad R_{-}(\vec{p}) \equiv \frac{\langle D(\vec{p})| \vec{V}|B(\overrightarrow{0})\rangle}{\langle D(\vec{p})| V^{4}|B(\overrightarrow{0})\rangle} \quad x_{f}(\vec{p}) \equiv \frac{\langle D(\vec{p})| \vec{V}|D(\overrightarrow{0})\rangle}{\langle D(\vec{p})| V^{4}|D(\overrightarrow{0})\rangle}$

- Need renormalization factors ( $\varrho$ ) for obtaining form factors at $q^{2}>0$


## Exploratory comparison of single vs. double ratios

- $32^{3} \times 64$ ensemble with $a^{-1}=2.383 \mathrm{GeV}$ and $a m_{\ell}=0.006\left(M_{\pi} \approx 362 \mathrm{MeV}\right)$
- Subset of data (1 source), only pseudoscalar final states ( $D$ and $D_{s}$ )
- Analyzed data for $B_{s} \rightarrow D_{s} \ell \nu$

- Relative error: 1.9\%
- Will it be worth $5 \times$ larger costs?
- Have to look at $B \rightarrow D$, nonzero momenta, fitting ranges, etc.


## Resources and Acknowledgements

- Simulations on $24^{3}, 32^{3}$, and the $48^{3}$ ensemble with physical pions

USQCD: kaon, J/psi, Ds, Bc, and pi0 cluster at Fermilab
12s at Jlab
RBRC/BNL and Columbia U: small local clusters

- Simulations on the $a^{-1} \sim 2.7 \mathrm{GeV} 48^{3}$ ensemble ARCHER UoE: Cray XC30
DiRAC UoE: BG/Q



## Cost for charm 3-point functions

- Single ratios
- $B \rightarrow D \ell \nu, B \rightarrow D^{*} \ell \nu$ (1 charm inversion)
- $B_{s} \rightarrow D_{s} \ell \nu, B_{s} \rightarrow D_{s}^{*} \ell \nu$ (0 additional charm inversions)
- Double ratios
- $B \rightarrow D \ell \nu$ and $B \rightarrow D^{*} \ell \nu$ (3 charm inversions)

$$
\begin{aligned}
& B \rightarrow D, B \rightarrow B, D \rightarrow B, D \rightarrow D \\
& B \rightarrow D^{*}, B \rightarrow B, D^{*} \rightarrow B, D^{*} \rightarrow D^{*}
\end{aligned}
$$

- $B_{s} \rightarrow D_{s} \ell \nu$ and $B_{s} \rightarrow D_{s}^{*} \ell \nu$ (2 additional charm inversions)

$$
\begin{aligned}
& B_{s} \rightarrow D_{s}, B_{s} \rightarrow B_{s}, D_{s} \rightarrow B_{s}, D_{s} \rightarrow D_{s} \\
& B_{s} \rightarrow D_{s}^{*}, B_{s} \rightarrow B_{s}, D_{s}^{*} \rightarrow B_{s}, D_{s}^{*} \rightarrow D_{s}^{*}
\end{aligned}
$$

- Since we are extrapolating (interpolating) to the physical charm quark pass, we encounter the factor 5 for 3 (2) used charm quark masses
- Total: $N_{\text {configurations }} \times N_{\text {sources }} \times 2 \times N_{\text {charm }} \times(5$ or 1$)$

