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Equation of state in (2+1)-flavor QCD with gradient flow

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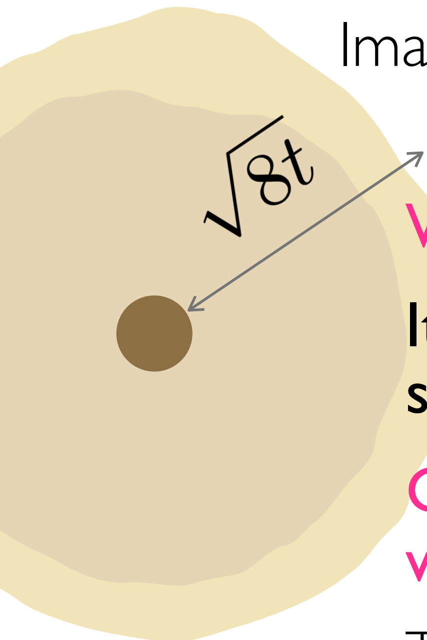
(WHOT-QCD Collaboration)

QCD Thermodynamics with Gradient Flow

Gradient flow

Lüscher(2009–), Narayanan-Neuberger(2006)



Imaginary evolution of the system into a fictitious "time" t preserving gauge sym. etc.:

(ex) pure gauge theory $\dot{B}_\mu = D_\nu G_{\nu\mu}$, $\quad B_\mu\big|_{t=0} = A_\mu \leftarrow$ original gauge field

We may view the flowed field $B\mu$ as a smeared $A\mu$ over a physical range of $\sqrt{(8t)}$.

It was shown that operators of flowed fields have no UV divergences nor short-dist. singularities at $t > 0$. Lüscher-Weisz(2011)

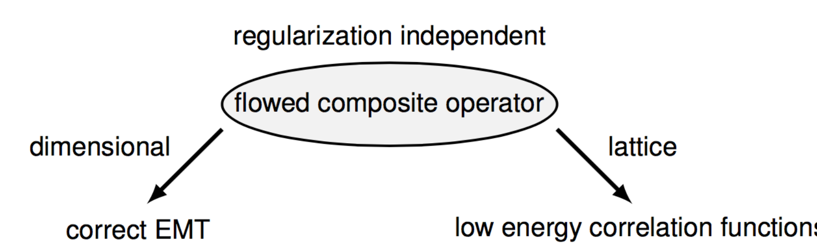
GF provides us with a new physical (i.e. non-perturbative) renormalization scheme, which is directly calculable on the lattice in the $a \to 0$ limit.

This opened many possibilities to drastically simplify lattice evaluation of physical observables.

Energy-momentum tensor from gradient flow

H.Suzuki(2013)

EMT = generator of continuous coord. trans. => not simple to define/evaluate on the lattice.



1) Define EMT by a W-T identity in a continuum scheme.

2) Relate it with a lattice operator through finite observable at $t > 0$ in the $a \to 0$ limit. By the GF evolution, however, unwanted operators can mix at $t > 0$.

3) Remove unwanted contributions using a small-t oper. expansion. The coeff's. c_i near the $t \to 0$ limit can be calculated by PT.

$$\tilde{O}(t,x) \xrightarrow[t\to 0]{} \sum_i c_i(t) O_i^{\mathrm{R}}(x)$$

=> We extract EMT, EOS etc. by $t \to 0$ & $a \to 0$ extrapolations. $\epsilon = -\langle T_{00} \rangle, \; p = \frac{1}{3}\sum_i \langle T_{ii} \rangle$

QCD Thermodynamics with Gradient Flow

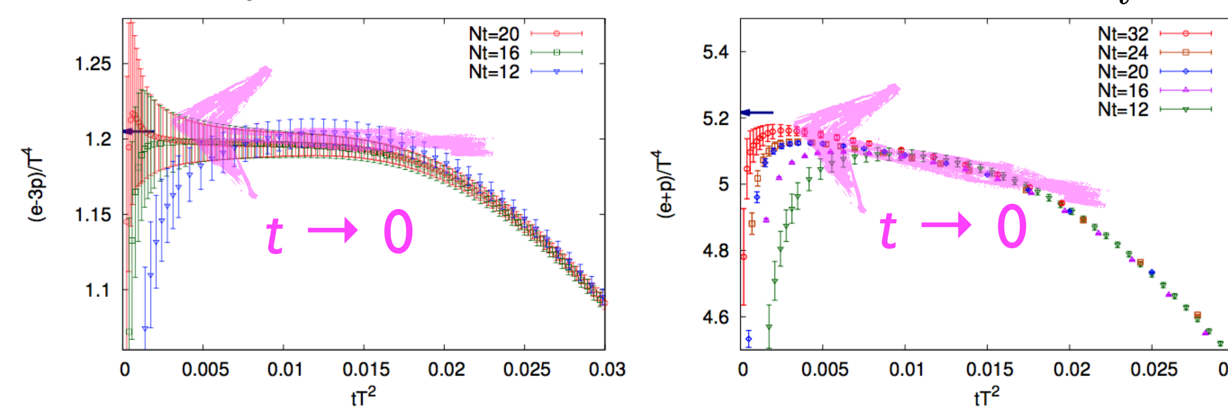
Previous test in quenched QCD

FlowQCD Collab. (2014-)

$$T^R_{\mu\nu}(x) = \lim_{t\to 0}\left\{ \frac{1}{\alpha_U(t)} U_{\mu\nu}(t,x) + \frac{\delta_{\mu\nu}}{4\alpha_E(t)}\left[E(t,x) - \langle E(t,x)\rangle_0\right]\right\}$$

$$\epsilon = -\langle T_{00}\rangle,\ p = \frac{1}{3}\sum_i \langle T_{ii}\rangle$$

The EOS' from the (T-)integration methods correctly reproduced in the $t \to 0$ and $a \to 0$ limit with less computational costs.



arXiv:1511.05235 (Lattice 2015)

Figure 2: Flow time dependence of the dimensionless interaction measure $(e-3p)/T^4$ (left panel) and the dimensionless entropy density $(e+p)/T^4$ (right panel) for different lattice spacings at $T/T_c = 1.66$. The continuum extrapolated result obtained in the integral method in Ref. [10] is indicated by the arrow at vertical axis.

Our project: Application to (2+1)-flavor QCD

GF with quarks : Lüscher, JHEP 1304, 123 (2013)

- We can adopt pure gauge actions for GF,
- at the price of a non-trivial field renormalization of quarks.

Full QCD EMT by GF : Makino-Suzuki, PTEP 2014, 063B02 (2014)

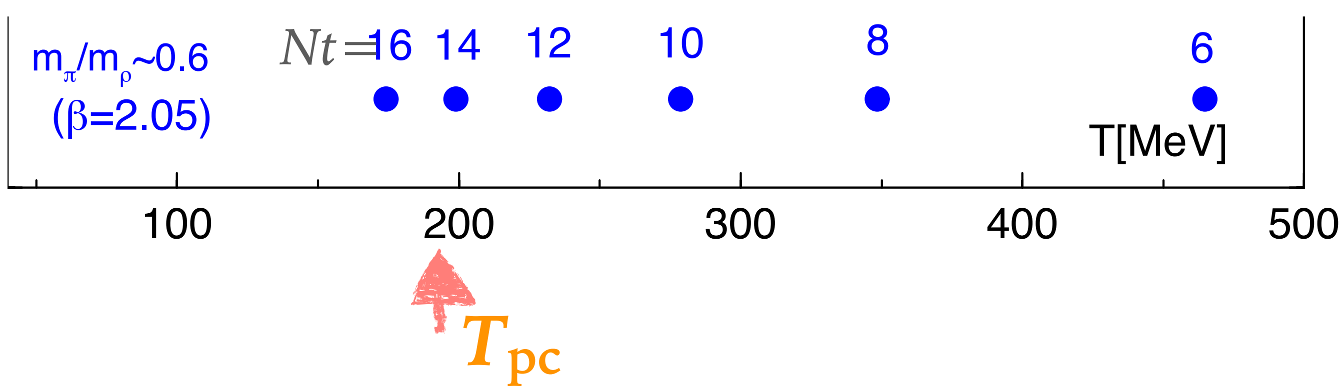
Chiral condensate by GF : Hieda-Suzuki, arXiv:1606.04193 (2016)

Topological charge / susceptibility by GF : => Talk by Taniguchi (June 29, Friday)

Simulation Parameters

WHOT-QCD, Phys.Rev.D85, 094508 (2012)

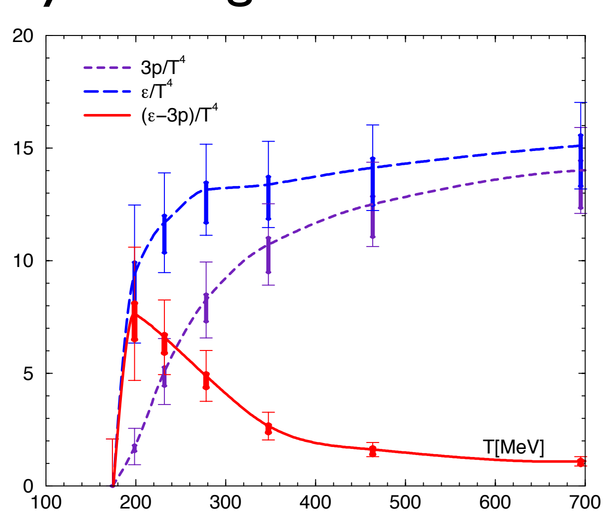
- ☑ Nf=2+1 QCD, Iwasaki gauge + NP-clover // fine lattice, physical *s* & heavy *ud*
- ☑ CP-PACS+JLQCD's $T = 0$ config. ($\beta = 2.05$, 28^3x56, $a \approx 0.07$fm, $m_{PS}/m_V \approx 0.63$) available on ILDG/JLDG
- ☑ $T > 0$ by fixed-scale approach, WHOT-QCD config.(32^3xNt, $Nt = 4, 6, 8, 10, 12, 14, 16$)
- ☑ gauge measurements at every config.
- ☑ quark measurements every 10 config's, using a noisy estimator method.
- ☐ continuum extrapolation => next step study



T (MeV)	T/T_{pc}	N_t	$t_{1/2}$	gauge confs.
0	0	56	24.5	650
174	0.92	16	8	1440
199	1.05	14	6.125	1270
232	1.22	12	4.5	1290
279	1.47	10	3.125	780
348	1.83	8	2	510
464	2.44	6	1.125	500
697	3.67	4	0.5	700

T_{pc} = 190 MeV assumed

EOS by *T*-integration method available



To avoid oversmearing. wrapping around the lattice:

$$\sqrt{(8t/a^2)} \leq \min(Ns/2, Nt/2)$$

i.e., $\boldsymbol{t/a^2 \leq t_{1/2}} = [\min(Ns/2, Nt/2)]^2/\sqrt{8}$

=> to be compared with GF!

Gauge and Quark Flows

Lüscher, JHEP 1008, 071 (2010); 1304, 123 (2013)

We adopt the simplest one suggested by Lüscher.

Gauge flow: standard Wilson flow

original gauge field at t = 0

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \qquad B_\mu(t=0,x) = A_\mu(x)$$

$$G_{\mu\nu}(t,x) = \partial_\mu B_\nu(t,x) - \partial_\nu B_\mu(t,x) + [B_\mu(t,x), B_\nu(t,x)],$$

$$D_\nu G_{\nu\mu}(t,x) = \partial_\nu G_{\nu\mu}(t,x) + [B_\nu(t,x), G_{\nu\mu}(t,x)],$$

Quark flow: as suggested by Lüscher

original quark field at t = 0

$$\partial_t \chi_f(t,x) = \Delta \chi_f(t,x), \qquad \chi_f(t=0,x) = \psi_f(x),$$

$$\partial_t \bar{\chi}_f(t,x) = \bar{\chi}_f(t,x) \overleftarrow{\Delta}, \qquad \bar{\chi}_f(t=0,x) = \bar{\psi}_f(x),$$

$$\Delta \chi_f(t,x) \equiv D_\mu D_\mu \chi_f(t,x), \qquad D_\mu \chi_f(t,x) \equiv [\partial_\mu + B_\mu(t,x)]\, \chi_f(t,x),$$

$$\bar{\chi}_f(t,x) \overleftarrow{\Delta} \equiv \bar{\chi}_f(t,x) \overleftarrow{D}_\mu \overleftarrow{D}_\mu, \qquad \bar{\chi}_f(t,x) \overleftarrow{D}_\mu \equiv \bar{\chi}_f(t,x) \left[\overleftarrow{\partial}_\mu - B_\mu(t,x)\right]$$

only gauge fields involved

Nf=2+1 QCD EMT by GF

Makino-Suzuki, PTEP 2014, 063B02 (2014)

EMT in full QCD

Operators on the lattice

$$\tilde{O}_{1\mu\nu}(t,x) \equiv G^a_{\mu\rho}(t,x)G^a_{\nu\rho}(t,x),$$
$$\tilde{O}_{2\mu\nu}(t,x) \equiv \delta_{\mu\nu}G^a_{\rho\sigma}(t,x)G^a_{\rho\sigma}(t,x),$$
$$\tilde{O}^f_{3\mu\nu}(t,x) \equiv \varphi_f(t)\bar{\chi}_f(t,x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\chi_f(t,x),$$
$$\tilde{O}^f_{4\mu\nu}(t,x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x),$$
$$\tilde{O}^f_{5\mu\nu}(t,x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\chi_f(t,x),$$

$$T_{\mu\nu}(x) = \lim_{t\to 0}\Big\{c_1(t)\left[\tilde{O}_{1\mu\nu}(t,x) - \frac{1}{4}\tilde{O}_{2\mu\nu}(t,x)\right] + c_2(t)\left[\tilde{O}_{2\mu\nu}(t,x) - \left\langle \tilde{O}_{2\mu\nu}(t,x)\right\rangle_0\right] + c_3(t)\sum_{f=u,d,s}\left[\tilde{O}^f_{3\mu\nu}(t,x) - 2\tilde{O}^f_{4\mu\nu}(t,x) - \left\langle \tilde{O}^f_{3\mu\nu}(t,x) - 2\tilde{O}^f_{4\mu\nu}(t,x)\right\rangle_0\right] + c_4(t)\sum_{f=u,d,s}\left[\tilde{O}^f_{4\mu\nu}(t,x) - \left\langle \tilde{O}^f_{4\mu\nu}(t,x)\right\rangle_0\right] + \sum_{f=u,d,s} c^f_5(t)\left[\tilde{O}^f_{5\mu\nu}(t,x) - \left\langle \tilde{O}^f_{5\mu\nu}(t,x)\right\rangle_0\right]\Big\},$$

VEV-subtraction
($T = 0$ subtraction)

Quark field renormalization

$$\varphi_f(t) \equiv \frac{-6}{(4\pi)^2 t^2 \left\langle \bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x)\right\rangle_0}.$$

Coefficients of Makino-Suzuki by PT.

$$\boxed{\begin{aligned}
c_1(t) &= \frac{1}{\bar{g}(1/\sqrt{8t})^2} - \frac{1}{(4\pi)^2}\left[9(\gamma - 2\ln 2) + \frac{19}{4}\right],\\
c_2(t) &= \frac{1}{(4\pi)^2}\frac{33}{16},\\
c_3(t) &= \frac{1}{4}\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[2 + \frac{4}{3}\ln(432)\right]\right\},\\
c_4(t) &= \frac{1}{(4\pi)^2}\bar{g}(1/\sqrt{8t})^2,\\
c^f_5(t) &= -\bar{m}_f(1/\sqrt{8t})\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[4(\gamma - 2\ln 2) + \frac{14}{3} + \frac{4}{3}\ln(432)\right]\right\}
\end{aligned}}$$

Physics extracted by $t \to 0$ extrapolation.

At $a > 0$

$$T_{\mu\nu}(t,x,a) = T_{\mu\nu}(t,x) + A_{\mu\nu}\frac{a^2}{t} + \sum_f B_{f\mu\nu}(am_f)^2 + C_{\mu\nu}(aT)^2 + D_{\mu\nu}(a\Lambda_{\rm QCD})^2 + a^2 S'_{\mu\nu}(x) + \mathcal{O}(a^4),$$

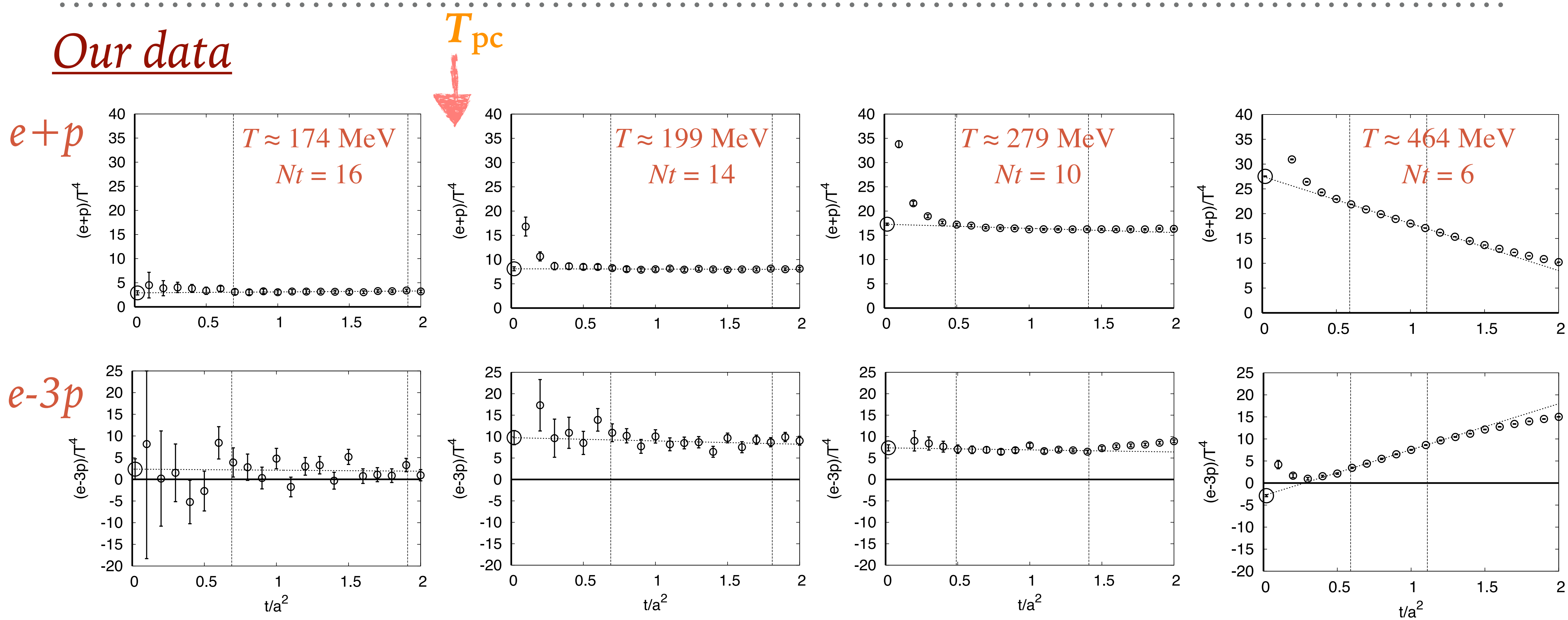
Singular term at $t \to 0$ due to mixing with D=4 ops.

Note: lattice artifacts of NP-clover is O(a^2).

Nf=2+1 QCD EoS by GF

$$\epsilon = -\langle T_{00} \rangle,\ p = \frac{1}{3}\sum_i \langle T_{ii} \rangle$$

Our data

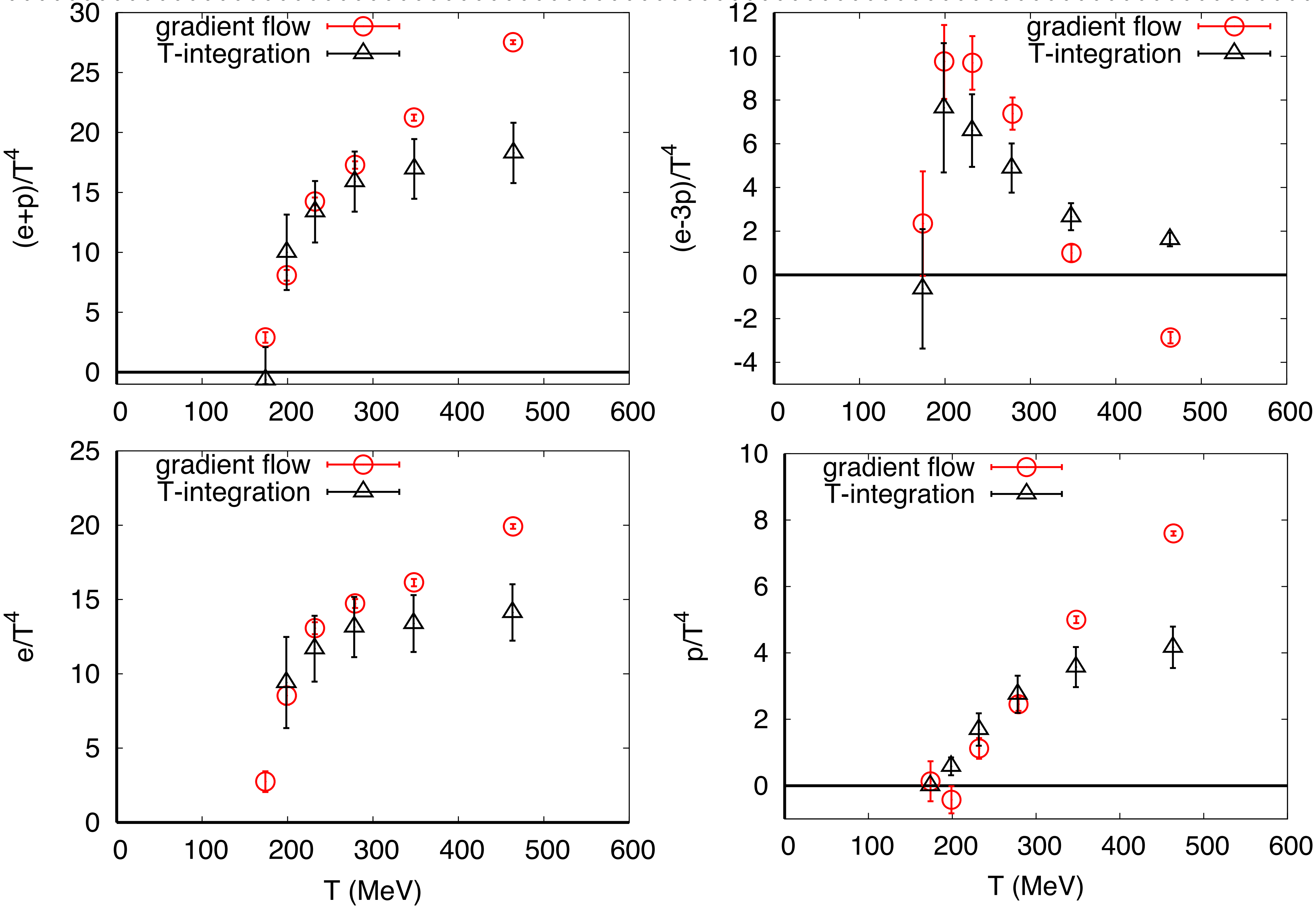


- a^2/t-like behavior close to t = 0.
- Wide linear behavior within meaningful range of t. <= $\sqrt{(8t/a^2)} \leq \min(Ns/2, Nt/2)$ to avoid oversmearing.
- a^2/t term suggested to be negligible in the windows <= confirmed to be so from non-linear fits including $1/t$.
- after several try & errors **=>** **Linear fit choosing linear window**
- At $T \approx 697$ MeV (Nt=4), no linear window found. We perform a non-linear fit, but data dominated by lattice artifacts within the meaningful range of t. Results at this T should not be taken seriously.

Nf=2+1 QCD EoS by GF

GF errors include statistical + syst. from c_i's
T-integration results from PRD85,094508('12)

Results



- Good agreement with the conventional method at $T \leq 300$ MeV ($Nt \geq 10$).
- Though a definite comparison possible only at $a \to 0$, GF results with similar amount are encouraging.

Chiral Condensate by GF

Hieda-Suzuki, arXiv:1606.04193 (2016)

From axial W-T identity $$\left\{\bar{\psi}_f\psi_f\right\}^{(0)}(t,x) = \left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[4(\gamma - 2\ln 2) + 8 + \frac{4}{3}\ln(432)\right]\right\} \times \frac{\bar{m}_f(1/\sqrt{8t})}{m_f}\left[\varphi_f(t)\bar{\chi}_f(t,x)\chi_f(t,x)\right]$$

At $m_f > 0$, chiral cond. in usual lattice simulation can have m_f/a^2 singularity.

With GF, such divergence is prohibited by the finiteness of flowed operators, but m_f/t can appear, instead.

In fact, to the lowest order of PT, we do encounter such m_f/t term.

$$\begin{aligned}&\sum_{f,f'=u,d,s}\sqrt{\varphi_f(t)}\sqrt{\varphi_{f'}(t)}\,\bar{\chi}_f(t,x)\,\{\{t^A,M\},t^B\}_{ff'}\chi_{f'}(t,x)\\ &\overset{t\to 0}{\sim}\left[-\frac{12}{(4\pi)^2}\sum_{f=u,d,s}\left(\{\{t^A,M\},t^B\}M\left\{\frac{1}{2t} + M^2\left[\gamma + \ln(2M^2t)\right] + \mathcal{O}(t)\right\}\right)_{ff} + \mathcal{O}(g^2)\right]\mathbb{1}\\ &\quad + [1 + \mathcal{O}(g^2)]\bar{\psi}(x)\{\{t^A,M\},t^B\}\psi(x) + \mathcal{O}(t).\end{aligned}$$

To remove this obstacle in the $t \to 0$ extrapolation, Hieda-Suzuki suggests a VEV-subtraction.

$$\left\{\bar{\psi}_f\psi_f\right\}(x) = \lim_{t\to 0}\left\{1 + \frac{\bar{g}(1/\sqrt{8t})^2}{(4\pi)^2}\left[4\left(\gamma - 2\ln 2\right) + 8 + \frac{4}{3}\ln(432)\right]\right\} \times \frac{\bar{m}_f(1/\sqrt{8t})}{m_f}\left[\varphi_f(t)\,\bar{\chi}_f(t,x)\,\chi_f(t,x) - \mathrm{VEV}\right].$$

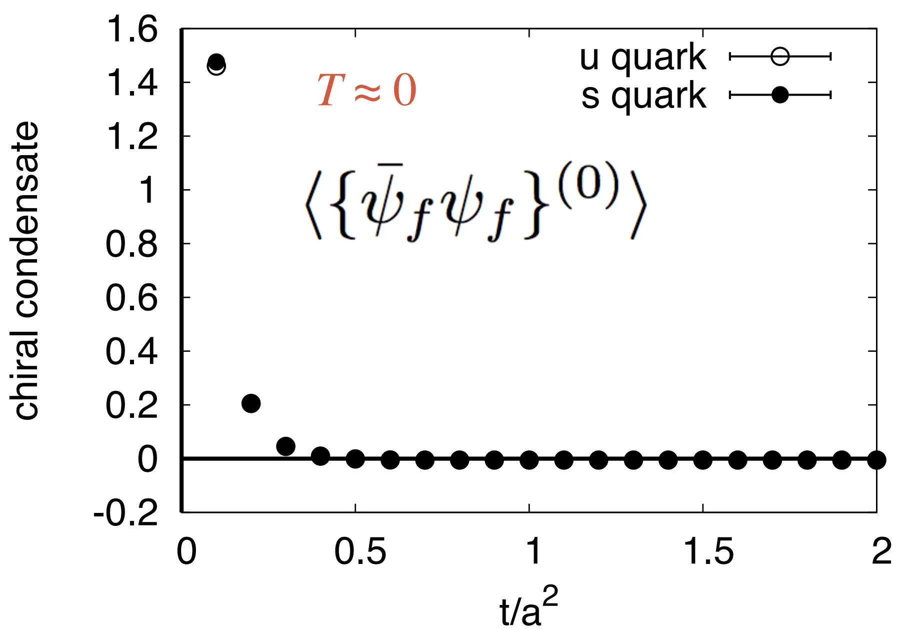
Chiral Condensate by GF

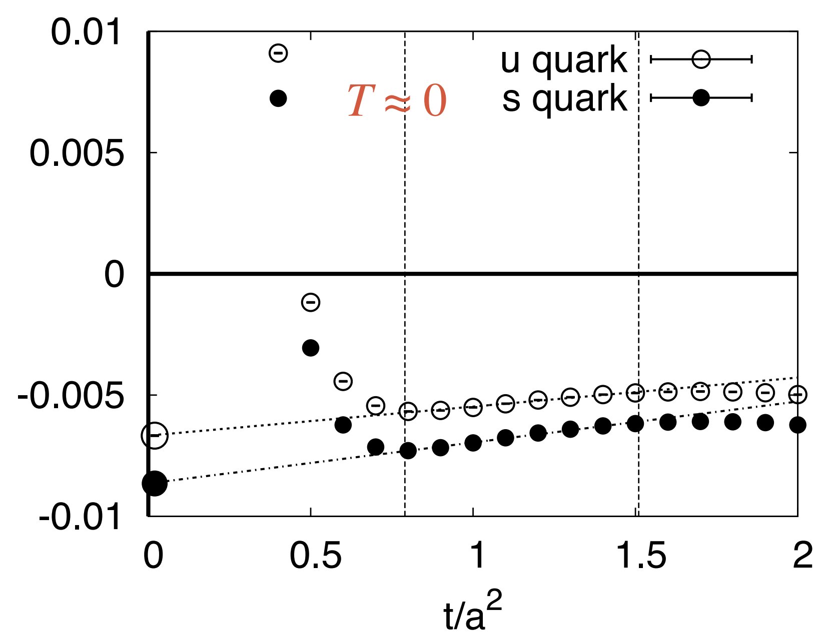
At a > 0

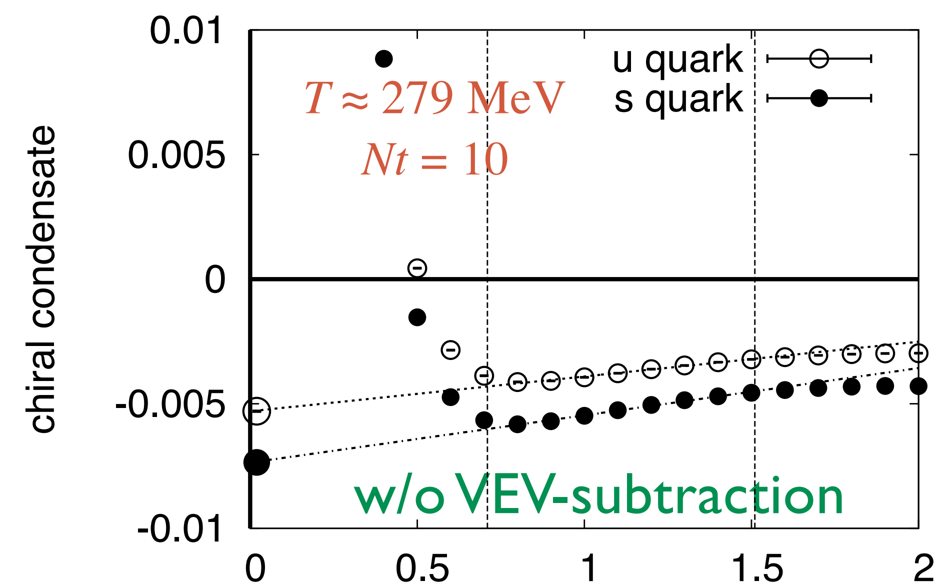
We have both m_f/t and a^2/t terms we should remove.

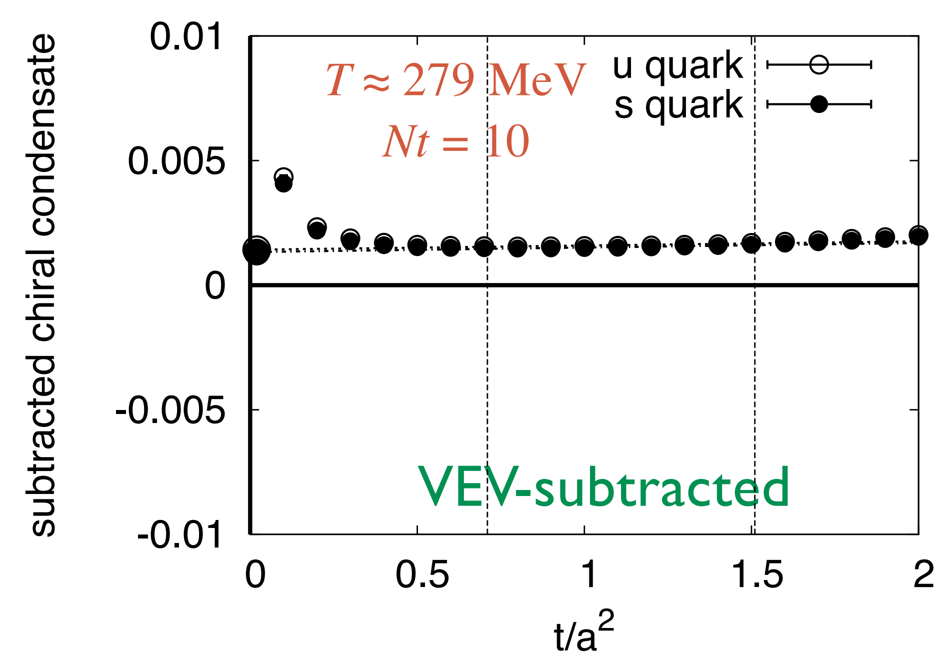
$$\left\{\bar{\psi}_f\psi_f\right\}^{(0)}(t,x,a) = \left\{\bar{\psi}_f\psi_f\right\}^{(0)}(t,x) + A\frac{a^2}{t} + \sum_f B_f(am_f)^2 + C(aT)^2 + D\left(a\Lambda_{\rm QCD}\right)^2 + a^2 S(x) + \mathcal{O}(a^4),$$

Our data







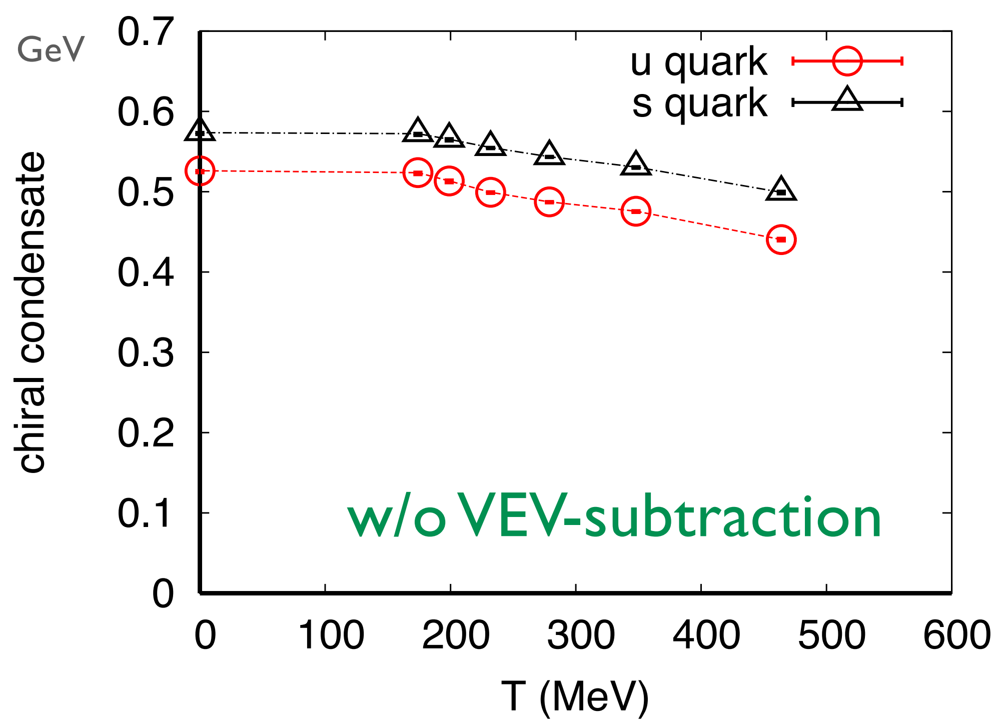


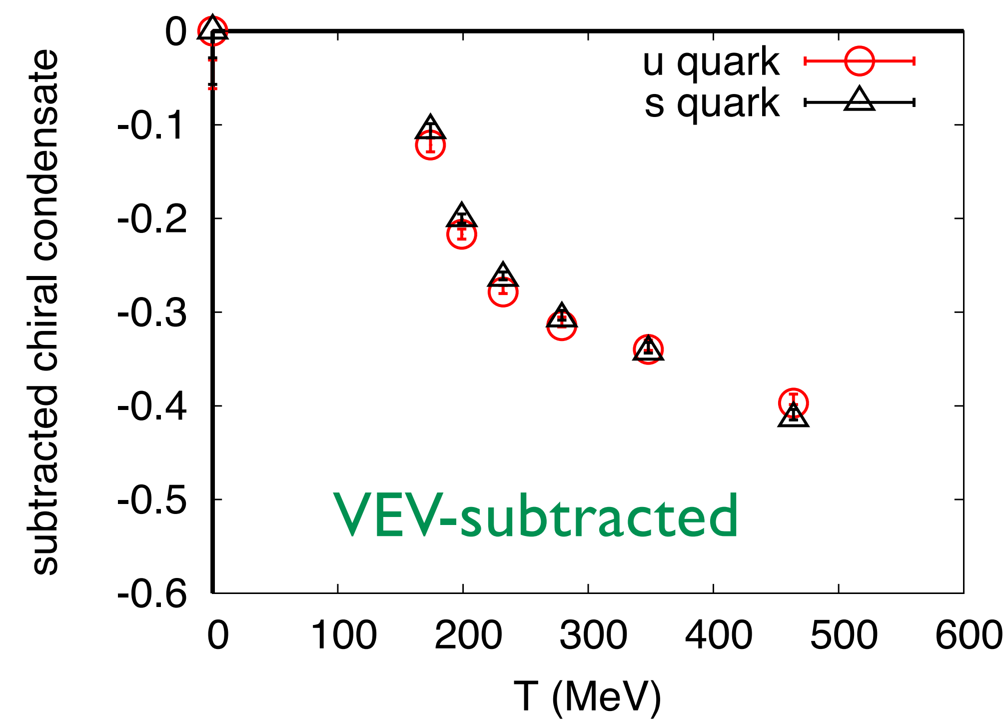
- Singular behavior at t ≈ 0, but m_f-dep. small.
- => linear fit as before.
- Wider linear region by VEV-subtraction
- <= Large part of a2/t. also removed by the VEV-subtraction.

Chiral Condensate by GF

$\overline{\text{MS}}$ scheme at $\mu = 2$ GeV
Errors include statistical + syst. from pert. coeff's

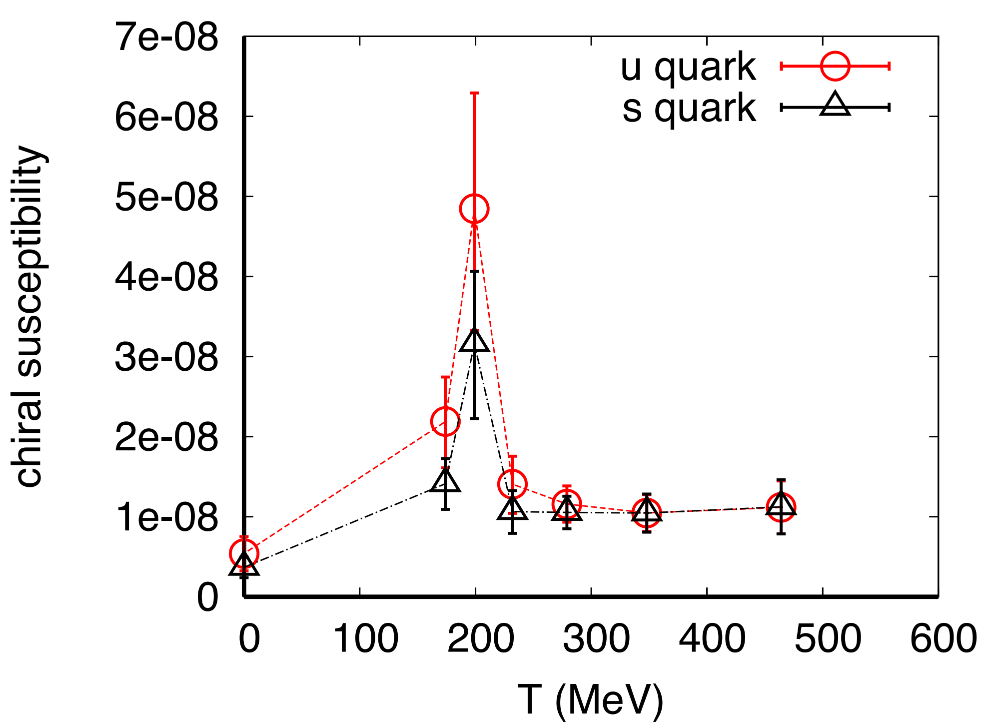
Results





- Crossover suggested at $T \approx 190$ MeV.

Disconnected chiral susceptibility



Disconnected part only.

VEV-subtraction no effects in this quantity.

Windows for linear fit clear, except for T≈697MeV.

- Clear peak at T≈190 MeV, as expected.
- Peak higher with decreasing m_q.

SUMMARY

LATTICE 2016

- We apply gradient flow ideas to investigate thermodynamics of (2+1)-flavor QCD. As the first test, we choose heavy ud quarks with physical s quark, on a fine lattice ($a \approx 0.07$fm, $m_{PS}/m_V \approx 0.63$), and adopt the fixed-scale approach.
- EOS agrees with conventional T-integration method at $T \leq 300$ MeV ($Nt \geq 10$).
- A definite comparison possible only after cont. extrapolation. The good agreement at $Nt \geq 10$ suggests that our a sufficiently small, but small-Nt artifact large at $Nt \leq 8$.
- Chiral condensate and its disconnected susceptibility also calculated. Even with the explicit chiral violation of Wilson-type quarks, we obtain reasonable results, reassuring the powerfulness of the GF method.
- Results for topological susceptibility. also encouraging. => *Taniguchi (June 29, Friday, 17:10-)*
- Further study needed to complete the continuum extrapolation.