A study of the radiative transition $\pi\pi \to \pi\gamma^*$ with lattice QCD

Luka Leskovec

in collaboration with:
Constantia Alexandrou, Giannis Koutsou, Stefan Meinel, John Negele, Srijit Paul, Marcus Petschlies, Andrew Pochinsky, Gumaro Rendon, Sergey Syritsyn

University of Southampton, July 24-30, 2016
34th International Symposium on Lattice Field Theory

Lellouch and Lüscher [hep-lat/0003023]:

... One might think that having a finite volume ... makes it even more difficult to extract the transition amplitudes. ... this is actually not so.

lattice QCD in the past:

- masses, resonance masses, strong decay widths
- form factors, structure properties
- radiative and weak transitions between (strongly) stable hadrons

Limits the physics we can do!

lattice QCD now:

- transition matrix elements between a stable and unstable hadron

Explore what the Briceño formalism can do!
[Briceño et al. 1406.5965]

A clear channel where a lot is already known: [Crisafulli & Lubicz PLB278, HadSpec 1507.06622, 1604.03530]

$$\pi^+\pi^0 \to \gamma^*\pi^+$$

part I - the ρ meson parameters

- Operators used to describe the ρ meson
- Wick contractions and how we calculate them
- Using the Lüscher method to determine the ρ parameters
- The Breit-Wigner fit

part II - the radiative transition

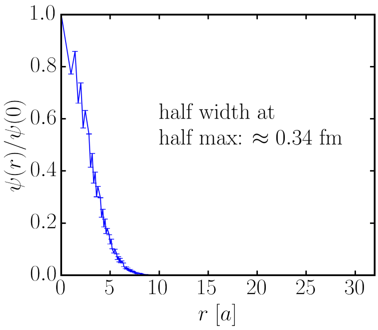
- The three-point functions
- Wick contractions and their calculation
- Optimized three-point functions
- Radiative transition natrix elements
- Mapping from the Finite Volume to the Infinite Volume

Lattice setup

- preliminary results
- $N_f = 2+1$ Clover fermions
- istotropic lattice by Orginos et al.
- non-polynomial FV effects very small: $e^{-m_\pi L} \approx 0.3\%$
- m_π low enough: ρ is unstable

Label	$N_s^3 \times N_t$	$a\,(\mathrm{fm})$	L (fm)	m_π (MeV)	N_{config}
C13	$32^3 \times 96$	0.11403	3.65	317	367/1050

- forward, sequential and stochastic propagators
- 8 sources/config
- Wuppertal smearing at source/sink: $(N, \alpha) = (20, 3.0)$



part I - the ρ meson parameters

ρ meson: $J^P = 1^-$: p-wave $\pi\pi$ scattering [a lot of previous studies]

- 3/4 operators with total momentum $\vec{p}_{\pi\pi} = \frac{2\pi}{L}\vec{d}$:

$$O_1 = \bar{u}\gamma_\mu d(\vec{d})$$
$$O_2 = \bar{u}\gamma_t\gamma_\mu d(\vec{d})$$
$$O_3 = \pi^+(\vec{p}_1)\pi^0(\vec{p}_2) - \pi^+ \leftrightarrow \pi^0$$
$$(*) O_4 = \pi^+(\vec{p}_1')\pi^0(\vec{p}_2') - \pi^+ \leftrightarrow \pi^0$$

$\vec{d}$	$(0,0,0)$
$\vec{d}$	$(0,0,1)$
$\vec{d}$	$(0,1,1)$

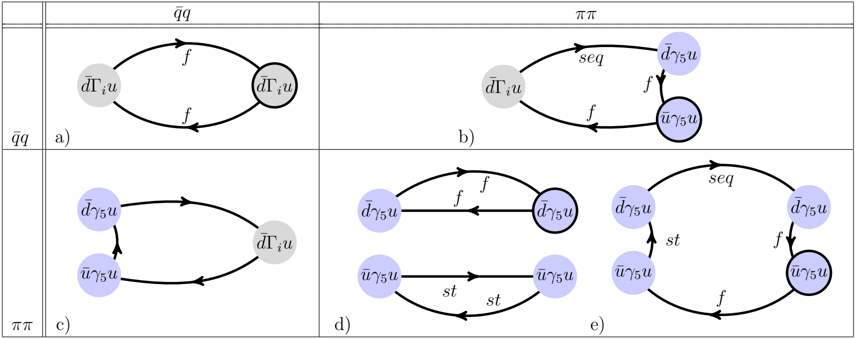
and permutations

- multiple moving frames:

$$\vec{p}_{\pi\pi} = \vec{p}_1^{(\prime)} + \vec{p}_2^{(\prime)}$$

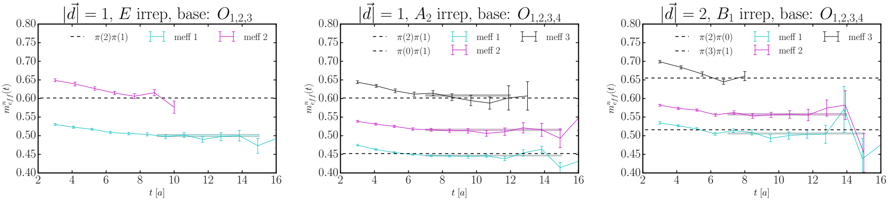
following Gottlieb & Rummukainen:
[Gottlieb & Rummukainen hep-lat/9503028]

- $\vec{d} = (0,0,0)$ O_h:
  T_1 irrep, 3 polarizations
- $\vec{d} = (0,0,1)$ D_{4h}:
  A_2 irrep, 1 polarization
  E irrep, 2 polarizations
- $\vec{d} = (0,1,1)$ D_{2h}:
  B_1 irrep, 1 polarization
  B_2 irrep, 1 polarization
  B_3 irrep, 1 polarization



- black line u/d quark

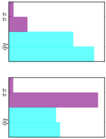
- f - forward propagator
- *seq* - sequential propagator
- *st* - stochastic propagator

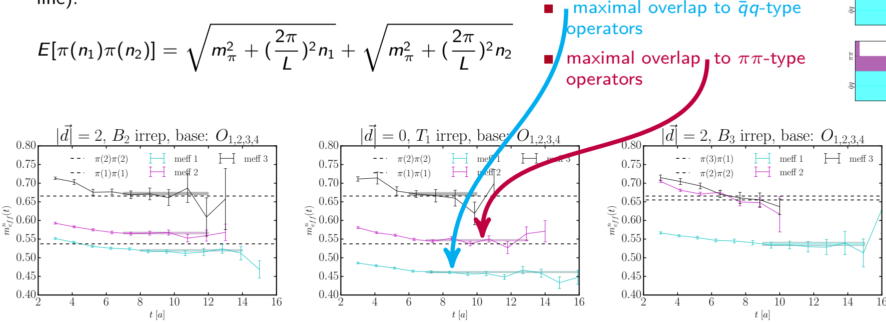


- non interacting energy level (dashed line):

$$E[\pi(n_1)\pi(n_2)] = \sqrt{m_\pi^2 + (\frac{2\pi}{L})^2 n_1} + \sqrt{m_\pi^2 + (\frac{2\pi}{L})^2 n_2}$$

- maximal overlap to $\bar{q}q$-type operators
- maximal overlap to $\pi\pi$-type operators





map from finite volume spectrum to infinite volume phase shifts

$$s_{\pi\pi} = E_n^2 - \left(\tfrac{2\pi}{L}\vec{d}\right)^2$$

$$\sqrt{s_{\pi\pi}} = 2\sqrt{m_\pi^2 + p^{*2}} \text{ and find } p^*$$

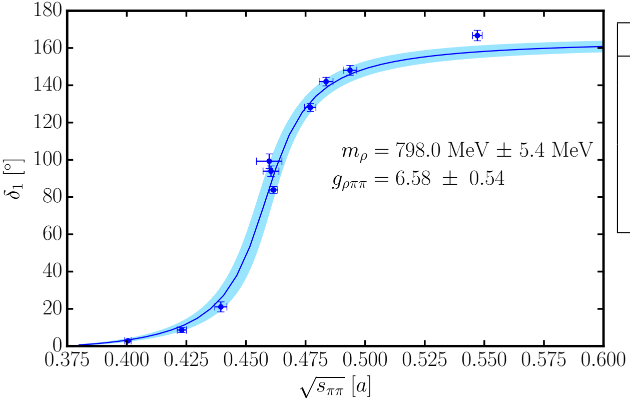
$$k = \tfrac{L}{2\pi} p^*$$

$$\tan \delta_1^{\vec{d}=(0,0,0),T_1} = \frac{\gamma \pi^{3/2} k}{Z_{00}^{\vec{d}}(1;k^2)}$$

$$\tan \delta_1^{\vec{d}=(0,0,1),A_2} = \frac{\gamma \pi^{3/2} k}{Z_{00}^{\vec{d}}(1;k^2) + \frac{2}{k^2\sqrt{5}} Z_{20}^{\vec{d}}(1;k^2)}$$

$$\dots$$

see e.g. [Briceño 1401.3312]



$\vert\vec{d}\vert$	irrep	n
0	T_1	1, 2
1	E	1
1	A_2	1, 2
2	B_1	1, 2
2	B_2	1, 2
2	B_3	1

- 10 points
- 3 frames
- 6 irreps

(Simple) Breit-Wigner fit:

$$\tan\delta_1 = \frac{\sqrt{s}\Gamma(s)}{m_\rho^2 - s} \qquad \Gamma(s) = \frac{g_{\rho\pi\pi}^2}{6\pi}\frac{p^{*3}}{s}$$

part II - the radiative transition

- initial state:
  $J^P = 1^-$: p-wave $\pi\pi$ scattering
  $\vec{d} = (0, 0, 0)$:

$$O_{\pi\pi}^{p=1} = \bar{u}\gamma_\mu d(\vec{d})$$
$$O_{\pi\pi}^{p=2} = \bar{u}\gamma_t\gamma_\mu d(\vec{d})$$
$$O_{\pi\pi}^{p=3} = \pi^+(\vec{p}_1)\pi^0(\vec{p}_2) - \pi^+ \leftrightarrow \pi^0$$
$$O_{\pi\pi}^{p=4} = \pi^+(\vec{p}_1')\pi^0(\vec{p}_2') - \pi^+ \leftrightarrow \pi^0$$

- final state: π:

$$O_\pi = \bar{u}\gamma_5 d(\vec{p}_\pi)$$

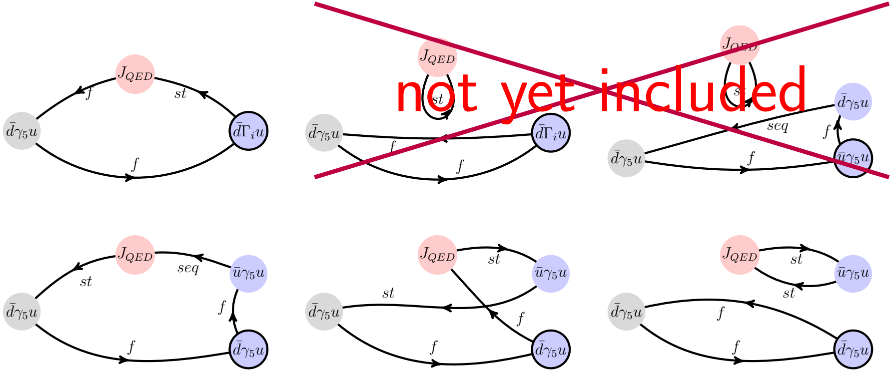
- local current operator:

$$J_{QED} = O_c = Z_V(\frac{2}{3}\bar{u}\gamma_\mu u(\vec{Q}) - \frac{1}{3}\bar{d}\gamma_\mu d(\vec{Q}))$$

- $Z_V = 0.79700(24)$ [LHPC internal communication]

Three point functions for each $O_{\pi\pi}^p$:

$$C_3^p(t_\rho, t_J, t_\pi) = \langle O_\pi(t_\pi, \vec{p}_\pi) O_c(t_J, \vec{Q}) O_{\pi\pi}^p(t_{\rho,\vec{p}_{\pi\pi}}) \rangle$$


not yet included
J_{QED}
$\bar{d}\gamma_5 u$
$\bar{d}\Gamma_i u$
$\bar{u}\gamma_5 u$
f
st
seq

three point function:

$$C_3^p(t_{src} = t_{\pi\pi}, t_J, t_{snk} = t_\pi) = C_3^p(t_J, \Delta t = t_{snk} - t_{src})$$
$$= \sum_{n \in T_1, \vec{d}=(0,0,0)} \langle 0 | O^p_{\pi\pi} | n, T_1, \nu \rangle \langle n, T_1, \nu, | J^\mu_{QED}, \vec{Q} | \pi, \vec{p}_\pi \rangle \langle \pi | O_\pi | 0 \rangle \times \frac{e^{-E_n(t_J - t_n)} e^{-E_\pi(t_J - t_\pi)}}{2E_\pi E_n}$$

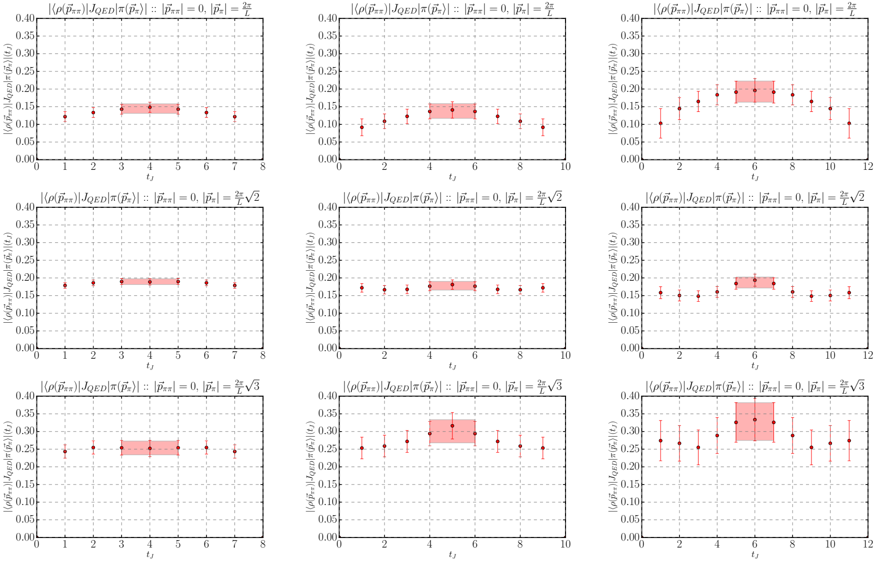
ratio:[Detmold et al. 1503.01421]

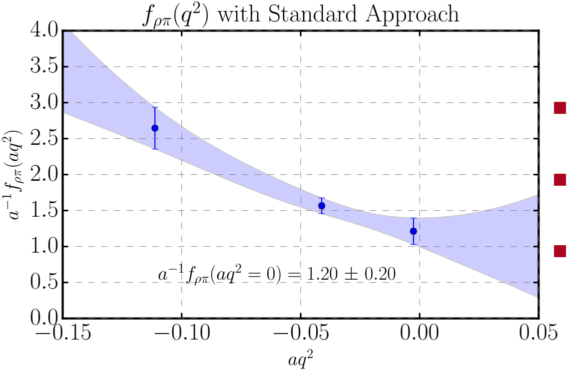
$$R^S(t_J) = \frac{C_3(t_J, \Delta t) C_3^*(\Delta t - t_J, \Delta t)}{C_2^{(n)}(\Delta t) C_2^{(\pi)}(\Delta t)} \rightarrow |\langle n, T_1, \nu, (\vec{p}_{\pi\pi}) | J^\mu_{QED}, \vec{Q} | \pi, \vec{p}_\pi \rangle|^2$$

decomposition:

$$|\langle n, T_1, \nu | J^\mu_{QED}, \vec{Q} | \pi, \vec{p}_\pi \rangle| = f_{\rho\pi}(q^2, s_{\pi\pi}) \epsilon_{\nu\mu\alpha\beta} (p_\rho)_\alpha (p_\pi)_\beta$$
$$q = p_\pi - p_{\pi\pi}$$

Δ t = 8 | Δ t = 10 | Δ t = 12





- ρ is not a particle - a resonance
- looks sensible (within approx)
- compare $f_{\rho\pi}(0)$ from branching ratios

us [GeV]	[Crisafulli] [GeV]	exp [GeV]
~0.69	~0.75@LFq^2	~0.73

To determine matrix elements from excited states use optimized three point function:
[HadSpec 0902.2241, Becirevic 1411.6426, HadSpec 1501.07457+]

$$\Omega_3(t_J, \Delta t) = v_p^{(n)} C_3^p(t_J, \Delta t = t_{snk} - t_{src})$$

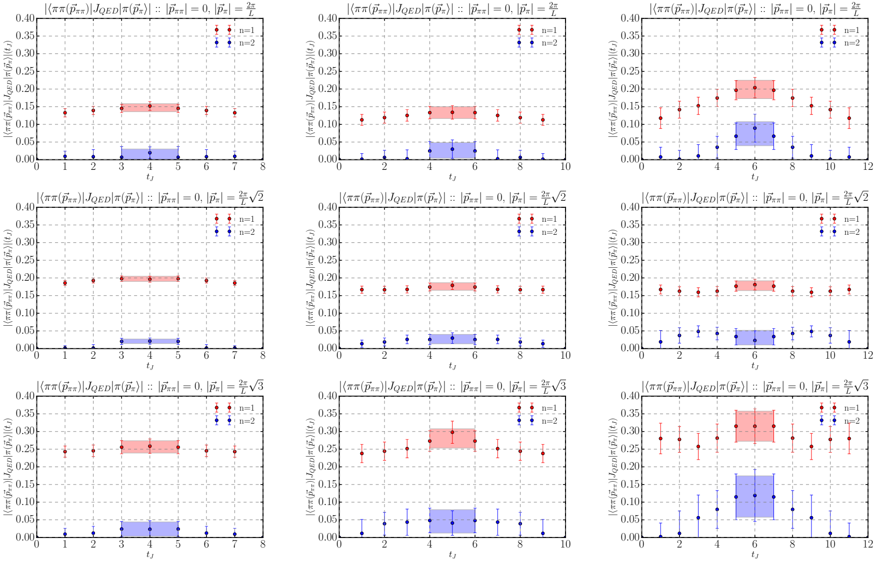
Use R^S to determine:

$$\langle n, T_1, \nu, \vec{p}_{\pi\pi} | J^{\mu}_{QED}, \vec{Q} | \pi, \vec{p}_{\pi} \rangle$$

The $\pi\pi \to \pi\gamma$ matrix element will have the same lorentz decomposition as the "form factor".

$\pi\pi \to \pi\gamma^*$ radiative transition
matrix elements in finite volume

Δ t = 8 | Δ t = 10 | Δ t = 12



The mapping described by the Lellouch-Lüscher factor
[Lellouch & Lüscher hep-lat/0003023, Briceño et al. 1406.5965]

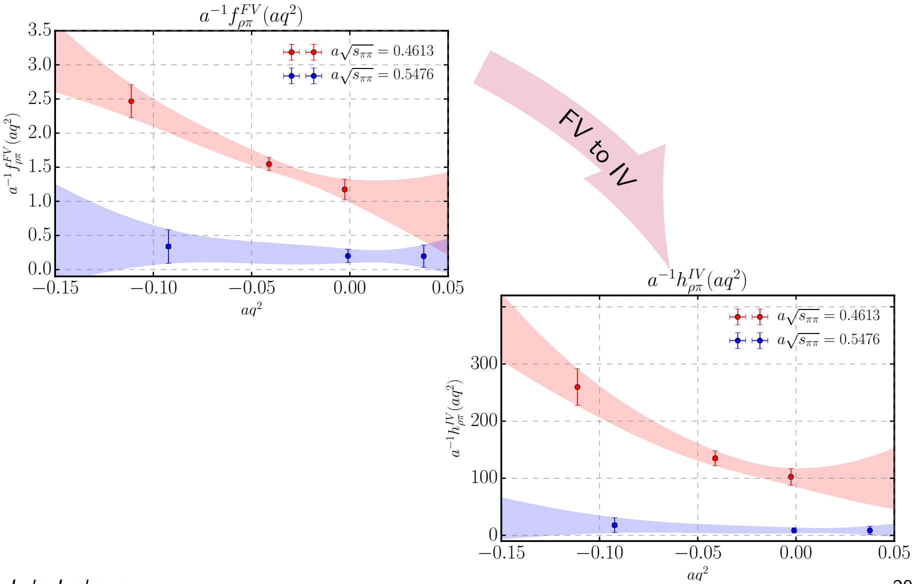
$$\frac{|H_{IV}(T_1, \nu; J^{\mu}_{QED}; q^2, s_{\pi\pi})|^2}{|\langle n, T_1, \nu, \vec{p}_{\pi\pi}| J^{\mu}_{QED}, \vec{Q}|\pi, \vec{p}_{\pi}\rangle|^2} = \frac{32\pi E_{\pi}\sqrt{s_{\pi\pi}}}{k}\left[\frac{\partial\delta_1(\sqrt{s_{\pi\pi}})}{\partial E_{\pi\pi}} + \frac{\partial\phi_1^{\vec{d}}(k)}{\partial E_{\pi\pi}}\right]$$

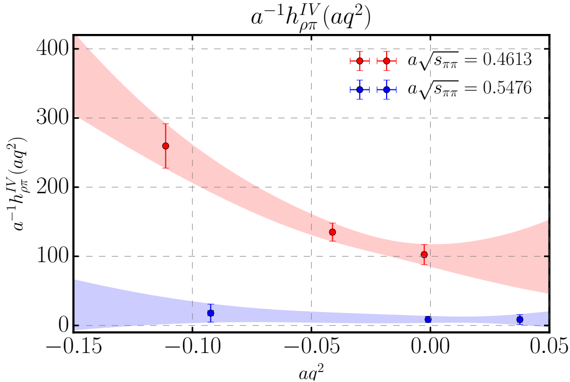
Map from FV matrix element to IV amplitude!

$$|H_{IV}(T_1, \nu; J^{\mu}_{QED}; q^2, s_{\pi\pi})| = h_{\rho\pi}(q^2, s_{\pi\pi})\epsilon_{\nu\mu\alpha\beta}(p_{\rho})_{\alpha}(p_{\pi})_{\beta}$$

$\pi\pi \to \pi\gamma^*$ radiative transition

mapping from the FV to IV at $\Delta t = 10$





- the amplitude for various q^2 and $\sqrt{s_{\pi\pi}}$
- looks promising for some interesting features
- qualitatively similar to the HadSpec study [Briceño et al. 1507.06622]
- still a long way, but theres a light at the end of the tunnel!

Conclusion

not real conclusions ... more like comments

- the ensemble C13:
  great to work with!
- method of forward, sequential and stochastic props:
  - two-point and three-point functions
    with the same setup of propagators
  - good for studying scattering of strong particles
  - good for determining three point functions
  - scales well with volume (@$L = 3.65$ fm)
- the Lüscher method ...
  no need to re#
- the Briceño formalism:
  - still much to explore and more to understand
  - difficult but fun - access to interesting physics
  - need to understand the interesting physics

Thank you for your attention!

:)