dark matter from one-flavor SU(2) gauge theory

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Outline

- motivation
- the basic theory
- preliminary lattice explorations
- coupling to the standard model
motivation

- a minimal non-Abelian dark sector
- dark matter stability exists naturally
**SU(2) with one Dirac flavor**

\[ \mathcal{L} = -\frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu} + i \bar{Q} \gamma^\mu D_\mu Q + m Q^T C E Q \]

where \[ Q = \begin{pmatrix} \chi^L \\ C \bar{\chi}^T \end{pmatrix} \]
\[ E = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \]

\( Q \) has no standard model quantum numbers.

\( \mathcal{L} \) has an unbroken global \( SU(2) \), \( Q \to e^{i \sum_{i=1}^{3} T_i \alpha_i} Q \), which is the generalization of baryon number.

For \( m=0 \), \( \mathcal{L} \) has an unbroken (but anomalous) global \( U(1) \), \( Q \to e^{i \beta} Q \), like the axial \( U(1) \) in QCD. We expect the \( U(1) \) to be broken dynamically by a mass-like vev.

For \( N_f > 1 \), the global \( SU(2) \) would be \( SU(2N_f) \).
the particle spectrum

The theory will have mesons, baryons and glueballs. Simple operators for mesons: $\bar{Q} \Gamma Q$
Simple operators for baryons: $Q^T C \Gamma E Q$

Recall $E = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

The spectrum forms multiplets of the global SU(2).

Examples:
$\bar{Q} \gamma_5 Q$ is a singlet. Let’s name it $\eta$.
$\bar{Q} \gamma_\mu Q$ and baryon and antibaryon form a triplet $\rho^{\pm,0}$. 
## lattice ensembles

plaqutte gauge action  
Wilson fermion action  

**HiRep code** [Del Debbio, Patella, Pica, PRD81, 094503 (2010)]

<table>
<thead>
<tr>
<th></th>
<th>$\beta$ = 2.2</th>
<th>$\beta$ = 2.309</th>
</tr>
</thead>
<tbody>
<tr>
<td>lattice dimensions</td>
<td>$20^3 \times 56$</td>
<td>$28^3 \times 56$</td>
</tr>
<tr>
<td>number of configurations</td>
<td>2000</td>
<td>1540</td>
</tr>
<tr>
<td>acceptance</td>
<td>73%</td>
<td>74%</td>
</tr>
<tr>
<td>unitary $m_{\text{bare}}$</td>
<td>-0.865</td>
<td>-0.76</td>
</tr>
<tr>
<td>partially quenched $m_{\text{bare}}$</td>
<td>-0.845, -0.855</td>
<td>-0.74, -0.75</td>
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<tr>
<td>average plaquette</td>
<td>0.5989</td>
<td>0.6255</td>
</tr>
<tr>
<td>$aw_0$</td>
<td>1.430(5)</td>
<td>1.956(7)</td>
</tr>
<tr>
<td>$m_V L$</td>
<td>9.0(2)</td>
<td>8.8(2)</td>
</tr>
</tbody>
</table>
preliminary lattice spectrum

\[ \beta = 2.309 \]
linear extrapolations

$\beta = 2.309$

![Graph showing linear extrapolations with data points and lines. The graph includes axes labeled 'am' and 'am_{bare}' with data points plotted at various values of am_{bare}. The graph also has labels for 'axial' and 'vector' data points.]
pseudoscalar correlators
connected part only, \( \beta = 2.309, \quad am_{\text{bare}} = -0.76 \)

\[ C(t) \]

\[ W=\text{wall (Coulomb gauge fixed), L=local} \]
vector correlators

$\beta = 2.309, \quad a m_{\text{bare}} = -0.76$

$W = \text{wall (Coulomb gauge fixed)}, \ L = \text{local}$
a large $N_c$ limit

For $N_c > 2$, one-flavor SU($N_c$) does not have the global SU(2).

But recall SU(2) = Sp(2).

The global SU(2) is present for Sp($N_c$). ($N_c$ is even.)

In Sp($N_c$),
the $\eta$ becomes massless as $N_c \to \infty$ and $m_Q \to 0$.
In this double limit, the global U(1) is only broken dynamically and the $\eta$ is its Goldstone boson.
particle spectrum for $N_c \to \infty$

How can the global SU(2) remain unbroken?

Mesons have this fermion content: $M = \sum_{i=1}^{N_c} \bar{Q}_i Q_i$

Baryons: ??? $X = \sum_{i,j,\ldots k=1}^{N_c} Q_i Q_j \ldots Q_k$ ??? No.

$\text{Sp}(N_c)$ baryons are $B = \sum_{i=1}^{N_c} \sum_{j=1}^{N_c} Q_i E_{ij} Q_j$

$X$ operators reduce to a collection of $B$ operators.

Thus the global SU(2) remains unbroken.
Baryons $B$ and mesons $M$ remain degenerate.
Higgs couplings to the dark sector

Can dim=5 BSM physics destabilize dark matter? No.

\[
\begin{align*}
\text{dim 5: } & \quad \delta \mathcal{L} \sim \bar{Q} \gamma_5 Q H^\dagger H \quad \text{(couples to } \eta) \\
\text{dim 6: } & \quad \delta \mathcal{L} \sim \bar{Q} \gamma^\mu Q H^\dagger \nabla_\mu H \quad \text{(couples to } \rho)
\end{align*}
\]

Dark matter, \( \rho \), is stable at dimension 5.

This feature is specific to the one-flavor theory.

Like SM proton decay, \( \rho \) decay is dimension 6. Thus \( \rho \) can be stable for the life of the universe.
fermion mass with parity violation

The dark fermion gets mass from 2 sources: BSM (dimension 4) and the SM Higgs (dimension 5):

\[
\delta L = -m_4 \cos \theta_4 \bar{Q}Q - m_4 \sin \theta_4 \bar{Q}i\gamma_5 Q \\
- \frac{v^2}{\Lambda} \cos \theta_5 \bar{Q}Q \left(1 + \frac{h}{v}\right)^2 - \frac{v^2}{\Lambda} \sin \theta_5 \bar{Q}i\gamma_5 Q \left(1 + \frac{h}{v}\right)^2
\]

\(\Lambda=\text{BSM scale.}\) \(v=246\ \text{GeV}.\) Parity violation allowed.

Mass terms: \(\delta L = m\bar{Q}_{tw}Q_{tw}\) where \(Q_{tw} \equiv e^{i\gamma_5 \alpha/2}Q.\)

Vector and axial hadrons are invariant under \(Q \rightarrow Q_{tw}.\)
invisible Higgs decays for SU(2)

The experimental bound $\Gamma(h \rightarrow Q\bar{Q}) < 1.2 \text{ MeV}$ gives
decays from the dark sector

The only $\text{dark} \rightarrow \text{SM}$ decays are through Higgs bosons. If $\theta_5 \neq 0$, then $\eta$ can decay through a single Higgs.

Recall: $\rho$ is essentially stable due to the global SU(2).

Example:
BBN $\Rightarrow$ lifetime of $\eta < 1$ second.
Use $\langle 0 | \bar{Q} \gamma_5 Q | \eta \rangle$ from lattice to bound $\frac{\sin \theta_5}{\Lambda}$.

For $m_\eta \ll m_H$,
$$
\Gamma_\eta = \left| \langle 0 | \bar{Q} \gamma_5 Q | \eta \rangle \right|^2 \frac{m_\eta \sin^2 \theta_5}{2\pi \Lambda^2 m_H^4} \sum_{f \in \text{SM}} m_f^2 \left( 1 - \frac{4m_f^2}{m_\eta^2} \right)^{3/2}.
$$
amplitude useful for $\eta$ decay

connected part only, $\beta=2.309$
also in progress

1. direct detection: $\rho$ scattering from a SM nucleon.

2. relic density:
   Options for $m_\eta < m_\rho$ include
   • $\rho\rho \rightarrow \eta\eta$
   • asymmetric dark matter

   Options for $m_\eta > m_\rho$ include
   • $\rho\rho\rho \rightarrow \rho\rho$

Future lattice simulations will reveal $m_\eta - m_\rho$ ordering. Disconnected diagrams are required.

For disconnected calculations in the 2-flavour theory, see Arthur, Drach, Hietanen, Pica, Sannino, 1607.06654 and Drach, Monday 14:15
1-flavor SU(2) is a minimal non-Abelian dark sector. It has a *global* SU(2) to stabilize dark matter. It has no dark matter decay at dimension 5. 1 Goldstone boson should emerge for Sp($N_c \to \infty$). SU(2)=Sp(2) lattice explorations are in progress.