Calculation of hadronic matrix elements contributing to the $B_s - \bar{B}_s$ width difference

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Work done in collaboration with
CTH Davies, CJ Monahan, GP Lepage, J Shigemitsu (HPQCD)

Outline

- B-mixing description and Heavy Quark Expansion (HQE)
- Status of Standard Model calculation
- Our calculation
- To-do list

B mixing

Wigner-Weisskopf approximation

$$i\frac{d}{dt}\begin{pmatrix}|B^0(t)\rangle\\|\bar{B}^0(t)\rangle\end{pmatrix}=\left(M-\frac{i}{2}\Gamma\right)\begin{pmatrix}|B^0(0)\rangle\\|\bar{B}^0(0)\rangle\end{pmatrix}$$

Flavour basis ≠ mass basis ⇒ M & Γ non-diagonal

Mixing governed by 3 parameters:

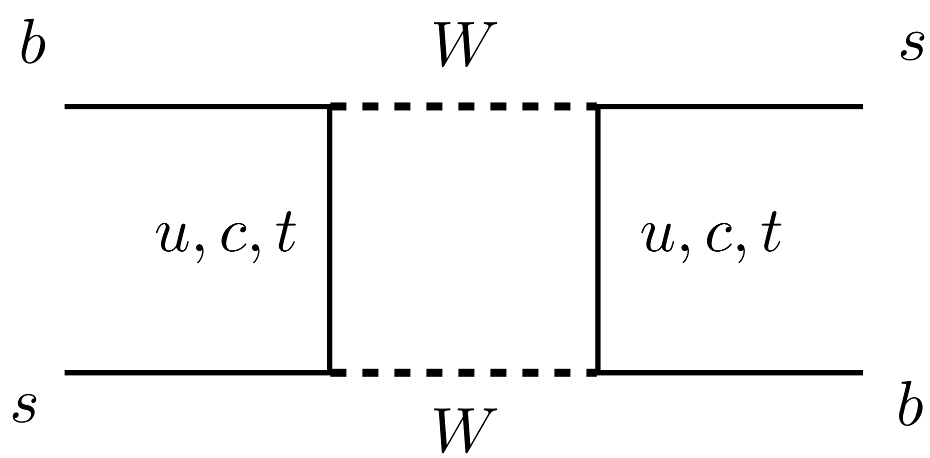
$$|M_{12}| \qquad |\Gamma_{12}| \qquad \phi=\arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$

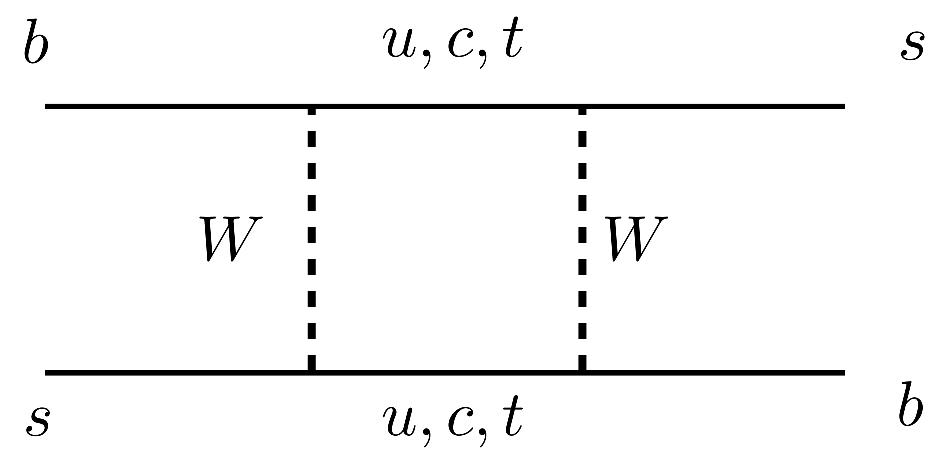
Observables

$$\Delta M=2|M_{12}| \qquad \Delta\Gamma=2|\Gamma_{12}|\cos\phi \qquad a_{\mathrm{fs}}=\frac{\Delta\Gamma}{\Delta M}\tan\phi$$

[up to corrections O(m_b^2/m_W^2)]

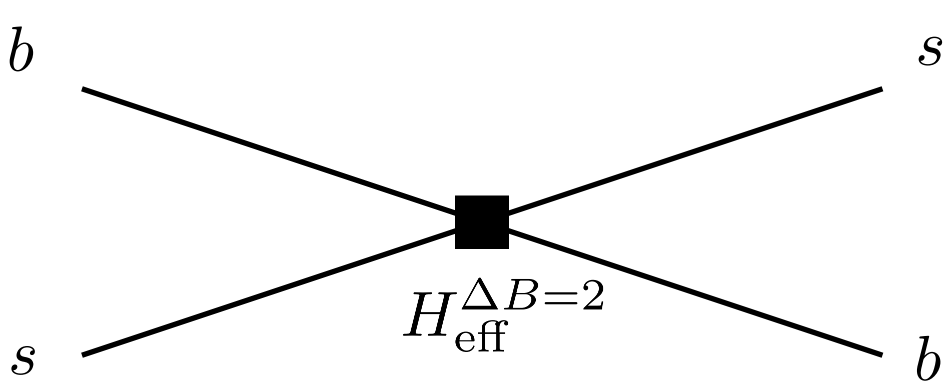
SM mixing

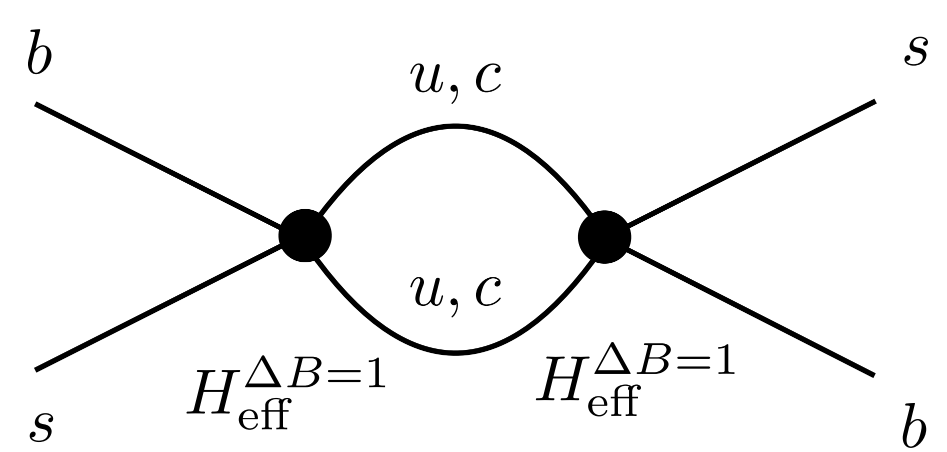




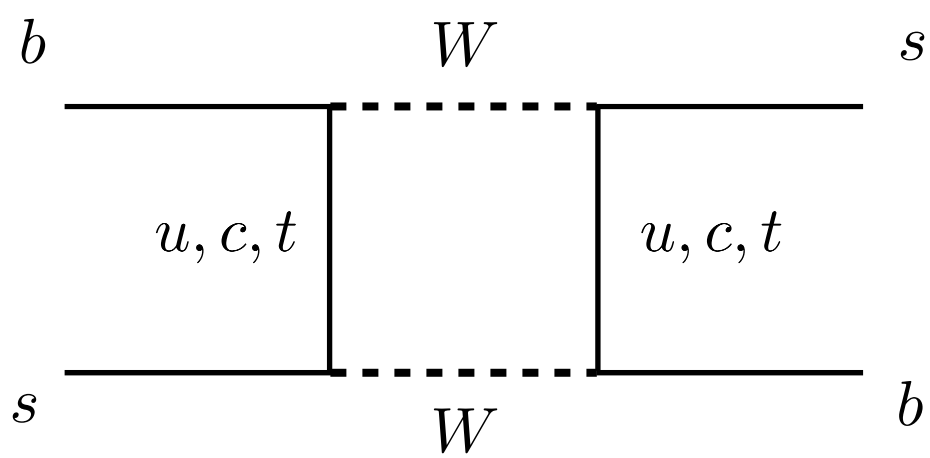
Integrate out *W* and *t*

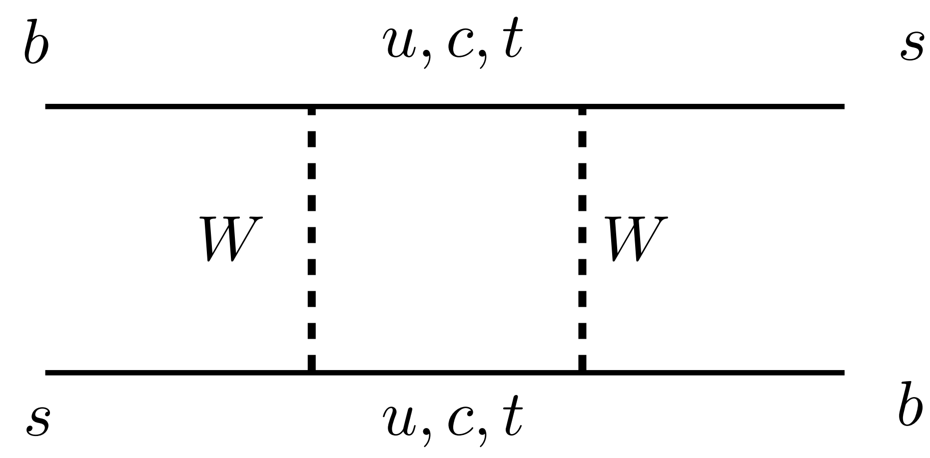
$\Downarrow$





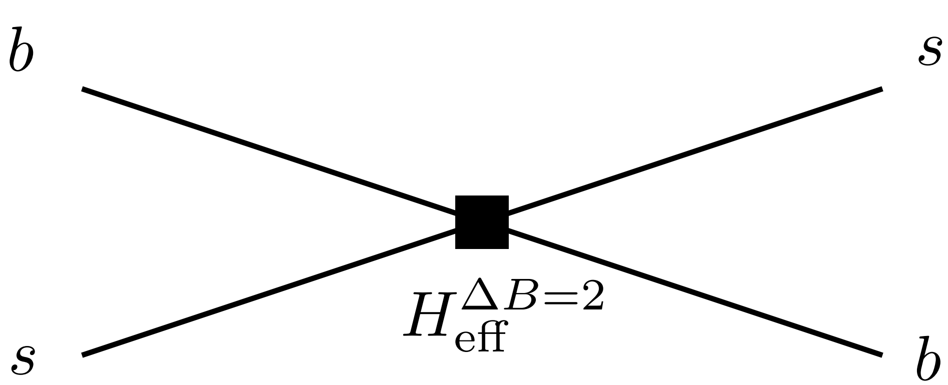
SM mixing

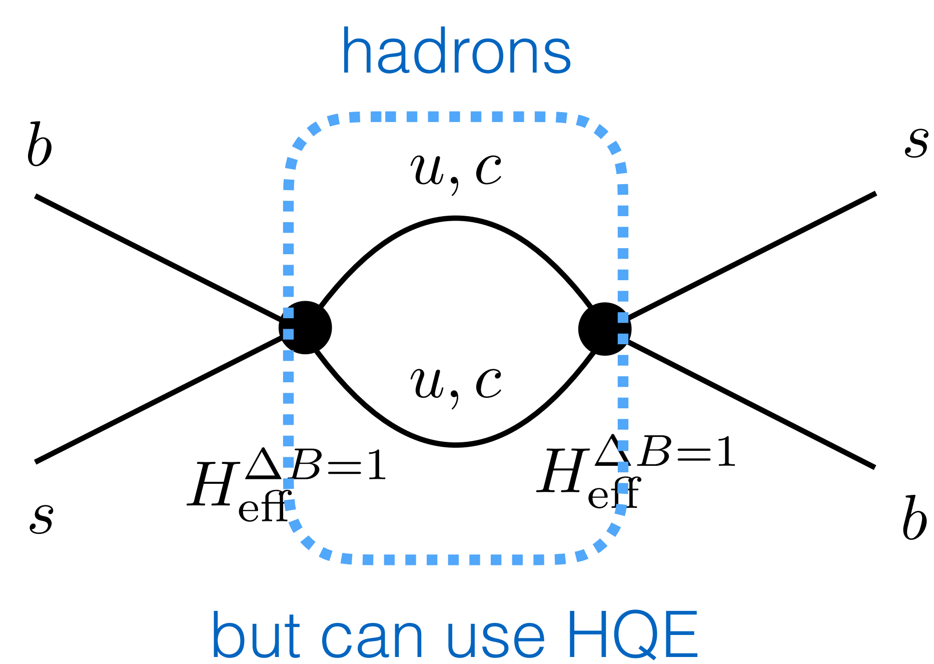




Integrate out W and t

$$\Downarrow$$





Mass difference

Oscillations governed by Hermitian part: dominated by local $\Delta B = 2$ matrix el.

$$\Delta M_s = \frac{1}{2m_{B_s}} \langle \bar{B}_s | H_{\rm eff}^{\Delta B=2} | B_s \rangle$$

$$H_{\rm eff}^{\Delta B=2} = \frac{G_F^2 m_W^2}{4\pi^2} (V_{ts}^* V_{tb})^2 \sum_{i=1}^{5} C_i Q_i$$

$$Q_1 = (\bar{b}^\alpha \gamma^\mu (1-\gamma^5) s^\alpha)(\bar{b}^\beta \gamma_\mu (1-\gamma^5) s^\beta)$$

$$Q_2 = (\bar{b}^\alpha (1-\gamma^5) s^\alpha)(\bar{b}^\beta (1-\gamma^5) s^\beta)$$

$$Q_3 = (\bar{b}^\alpha (1-\gamma^5) s^\beta)(\bar{b}^\beta (1-\gamma^5) s^\alpha)$$

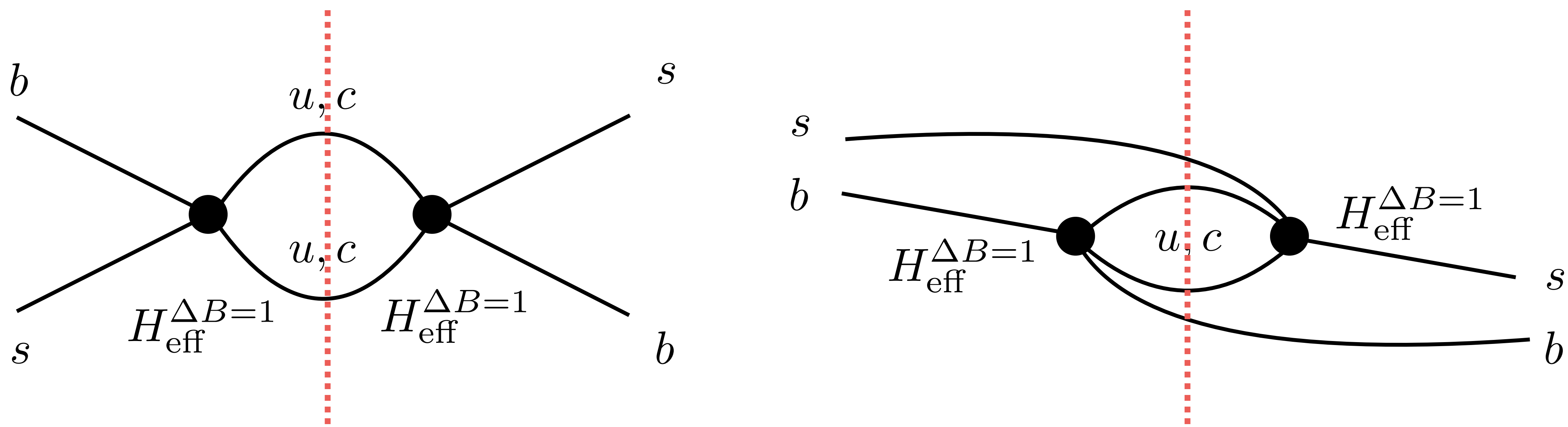
$$Q_4 = (\bar{b}^\alpha (1-\gamma^5) s^\alpha)(\bar{b}^\beta (1+\gamma^5) s^\beta)$$

$$Q_5 = (\bar{b}^\alpha (1-\gamma^5) s^\beta)(\bar{b}^\beta (1+\gamma^5) s^\alpha)$$

In the Standard Model only Q_1 enters ΔM_s.

Recent work: HPQCD (prelim) arXiv:1411.6989, FNAL/MILC arXiv:1602.03560v2, HPQCD (in progress)

Lifetime difference & HQE



- Γ_{12} from imaginary part (optical theorem)
- Large momentum through loop
- Operator product expansion: Heavy Quark Expansion (HQE), quark-hadron duality

$$\Gamma_{21} = \frac{1}{2m_{B_s}} \langle \bar{B}_s | \mathscr{T} | B_s \rangle$$

$$\mathscr{T} = \mathrm{Im}\, i \int d^4x\, \mathcal{T}\, H_{\text{eff}}^{\Delta B=1}(x)\, H_{\text{eff}}^{\Delta B=1}(0)$$

$$\Downarrow \quad \text{HQE @ LO}$$

$$\mathscr{T} = -\frac{G_F^2 m_b^2}{12\pi} (V_{cb}^* V_{cs})^2 [F(z, \mu_2) Q_1(\mu_2) + F_S(z, \mu_2) Q_2(\mu_2)]$$

HQE expressions

$$\Gamma_{12}^{s} = -\left[\,\lambda_c^2\,\Gamma_{12}^{cc} \;+\; 2\,\lambda_c\,\lambda_u\,\Gamma_{12}^{uc} \;+\; \lambda_u^2\,\Gamma_{12}^{uu}\,\right]$$ with $\lambda_i = V_{is}^{*} V_{ib}$

Leading order:

$$\Gamma_{12}^{cc} = \frac{G_F^2 m_b^2}{24\pi m_{B_s}} \left[(G + \frac{1}{2}\alpha_2 G_S)\langle \bar{B}_s | Q_1 | B_s\rangle + \alpha_1 G_S \langle \bar{B}_s | Q_3 | B_s\rangle\right] \;+\; \tilde{\Gamma}_{12,1/m_b}^{cc}$$

where lattice QCD gives the matrix elements of Q_1 and Q_3.

G's depend on α_s, m_b, m_c/m_b, μ_1, μ_2 Beneke, et al. PLB459, hep-ph/9808385

Expressions from Lenz & Nierste, JHEP 06 (2007), hep-ph/0612167

HQE expressions

Dominant

$$\Gamma_{12}^{s} = -\left[\boxed{\lambda_c^2}\,\Gamma_{12}^{cc} + 2\,\lambda_c\,\lambda_u\,\Gamma_{12}^{uc} + \lambda_u^2\,\Gamma_{12}^{uu}\right]$$ with $\lambda_i = V_{is}^* V_{ib}$

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NLO

$$\tilde{\Gamma}^{cc}_{12,1/m_b} = \frac{G_F^2 m_b^2}{24\pi m_{B_s}} \left\{ g_0^{cc} \langle \bar{B}_s | R_0 | B_s \rangle + \sum_{j=1}^{3} \left[g_j^{cc} \langle \bar{B}_s | R_j | B_s \rangle + \tilde{g}_j^{cc} \langle \bar{B}_s | \tilde{R}_j | B_s \rangle \right] \right\}$$

g^{cc}'s depend on m_c/m_b

$$R_0 = Q_2 + \alpha_1 Q_3 + \frac{1}{2}\alpha_2 Q_1$$

$$R_1 = \frac{m_s}{m_b} (\bar{b}^\alpha (1-\gamma^5) s^\alpha)(\bar{b}^\beta (1+\gamma^5) s^\beta) = \frac{m_s}{m_b} Q_4$$

$$\boxed{R_2 = \frac{1}{m_b^2} (\bar{b}^\alpha \overleftarrow{D}_\rho \gamma^\mu (1-\gamma^5) D^\rho s^\alpha)(\bar{b}^\beta \gamma_\mu (1-\gamma^5) s^\beta)}$$

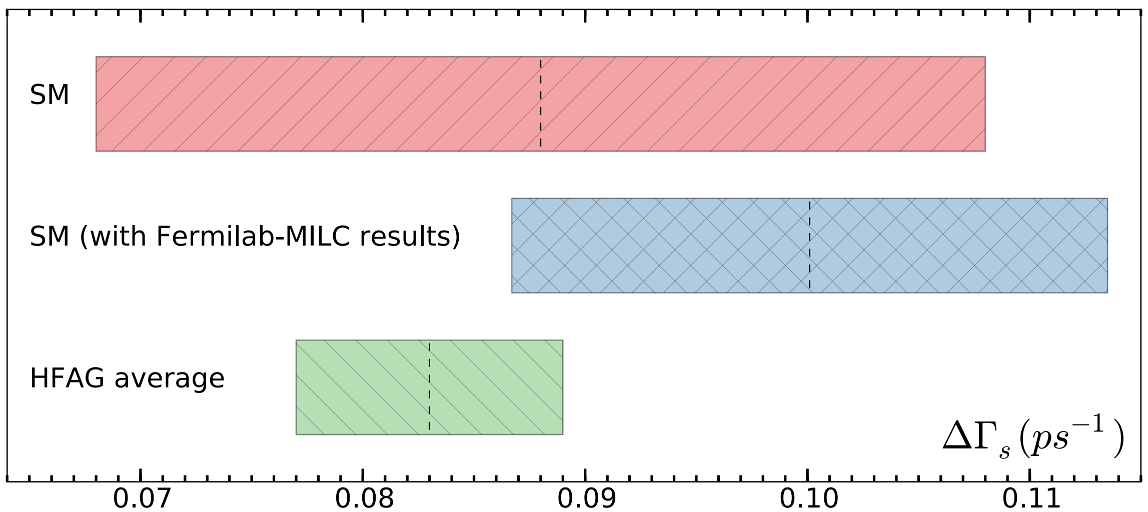
$$R_3 = \frac{1}{m_b^2} (\bar{b}^\alpha \overleftarrow{D}_\rho (1-\gamma^5) D^\rho s^\alpha)(\bar{b}^\beta (1-\gamma^5) s^\beta)$$

Tilde'd operators correspond to mixing color indices

Status

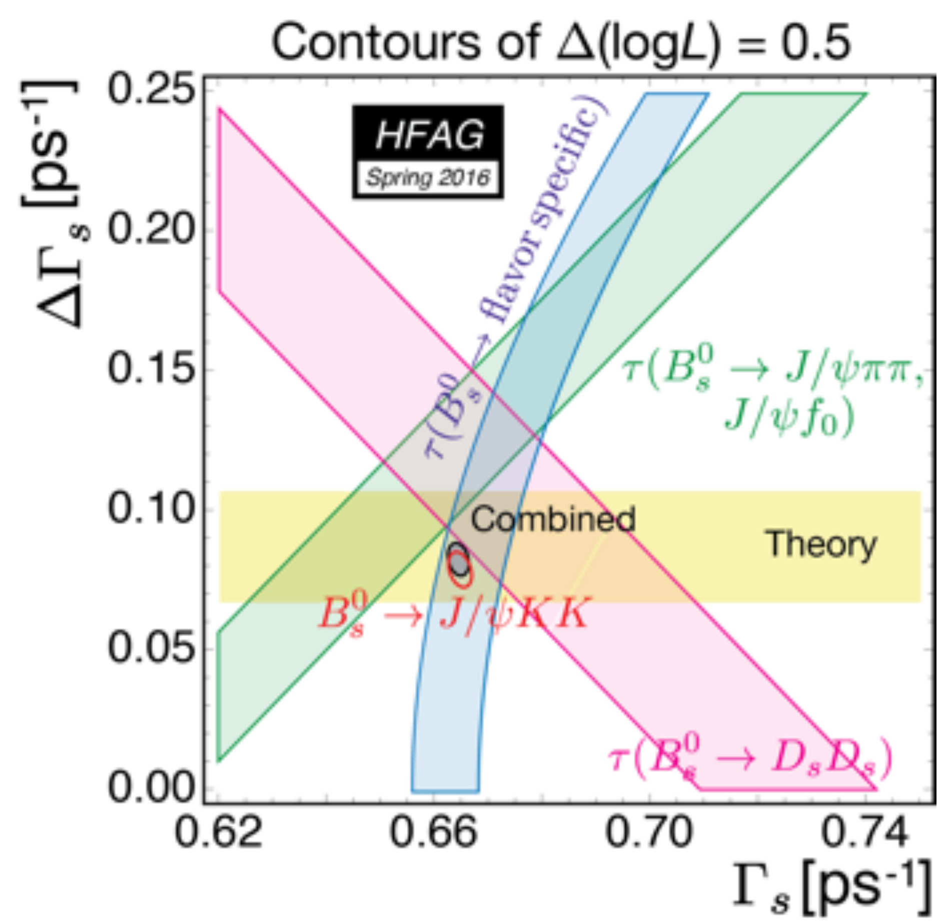
Artuso, Borissov, Lenz, arXiv:1511.09466v1

$\Delta\Gamma_s^{\mathrm{SM},2015} = 0.088(20)\,\mathrm{ps}^{-1}$



Plot and updated SM prediction from MJ Kirk, Lattice 2016 poster

Heavy Flavour Averaging Group



Dominent SM uncertainties:

- 15% due to matrix element of R_2 (bag factor = 1.0 ± 0.5, for one definition of m_b)
- 14% due to matrix element of Q_1 (FLAG, but see new FNAL/MILC)
- 8% due to renormalization scale

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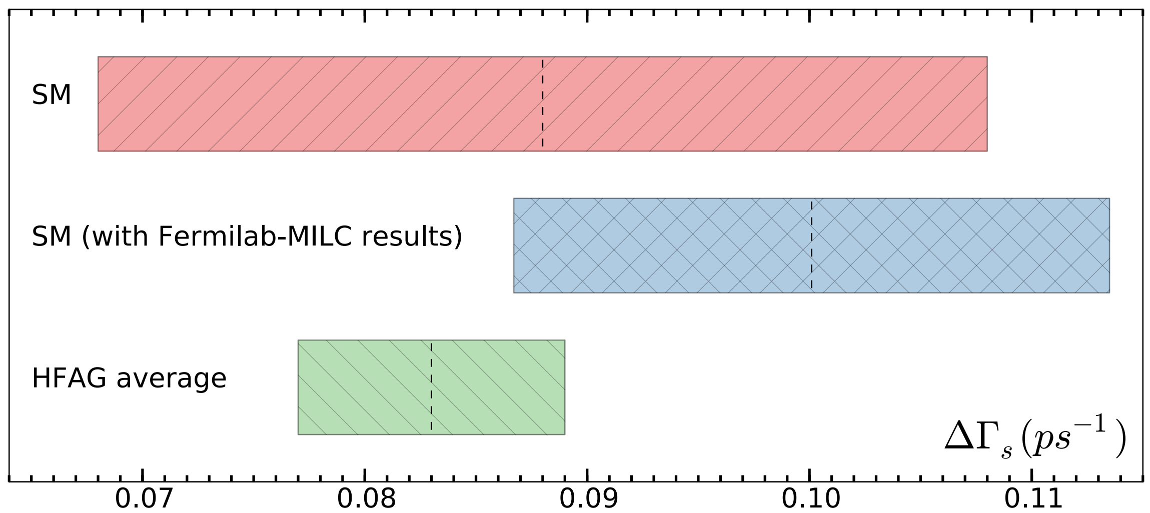

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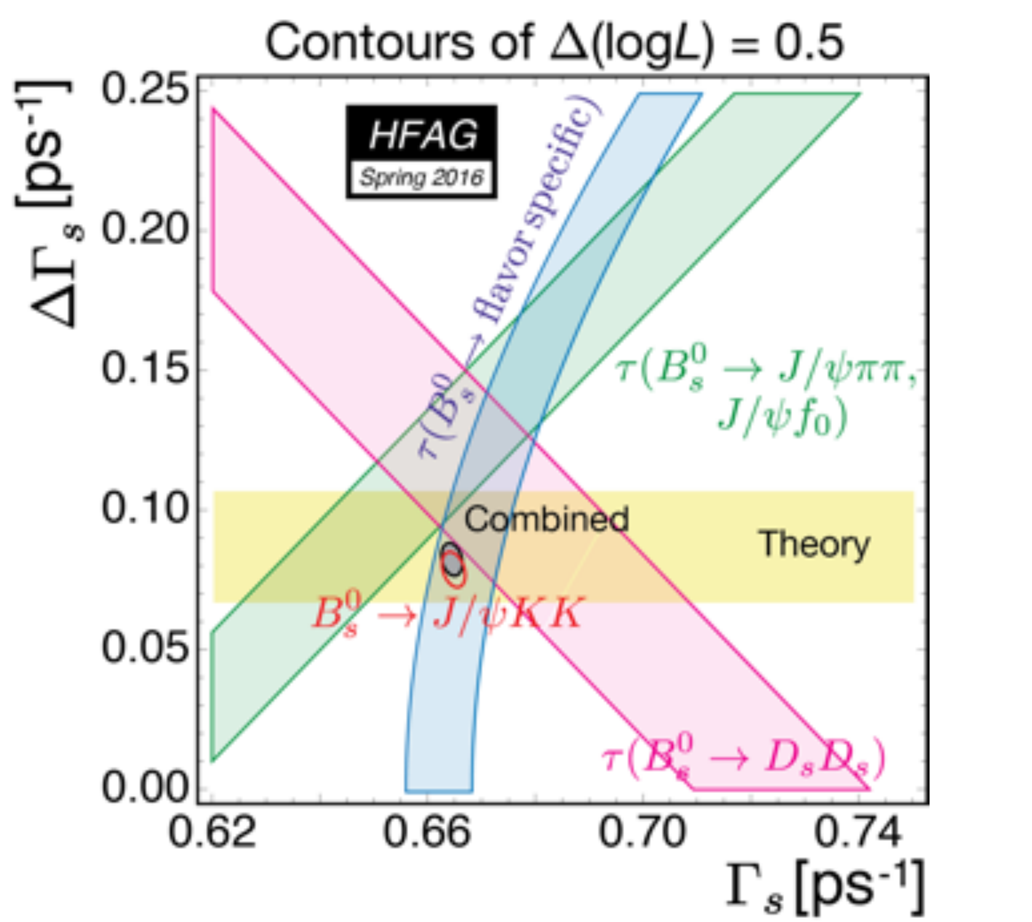

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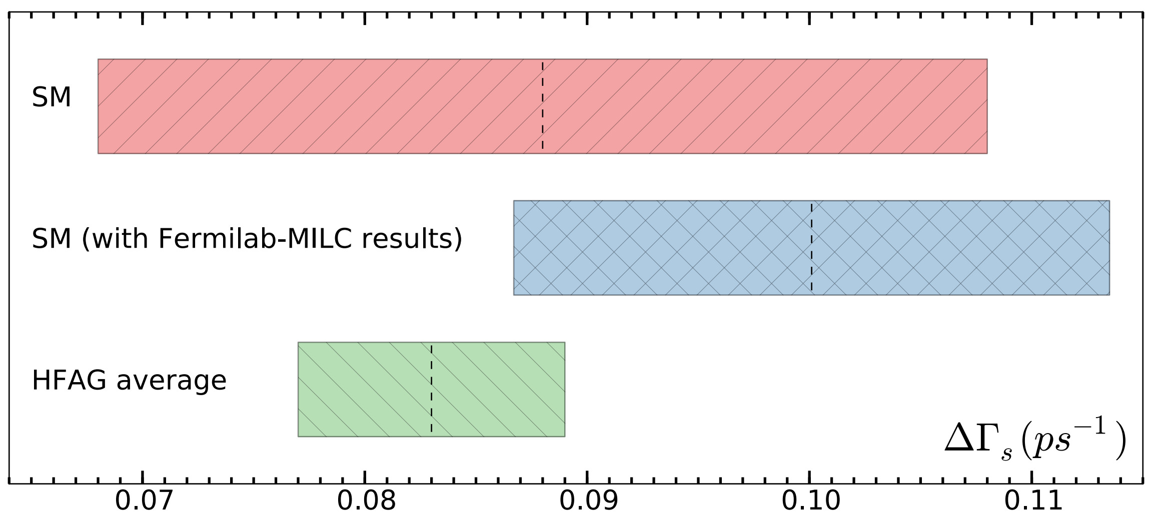

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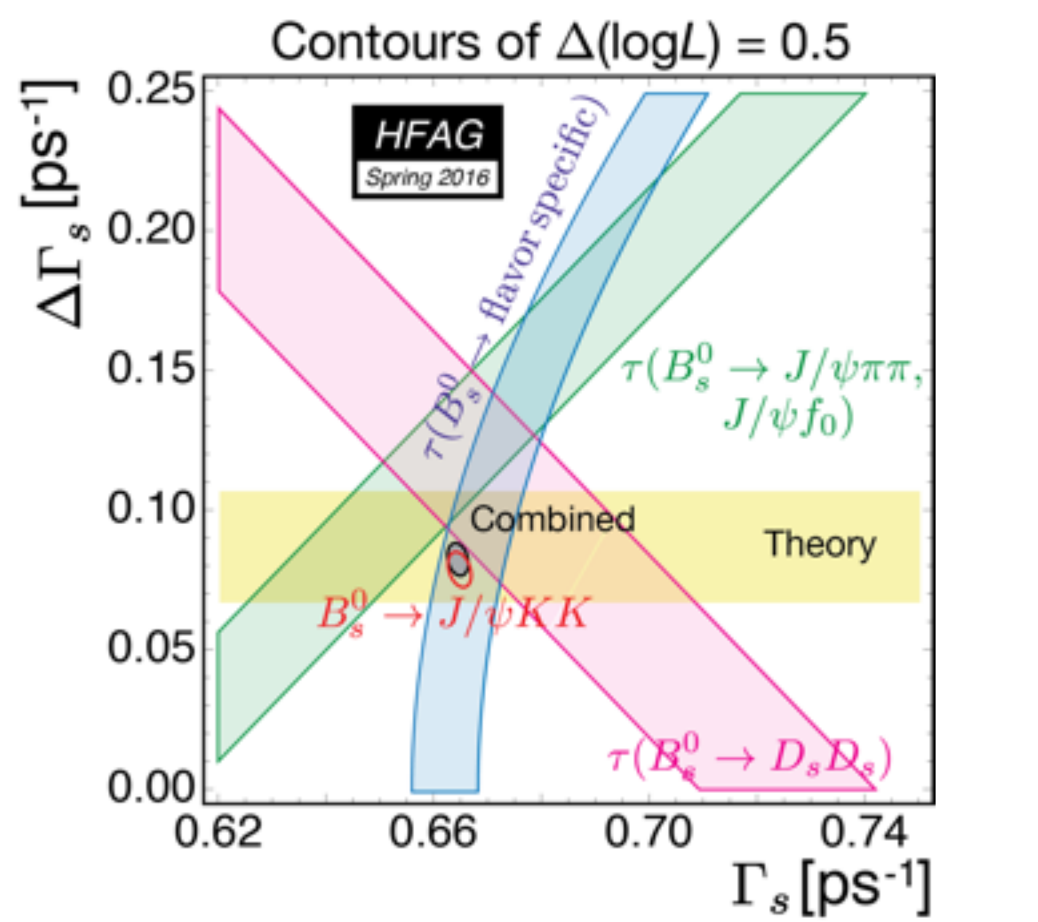

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This work

HPQCD calculation

- Extends ongoing HPQCD calculation of matrix elements of dimension-6 ΔB=2 operators ($Q_1 \ldots Q_5$)
- MILC highly improved staggered quark (HISQ) gauge field configurations (2+1+1 sea quarks)
- Nonrelativistic bottom quark, HISQ strange quark

Matching schemes

Continuum QCD

Lattice NRQCD

$$\langle Q_i \rangle_{\overline{\mathrm{MS}}} = \langle \hat{Q}i \rangle_L + \langle \hat{Q}i1 \rangle_L + \ldots$$

where lattice NRQCD is a 1/M expansion.

$$\hat{Q}i = (\bar{\Psi}_Q \Gamma_1 \Psi_q)(\bar{\Psi}_{\bar{Q}} \Gamma_2 \Psi_q) + (\bar{\Psi}_{\bar{Q}} \Gamma_1 \Psi_q)(\bar{\Psi}_Q \Gamma_2 \Psi_q).$$

$$\begin{aligned}
\hat{Q}i1 = \frac{1}{2M} [&(\vec{\nabla}\bar{\Psi}_Q \cdot \boldsymbol{\gamma}\Gamma_1 \Psi_q)(\bar{\Psi}_{\bar{Q}} \Gamma_2 \Psi_q) \\
&+ (\bar{\Psi}_Q \Gamma_1 \Psi_q)(\vec{\nabla}\bar{\Psi}_{\bar{Q}} \cdot \boldsymbol{\gamma}\Gamma_2 \Psi_q) \\
&+ (\vec{\nabla}\bar{\Psi}_{\bar{Q}} \cdot \boldsymbol{\gamma}\Gamma_1 \Psi_q)(\bar{\Psi}_Q \Gamma_2 \Psi_q) \\
&+ (\bar{\Psi}_{\bar{Q}} \Gamma_1 \Psi_q)(\vec{\nabla}\bar{\Psi}_Q \cdot \boldsymbol{\gamma}\Gamma_2 \Psi_q)].
\end{aligned}$$

Perturbative matching

Match continuum and lattice at $O(\alpha_s)$

$$\langle Q_i \rangle_{\overline{\rm MS}} = \langle \hat{Q}i \rangle + \alpha_s \rho_{ij} \langle \hat{Q}j \rangle + \langle \hat{Q}i1 \rangle^{\rm sub}$$

taking into account power-law “mixing down” at $O\left(\frac{\alpha_s}{aM}\right)$

$$\langle \hat{Q}i1 \rangle^{\rm sub} = \langle \hat{Q}i1 \rangle - \alpha_s \zeta_{ij} \langle \hat{Q}j \rangle$$

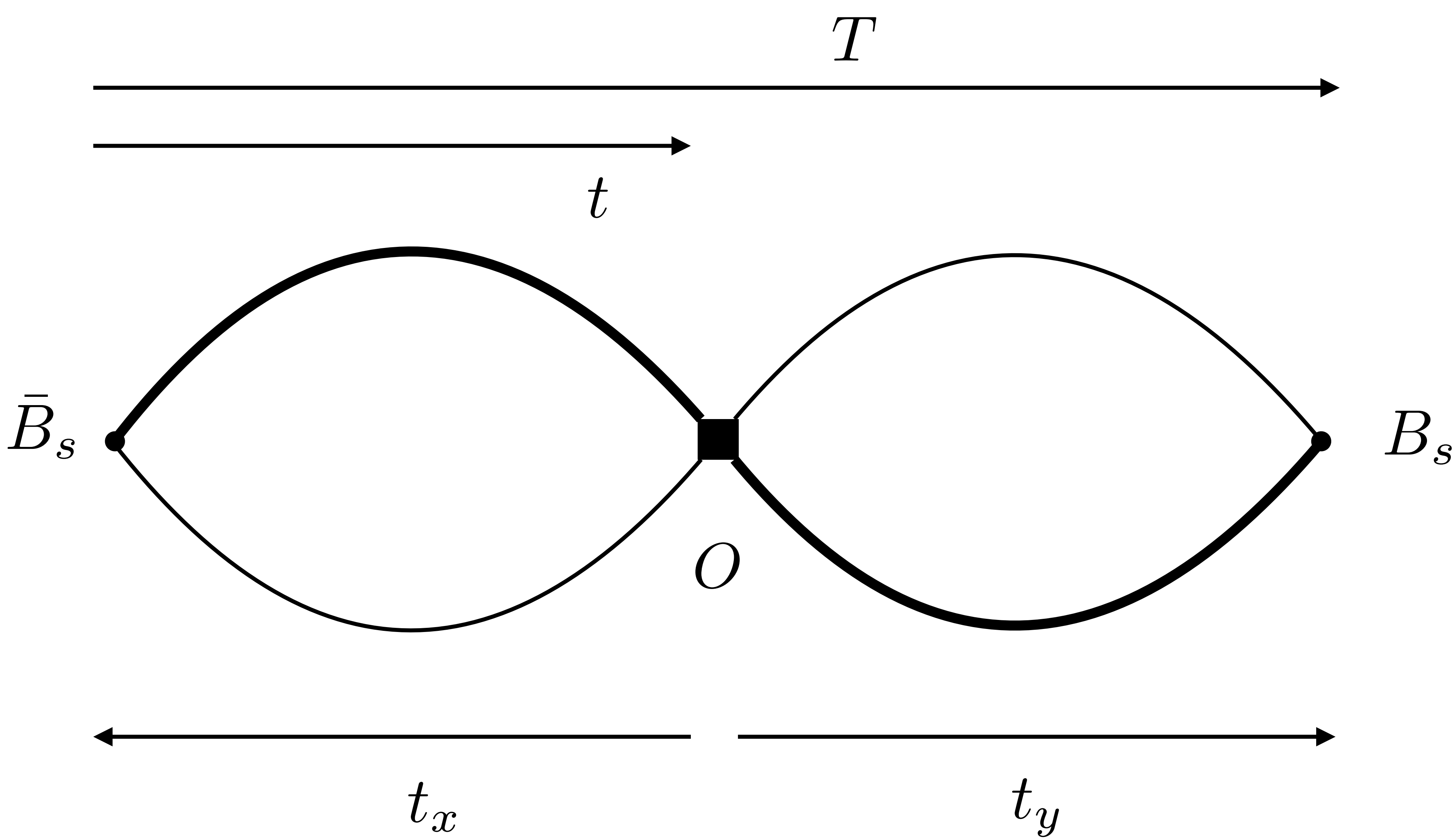
Monahan, Gámiz, Horgan, Shigemitsu, PRD90 (2014), arXiv:1407.4040

Similarly we have now computed coefficients in

$$\langle \hat{R}_i \rangle^{\rm sub} = \langle \hat{R}_i \rangle - \alpha_s \xi_{ij} \langle \hat{Q}j \rangle$$

In fact, $|\rho_{ij}|, |\zeta_{ij}|, |\xi_{ij}| < 1$ for lattices in use here.

Correlation functions



Strange quark "source" at operator *O.*
Derivative source (finite difference) for *R* operators

Dimension-7

Derivative part of R_2 and R_3

$$\frac{1}{m_b^2}(\bar{b}^\alpha \overleftarrow{D}_\rho \Gamma D^\rho s^\alpha) = \frac{1}{m_b^2}(\bar{b}^\alpha \overleftarrow{D}_0 \Gamma D^0 s^\alpha) + \mathcal{O}\left(\frac{1}{m_b^2}\right)$$

Using EOM

$$\bar{b}\overleftarrow{D}_0 = \pm m_b \bar{b}\,\gamma_0 \quad \text{and} \quad i\gamma_0 D^0 s = -i(\vec{\gamma}_M \cdot \vec{D})s = (\vec{\gamma}_E \cdot \vec{D})s$$

we have

$$R_{2,3} = \pm\frac{1}{m_b}(\bar{b}_\alpha \Gamma \gamma_0 (\vec{\gamma} \cdot \vec{D}) s_\alpha)(\bar{b}_\beta \Gamma s_\beta)$$

± correspond to outgoing b quark/incoming anti-b quark

Computation

Staggered & naive propagators from local source

$$K(x,y)g(y,z) = \delta(x,z)$$

$$G(y,z) = \Omega(x)g(y,z)\Omega^\dagger(z) \qquad \Omega(x) = \prod_{\mu=0}^{3}(\gamma_\mu)^{x_\mu/a}$$

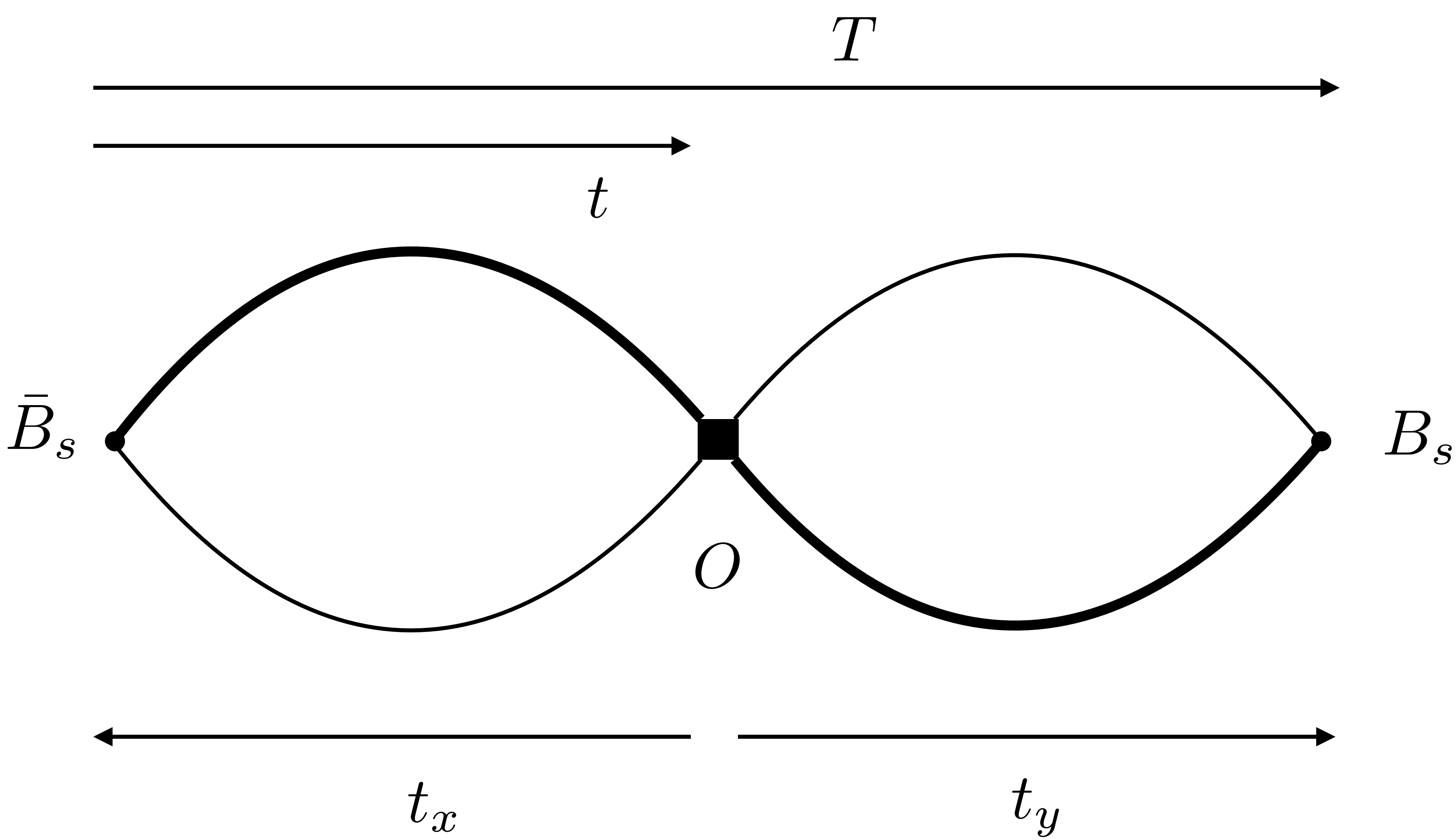
Staggered & naive propagators from derivative source

$$K(x,y)g^{(k)}(y,z) = \frac{1}{2}\left[\delta(x,z+\hat{k})U_k^\dagger(z) - \delta(x,z-\hat{k})U_k(z-\hat{k})\right]$$

$$G^{(k)}(y,z) = \Omega(x)g^{(k)}(y,z)\Omega^\dagger(z\pm\hat{k}) \qquad \hat{k} = e_1, e_2, e_3$$

4 inversions to get necessary strange quark propagators

Correlation functions



$$C_{ab}^{3\mathrm{pt}}(t,T) = \sum_{i,j} X_{a,i} \boxed{V_{nn,ij}} X_{b,j} \exp(-E_i t) \exp(-E_j (T-t))$$

Correlation functions

$$C_{ab}^{3\mathrm{pt}}(t,T) = \sum_{i,j} X_{a,i} V_{nn,ij} X_{b,j} \exp(-E_i t) \exp(-E_j(T-t))$$

$$X_{a,0} V_{nn,00} X_{b,0} = \frac{\langle 0|\Phi_a|B_s\rangle \boxed{\langle \bar{B}_s|a^6 O_{comp}|B_s\rangle} \langle B_s|\Phi_b|0\rangle}{(2m_{B_s}a^3)^2}$$

Remove unwanted factors using 2-point functions

$$C_{ab}^{2\mathrm{pt}}(t) = \sum_{i} X_{a,i} X_{b,i} \exp(-E_i t)$$

$$X_{a,0} X_{b,0} = \frac{\langle 0|\Phi_a|B_s\rangle \langle B_s|\Phi_b|0\rangle}{2m_{B_s}a^3}$$

Status

Set	Label	β	a/fm	am_l	am_s	am_c	$N_s^3 \times N_t$	#
1	VC5	5.8	0.1474(5)(14)(2)	0.013	0.0650	0.838	$16^3 \times 48$	1020
3	VCp	5.8	0.1450(3)(14)(2)	0.00235	0.0647	0.831	$32^3 \times 48$	1000
4	C5	6.0	0.1219(2)(9)(2)	0.0102	0.0509	0.635	$24^3 \times 64$	1052
6	Cp	6.0	0.1189(2)(9)(2)	0.00184	0.0507	0.628	$48^3 \times 64$	1000
7	F5	6.3	0.0873(2)(5)(1)	0.0074	0.037	0.440	$32^3 \times 96$	1008

- Matrix elements computed on 2 ensembles (VC5, C5)
    - will do 5 ensembles: 3 lattice spacings, including some with physically light quark masses
- Statistical errors about 10% for $\langle R_i \rangle^{\mathrm{sub}}$
- Systematic uncertainty dominated by tree-level matching between lattice and continuum: 20-30%

Rough numerics

$$\Delta\Gamma_s = \left[0.071(11)\left(\frac{\langle Q_1\rangle}{3\,\mathrm{GeV}^4}\right) + 0.035(6)\left(\frac{\langle Q_3\rangle}{0.8\,\mathrm{GeV}^4}\right) - 0.027(4)\left(\frac{\langle R_2\rangle}{-0.3\,\mathrm{GeV}^4}\right)\right]\,\mathrm{ps}^{-1}$$

Derived from Lenz & Nierste, JHEP 06 (2007), arXiv:hep-ph/0612167

FNAL/MILC arXiv:1602.03560v2: $\langle Q_1\rangle$ @ 6% $\langle Q_3\rangle$ @ 13%

Reducing uncertainty* on $\langle R_2\rangle$ @ $50\% \to 25\%$ $\Longrightarrow$ $\Delta\Gamma_s$ @ $25\% \to 18\%$

[Figures here are rough, e.g. prefactors above may be out of date.]

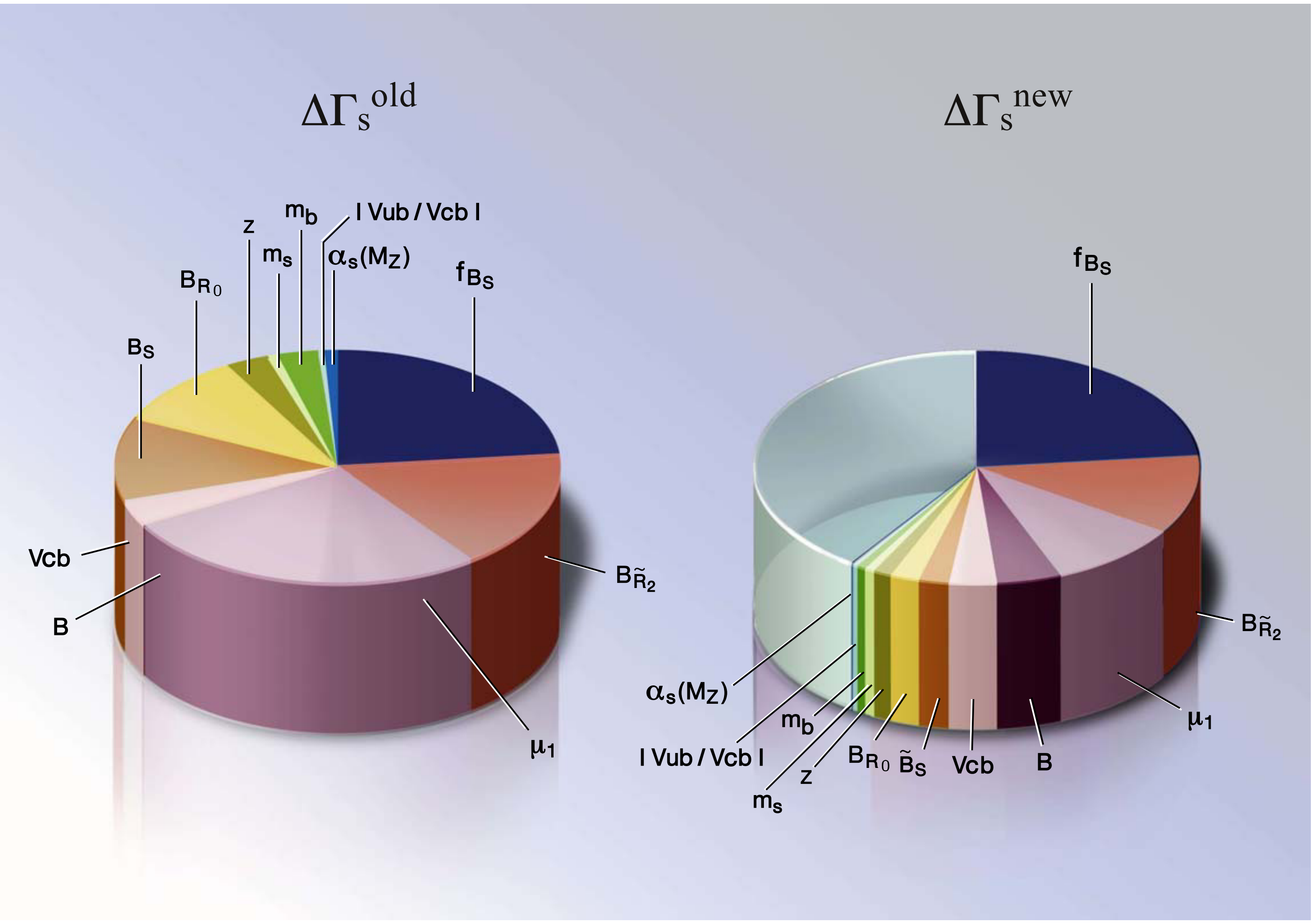
* 50% estimate from VSA, to be replaced by 25% LQCD calculation

To do list

- Double-check everything
- Extend to fine lattice, physical mass sea quarks
- Full error analysis

Stay tuned

Error reduction 1999 ➔ 2006



Lenz & Nierste, JHEP 06 (2007) 072