Leading electromagnetic corrections to meson masses and the HVP

Vera Gülpers

James Harrison, Andreas Jüttner, Antonin Portelli, Christopher Sachrajda

School of Physics and Astronomy
University of Southampton

July 26, 2016

RBC/UKQCD Collaboration

BNL and RBRC

Mattia Bruno
Tomomi Ishikawa
Taku Izubuchi
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Taichi Kawanai
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni
Sergey Syritsyn

CERN

Marina Marinkovic

Columbia University

Ziyuan Bai
Norman Christ
Luchang Jin
Christopher Kelly
Bob Mawhinney
Greg McGlynn
David Murphy
Jiqun Tu

University of Connecticut

Tom Blum

Edinburgh University

Peter Boyle
Guido Cossu
Luigi Del Debbio
Richard Kenway
Julia Kettle
Ava Khamseh
Brian Pendleton
Antonin Portelli
Oliver Witzel
Azusa Yamaguchi

KEK

Julien Frison

Peking University

Xu Feng

Plymouth University

Nicolas Garron

University of Southampton

Jonathan Flynn
Vera Gülpers
James Harrison
Andreas Jüttner
Andrew Lawson
Edwin Lizarazo
Chris Sachrajda
Francesco Sanfilippo
Matthew Spraggs
Tobias Tsang

York University (Toronto)

Renwick Hudspith

Outline

Introduction

QED correction to meson masses

QED correction to the HVP

Introduction

- Isospin breaking corrections
  — different masses of $\mathbf{u}$ and $\mathbf{d}$ quark
  — QED corrections
- expected to be of order of $\mathbf{1\%}$
- e.g. $\mathbf{a_\mu}$, isospin breaking effects crucial to be competitive with determination from $\mathbf{e^+e^-} \rightarrow$ hadrons

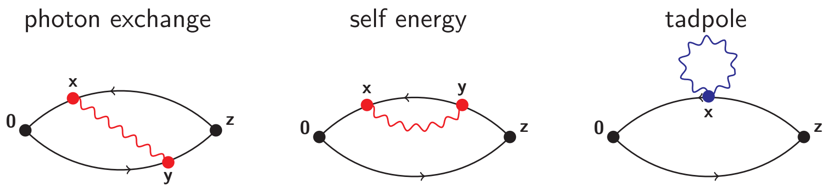
- QED effects
- stochastic QED using $\mathbf{U(1)}$ gauge configurations [J. Harrison, Tue 15:00]
- expansion of the path integral in α [RM123 Collaboration, Phys.Rev. **D87**, 114505 (2013)]

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U]\, \mathcal{D}[A]\, \mathcal{D}[\Psi, \overline{\Psi}]\ \mathcal{O}\ e^{-S_F[\Psi,\overline{\Psi},A,U]}\ e^{-S_A[A]}\ e^{-S_G[U]}$$

$\rightarrow$ compute the leading order QED corrections

Diagrams at $\mathcal{O}(\alpha)$

- two insertions of the conserved vector current or one insertion of the tadpole operator at $\mathcal{O}(\alpha)$
- three different types of (connected) diagrams



- e.g. photon exchange diagram for a charged Kaon

$$\mathbf{C}(\mathbf{z}_0) = \sum_{\vec{z}} \sum_{\mathbf{x},\mathbf{y}} \mathrm{Tr}\left[\mathbf{S}^{s}(\mathbf{z},\mathbf{x})\,\Gamma^{c}_{\nu}\,\mathbf{S}^{s}(\mathbf{x},0)\,\gamma_5\,\mathbf{S}^{u}(0,\mathbf{y})\,\Gamma^{c}_{\mu}\,\mathbf{S}^{u}(\mathbf{y},\mathbf{z})\,\gamma_5\right]\boldsymbol{\Delta}_{\mu\nu}(\mathbf{x}-\mathbf{y})$$

photon propagator

- photon propagator (Feynman gauge)

$$\Delta_{\mu\nu}(x-y) = \delta_{\mu\nu}\frac{1}{V}\sum_{k,\vec{k}\neq 0}\frac{e^{ik\cdot(x-y)}}{4\sum_\rho \sin^2\frac{k_\rho}{2}}$$

- subtract all spatial zero modes $\longrightarrow$ $\mathrm{QED_L}$ [Borsanyi et al., Science **347** (2015) 1452-1455]
- rewrite photon propagator

$$\Delta_{\mu\nu}(x-y) \approx \sum_u \Delta_{\mu\nu}(x-u)\eta(u)\eta^\dagger(y) = \tilde{\Delta}_{\mu\nu}(x)\eta^\dagger(y)$$

with a stochastic source (e.g. $\mathbf{Z_2}$)

$$\frac{1}{N}\sum_{i=1}^{N}\eta_i(u)\eta_i^\dagger(y) \approx \delta_{u,y}$$

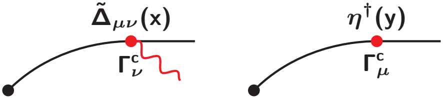
- calculate $\tilde{\Delta}_{\mu\nu}(x) = \sum_u \Delta_{\mu\nu}(x-u)\eta(u)$ using Fast Fourier Transform

construction of the correlators

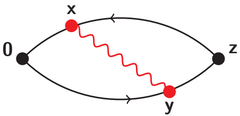
- photon exchange for a charged Kaon

$$\mathbf{C(z_0)} = \sum_{\vec{z}} \sum_{x,y} \mathrm{Tr}\left[\mathbf{S^s(z,x)\,\Gamma^c_\nu\,S^s(x,0)}\,\gamma_5\,\mathbf{S^u(0,y)\,\Gamma^c_\mu\,S^u(y,z)}\,\gamma_5 \right] \tilde{\mathbf{\Delta}}_{\mu\nu}\mathbf{(x)}\eta^\dagger\mathbf{(y)}$$

- sequential propagators



- contraction



- similar for the self energy using a double sequential propagator

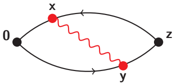
setup of the run

- $\mathbf{N_f = 2 + 1}$ Domain Wall Fermions
- $\mathbf{64 \times 24^3}$ lattice with $\mathbf{a^{-1} = 1.78}$ GeV
- $\mathbf{L_s = 16}$, $\mathbf{M_5 = 1.8}$
- **87** gauge configurations
- pion mass $\mathbf{m_\pi = 340}$ Mev
- different masses for valence **u** and **d** quarks
  $\approx$ physical mass difference [BMW Collaboration, 1604.07112]
- physical valence strange quark mass [T. Blum *et al*, Phys. Rev. D93, 074505 (2016)]

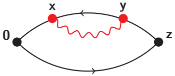
- one $\mathbf{Z_2}$ noise for the stochastic insertion of the photon propagator per gauge configuration and source position
- **3** source positions
- computational cost:
  17 inversions per valence quark and source position

results - correlators

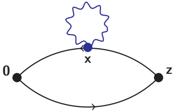
- two quarks with $\mathbf{m_u}$
- photon exchange

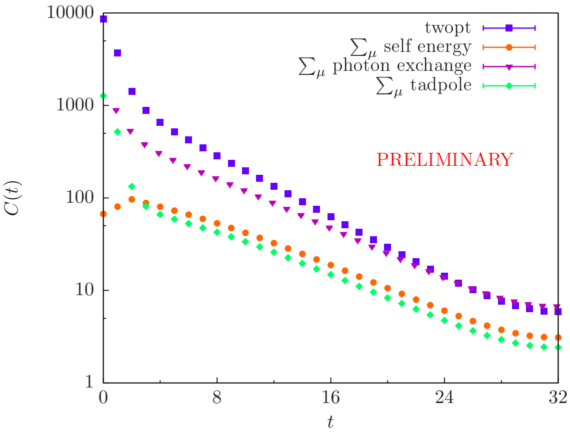


- self energy



- tadpole





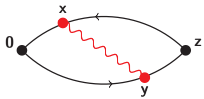
corrections to meson masses

- extract mass correction from an $\mathcal{O}(\alpha)$ diagram by
  [RM123 Collaboration, Phys.Rev. **D87**, 114505 (2013)]

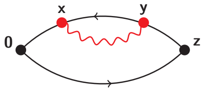
$$\mathbf{C(t) = C_{2pt}(t) + C_{\mathcal{O}(\alpha)}(t) = A\, e^{-(m+\delta m)\cdot t}} \quad \Rightarrow \quad \boldsymbol{\delta m = -\partial_t \frac{C_{\mathcal{O}(\alpha)}(t)}{C_{2pt}(t)}}$$

- photon exchange — self energy — tadpole

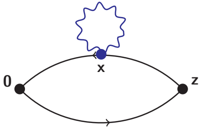
photon exchange



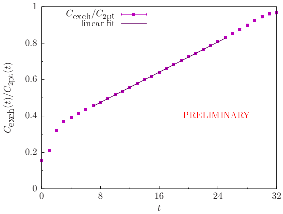
self energy

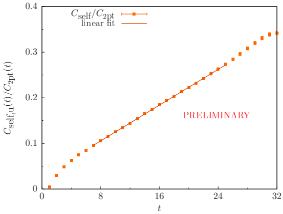


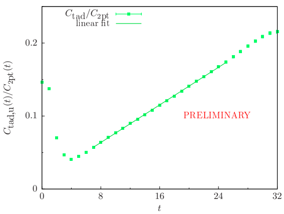
tadpole



- example: charged Kaon







results QED corrections to meson masses

- some (very preliminary) results for QED corrections to meson masses (w/o finite volume correction)

Quantity	this work	stochastic QED [Tue, 15:00]
$\mathbf{M}^{\gamma}_{\pi^+}$	$\mathbf{2.70} \pm \mathbf{0.02}$ MeV	$\mathbf{3.42} \pm \mathbf{0.02}$ MeV
$\mathbf{M}^{\gamma}_{\pi^0}$	$\mathbf{0.70} \pm \mathbf{0.02}$ MeV	$\mathbf{1.52} \pm \mathbf{0.01}$ MeV
$\mathbf{M}_{\pi^+} - \mathbf{M}_{\pi^0}$	$\mathbf{2.00} \pm \mathbf{0.03}$ MeV	$\mathbf{1.90} \pm \mathbf{0.02}$ MeV
$\mathbf{M}^{\gamma}_{K^+}$	$\mathbf{2.12} \pm \mathbf{0.02}$ MeV	$\mathbf{2.70} \pm \mathbf{0.02}$ MeV
$\mathbf{M}^{\gamma}_{K^0}$	$\mathbf{0.28} \pm \mathbf{0.01}$ MeV	$\mathbf{0.55} \pm \mathbf{0.01}$ MeV

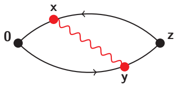
results QED corrections to meson masses

- some (very preliminary) results for QED corrections to meson masses (w/o finite volume correction)

Quantity	this work	stochastic QED [Tue, 15:00]
$\mathbf{M}^{\gamma}_{\pi^+}$	$\mathbf{2.70 \pm 0.02}$ MeV	$\mathbf{3.42 \pm 0.02}$ MeV
$\mathbf{M}^{\gamma}_{\pi^0}$	$\mathbf{0.70 \pm 0.02}$ MeV	$\mathbf{1.52 \pm 0.01}$ MeV
$\mathbf{M}_{\pi^+} - \mathbf{M}_{\pi^0}$	$\mathbf{2.00 \pm 0.03}$ MeV	$\mathbf{1.90 \pm 0.02}$ MeV
$\mathbf{M}^{\gamma}_{K^+}$	$\mathbf{2.12 \pm 0.02}$ MeV	$\mathbf{2.70 \pm 0.02}$ MeV
$\mathbf{M}^{\gamma}_{K^0}$	$\mathbf{0.28 \pm 0.01}$ MeV	$\mathbf{0.55 \pm 0.01}$ MeV

- pion mass splitting is a special case [RM123 Collaboration, Phys.Rev. **D87**, 114505 (2013)]
  $\longrightarrow$ depends only on photon exchange diagram

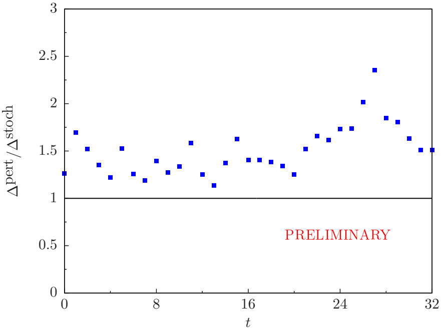
$$\mathbf{M}_{\pi^+} - \mathbf{M}_{\pi^0} = \frac{(q_u - q_d)^2}{2} e^2 \, \partial_t \, \frac{C_{\mathrm{exch}}(t)}{C_{\mathrm{2pt}}(t)}$$



- problem in the self energy and/or the tadpole diagram?
  $\longrightarrow$ needs to be resolved

Comparison of statistical precision

- computational cost
  - perturbative method **17** inversions per quark flavor
  - stochastic method **3** inversions per quark flavor
- statistical error **Δ** of QED contribution to effective Kaon mass
- scaled by $\sqrt{\#\,\text{inversions}}$



The hadronic vacuum polarisation

- vacuum polarisation tensor

$$\Pi_{\mu\nu}(Q) = \sum_x e^{i\,Q\cdot x} \left\langle j^\gamma_\mu(x)\; j^\gamma_\nu(0) \right\rangle = (Q_\mu Q_\nu - \delta_{\mu\nu} Q^2)\,\Pi(Q^2)$$

- correlator

$$C_{\mu\nu}(x) = Z_v\, q_f^2 \left\langle V^c_\mu(x) V^\ell_\nu(0) \right\rangle$$

- construction of the HVP tensor, see eg. [RBC/UKQCD, JHEP 1604 (2016) 063], [M. Spraggs, Tue 17:10]

$$\Pi_{\mu\nu}(Q) = \sum_x e^{-iQ\cdot x} C_{\mu\nu}(x) - \sum_x C_{\mu\nu}(x)$$

(with zero mode subtraction)

- vacuum polarisation

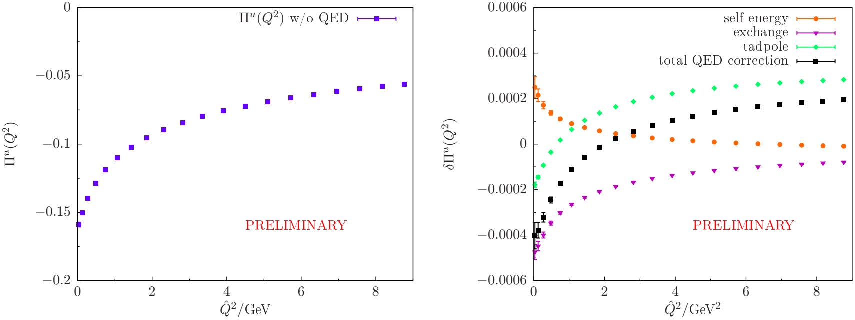
$$\Pi(\hat{Q}^2) = \frac{1}{3} \sum_j \frac{\Pi_{jj}(Q)}{\hat{Q}^2}$$

$$\Pi(\hat{Q}^2) \equiv \frac{4}{9}\Pi^u(\hat{Q}^2) + \frac{1}{9}\Pi^d(\hat{Q}^2) + \frac{1}{9}\Pi^s(\hat{Q}^2)$$

First look at HVP

- hadronic vacuum polarisation for the **u** quark
- left: HVP without QED $\mathbf{\Pi_0^u(\hat{Q}^2)}$, right: QED corrections to HVP $\boldsymbol{\delta\Pi^u(\hat{Q}^2)}$

$$\mathbf{\Pi^u(\hat{Q}^2) = \Pi_0^u(\hat{Q}^2) + \delta\Pi^u(\hat{Q}^2)}$$



Summary

- Leading order QED corrections by expansion of the path integral
- corrections to meson masses and HVP
- exploratory study
- currently, discrepancy between results from stochastic and perturbative approach
  $\longrightarrow$ needs to be resolved

Outlook

- Coulomb gauge for the photon propagator
- more gauge ensembles
- matrix elements [N. Carrasco *et al*, Phys. Rev. D91 (2015) 074506], [N. Tantalo, Wed 11:50], [S. Simula, Wed 12:10]

Backup

expansion of the Wilson-Dirac operator in **e**

- Including QED link variables the action is

$$\mathbf{S}_W^e = \sum_x \Big[\overline{\Psi}(x)\,(M+4)\,\Psi(x) - \frac{1}{2}\,\overline{\Psi}(x)(1-\gamma_\mu)E_\mu(x)U_\mu(x)\Psi(x+\mu) - \frac{1}{2}\,\overline{\Psi}(x+\mu)(1-\gamma_\mu)U_\mu^\dagger(x)E_\mu^\dagger(x)\Psi(x) \Big]$$

  with QED link variables

$$E_\mu(x) = e^{-iee_f A_\mu(x)} = 1 - iee_f A_\mu(x) + \frac{1}{2}(ee_f)^2 A_\mu(x)A_\mu(x) + \ldots$$

- Expanding the action in **e** one finds

$$S_W^e - S_W^0 = \sum_{x,\mu} \left\{ -iee_f\, A_\mu(x)\, V_\mu^c(x) + \frac{(ee_f)^2}{2} A_\mu(x)A_\mu(x)\; T_\mu(x) \right\}$$

  with the conserved vector current $\mathbf{V}_\mu^c(x)$ and the tadpole operator $\mathbf{T}_\mu(x)$

$$V_\mu^c(x) = \frac{1}{2}\Big[\overline{\Psi}(x+\mu)(1+\gamma_\mu)U_\mu^\dagger(x)\Psi(x) - \overline{\Psi}(x)(1-\gamma_\mu)U_\mu(x)\Psi(x+\mu)\Big]$$

$$T_\mu(x) = \frac{1}{2}\Big[\overline{\Psi}(x)(1-\gamma_\mu)U_\mu(x)\Psi(x+\mu) + \overline{\Psi}(x+\mu)(1+\gamma_\mu)U_\mu^\dagger(x)\Psi(x)\Big]$$

photon propagator with FFT

- $\tilde{\mathbf{\Delta}}_{\mu\nu}(\mathbf{x}) = \sum_{\mathbf{u}} \mathbf{\Delta}_{\mu\nu}(\mathbf{x}-\mathbf{u})\eta(\mathbf{u})$ with $\mathbf{\Delta}_{\mu\nu}(\mathbf{x}-\mathbf{y}) = \delta_{\mu\nu}\frac{1}{\mathbf{V}}\sum_{\mathbf{k},\vec{\mathbf{k}}\neq 0} \frac{\mathbf{e}^{\mathbf{ik}\cdot(\mathbf{x}-\mathbf{y})}}{\hat{\mathbf{k}}^2}$
- Fourier Transform of the stochastic source

$$\eta(\mathbf{u}) \quad \xrightarrow{\text{FFT}} \quad \hat{\eta}(\mathbf{k})$$

- divide by $\hat{\mathbf{k}}^2$ and subtract the zero mode

$$\frac{\hat{\eta}(\mathbf{k})}{\hat{\mathbf{k}}^2} \quad \longrightarrow \quad \left.\frac{\hat{\eta}(\mathbf{k})}{\hat{\mathbf{k}}^2}\right|_{\vec{\mathbf{k}}=0} = 0$$

- Fourier Transform

$$\xrightarrow{\text{FFT}} \quad \tilde{\mathbf{\Delta}}(\mathbf{x}) = \sum_{\mathbf{k},\vec{\mathbf{k}}\neq 0} \frac{\hat{\eta}(\mathbf{k})}{\hat{\mathbf{k}}^2}\mathbf{e}^{\mathbf{ik}\cdot\mathbf{x}} = \sum_{\mathbf{u}} \mathbf{\Delta}(\mathbf{x}-\mathbf{u})\eta(\mathbf{u})$$