Electromagnetic pion form factor near physical point in $N_f = 2 + 1$ lattice QCD

Junpei Kakazu
University of Tsukuba

K.-I. Ishikawa, N. Ishizuka, Y. Kuramashi,Y. Nakamura,
Y. Namekawa, Y. Taniguchi, N. Ukita,T. Yamazaki, and T. Yoshie

(PACS Collaboration)

Lattice 2016 @ Univ. of Southampton
29/7/2016

Motivation

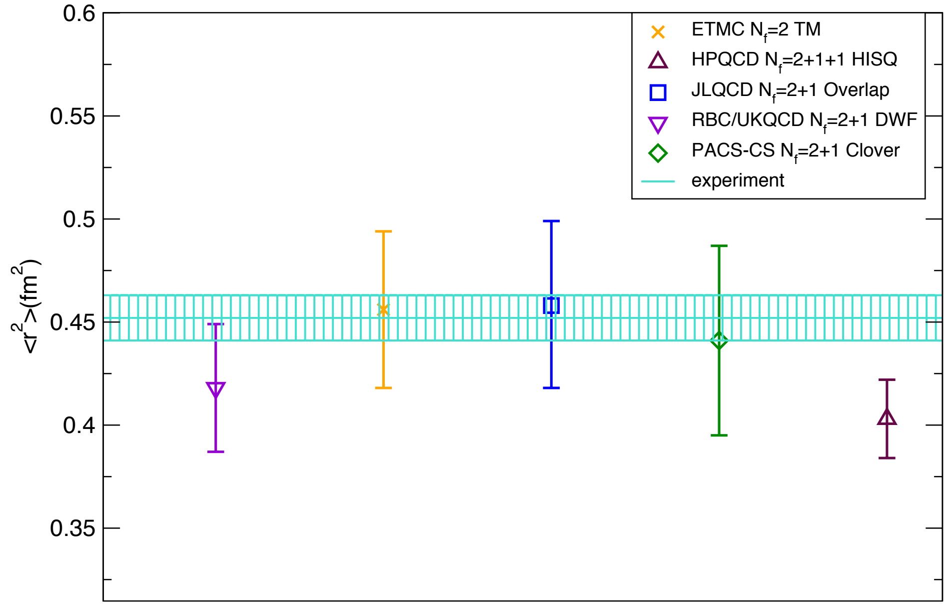
- Electromagnetic form factor

=deviation from charged point particle

→Hadron's structure (cf. charge radius)

However, errors of charge radii in lattice calculation are larger than experimental one

- chiral extrapolation → error increases
- small momentum transfer and suppressing finite size effect → need large box size



previous results at physical point and experimental value

Purpose

calculation near physical point

$$m_\pi = 0.145(\mathrm{GeV}), L = 8.1(\mathrm{fm})$$

Electromagnetic form factor(ff)

Definition $\langle \pi^+(p_f) | V_\mu | \pi^+(p_i) \rangle = (p_f + p_i)_\mu f_{\pi\pi}(q^2)$

electromagnetic current $V_\mu = \sum_f Q_f \bar{\psi}_f \gamma_\mu \psi_f$ $\psi_f = u, d, s$ Q_f:charge of flavor f

$f_{\pi\pi}(0) = 1$ (normalization condition relating to pion's charge)

$q^2 = -(p_f - p_i)^2 \geq 0$ (space-like momentum transfer →calculating ff directly)

calculating 3-pt function and 2-pt function by Lattice QCD

$$
\begin{aligned}
C_{\pi V \pi} &= Z_V \left\langle 0 \middle| O_\pi(t_f, \vec{p}_f = \vec{0}) V_4(t, \vec{p} = \vec{p}_f - \vec{p}_i) O_\pi^\dagger(0, \vec{p}_i) \middle| 0 \right\rangle \\
&= Z_V \frac{Z_\pi(0) Z_\pi(p)}{4E_\pi(p) m_\pi} \langle \pi(0) | V_4(0,p) | \pi(p) \rangle e^{-E_\pi(p)t} e^{-m_\pi(t_f - t)} \\
&= Z_V \frac{Z_\pi(0) Z_\pi(p)}{4E_\pi(p) m_\pi} (E_\pi(p) + m_\pi) f_{\pi\pi}(q^2) e^{-E_\pi(p)t} e^{-m_\pi(t_f - t)}
\end{aligned}
$$

$$|Z_\pi(p)|^2 (e^{-E_\pi(p)t} + e^{-E_\pi(p)(T-t)})$$

$$Z_\pi(p) = \langle 0 | O_\pi(0,0) | \pi(p) \rangle$$

$$\rightarrow \quad R(t,p) = \frac{2m_\pi Z_\pi(0)}{(E_\pi(p) + m_\pi) Z_\pi(p)} \frac{C_{\pi V \pi}(t, t_f; p)}{C_{\pi V \pi}(t, t_f, 0)} e^{(E_\pi(p) - m_\pi)t}$$

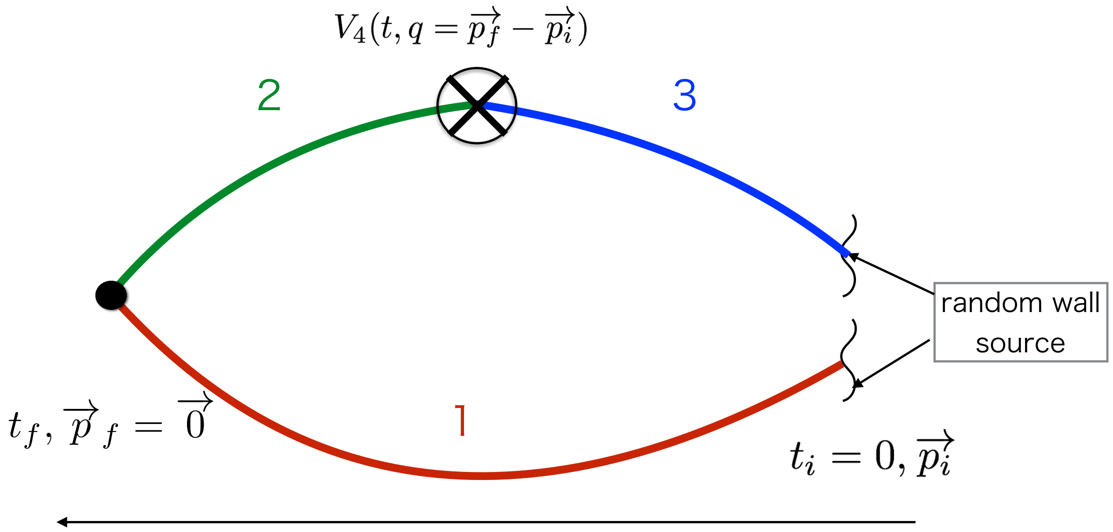
in $0 \ll t \ll t_f$ we can extract form factor from R

Calculation 3-point function

- connected 3-point function is consist of 3 quark propagators

$$C_{\pi V \pi} = Z_V \left\langle 0 \middle| O_\pi(t_f, \vec{p}_f) V_4(t, \vec{q}) O_\pi^\dagger(t_i = 0, \vec{p}_i) \middle| 0 \right\rangle$$

- 1 random wall source $\vec{p}_i = \vec{0}$
- 2 sequential source $\vec{p}_f = \vec{0}$
- 3 random wall source $\vec{p}_i \neq \vec{0}$



random wall source (A,B:color&spinor index)

$$\eta_B(\vec{y}, t_i) = \left\{ \frac{1+i}{\sqrt{2}}, \frac{1-i}{\sqrt{2}}, \frac{-1+i}{\sqrt{2}}, \frac{-1-i}{\sqrt{2}} \right\} \in \mathbb{Z}(2) \otimes \mathbb{Z}(2)$$

$$\lim_{N \to \infty} \frac{1}{N} \sum_{j=0}^{N} \eta_A^j(\vec{x}, t_i) \eta_B^{\dagger j}(\vec{y}, t_i) = \delta(\vec{x} - \vec{y}) \delta_{AB}$$

calculation cost is reduced by random wall source

RBC&UKQCD:JHEP(0807 (2008)112)

disconnected term is vanished by charge symmetry

Simulation details

All the results are preliminary

gauge configuration (HPCI Strategic Program Field 5)

$m_\pi = 0.145(\mathrm{GeV}), L = 8.1(\mathrm{fm})$ $m_\pi L \approx 6$ $(\kappa_{ud}, \kappa_s) = (0.126117, 0.124790)$

$L^3 \times T = 96^3 \times 96$ $a^{-1} = 2.333(\mathrm{GeV}) \to a = 0.084(\mathrm{fm})$ $m_K \approx 0.525\mathrm{GeV}$

$N_f = 2+1$ Iwasaki gauge + stout smeared link Wilson clover action

$\beta = 1.82, n_{stout} = 6, \rho = 0.1, c_{sw} = 1.11$

measurement parameter

40 configs in total at present

4 sources × 4 directions (x, y, z, t) × 2 random sources = 32 meas. per config.

periodic boundary condition for all directions

$t_f - t_i = 36$

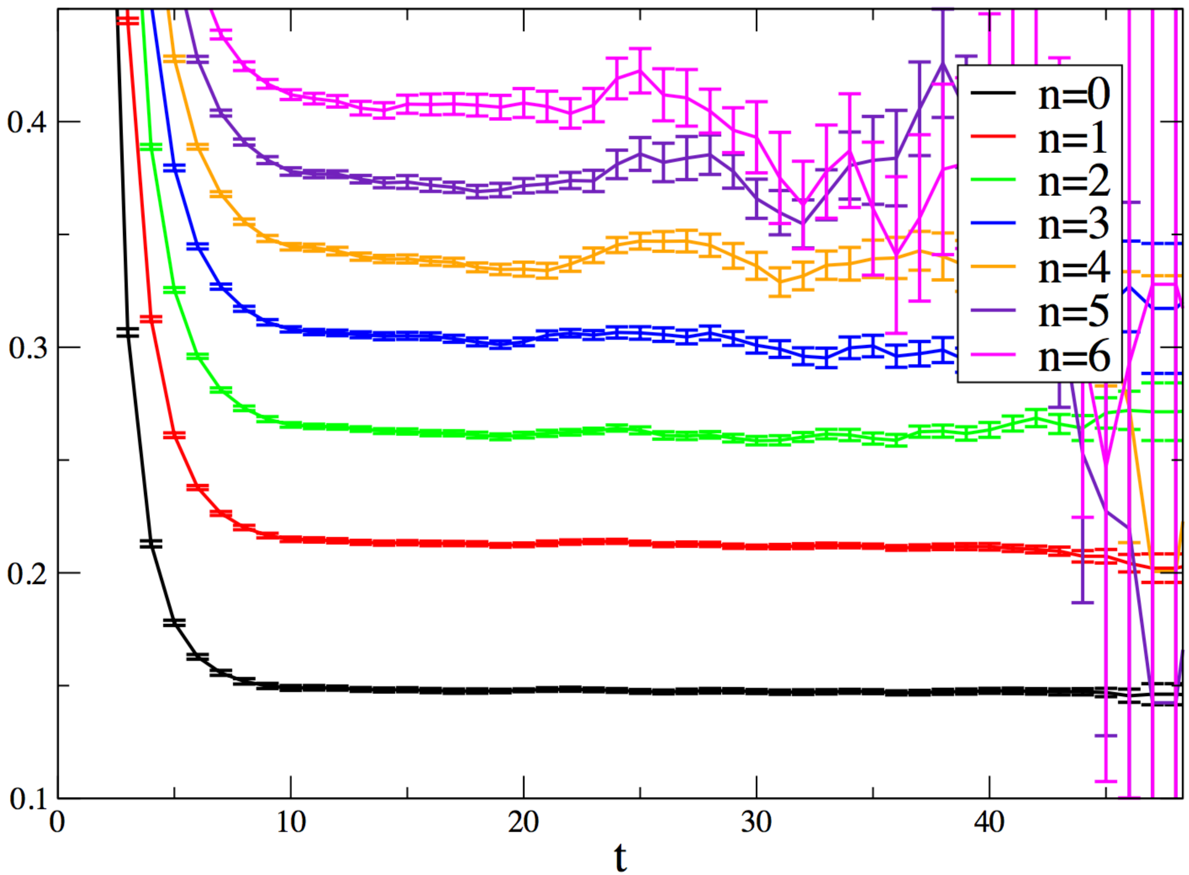
directions of source momenta $\vec{n}$

$(1,0,0)$	$(0,1,0)$	$(0,0,1)$
$(1,1,0)$	$(1,0,1)$	$(1,1,0)$
$(1,1,-1)$	$(1,-1,1)$	$(1,1,-1)$
$(2,0,0)$	$(0,2,0)$	$(0,0,2)$
$(2,1,0)$	$(0,2,1)$	$(1,0,2)$
$(1,1,2)$	$(1,2,1)$	$(1,1,2)$

$$\vec{p} = \frac{2\pi}{L}\vec{n}$$

resources
PRIMRGY cx400 (tatara),RIIT, Kyusyu University and HAPACS ,CCS,University of Tsukuba

preliminary result: effective energy



$$q^2 = 2m_\pi(E_\pi - m_\pi)$$

	$q^2(GeV^2)$
$n=1$	0.01909(7)
$n=2$	0.03363(22)
$n=3$	0.04580(45)
$n=4$	0.05633(54)
$n=5$	0.06688(62)
$n=6$	0.07593(115)

previous studies used Twisted Boundary Condition(TBC)
for small momentum transfers
→we obtain small momentum transfers without TBC
(no extra finite size effect by TBC and not partially quenched QCD)

Extraction of form factor

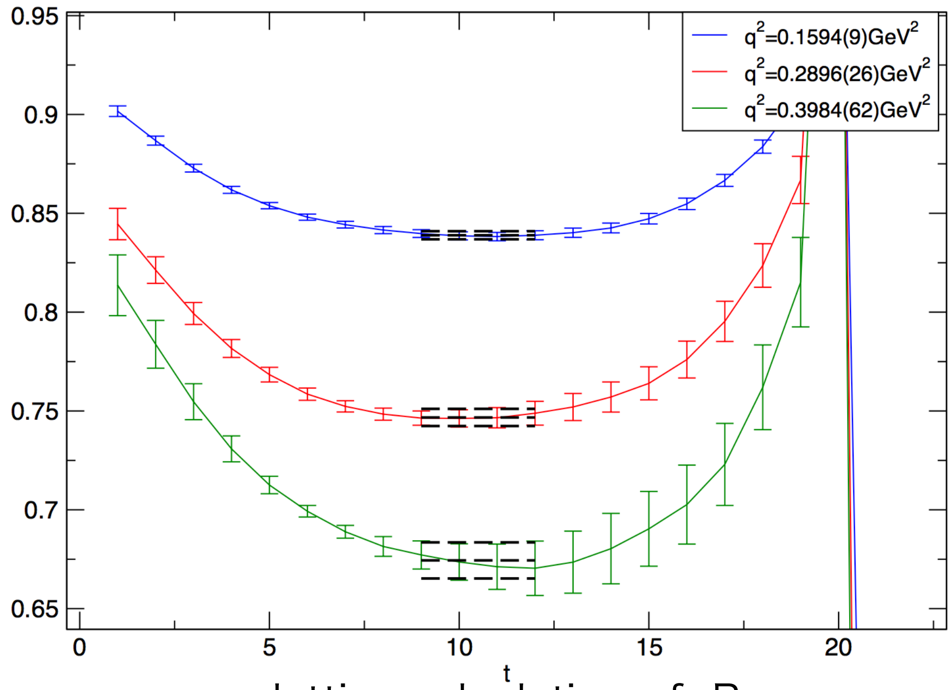
heavy pion mass

$m_\pi = 0.51(\mathrm{GeV}), L = 2.9(\mathrm{fm})$

when $0 \ll t \ll t_f$

ff is extracted by fitting

plateau of R as constant

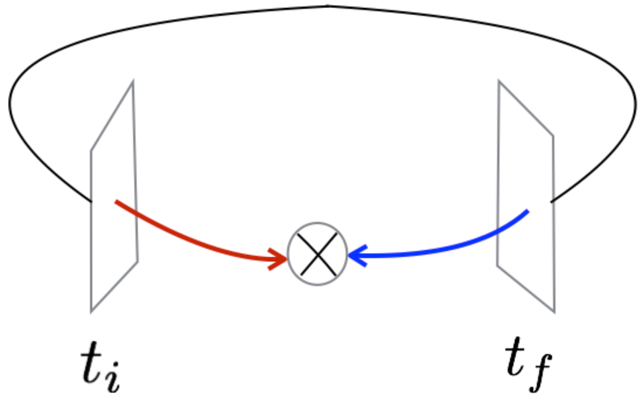


lattice calculation of R

Diagrammatically,

one pion from source

the other pion from sink



(using periodic boundary condition in temporal direction)

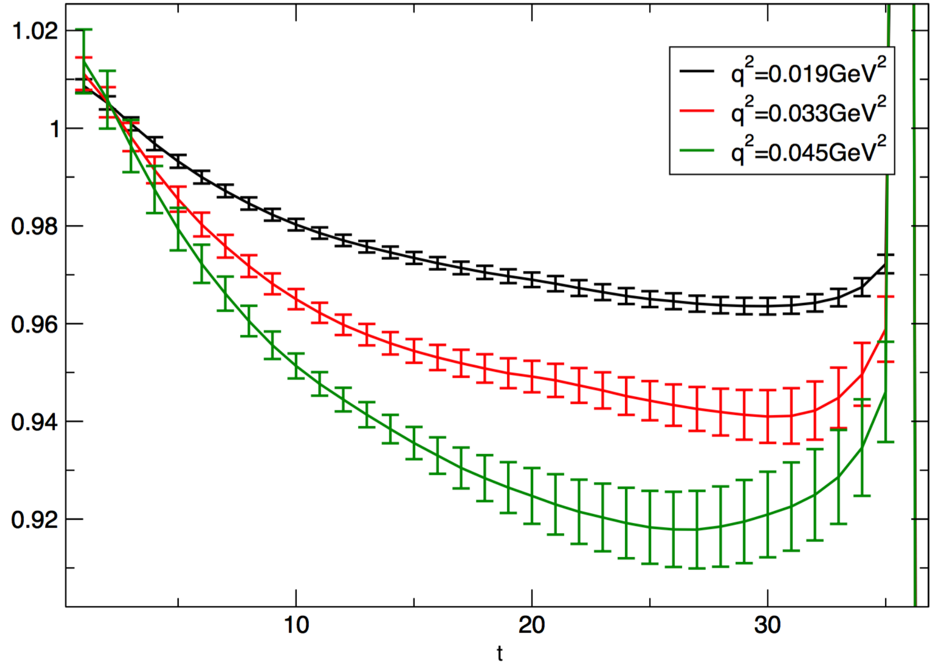
While, light pion mass

$$m_\pi = 0.145(\mathrm{GeV}), L = 8.1(\mathrm{fm})$$

t dependence appears in R

no plateau in R

we should not only take
usual form factor but also
other effect into account



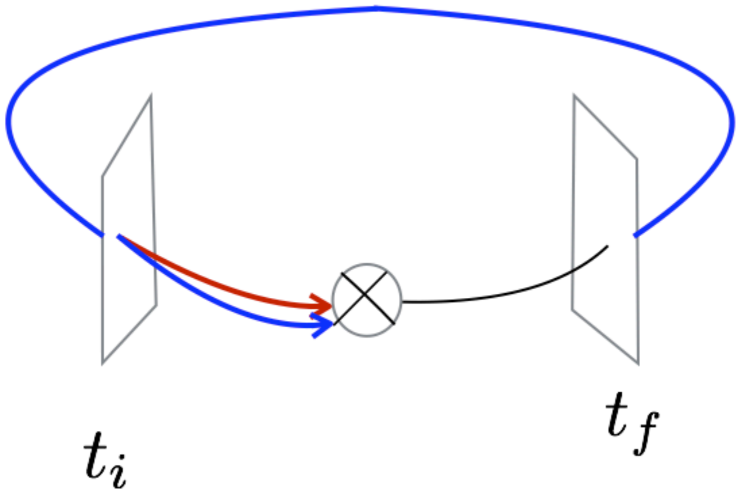
lattice calculation of R

pion wrapping around effect

the opposite direction pion from sink
+pion from source +finite size effect of 2 pions

$$e^{-E_{\pi_f}\times(T-t_f+t)} e^{-E_{\pi_i}(t-t_i)} e^{-\Delta E(t-t_i)}$$

another 2pions effect; source ⇔ sink



Phys.Rev. D77 (2008) 094503 [arXiv:0801.4186] we fit R with

$$f_0 + f_1 \times \frac{e^{-E_{2\pi}\times t}e^{-E_{\pi_f}\times(T-t_f)} - e^{-E_{2\pi}\times(t_f-t)}e^{-E_{\pi_i}\times(T-t_f)}}{e^{-E_{\pi_i}\times t}e^{-E_{\pi_f}\times(t_f-t)}}$$

form factor

2pions effect

$$E_{2\pi} = E_{\pi_i} + E_{\pi_f} + \Delta E$$

E_{π_i} E_{π_f} :single pion energy

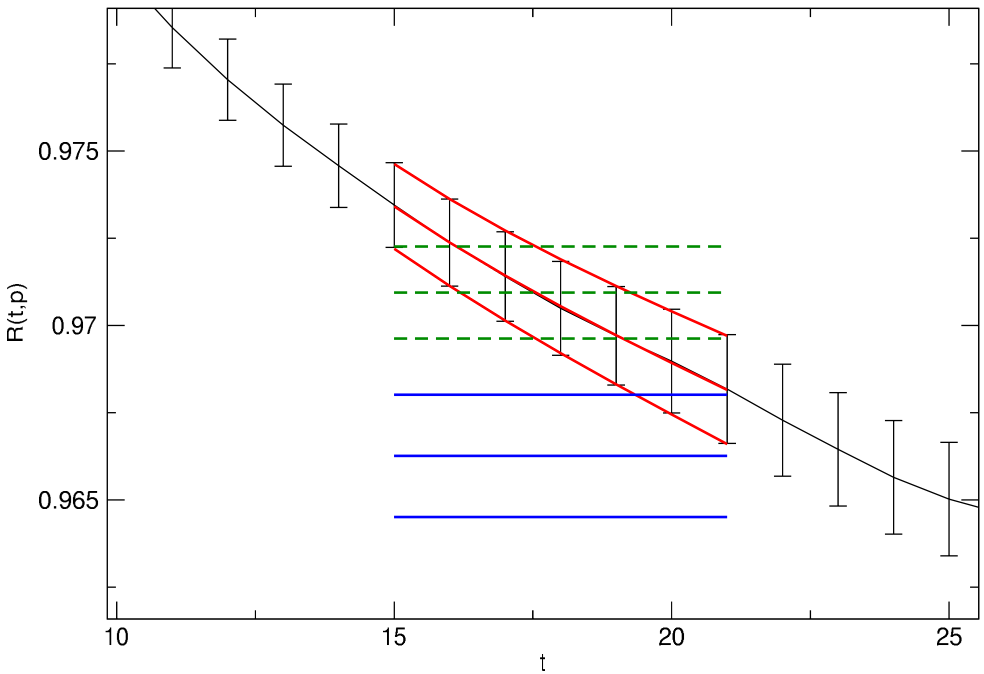
ΔE :finite size effect by 2 pions interaction

thanks to large box calculation, we assume that finite size effect is negligible $\rightarrow \Delta E = 0$

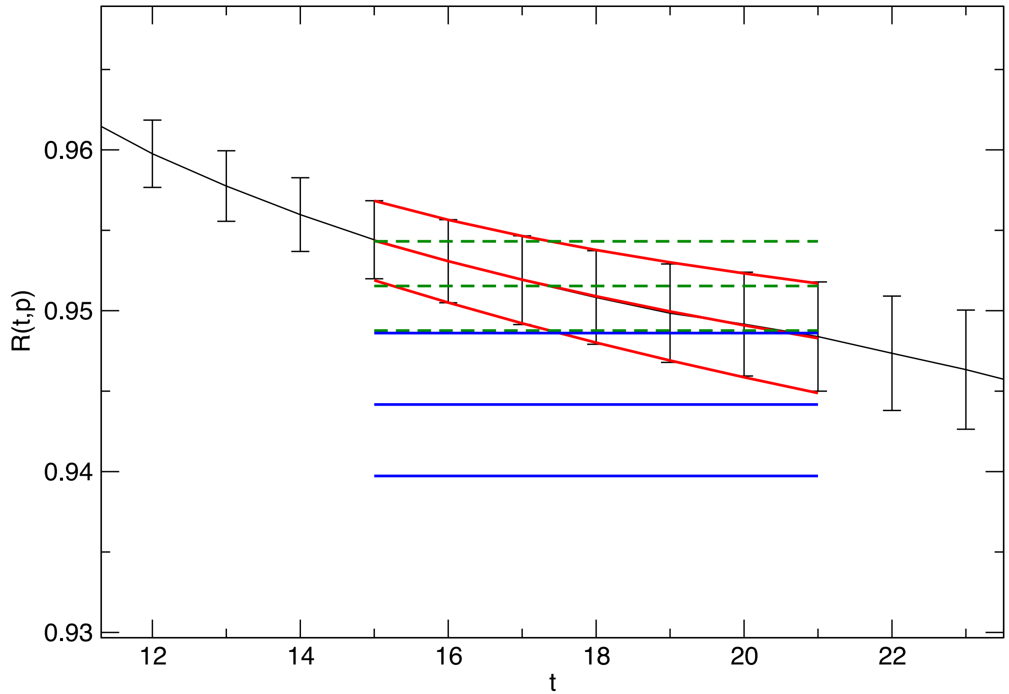
preliminary result: analysis of form factor

fit form $\boxed{f_0} + f_1 \times \frac{e^{-E_{2\pi}\times t}e^{-E_{\pi_f}\times(T-t_f)} - e^{-E_{2\pi}\times(t_f-t)}e^{-E_{\pi_i}\times(T-t_f)}}{e^{-E_{\pi_i}\times t}e^{-E_{\pi_f}\times(t_f-t)}}$

$$R(t,p) = \frac{2m_\pi Z_\pi(0)}{(E_\pi(p)+m_\pi)Z_\pi(p)} \frac{C_{\pi V\pi}(t,t_f;\overrightarrow{p})}{C_{\pi V\pi}(t,t_f;\overrightarrow{0})} e^{(E_\pi(p)-m_\pi)t}$$



$q^2 = 0.01909 GeV^2$



$q^2 = 0.03363 GeV^2$

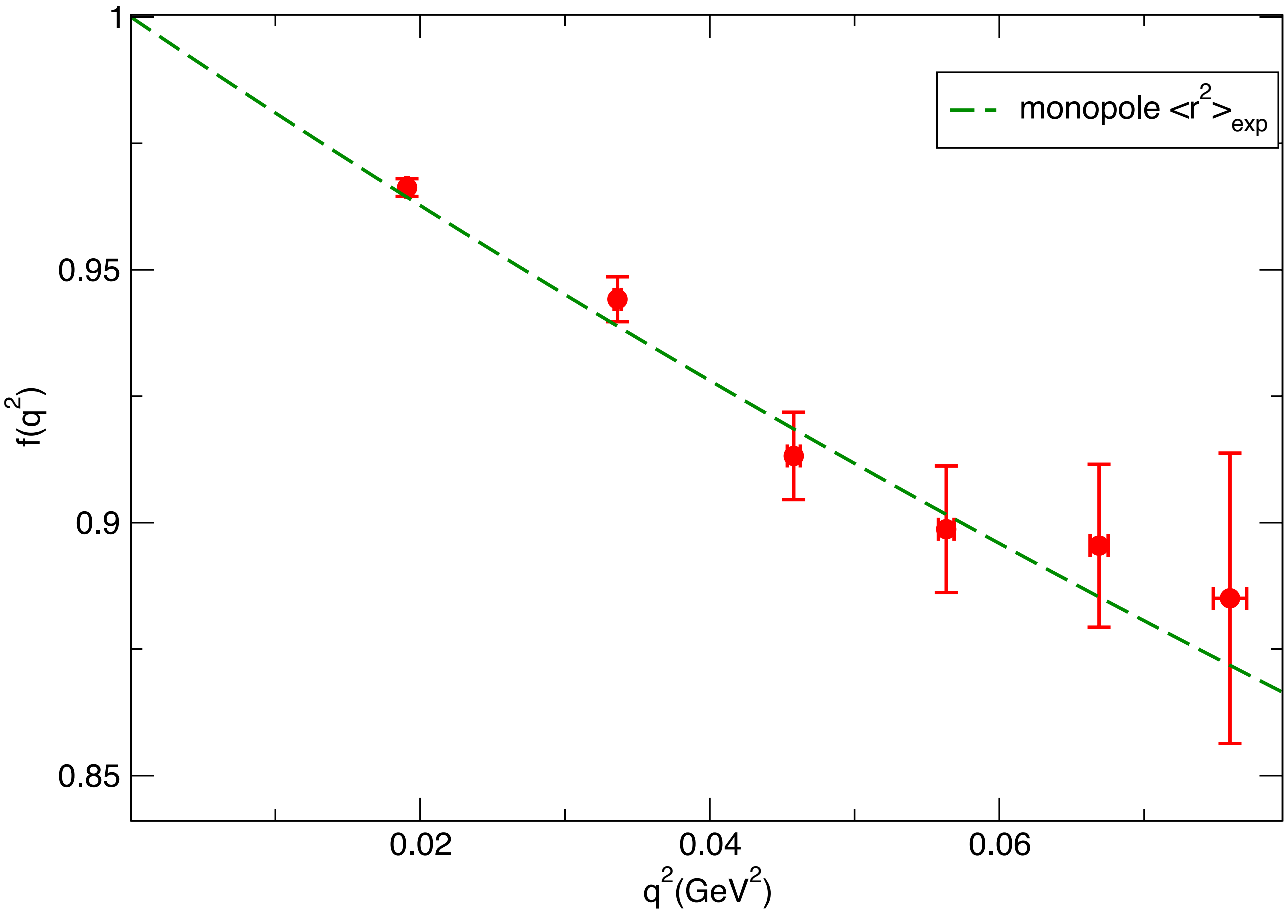
red : fit including 2 pions effect

blue : form factor including 2pions effect(f_0)

green : form factor with constant fit(f_0 only)

fits including 2pions effect are better than constant ones

preliminary result: form factor vs q^2



monopole form

$$f_{\pi\pi}(q^2) = \frac{1}{1+q^2/M_{pole}^2}$$

$$\langle r^2 \rangle = -6\frac{d}{dq^2} f_{\pi\pi}(q^2)\Big|_{q^2=0} = \frac{6}{M_{\rm pole}^2}$$

$$\langle r^2 \rangle_{exp} = 0.451(11)(fm^2)$$

(from PDG)

roughly consistent with experiment

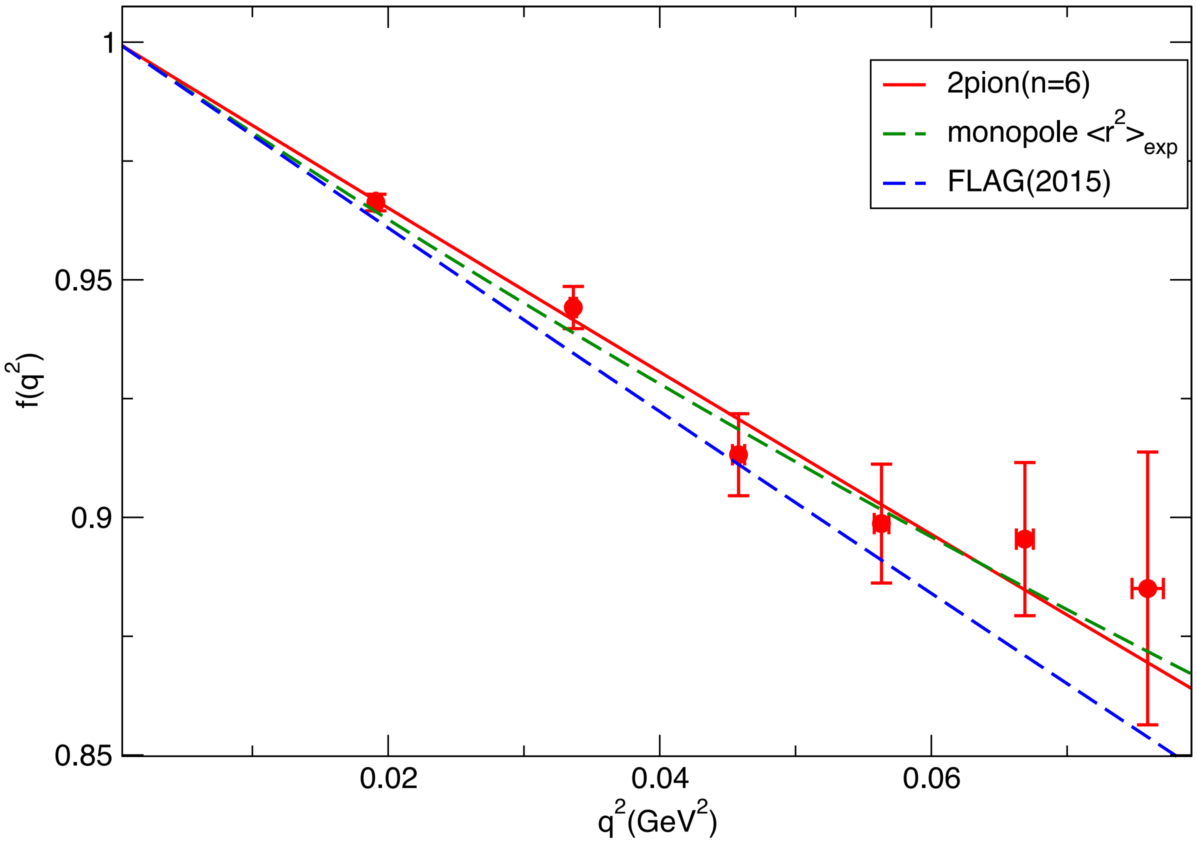
preliminary result: NLO SU(2) ChPT fit

$$f_{\pi\pi}(q^2) = 1 + \frac{1}{f^2}\left[2l_6 q^2 + \frac{m^2 H(x)}{8\pi^2} + \frac{q^2}{48\pi^2}\log\left(\frac{m^2}{\mu^2}\right)\right] \qquad \langle r^2\rangle = \frac{-12 l_6}{f^2} - \frac{1}{8\pi^2 f^2}\left(\log\left(\frac{m_\pi^2}{\mu^2}\right) + 1\right)$$

$$H(x) = -\frac{4}{3} + \frac{5}{18}x - \frac{x-4}{6}\sqrt{\frac{x-4}{x}}\log\left(\frac{\sqrt{\frac{x-4}{x}}+1}{\sqrt{\frac{x-4}{x}}-1}\right) \quad (x = -\frac{q^2}{m^2})$$

$$\mu = m_\rho = 0.77(\mathrm{GeV})$$

$$f = 0.12925(\mathrm{GeV}) \ (m_u, m_d \to 0)$$



there is one fit parameter

: Low Energy Constant l_6

	l_6	$\chi^2/d.o.f$
2π fit(n=6)	-0.01223(75)	0.42
2π fit(n=3)	-0.01234(72)	0.58
FLAG(2015)	-0.01233(127)	

fit n=6: all 6 momentum transfers

n=3: smaller 3 momentum transfers

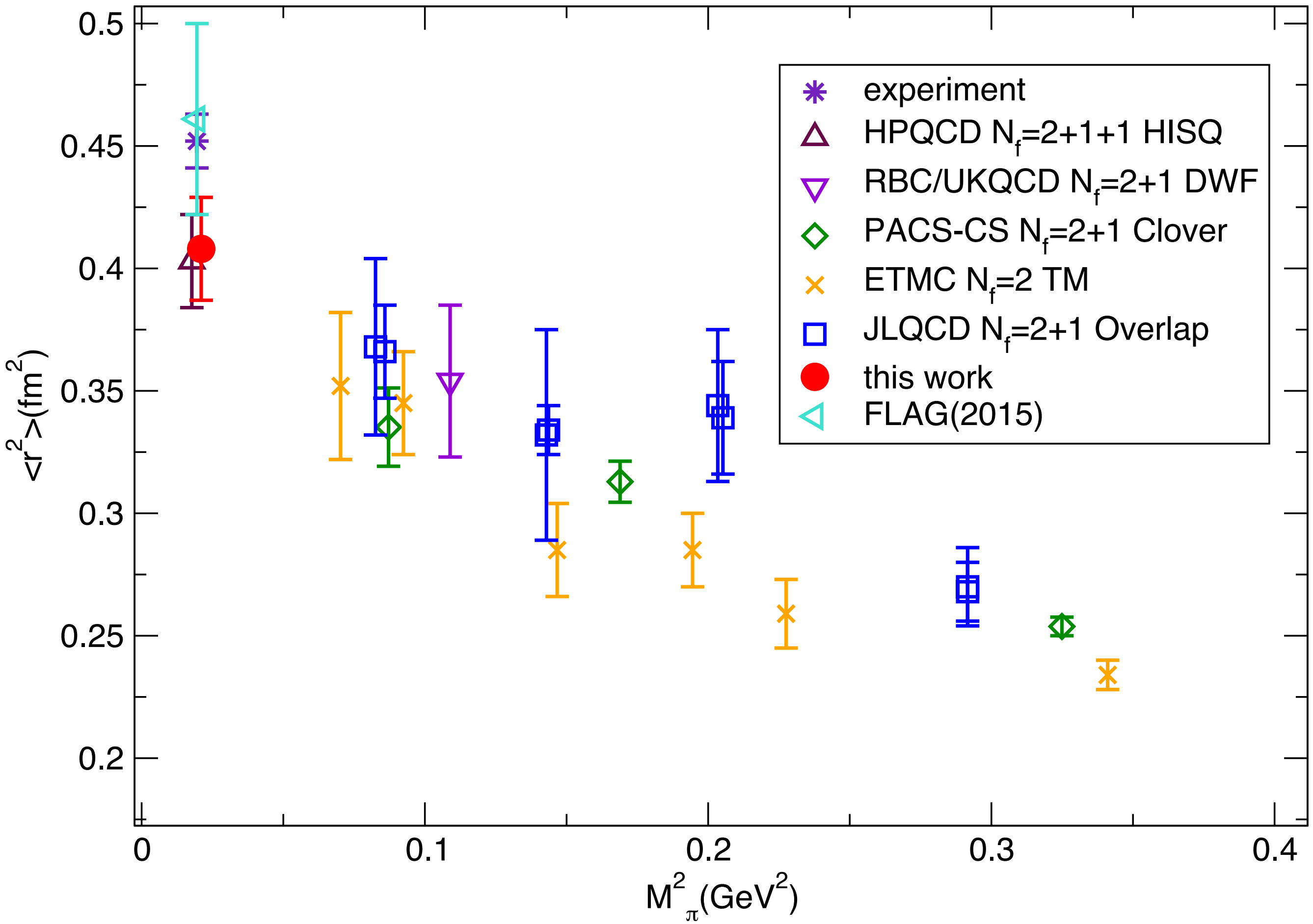
in FLAG's result ,we used

$$f = 0.122553(GeV) \ (m_u, m_d \to 0)$$

ChPT fit works well

thanks to small q^2 and almost physical pion mass

Pion mass dependence of $\langle r^2 \rangle$



FLAG's radius is estimated by SU(2)ChPT form with physical pion mass and

$$f = 0.122553(GeV)$$

and

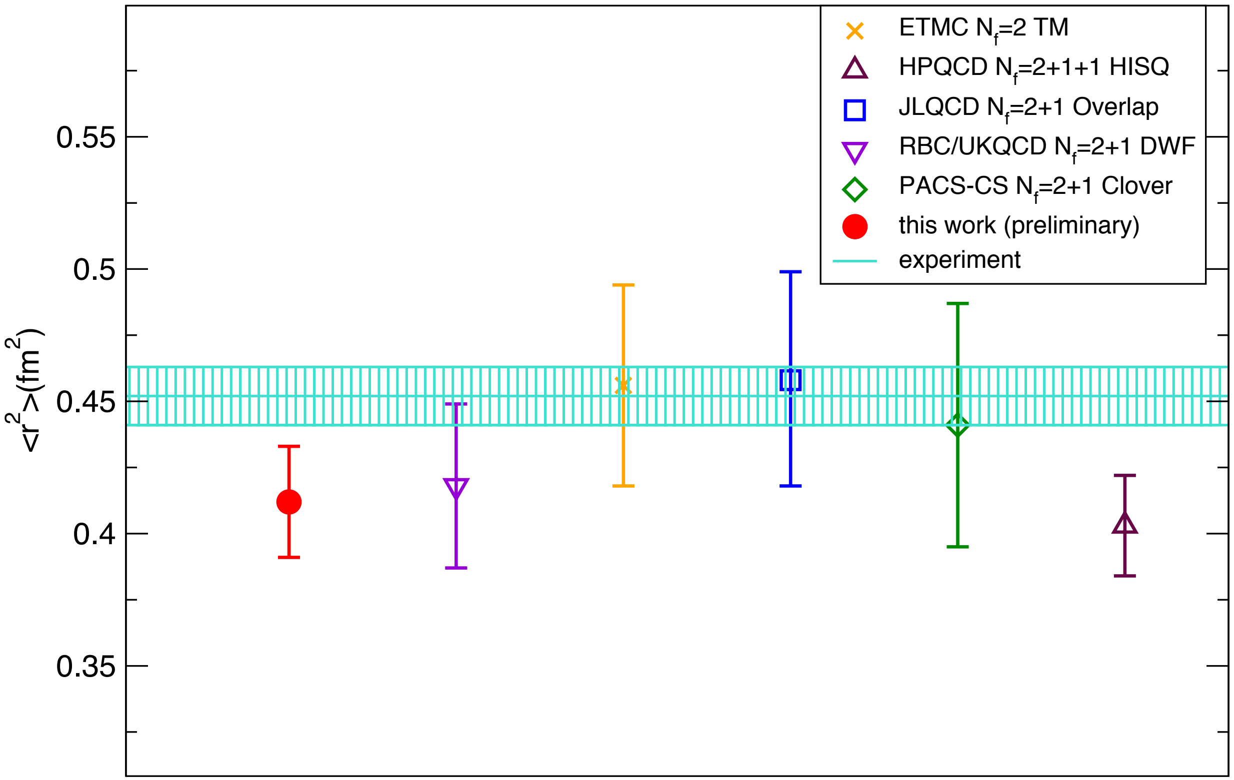
$$l_6 = -0.01234(127)$$

preliminary result $\langle r^2 \rangle = 0.408(21)(\mathrm{fm}^2)$ (NLO SU(2) ChPT)

consistent with HPQCD and FLAG(2015)

comparison with other calculation at physical point

$$m_\pi = 0.13957(\mathrm{GeV})$$



preliminary result

$$\langle r^2 \rangle = 0.412(21)(\mathrm{fm}^2)$$

extrapolation with
NLO SU(2) ChPT formula

$$\langle r^2 \rangle = \frac{-12 l_6}{f^2} - \frac{1}{8\pi^2 f^2}\left(\log\left(\frac{m_\pi^2}{\mu^2}\right) + 1\right)$$

consistent with previous results and
roughly consist with experiment

conclusion

We calculated pion electromagnetic form factor

in N=2+1 Lattice QCD

$m_\pi = 0.145\text{GeV}$

- smaller momentum transfers by large box size
- fits including 2pions effect to extract form factor

preliminary result: $\langle r^2 \rangle = 0.412(21)(\text{fm}^2)$

(NLO SU(2) ChPT at physical point)

roughly consistent with experiment and consistent previous results

future works

- increasing statistics (~100 configs)
- analysis with other fit forms
    - NNLO SU(2) ChPT
    - NLO SU(3) ChPT
- reweighting strange quark mass +small extrapolation

$m_K = 0.525(\text{GeV}) \to m_K = 0.493(\text{GeV})$

back up

Charge radius

In non-relativistic limit , ff is regarded as <u>3D Fourier transformation of charge density</u>

$$f_{\pi\pi}(q^2) = \int d^3x e^{i\vec{q}\cdot\vec{x}} \rho_\pi(\vec{x})$$

Assuming spherical symmetry of density and $\int d^3x \rho_\pi(x) = 1$

Expand ff by $\vec{q}\cdot\vec{x} \ll 1$

$$\begin{aligned} f_{\pi\pi}(q^2) &= \int_0^\infty r^2 dr \int_{-1}^{1} d(cos\theta) \int_0^{2\pi} d\phi (1 - \frac{1}{2} q^2 \cdot r^2 cos^2\theta + \ldots) \rho_\pi(r) \\ &= 1 - \frac{1}{6} q^2 \int_0^\infty 4\pi r^2 dr r^2 \rho_\pi(r) + \ldots \end{aligned}$$

$$f_{\pi\pi}(q^2) = 1 - \frac{\langle r_\pi^2 \rangle}{6} q^2 + \ldots$$

We can consider mean square of charge radius as <u>1st differential coefficient of ff</u> →

$$\langle r_\pi^2 \rangle = -6 \frac{d}{dq^2} f_{\pi\pi}(q^2) \Big|_{q^2=0}$$

(so we need large box size for small momentum)

disconnect diagram of 3-point function

DRAPER and WOLOSHYN,Nucl. Phys., B318 (1989), p. 319-336

$$C^{disc}_{\pi J_\mu \pi}(U) = \frac{1}{2}C^{disc}_{\pi J_\mu \pi}(U) + \frac{1}{2}C^{disc}_{\pi J_\mu \pi}(U^*)$$

from charge conjugation

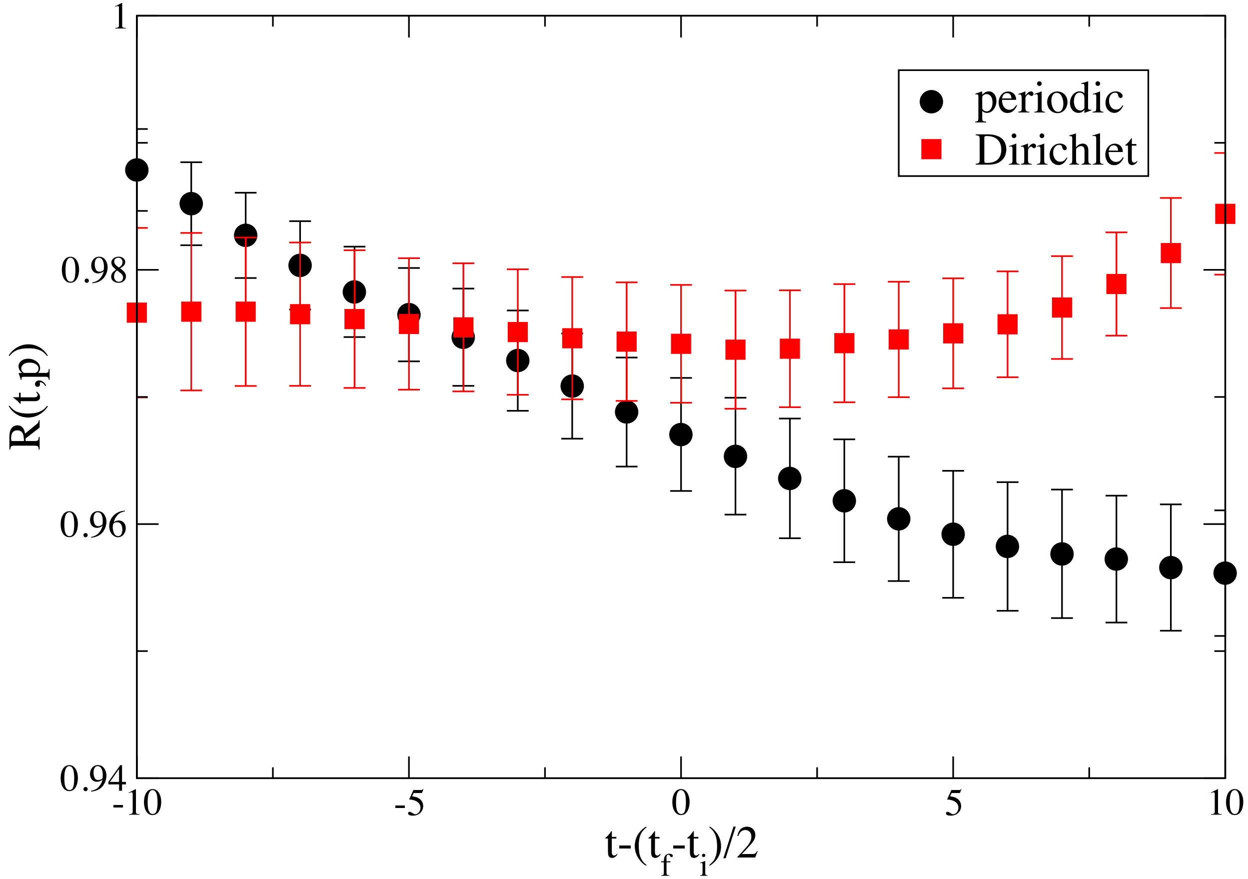
$$C\gamma_\mu C^{-1} = -\gamma_\mu^T, CUC^{-1} = U^* = (U^\dagger)^T$$

$$C^{disc}_{\pi J_\mu \pi}(U) = -C^{disc}_{\pi J_\mu \pi}(U^*)$$

$$C^{disc}_{\pi J_\mu \pi}(U) = \frac{1}{2}C^{disc}_{\pi J_\mu \pi}(U) - \frac{1}{2}C^{disc}_{\pi J_\mu \pi}(U) = 0$$

disconnected term is vanished by charge symmetry

R in Dirichlet BC



Dirichlet

$$11 conf, q^2 \approx 0.019(\mathrm{GeV}^2)$$

$$|t_f - t_i| = 25$$

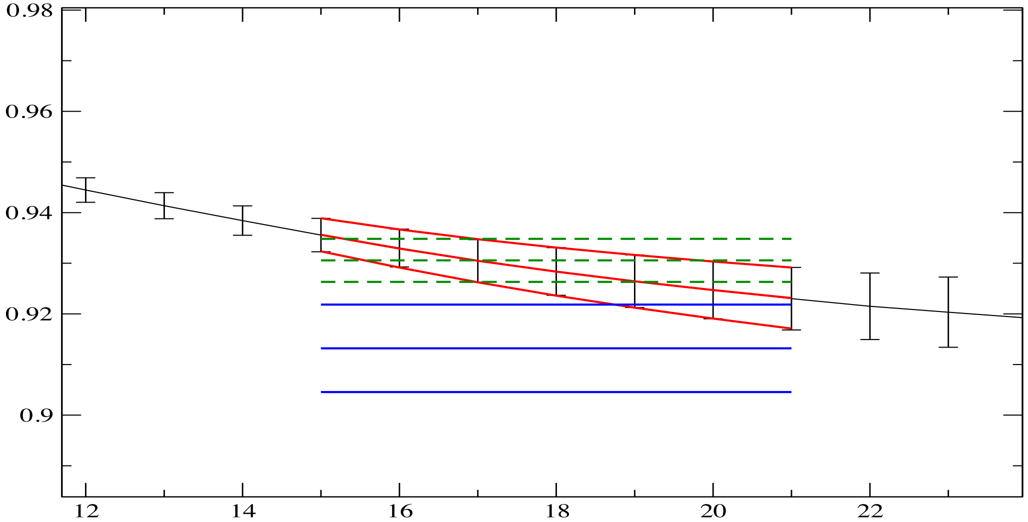
periodic

$$19 conf, q^2 \approx 0.019(\mathrm{GeV}^2)$$

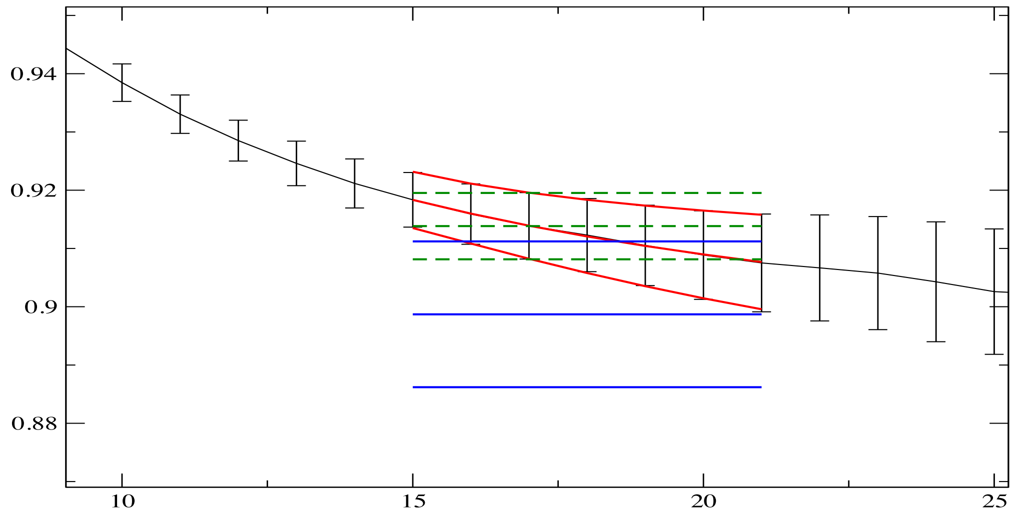
$$|t_f - t_i| = 36$$

there is no t dependence in Dirichlet BC

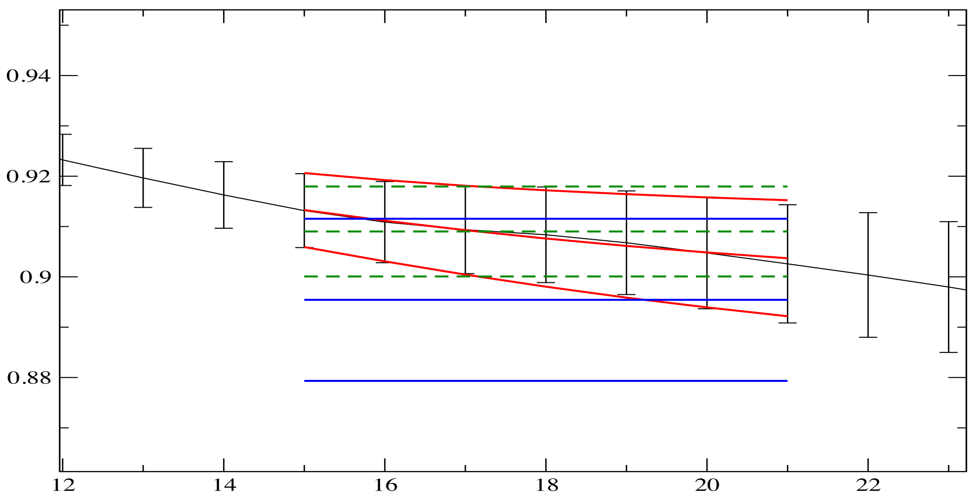
preliminary result: analysis of form factor



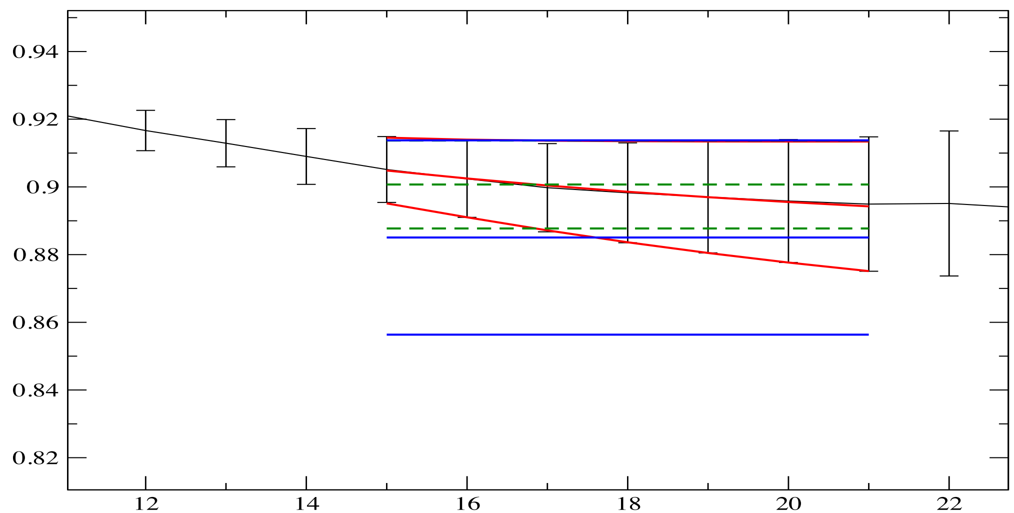
$$q^2 = 0.04580(45)$$



$$q^2 = 0.05633(54)$$



$$q^2 = 0.06688(62)$$



$$q^2 = 0.07593(115)$$

preliminary result:charge radii & LECs

	$<r^2>_{mono}\ (fm^2)$	$<r^2>^{n=6}_{su(2)chpt}\ (fm^2)$	$<r^2>^{n=6}_{su(2)chptphys}\ (fm^2)$
2π fit	0.429(23)	0.408(21)	0.412(21)
const fit	0.366(16)	0.349(14)	0.352(14)
	$<r^2>_{mono}\ (fm^2)$	$<r^2>^{n=3}_{su(2)chpt}\ (fm^2)$	$<r^2>^{n=3}_{su(2)chptphys}\ (fm^2)$
2π fit	0.428(22)	0.411(20)	0.415(20)
const fit	0.364(15)	0.352(14)	0.355(14)

$\langle r^2 \rangle_{exp} = 0.451(11)(fm^2)$

$\langle r^2 \rangle_{FLAG} = 0.461(39)(fm^2)$

FLAG's radius is estimated by SU(2)ChPT form and $f = 0.122553(GeV)$

	l_6	$\chi^2/d.o.f$
2π fit(n=6)	-0.01223(75)	0.42
2π fit(n=3)	-0.01234(72)	0.58
const fit(n=6)	-0.01009(50)	0.31
const fit(n=3)	-0.01019(48)	0.56
FLAG(2015)	-0.01233(127)	
	$\bar{l_6}$	$\chi^2/d.o.f$
2π fit(n=6)	15.1(0.7)	0.42
2π fit(n=3)	15.0(0.7)	0.54
const fit(n=6)	13.1(0.5)	0.31
const fit(n=3)	13.0(0.5)	0.56
FLAG(2015)	15.1(1.2)	
phenomenological	16.0(0.9)	

the phenomenological result is

estimated by $\bar{l_6} = 16.0(0.9)$

JHEP 9805, 014 (1998)

with conversion formula

$$l_6 = -\frac{1}{6 \times (4\pi)^2}\left(\bar{l_6} + ln\left[\left(\frac{m_\pi^{phys}}{\mu}\right)^2\right]\right)$$

and using $f = 0.12275(GeV)$

fit n=3:smaller 3 momentum transfers

n=6:all the data (6 momentum transfers)