Mon 25th July, Lattice 2016

Hindered M1 Radiative Decay of $\Upsilon(2S)$ and $\eta_b(2S)$ from Lattice NRQCD

arXiv:1508.01694

C. Hughes, R. Dowdall, C. Davies,

R. Horgan, G. Von Hippel, M. Wingate

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- M1 $\bar{b}b$ Radiative Decay \Longrightarrow Requires Spin Flip

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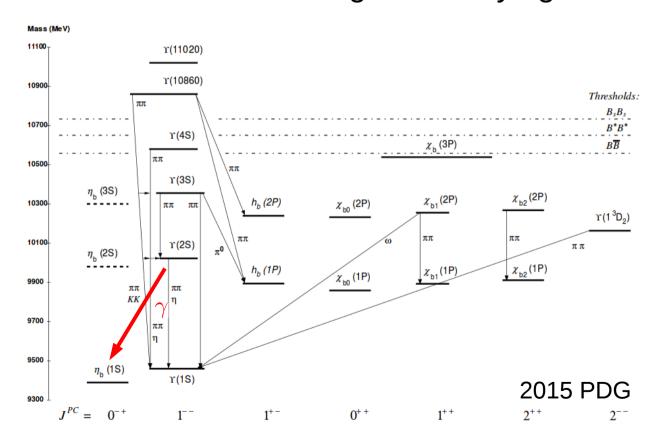
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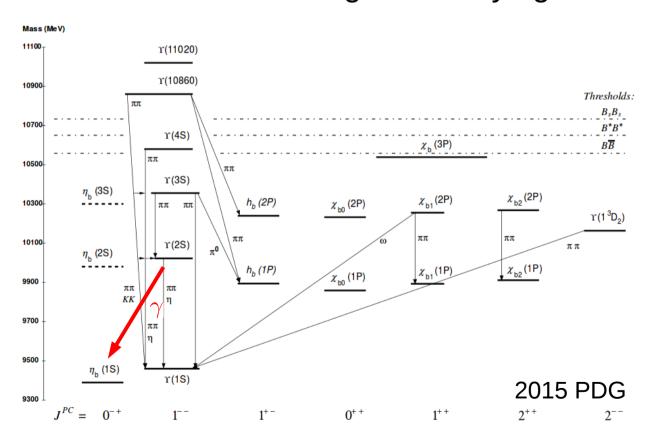
• Hindered M1: $n'^3S_1 \rightarrow n^1S_0\gamma$, $n' \neq n$

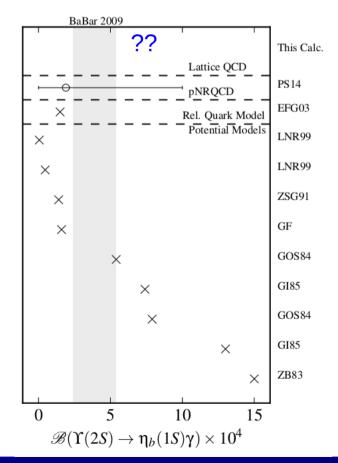
• Full Understanding of Low-Lying States/Decays

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• Production of Spin Singlets: η_b

Full Understanding of Low-Lying States/Decays

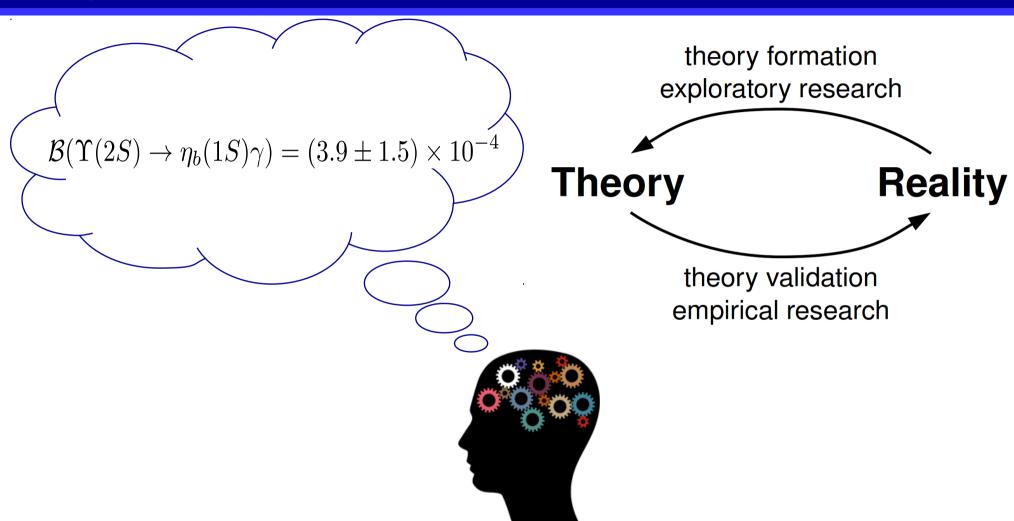
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Exclusion of parity odd Higgs

- Full Understanding of Low-Lying States/Decays
- Production of Spin Singlets: η_b

- Exclusion of parity odd Higgs
- Laboratory to test relativistic effects: NRQCD =? Experiment

Decay Rate: $\Upsilon(2S) \to \eta_b(1S)\gamma$



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$$\langle \eta_{b(mS)}(p_i)|J^{\mu}(0)|\Upsilon_{(nS)}(p_f,s_{\Upsilon})\rangle = \frac{2\mathbf{V}_{\mathbf{21}}^{\Upsilon\eta_{\mathbf{b}}}(\mathbf{q}_{\mathbf{2}}^{2})}{M_{\Upsilon}+M_{\eta_{b}}}\varepsilon^{\mu\alpha\beta\tau}p_{i,\alpha}p_{f,\beta}\epsilon_{\tau}(p_f,s_{\Upsilon})$$

$$\Gamma_{\Upsilon\to\eta_{b}\gamma} = \alpha_{QED}e_{q}^{2}\frac{16}{3}\frac{|\mathbf{q}|^{3}}{(M_{\Upsilon}+M_{\eta_{b}})^{2}}\left|\mathbf{V}_{\mathbf{21}}^{\Upsilon\eta_{\mathbf{b}}}(\mathbf{q}_{\mathbf{2}}=\mathbf{0})|_{\text{lat.}}\right|^{2}$$

NRQCD Evolution: $v^2 \sim 0.1$

$$aH = aH_0 + a\delta H_{v^4} + a\delta H_{v^6}$$

$$aH_0 = -\frac{\Delta^{(2)}}{2am_b},$$

$$a\delta H_{v^4} = -c_1 \frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2 \frac{i}{8(am_b)^2} \left(\nabla \cdot \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \cdot \nabla\right)$$

$$-c_3 \frac{1}{8(am_b)^2} \sigma \cdot \left(\tilde{\nabla} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \tilde{\nabla}\right)$$

$$-c_4 \frac{1}{2am_b} \sigma \cdot \tilde{\mathbf{B}} + c_5 \frac{\Delta^{(4)}}{24am_b} - c_6 \frac{(\Delta^{(2)})^2}{16n(am_b)^2},$$

$$a\delta H_{v^6} = -c_7 \frac{1}{8(am_b)^3} \left\{\Delta^{(2)}, \sigma \cdot \tilde{\mathbf{B}}\right\}$$

$$-c_8 \frac{3i}{64(am_b)^4} \left\{\Delta^{(2)}, \sigma \cdot \left(\tilde{\nabla} \times \tilde{\mathbf{E}} - \tilde{\mathbf{E}} \times \tilde{\nabla}\right)\right\}$$

$$+c_9 \frac{1}{8(am_b)^3} \sigma \cdot \tilde{\mathbf{E}} \times \tilde{\mathbf{E}}$$

$$|\mathbf{q}_{\gamma}| \sim 0.6 \text{GeV} \sim mv^2, \ v^2 \sim 0.1$$

$$\mathcal{O}_F: \quad \omega_F \frac{ee_b}{2m_b} \psi_b^{\dagger} \sigma \cdot \mathbf{B}^{\mathbf{QED}} \psi_b \sim \mathbf{v}^4$$

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$$O_{W1}: \omega_{W1} \frac{e e_{b}}{8 m_{b}^{3}} \psi_{b}^{\dagger} \{\mathbf{D}^{2}, \sigma \cdot \mathbf{B}^{\mathbf{QED}}\} \psi_{b} \sim v^{6}$$

$$\mathcal{O}_{S}: \quad \omega_{S} \frac{i e e_{b}}{8 m_{b}^{2}} \psi_{b}^{\dagger} \sigma \cdot [\mathbf{D} \times, \mathbf{E}^{\mathbf{QED}}] \psi_{b} \sim v^{5}$$

$$\mathcal{O}_{S2}: \quad \omega_{S2} \frac{3i e e_{b}}{64 m_{s}^{4}} \psi_{b}^{\dagger} \sigma \cdot \{\mathbf{D}^{2}, [\mathbf{D} \times, \mathbf{E}^{\mathbf{QED}}]\} \psi_{b} \sim v^{7}$$

Need matching coefficient of L.O. Current Operator:

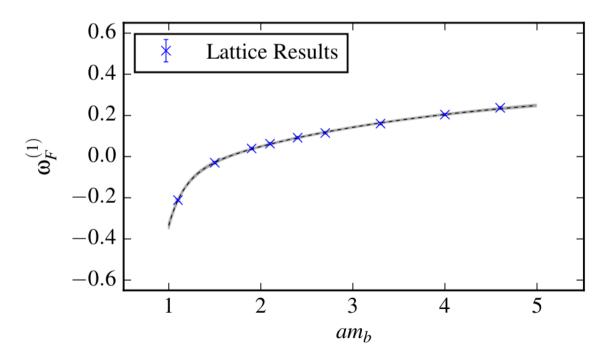
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with
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Lattice Methodology





1. Get one of these:



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2. Compute the 2pt functions: $C_{2pt}(n_{src}, n_{skn}; T) = \langle \mathcal{O}(n_{snk}, T + t_0) \mathcal{O}^{\dagger}(n_{src}, t_0) \rangle$

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- 2. Compute the 2pt functions: $C_{2pt}(n_{src}, n_{skn}; T) = \langle \mathcal{O}(n_{snk}, T + t_0) \mathcal{O}^{\dagger}(n_{src}, t_0) \rangle$
- 3. Compute the 3pt functions:

$$C_{3pt}(n_{src}, n_{skn}; T, t) = \langle \mathcal{O}(n_{snk}, T + t_0) \mathcal{O}_C(t + t_0) \mathcal{O}^{\dagger}(n_{src}, t_0) \rangle$$

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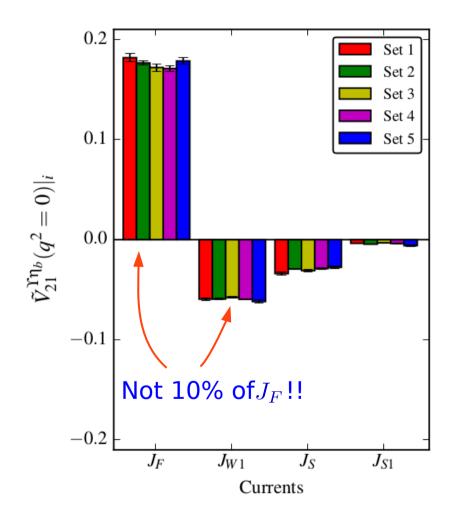
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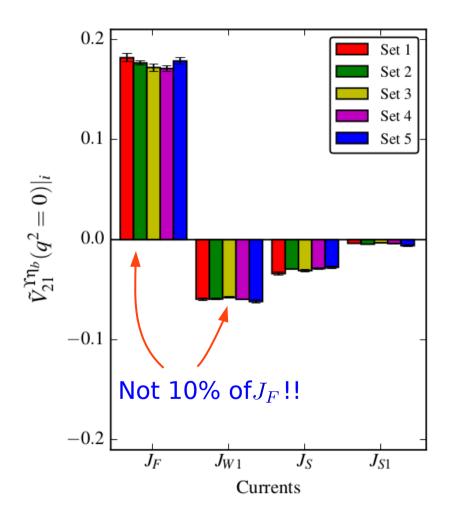
$$C_{3pt}(n_{src}, n_{skn}; T, t) = \langle \mathcal{O}(n_{snk}, T + t_0) \mathcal{O}_C(t + t_0) \mathcal{O}^{\dagger}(n_{src}, t_0) \rangle$$

4. Fit the data in your favourite way!

NB: Details swept under the rug, ask if interested!

For this talk, only necessary to know that it is possible to accurately extract energies and matrix elements from lattice QCD

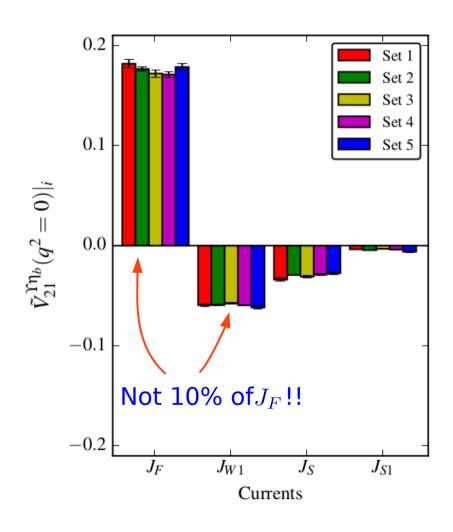




$$\langle \eta_b(mS)|J_F|\Upsilon(nS)\rangle = S_{fi} \int_0^\infty dr \ r^2 R_{m,\eta_b}^*(r) j_0\left(\frac{|q|r}{2}\right) R_{n,\Upsilon}(r)$$

$$\int_{0}^{\infty} dr \ r^{2} R_{m,\eta_{b}}^{*}(r) j_{0}\left(\frac{|q|r}{2}\right) R_{n,\Upsilon}(r) =$$

$$\delta_{nm} + a_{2}|q_{\gamma}|^{2} r_{0}^{2} + a_{4}|q_{\gamma}|^{4} r_{0}^{4} + \cdots$$



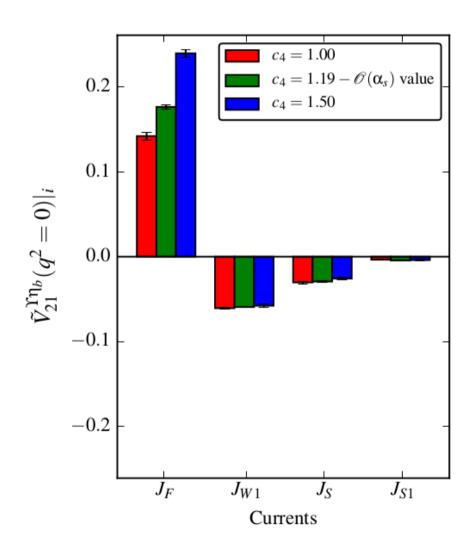
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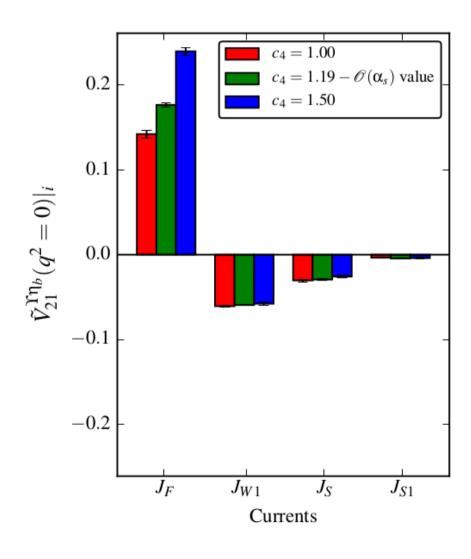
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$$\delta_{nm} + a_{2} |q_{\gamma}|^{2} r_{0}^{2} + a_{4} |q_{\gamma}|^{4} r_{0}^{4} + \cdots$$

N.B., In hindered decays ($n \neq m$) the leading order matrix element is suppressed, making sub-leading currents appreciable

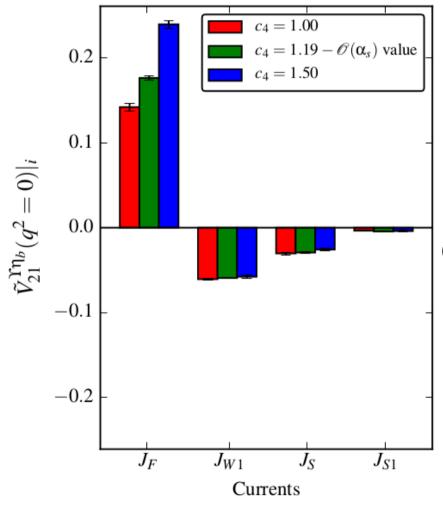
N.B., Destructive Interference occurs in the $\Upsilon(2S) \to \eta_b(1S) \gamma$ decay





$$|\eta_b(1S)\rangle^{(1)} = |\eta_b(1S)\rangle^{(0)} - \sum_{m \neq 1} |\eta_b(mS)\rangle^{(0)} \frac{V_{m1}^{\eta_b}}{E_{m1}^{\eta_b}} |\Upsilon(2S)\rangle^{(1)} = |\Upsilon(2S)\rangle^{(0)} - \sum_{n \neq 2} |\Upsilon(nS)\rangle^{(0)} \frac{V_{n2}^{\Upsilon}}{E_{n2}^{\Upsilon}}.$$

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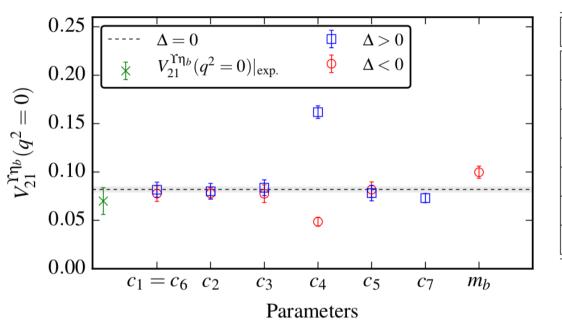
Overlap Suppressed

$$^{(1)}\langle\eta_{b}(1S)|J_{i}|\Upsilon(2S)\rangle^{(1)}\approx$$

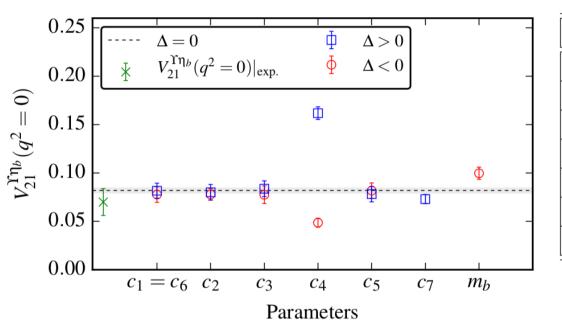
$$^{(0)}\langle\eta_{b}(1S)|J_{i}|\Upsilon(2S)\rangle^{(0)}$$

$$-\frac{V_{21}^{\eta_{b}*}}{E_{21}^{\eta_{b}}}^{(0)}\langle\eta_{b}(2S)|J_{i}|\Upsilon(2S)\rangle^{(0)}\longrightarrow \mathbf{X}$$

$$-\frac{V_{12}^{\Upsilon}}{E_{12}^{\Upsilon}}^{(0)}\langle\eta_{b}(1S)|J_{i}|\Upsilon(1S)\rangle^{(0)}\longrightarrow \mathbf{X}$$
N.B



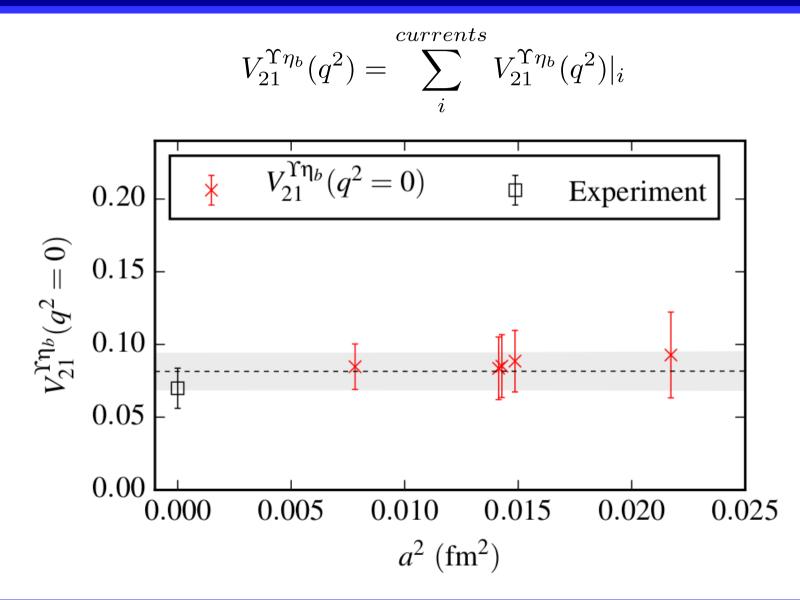
Parameter	p^{test} for $\Delta < 0$	$p^{\mathcal{O}(\alpha_s)}$ for $\Delta = 0$	p^{test} for $\Delta > 0$
$c_1 = c_6$	1.00	1.31	1.50
c_2	0.75	1.02	1.25
c_3	0.75	1.00	1.25
c_4	1.00	1.19	1.50
c_5	1.00	1.16	1.50
c_7		1.00	1.50
m_b	2.5935	2.73	

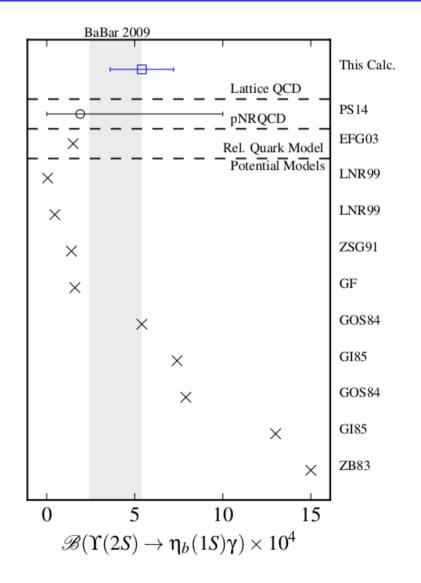


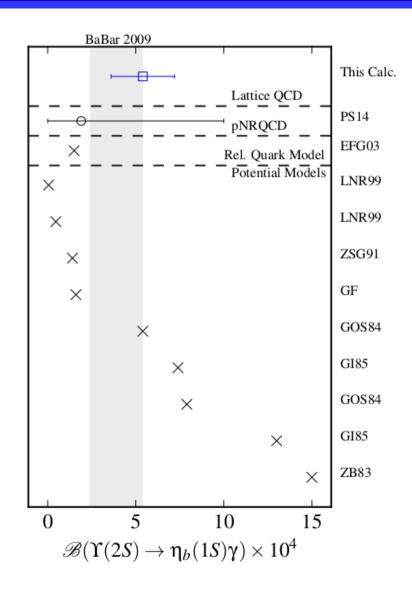
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N.B., In hindered decays the leading order matrix element is suppressed, making particular relativistic corrections due to perturbative potentials (arising from terms in the Hamiltonian) appreciable

Extrapolation

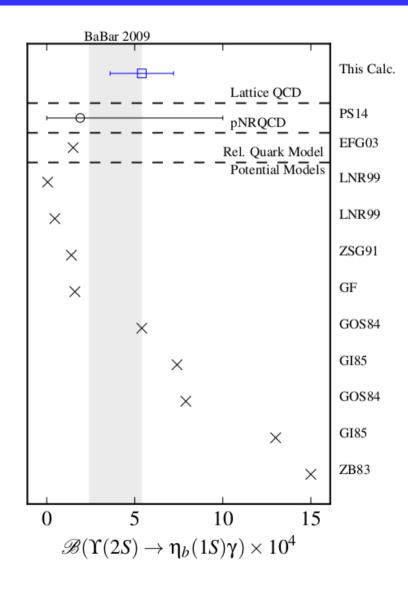






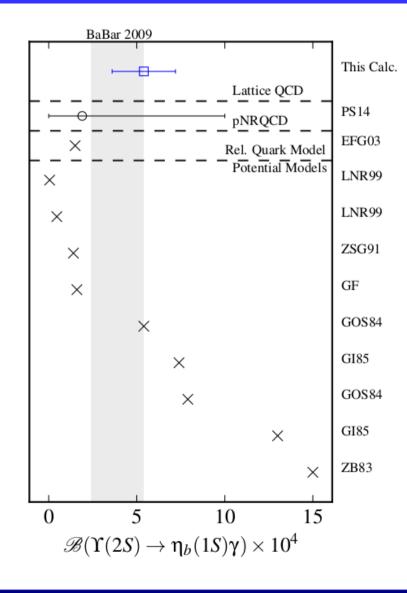
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 Relativistic corrections in current (Need multiple current corrections)



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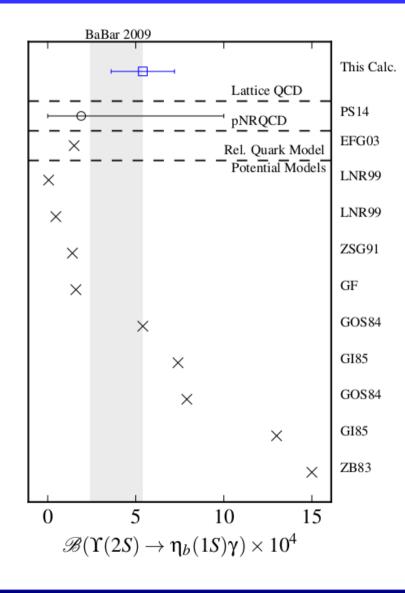
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- Radiative corrections in action (Need precise matching coefficients)

What did we learn?



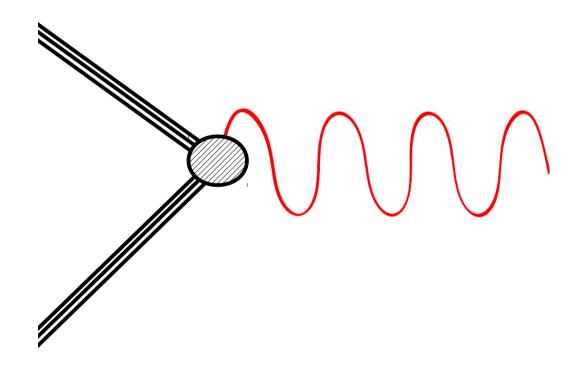
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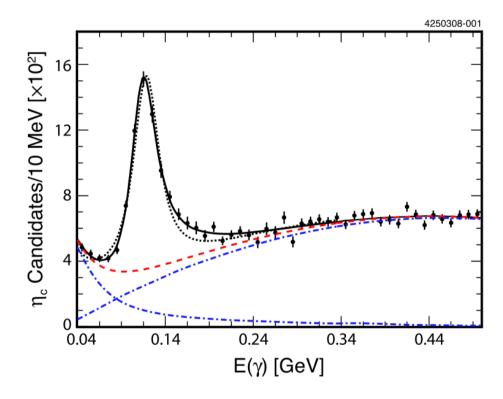
We (HPQCD) Have Done THIS!!!

Radiative Decays

What do experimentalists see?!

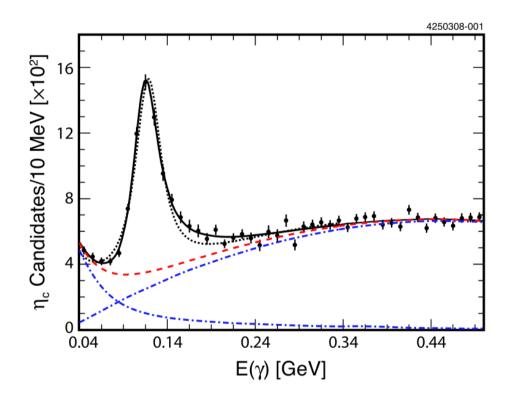


Line Shape from $J/\psi \to \eta_c \gamma$



CLEO:ArXiv:0805.0252

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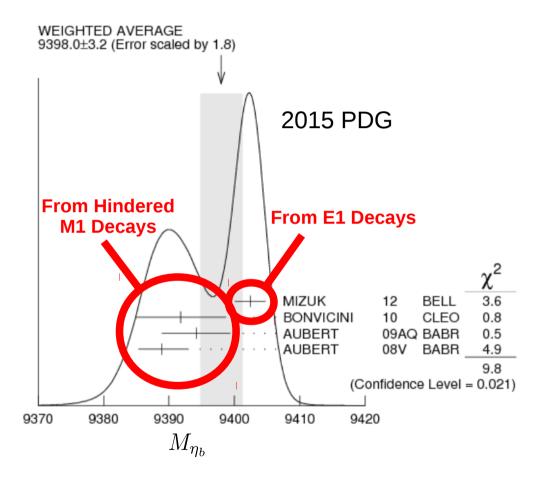
KEDR:ArXiv:1002.2071

$$\frac{dN_{\gamma}}{d\omega} = N_{\psi} \mathcal{B} \int_{0}^{M_{\psi}/2} d\omega' \frac{d\Gamma(\omega')}{d\omega'} \frac{\epsilon(\omega')g(\omega,\omega')}{\Gamma_{\eta_{c}\gamma}}$$

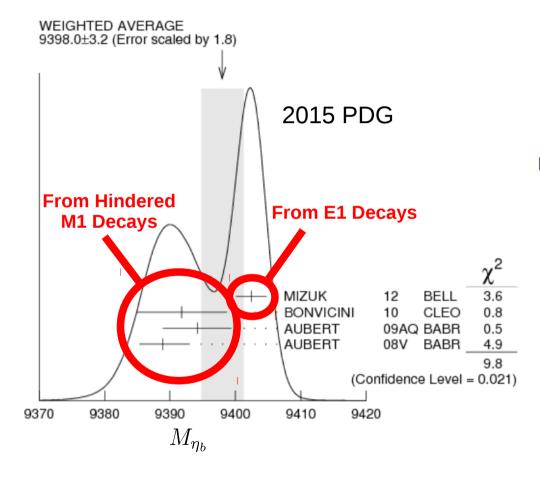
$$\frac{d\Gamma(\omega)}{d\omega} = \frac{4}{3} \alpha \frac{e_{\rm c}^2}{m_{\rm c}^2} \omega^3 |M|^2 BW(\omega)$$

N.B., Need Energy Dependence of matrix element ("damping" function) to fit line shape correctly.

Mass of the η_b



Mass of the $\overline{\eta_b}$



Reason for tension here:

Correct damping function (matrix element including suppression effects) needs to be used when fitting line shape from hindered M1 decays???

The Hindered M1 $\eta_b(2S) \to \Upsilon(1S)\gamma$ Decay



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- This produces sensitivity to relativistic and radiative corrections
- Yet, it is possible to accurately and reliably calculate from first principles (using LQCD)
- A damping function (including suppression effects) might be needed when fitting the experimental line-shape from hindered M1 decays

Future/Questions

- Get $h_b(1P)$ width from $h_b(1P) \to \eta_b(1S)\gamma$
- $B_s^* \to B_s \gamma$ needed for new-physics search in $B_s^* \to \ell\ell$ (arXiv:1509.05049)

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Questions!?! (ch558@cam.ac.uk)

Back Up Slides



Two Point Calculation

- 1. Build interpolating operators $\mathcal{O}(n,t_0)$, which overlap with states having specific quantum numbers J^{PC} , e.g., Υ, η_b
- 2. Calculate $C_{2pt}(n_{src}, n_{skn}; T) = \langle \mathcal{O}(n_{snk}, T + t_0) \mathcal{O}^{\dagger}(n_{src}, t_0) \rangle$ numerically on the lattice

Three Point Calculation

- 1. Build current operators which we are interested in: $\mathcal{O}_C(t+t_0)$
- 2. Calculate $C_{3pt}(n_{src}, n_{skn}; T, t) = \langle \mathcal{O}(n_{snk}, T + t_0) \mathcal{O}_C(t + t_0) \mathcal{O}^{\dagger}(n_{src}, t_0) \rangle$ numerically with the same twist as in the two point calculation

Bayesian Fitting

• Simultaneously fit two point correlator for Υ , η_b data to

$$C_{2pt}(n_{src}, n_{snk}) = \sum_{i}^{m} a_i(n_{src}) a_i(n_{snk}) \exp(-E_i t)$$

and three point correlator data in order to

$$C_{3pt}(n_{src}, n_{snk}) = \sum_{i,f}^{m} a_i(n_{src}) V_{i,f} b_f(n_{snk}) \exp(-E_i t) \exp(-E_f (T - t))$$

and extract what we need: $V_{i,f}$

Coulomb Gauge Fixed Ensembles

MILC Configurations ($n_f = 2 + 1 + 1 \text{ HISQ}$)

Set	β	$a_{\Upsilon}(\mathrm{fm})$	am_l	am_s	am_c	$N_s \times N_T$	$n_{\rm cfg}$
1	5.8	0.1474(15)	0.013	0.065	0.838	16×48	1020
2	6.0	0.1219(9)	0.0102	0.0509	0.635	24×64	1052
3	6.0	0.1195(10)	0.00507	0.0507	0.628	32×64	1000
4	6.0	0.1189(9)	0.00184	0.0507	0.628	48×64	1000
5	6.3	0.0884(6)	0.0074	0.037	0.440	32×96	1008
			ı				

Fitting

$$V(a^2, am_b) = V_{\text{phys}}$$

$$\times \left[1 + \sum_{j=1,2} k_j (a\Lambda)^{2j} (1 + k_{jb} \delta x_m + k_{jbb} (\delta x_m)^2) \right]. \quad (4)$$

The lattice spacing dependence is set by a scale $\Lambda = 500$ MeV, and $\delta x_m = (am_b - 2.7)/1.5$ allows for mild dependence on the effective theory cutoff am_b . We take priors of 0(1) on all the coefficients except k_1 which is 0.0(3) since the action includes radiatively improved a^2 lattice spacing corrections. We have tested that our results are not sensitive to the fit form or the priors.

Potential Model for L.O. Matrix Element

$$\Gamma_{\Upsilon \to \eta_b \gamma} = \alpha_{QED} e_q^2 \frac{4}{3m_b^2} |\mathbf{q}_{\gamma}|^3 \left| \int r^2 dr R_{\eta_b}^* (1S) j_0(\frac{|\mathbf{q}_{\gamma}|r}{2}) R_{\Upsilon}(2S) \right|^2$$

$$V(q^2)_{nm} \propto \int r^2 dr R_{\eta_b}^*(mS) j_0(\frac{|\mathbf{q}_{\gamma}|r}{2}) R_{\Upsilon}(nS)$$

Potential Model for L.O. Matrix Element

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•
$$V(q^2)_{11}^{\mathrm{Hyd}} \propto \left(1 + \frac{a_0^2 |\mathbf{q}|^2}{16}\right)^{-2} \xrightarrow{|\mathbf{q}| \to 0} 1$$

•
$$V(q^2)_{21}^{\text{Hyd}} \propto \frac{a_0^2 |\mathbf{q}|^2}{16} \left(1 + \frac{a_0^2 |\mathbf{q}|^2}{16}\right)^{-3} \xrightarrow{|\mathbf{q}| \to 0} 0$$

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$$\stackrel{\downarrow}{v^2} \longrightarrow \stackrel{\mathsf{Suppressed.}}{\mathsf{Difficult to predict.}}$$

L.O. Matrix Element dependence on spin-spin potential

ArXiv:1302.3528

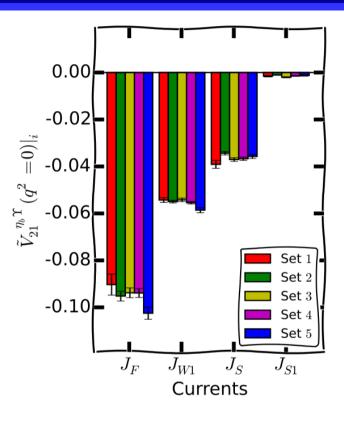
$$\Gamma(\psi(2S) \to \eta_c \gamma) = \frac{16\alpha}{27m_c^2} \tilde{q}_{\gamma}^3 \left[\frac{\tilde{q}_{\gamma}^2}{24} \eta_c \langle r^2 \rangle_{\psi(2S)} + \frac{5}{6} \frac{\eta_c \langle p^2 \rangle_{\psi(2S)}}{m_c^2} - \frac{2}{m_c^2} \frac{\eta_c \langle V_{S^2}(\vec{r}) \rangle_{\psi(2S)}}{E_{\psi(2S)} - E_{\eta_c}} \right]^2$$

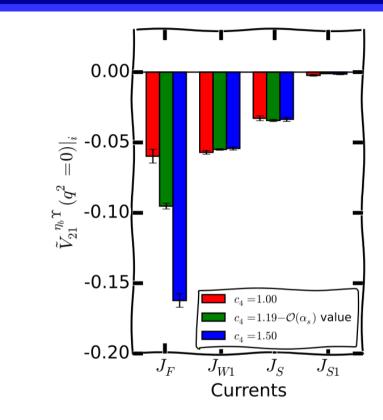
$$\Gamma(\eta_c(2S) \to J/\psi\gamma) = \frac{16\alpha}{9m_c^2} q_{\gamma}^3 \left[\frac{q_{\gamma}^2}{24} J/\psi \langle r^2 \rangle_{\eta_c(2S)} + \frac{5}{6} \frac{J/\psi \langle p^2 \rangle_{\eta_c(2S)}}{m_c^2} + \frac{2}{m_c^2} \frac{J/\psi \langle V_{S^2}(\vec{r}) \rangle_{\eta_c(2S)}}{E_{\eta_c(2S)} - E_{J/\psi}} \right]^2$$

The Hindered M1 $\eta_b(2S) \to \Upsilon(1S)\gamma$ Decay

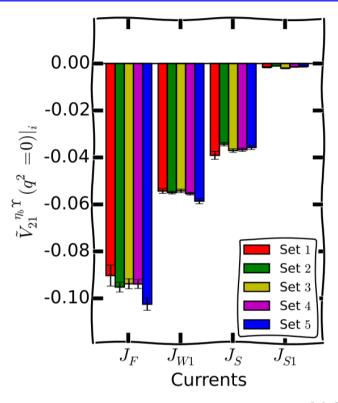


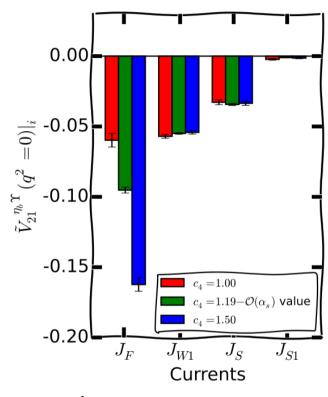
The Hindered M1 $\eta_b(2S) \to \Upsilon(1S)\gamma$ Decay





The Hindered M1 $\eta_b(2S) \rightarrow \Upsilon(1S)\gamma$ Decay





- N.B., Matrix element dependence on spinspin potential has opposite sign in this decay relative to $\Upsilon(2S) \to \eta_b(1S) \gamma$ (backup slides)
- N.B., spin-spin potential contribution dominates and L.O. matrix element becomes negative

The Hindered M1 $\eta_b(2S) \to \Upsilon(1S)\gamma$ Decay

