Hindered M1 Radiative Decay of $\Upsilon(2S')$ and $\eta_b(2S')$ from Lattice NRQCD

arXiv:1508.01694

C. Hughes, R. Dowdall, C. Davies,
R. Horgan, G. Von Hippel, M. Wingate
- M1 $\bar{b}b$ Radiative Decay $\implies$ Requires Spin Flip
M1 $\bar{b}b$ Radiative Decay $\implies$ Requires Spin Flip

- Allowed M1: $n' ^3 S_1 \rightarrow n ^1 S_0 \gamma$, $n' = n$
• M1 $\bar{b} b$ Radiative Decay $\rightarrow$ Requires Spin Flip

• Allowed M1: $n'{}^3 S_1 \rightarrow n^1 S_0 \gamma, \quad n' = n$

• Hindered M1: $n'{}^3 S_1 \rightarrow n^1 S_0 \gamma, \quad n' \neq n$
Inspirational Motivation!!

- Full Understanding of Low-Lying States/Decays
Motivation

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- Full Understanding of Low-Lying States/Decays

2015 PDG
Motivation

Inspirational Motivation!!

- Full Understanding of Low-Lying States/Decays

\[ \mathcal{B}(\Upsilon(2S) \to \eta_b(1S)\gamma) \times 10^4 \]

C. Hughes  
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Motivation

Inspirational Motivation!!

- Full Understanding of Low-Lying States/Decays
- Production of Spin Singlets: $\eta_b$
• Full Understanding of Low-Lying States/Decays

• Production of Spin Singlets: $\eta_b$

• Exclusion of parity odd Higgs
Inspirational Motivation!!

- Full Understanding of Low-Lying States/Decays
- Production of Spin Singlets: $\eta_b$
- Exclusion of parity odd Higgs
- Laboratory to test relativistic effects: NRQCD =? Experiment
Decay Rate: $\Upsilon(2S) \rightarrow \eta_b(1S)\gamma$

$\mathcal{B}(\Upsilon(2S) \rightarrow \eta_b(1S)\gamma) = (3.9 \pm 1.5) \times 10^{-4}$
Decay Rate: $\Upsilon(2S) \rightarrow \eta_b(1S)\gamma$

$$\langle \eta_{b(mS)}(p_i)|J^{\mu}(0)|\Upsilon_{(nS)}(p_f, s_\Upsilon)\rangle = \frac{2V_{21}^{\Upsilon\eta_b}(q^2)}{M_\Upsilon + M_{\eta_b}} \varepsilon^{\mu\alpha\beta\tau} p_{i,\alpha} p_{f,\beta} \varepsilon_\tau(p_f, s_\Upsilon)$$

$$\Gamma_{\Upsilon \rightarrow \eta_b \gamma} = \alpha_{QED} e_q^2 \frac{16}{3} \frac{|q|^3}{(M_\Upsilon + M_{\eta_b})^2} \left| V_{21}^{\Upsilon\eta_b}(q^2 = 0) \right|_{\text{lat.}}^2$$
NRQCD Evolution: $v^2 \sim 0.1$

\[ aH = aH_0 + a\delta H_{v^4} + a\delta H_{v^6} \]

\[ aH_0 = -\frac{\Delta^{(2)}}{2am_b}, \]

\[ a\delta H_{v^4} = -c_1 \frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2 \frac{i}{8(am_b)^2} \left( \nabla \cdot \tilde{E} - \tilde{E} \cdot \nabla \right) \]

\[ -c_3 \frac{1}{8(am_b)^2} \sigma \cdot \left( \tilde{\nabla} \times \tilde{E} - \tilde{E} \times \tilde{\nabla} \right) \]

\[ -c_4 \frac{1}{2am_b} \sigma \cdot \tilde{B} + c_5 \frac{\Delta^{(4)}}{24am_b} - c_6 \frac{(\Delta^{(2)})^2}{16n(am_b)^2}, \]

\[ a\delta H_{v^6} = -c_7 \frac{1}{8(am_b)^3} \left\{ \Delta^{(2)}, \sigma \cdot \tilde{B} \right\} \]

\[ -c_8 \frac{3i}{64(am_b)^4} \left\{ \Delta^{(2)}, \sigma \cdot \left( \tilde{\nabla} \times \tilde{E} - \tilde{E} \times \tilde{\nabla} \right) \right\} \]

\[ +c_9 \frac{1}{8(am_b)^3} \sigma \cdot \tilde{E} \times \tilde{E} \]
$|q_\gamma| \sim 0.6 \text{GeV} \sim m v^2, \quad v^2 \sim 0.1$

$O_F : \quad \omega_F \frac{ee_b}{2m_b} \psi_b^\dagger \sigma \cdot B^{\text{QED}} \psi_b \sim v^4$
Currents and Power Counting from NRQCD + NRQED

\[ |q_\gamma| \sim 0.6 \text{GeV} \sim mv^2, \quad v^2 \sim 0.1 \]

\[ \mathcal{O}_F : \quad \omega_F \frac{ee_b}{2m_b} \psi_b^\dagger \sigma \cdot \mathbf{B}^{\text{QED}} \psi_b \sim v^4 \]

\[ \mathcal{O}_{W1} : \quad \omega_{W1} \frac{ee_b}{8m_b^3} \psi_b^\dagger \{ \mathbf{D}^2, \sigma \cdot \mathbf{B}^{\text{QED}} \} \psi_b \sim v^6 \]

\[ \mathcal{O}_S : \quad \omega_S \frac{i ee_b}{8m_b^2} \psi_b^\dagger \sigma \cdot [\mathbf{D} \times, \mathbf{E}^{\text{QED}}] \psi_b \sim v^5 \]

\[ \mathcal{O}_{S2} : \quad \omega_{S2} \frac{3i ee_b}{64m_b^4} \psi_b^\dagger \sigma \cdot \{ \mathbf{D}^2, [\mathbf{D} \times, \mathbf{E}^{\text{QED}}] \} \psi_b \sim v^7 \]
Need matching coefficient of L.O. Current Operator:

\[ \mathcal{O}_F : \omega_F \frac{ee_b}{2m_b} \psi_b^{\dagger} \sigma \cdot \mathbf{B}^{\text{QED}} \psi_b \sim \nu^4 \]

with \( \omega_F = 1 + \omega_F^{(1)} \alpha_s + \mathcal{O}(\alpha_s^2) \).
Need matching coefficient of L.O. Current Operator:

\[ \mathcal{O}_F : \quad \omega_F \frac{e e_b}{2 m_b} \psi_b^\dagger \sigma \cdot B^{\text{QED}} \psi_b \sim \nu^4 \]

with \( \omega_F = 1 + \omega_F^{(1)} \alpha_s + \mathcal{O}(\alpha_s^2) \),

![Graph showing lattice results](image)
Lattice Methodology

Down the Rabbit Hole
1. Get one of these:
Lattice Methodology on one slide

1. Get one of these:

2. Compute the 2pt functions: $C_{2pt}(n_{src}, n_{skn}; T) = \langle \mathcal{O}(n_{snk}, T + t_0)\mathcal{O}^\dagger(n_{src}, t_0) \rangle$
1. Get one of these:

2. Compute the 2pt functions: \( C_{2pt}(n_{src}, n_{skn}; T) = \langle \mathcal{O}(n_{skn}, T + t_0)\mathcal{O}^\dagger(n_{src}, t_0) \rangle \)

3. Compute the 3pt functions:

\[
C_{3pt}(n_{src}, n_{skn}; T, t) = \langle \mathcal{O}(n_{skn}, T + t_0)\mathcal{O}_C(t + t_0)\mathcal{O}^\dagger(n_{src}, t_0) \rangle
\]
1. Get one of these:

2. Compute the 2pt functions: 
\[ C_{2pt}(n_{src}, n_{skn}; T) = \langle \mathcal{O}(n_{skn}, T + t_0) \mathcal{O}^{\dagger}(n_{src}, t_0) \rangle \]

3. Compute the 3pt functions:
\[ C_{3pt}(n_{src}, n_{skn}; T, t) = \langle \mathcal{O}(n_{skn}, T + t_0) \mathcal{O}_C(t + t_0) \mathcal{O}^{\dagger}(n_{src}, t_0) \rangle \]

4. Fit the data in your favourite way!
NB: Details swept under the rug, ask if interested!

For this talk, only necessary to know that it is possible to accurately extract **energies** and **matrix elements** from lattice QCD.
Results for Form Factors

Not 10% of $J_F$!!
\[
\langle \eta_b (mS) | J_F | \Upsilon(nS) \rangle = S_{fi} \int_0^\infty dr \ r^2 R_{m, \eta_b}^*(r) j_0 \left( \frac{|q|r}{2} \right) R_{n, \Upsilon}(r)
\]

\[
\int_0^\infty dr \ r^2 R_{m, \eta_b}^*(r) j_0 \left( \frac{|q|r}{2} \right) R_{n, \Upsilon}(r) = \delta_{nm} + a_2 |q_\gamma|^2 r_0^2 + a_4 |q_\gamma|^4 r_0^4 + \cdots
\]
Lattice NRQCD

Results for Form Factors

\[
\langle \eta_b(mS) | J_F | \Upsilon(nS) \rangle = S_{fi} \int_0^\infty dr \ r^2 R^*_{m, \eta_b}(r) j_0 \left( \frac{|q|r}{2} \right) R_{n, \Upsilon}(r)
\]

\[
\int_0^\infty dr \ r^2 R^*_{m, \eta_b}(r) j_0 \left( \frac{|q|r}{2} \right) R_{n, \Upsilon}(r) = \\
\delta_{nm} + a_2 |q_\Upsilon|^2 r_0^2 + a_4 |q_\Upsilon|^4 r_0^4 + \cdots
\]

N.B., In hindered decays \((n \neq m)\) the leading order matrix element is suppressed, making sub-leading currents appreciable

N.B., Destructive Interference occurs in the \(\Upsilon(2S) \rightarrow \eta_b(1S)\gamma\) decay
Results for Form Factors

\[ V_{21}^{\eta_b} (g^2 = 0) \]

- \( c_4 = 1.00 \)
- \( c_4 = 1.19 - \mathcal{O}(\alpha_s) \) value
- \( c_4 = 1.50 \)
Results for Form Factors

\[ |\eta_b(1S)\rangle^{(1)} = |\eta_b(1S)\rangle^{(0)} - \sum_{m \neq 1} |\eta_b(mS)\rangle^{(0)} \frac{V_{m1}^{\eta_b}}{E_{m1}^{\eta_b}} \]

\[ |\Upsilon(2S)\rangle^{(1)} = |\Upsilon(2S)\rangle^{(0)} - \sum_{n \neq 2} |\Upsilon(nS)\rangle^{(0)} \frac{V_{n2}^{\Upsilon}}{E_{n2}^{\Upsilon}}. \]
Results for Form Factors

\[ |\eta_b(1S)\rangle^{(1)} = |\eta_b(1S)\rangle^{(0)} - \sum_{m \neq 1} |\eta_b(mS)\rangle^{(0)} \frac{V_{m1}^{\eta_b}}{E_{m1}^{\eta_b}} \]

\[ |\Upsilon(2S)\rangle^{(1)} = |\Upsilon(2S)\rangle^{(0)} - \sum_{n \neq 2} |\Upsilon(nS)\rangle^{(0)} \frac{V_{n2}^{\Upsilon}}{E_{n2}^{\Upsilon}}. \]

Overlap Suppressed

\[ (1) \langle \eta_b(1S) | J_i | \Upsilon(2S) \rangle^{(1)} \approx (0) \langle \eta_b(1S) | J_i | \Upsilon(2S) \rangle^{(0)} \]

\[ - \frac{V_{21}^{\eta_b*}}{E_{21}^{\eta_b}} (0) \langle \eta_b(2S) | J_i | \Upsilon(2S) \rangle^{(0)} \]

\[ - \frac{V_{12}^{\Upsilon}}{E_{12}^{\Upsilon}} (0) \langle \eta_b(1S) | J_i | \Upsilon(1S) \rangle^{(0)} \]

N.B.
Results for Form Factors

\[ V_{21}^{\eta_b}(q^2 = 0) |_{\text{exp.}} \]

\( \Delta = 0 \)  \( \Delta > 0 \)  \( \Delta < 0 \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( p_{\text{test}} ) for ( \Delta &lt; 0 )</th>
<th>( p_{\text{O}(\alpha_s)} ) for ( \Delta = 0 )</th>
<th>( p_{\text{test}} ) for ( \Delta &gt; 0 )</th>
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<td>( c_1 = c_6 )</td>
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<td>( c_3 )</td>
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<td>( c_4 )</td>
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<td>( c_5 )</td>
<td>1.00</td>
<td>1.16</td>
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<td>( c_7 )</td>
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<td>1.00</td>
<td>1.50</td>
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<tr>
<td>( m_b )</td>
<td>2.5935</td>
<td>2.73</td>
<td>\text{——}</td>
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N.B., In hindered decays the leading order matrix element is suppressed, making particular relativistic corrections due to perturbative potentials (arising from terms in the Hamiltonian) appreciable.
Extrapolation

\[ V_{21}^{\gamma\eta_b}(q^2) = \sum_i V_{21}^{\gamma\eta_b}(q^2)|_i \]

\[ V_{21}^{\gamma\eta_b}(q^2 = 0) \]

Experiment
Wrapping Up

What did we learn?

\[ \mathcal{B}(\Upsilon(2S) \rightarrow \eta_b(1S)\gamma) \times 10^4 \]
Due to suppression of L.O. matrix element in hindered M1 decays, in order to accurately predict one needs:

- Relativistic corrections in current (Need multiple current corrections)
Wrapping Up

What did we learn?

Due to suppression of L.O. matrix element in hindered M1 decays, in order to accurately predict one needs:

- Relativistic corrections in current (Need multiple current corrections)
- Relativistic corrections in action (Need relativistic corrections in action)
Wrapping Up

What did we learn?

Due to suppression of L.O. matrix element in hindered M1 decays, in order to accurately predict one needs:

- Relativistic corrections in current (Need multiple current corrections)
- Relativistic corrections in action (Need relativistic corrections in action)
- Radiative corrections in action (Need precise matching coefficients)
Due to suppression of L.O. matrix element in hindered M1 decays, in order to accurately predict one needs:

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- Radiative corrections in action (Need precise matching coefficients)

We (HPQCD) Have Done THIS!!!
Radiative Decays

What do experimentalists see?!
Line Shape from $J/\psi \rightarrow \eta_c \gamma$

CLEO:ArXiv:0805.0252
Experimental Aside

Line Shape from $J/\psi \rightarrow \eta_c \gamma$

\[ \frac{dN_{\gamma}}{d\omega} = N_{\psi} B \int_{0}^{M_{\psi}/2} d\omega' \frac{d\Gamma'_{\omega'}}{d\omega'} \frac{\epsilon(\omega') g(\omega, \omega')}{\Gamma_{\eta_c \gamma}} \]

\[ \frac{d\Gamma(\omega)}{d\omega} = \frac{4}{3} \alpha \frac{e_c^2}{m_c^2} \omega^3 |M|^2 BW(\omega) \]

N.B., Need Energy Dependence of matrix element ("damping" function) to fit line shape correctly.

CLEO:ArXiv:0805.0252

KEDR:ArXiv:1002.2071
Mass of the $\eta_b$

[Graph showing mass distribution with annotations for 2015 PDG from hindered M1 decays and E1 decays.]
Experimental Aside

Mass of the $\eta_b$

Reason for tension here: Correct damping function (matrix element including suppression effects) needs to be used when fitting line shape from hindered M1 decays???
The Hindered M1 $\eta_b(2S) \rightarrow \Upsilon(1S)\gamma$ Decay
What Did We learn?

- Hindered M1 decays are difficult to predict as the L.O. matrix element is suppressed
What Did We learn?

- Hindered M1 decays are difficult to predict as the L.O. matrix element is suppressed
- This produces sensitivity to relativistic and radiative corrections
Hindered M1 decays are difficult to predict as the L.O. matrix element is suppressed.

This produces sensitivity to relativistic and radiative corrections.

Yet, it is possible to accurately and reliably calculate from first principles (using LQCD).
Hindered M1 decays are difficult to predict as the L.O. matrix element is suppressed. This produces sensitivity to relativistic and radiative corrections. Yet, it is possible to accurately and reliably calculate from first principles (using LQCD). A damping function (including suppression effects) might be needed when fitting the experimental line-shape from hindered M1 decays.
Future/Questions

- Get $h_b(1P)$ width from $h_b(1P) \to \eta_b(1S)\gamma$

- $B_s^* \to B_s\gamma$ needed for new-physics search in $B_s^* \to \ell\ell$ (arXiv:1509.05049)
Future/Questions

- Get $h_b(1P)$ width from $h_b(1P) \rightarrow \eta_b(1S)\gamma$

- $B_s^* \rightarrow B_s\gamma$ needed for new-physics search in $B_s^* \rightarrow \ell\ell$ (arXiv:1509.05049)

Questions!?! (ch558@cam.ac.uk)
Back Up Slides
Two Point Calculation

1. Build interpolating operators \( \mathcal{O}(n, t_0) \), which overlap with states having specific quantum numbers \( J^{PC} \), e.g., \( \Upsilon, \eta_b \)

2. Calculate
\[
C_{2pt}(n_{src}, n_{skn}; T) = \langle \mathcal{O}(n_{skn}; T + t_0) \mathcal{O}^\dagger(n_{src}, t_0) \rangle \]
numerically on the lattice
Three Point Calculation

1. Build current operators which we are interested in: $\mathcal{O}_C(t + t_0)$

2. Calculate $C_{3pt}(n_{src}, n_{skn}; T, t) = \langle \mathcal{O}(n_{snk}, T + t_0)\mathcal{O}_C(t + t_0)\mathcal{O}^\dagger(n_{src}, t_0) \rangle$ numerically with the same twist as in the two point calculation
Simultaneously fit two point correlator for $\Upsilon, \eta_b$ data to

$$C_{2pt}(n_{src}, n_{snk}) = \sum_{i}^{m} a_i(n_{src})a_i(n_{snk})\exp(-E_i t)$$

and three point correlator data in order to

$$C_{3pt}(n_{src}, n_{snk}) = \sum_{i, f}^{m} a_i(n_{src})V_{i,f}b_f(n_{snk})\exp(-E_i t)\exp(-E_f(T - t))$$

and extract what we need: $V_{i,f}$
Coulomb Gauge Fixed Ensembles

MILC Configurations \((n_f = 2 + 1 + 1 \text{ HISQ})\)

<table>
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<tr>
<th>Set</th>
<th>(\beta)</th>
<th>(a_Y) (fm)</th>
<th>(am_l)</th>
<th>(am_s)</th>
<th>(am_c)</th>
<th>(N_s \times N_T)</th>
<th>(n_{cfg})</th>
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<td>32 \times 96</td>
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$V(a^2, a m_b) = V_{\text{phys}}$
\[
\times \left[ 1 + \sum_{j=1,2} k_j (a \Lambda)^{2j} (1 + k_{jb} \delta x_m + k_{jbb} (\delta x_m)^2) \right]. \quad (4)
\]

The lattice spacing dependence is set by a scale $\Lambda = 500$ MeV, and $\delta x_m = (a m_b - 2.7)/1.5$ allows for mild dependence on the effective theory cutoff $a m_b$. We take priors of 0(1) on all the coefficients except $k_1$ which is 0.0(3) since the action includes radiatively improved $a^2$ lattice spacing corrections. We have tested that our results are not sensitive to the fit form or the priors.
Potential Model for L.O. Matrix Element

\[
\Gamma_{\gamma \to \eta_b \gamma} = \alpha_{QED} e_q^2 \frac{4}{3m_b^2} |q_\gamma|^3 \left| \int r^2 dr R^*_\eta_b (1S) j_0 \left( \frac{|q_\gamma| r}{2} \right) R_\gamma (2S) \right|^2
\]

\[
V(q^2)_{nm} \propto \int r^2 dr R^*_\eta_b (mS) j_0 \left( \frac{|q_\gamma| r}{2} \right) R_\gamma (nS)
\]
Potential Model for L.O. Matrix Element

\[ V(q^2)_{nm} \propto \int r^2 dr R^*_{\eta_b}(mS)j_0\left(\frac{|q|r}{2}\right)R_\chi(nS) \]

- \( V(q^2)^{\text{Hyd}}_{11} \propto \left(1 + \frac{a_0^2|q|^2}{16}\right)^{-2} \quad |q| \to 0 \rightarrow 1 \)

- \( V(q^2)^{\text{Hyd}}_{21} \propto \frac{a_0^2|q|^2}{16} \left(1 + \frac{a_0^2|q|^2}{16}\right)^{-3} \quad |q| \to 0 \rightarrow 0 \)
Potential Model for L.O. Matrix Element

\[ V(q^2)_{nm} \propto \int r^2 dr R_{\eta_b}^*(mS) j_0\left(\frac{|q|r}{2}\right) R_{\chi}(nS) \]

- \( V(q^2)_{11}^{\text{Hyd}} \propto \left(1 + \frac{a_0^2|q|^2}{16}\right)^{-2} \rightarrow 0 \) as \( |q| \rightarrow 0 \)

- \( V(q^2)_{21}^{\text{Hyd}} \propto \frac{a_0^2|q|^2}{16} \left(1 + \frac{a_0^2|q|^2}{16}\right)^{-3} \rightarrow 0 \) as \( |q| \rightarrow 0 \)

\( u^2 \) \( \Rightarrow \) Suppressed. Difficult to predict.
L.O. Matrix Element dependence on spin-spin potential

\[ \Gamma(\psi(2S) \to \eta_c \gamma) = \frac{16\alpha}{27m_c^2} q_\gamma^3 \left[ \frac{q_\gamma^2}{24} \eta_c \langle r^2 \rangle_{\psi(2S)} + \frac{5}{6} \frac{\eta_c \langle p^2 \rangle_{\psi(2S)}}{m_c^2} - \frac{2}{m_c^2} \frac{\eta_c \langle V_{S^2} \rangle_{\psi(2S)}}{E_{\psi(2S)} - E_{\eta_c}} \right]^2 \]

\[ \Gamma(\eta_c(2S) \to J/\psi \gamma) = \frac{16\alpha}{9m_c^2} q_\gamma^3 \left[ \frac{q_\gamma^2}{24} \frac{J/\psi \langle r^2 \rangle_{\eta_c(2S)}}{m_c^2} + \frac{5}{6} \frac{J/\psi \langle p^2 \rangle_{\eta_c(2S)}}{m_c^2} + \frac{2}{m_c^2} \frac{J/\psi \langle V_{S^2} \rangle_{\eta_c(2S)}}{E_{\eta_c(2S)} - E_{J/\psi}} \right]^2 \]
The Hindered M1 $\eta_b(2S) \rightarrow \Upsilon(1S)\gamma$ Decay
The Hindered M1 $\eta_b(2S) \rightarrow \Upsilon(1S)\gamma$ Decay
Current Work

The Hindered M1 $\eta_b(2S) \rightarrow \Upsilon(1S)\gamma$ Decay

- N.B., Matrix element dependence on spin-spin potential has opposite sign in this decay relative to $\Upsilon(2S) \rightarrow \eta_b(1S)\gamma$ (backup slides)
- N.B., spin-spin potential contribution dominates and L.O. matrix element becomes negative
Current Work

The Hindered M1 $\eta_b(2S) \rightarrow \Upsilon(1S)\gamma$ Decay

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Graph showing $V_{21}^{\eta_b}(q^2 = 0)$ versus $a^2$ (fm$^2$) with data points and error bars.}
\end{figure}