The HVP contribution to the anomalous magnetic moment of the muon

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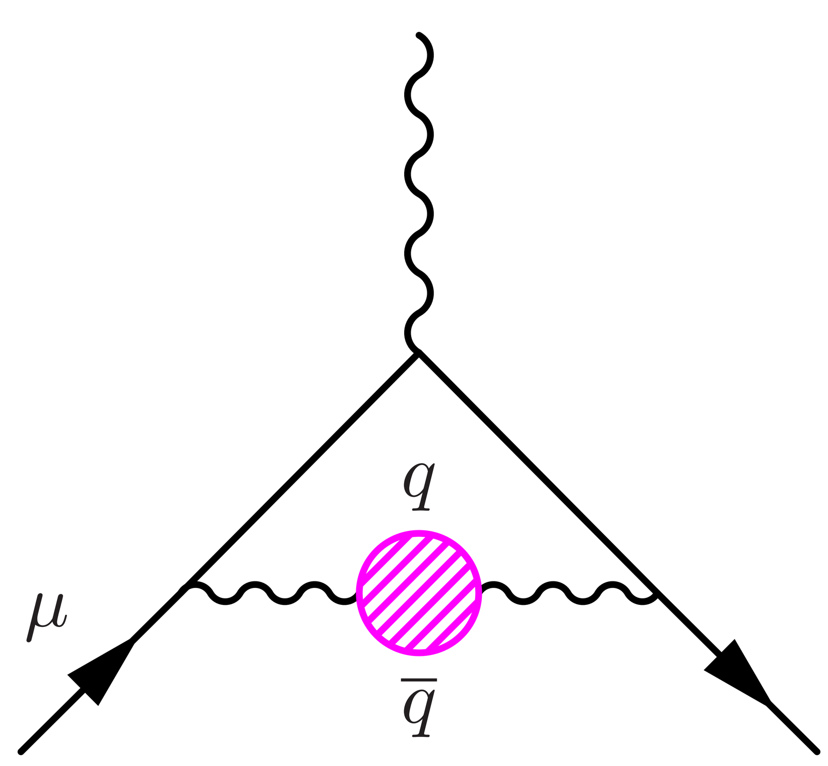
Abstract

New particles can reveal their existence indirectly through tiny discrepancies in the properties of known particles from that expected in the Standard Model. The magnetic moment of the muon shows such a discrepancy, a tantalising 25 parts in 10^{10}, but with 3σ significance. The magnetic moment, μ, is given in terms of the spin, S, by:

$$\vec{\mu} = g\frac{e}{2m}\vec{S} \qquad a_\mu = \frac{g-2}{2}$$

The difference of g from the naive value of 2 is called the 'anomalous magnetic moment', a_μ.
The anomalous magnetic moment is determined directly by measuring the spin precession as polarised muons circulate in a ring with a perpendicular magnetic field. Experiment E989 at Fermilab will start data-taking in 2017 and aims to reduce the experimental uncertainty by a factor of 4.
An improved theoretical uncertainty from the Standard Model is needed to match this. The largest uncertainty comes from the diagram containing a quark loop (see Fig. below): the Leading Order Hadronic Vacuum Polarisation (HVP) contribution.



Lattice Calculation

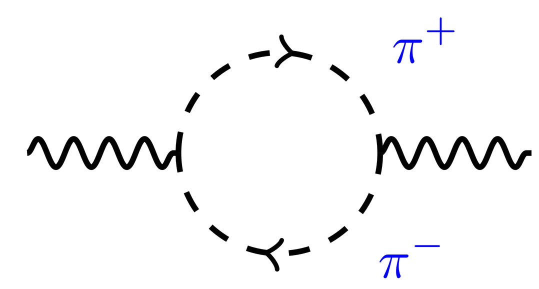
We use MILC gluon configurations that include u, d, s and c HISQ sea quarks at 3 lattice spacing values (0.15, 0.12 and 0.09 fm), 3 u/d quark masses going down to the physical u/d quark mass and 3 volumes (for one parameter set). This allows us to do the most realistic calculations to date [1].
On these configurations we generate u/d HISQ propagators and combine them to give the correlation function between two local vector currents (normalised nonperturbatively), separated by a time interval, t. Taking t-moments of the correlation function allows us to reconstruct the renormalised vacuum polarisation function [2] and determine, by integrating over k^2, its contribution to a_μ [3].
For the u/d case we reduce statistical errors by using fitting local and smeared correlators and using fitted local results at large t, rather than the data.

$$G_{2j} \equiv \sum_{t,\vec{x}} t^{2j} Z_V^2 \langle j^i(\vec{x},t) j^i(0,0)\rangle$$

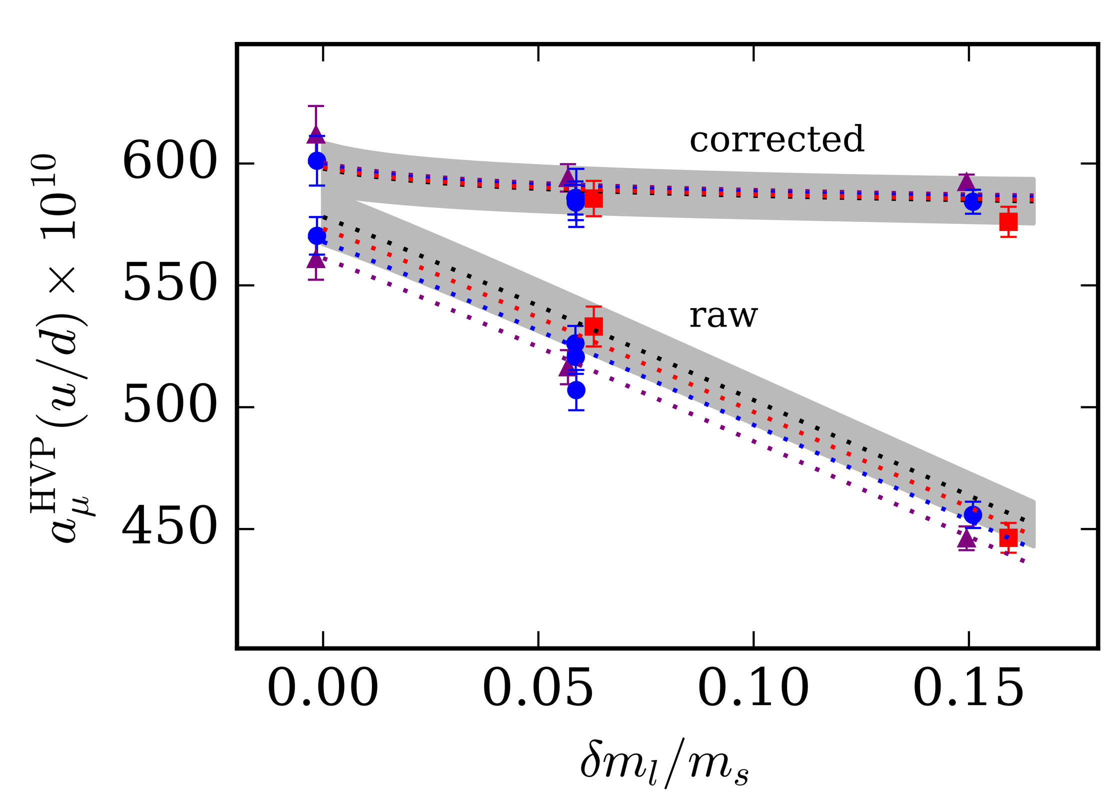
$$G_{2j} = (-1)^j (2j)! \Pi_{j-1};\ \hat{\Pi}(k^2) = \sum_j k^{2j}\Pi_j$$

$$a_\mu^{\mathrm{HVP,LO}}(\mathrm{f}) = \frac{\alpha}{\pi}\int_0^\infty dk^2 f(k^2)(4\pi\alpha Q_{\mathrm{f}}^2)\hat{\Pi}_{\mathrm{f}}(k^2)$$

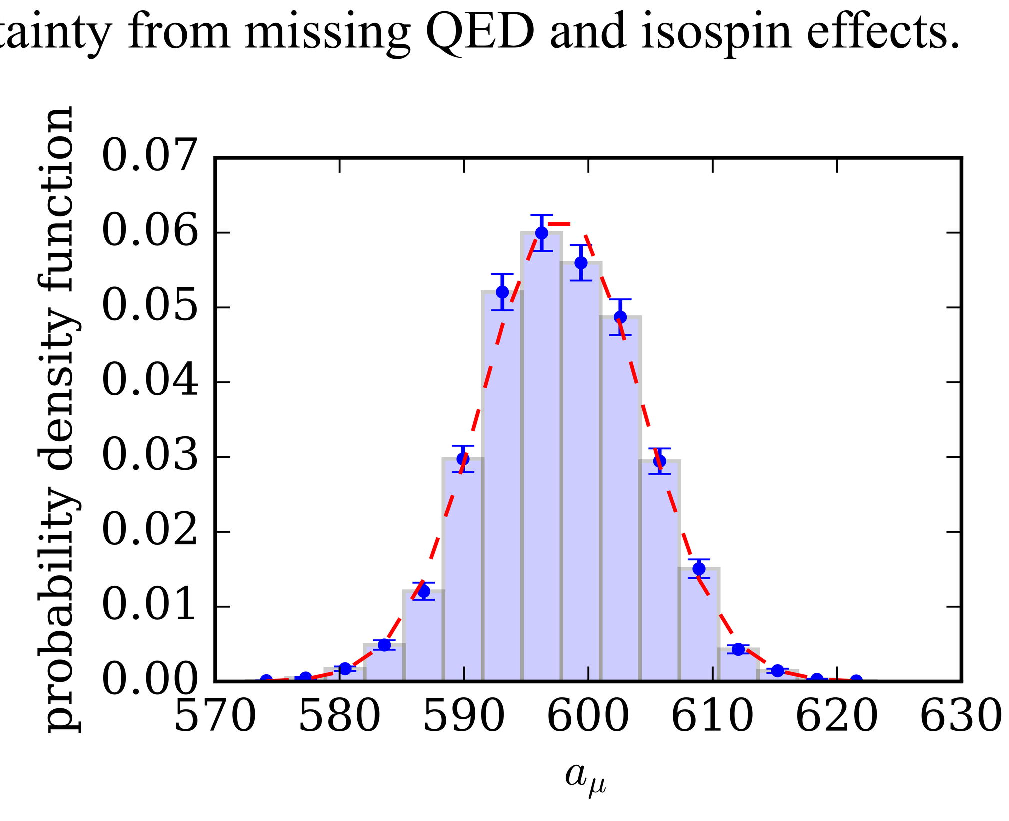
The ρ pole dominates the calculation, giving ~80% of the result. ρ properties obtained from our correlators at large t can be compared to experiment (see Fig. top right). At smaller t, and in the t-moments, additional states contribute, including $\pi\pi$. The $\pi\pi$ contribution is sensitive to the volume and the π mass. A specific staggered quark issue is that different tastes of π meson have masses that differ at $O(\alpha_s a^2)$. The leading $\pi\pi$ terms are readily calculated in scalar QED.
Corrections to this can be estimated by coupling in an explicit ρ field. Parameters m_π, e_π, m_ρ, f_ρ, $f_{\rho\pi\pi}$ are well-known. Our correction is 7.0(7)% at physical u/d masses.

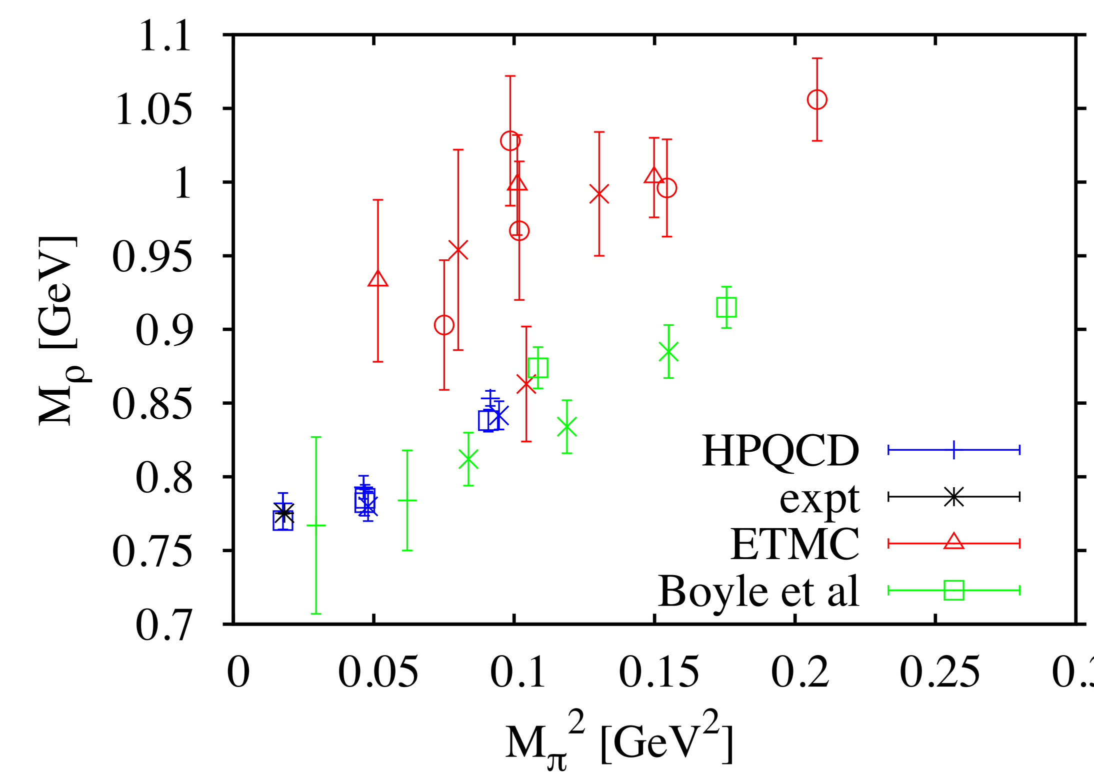


Sensitivity to the u/d quark mass is almost eliminated by rescaling the moments (with $\pi\pi$ removed) by appropriate powers of the experimental ρ mass [4].



The plot above shows corrected and raw data as a function of u/d quark mass. Note how the corrected data have reduced volume, m_{ud} and a-dependence. A simple fit (with distribution below) gives value 598(6)(8) $\times 10^{-10}$ where 8 is the systematic uncertainty from missing QED and isospin effects.



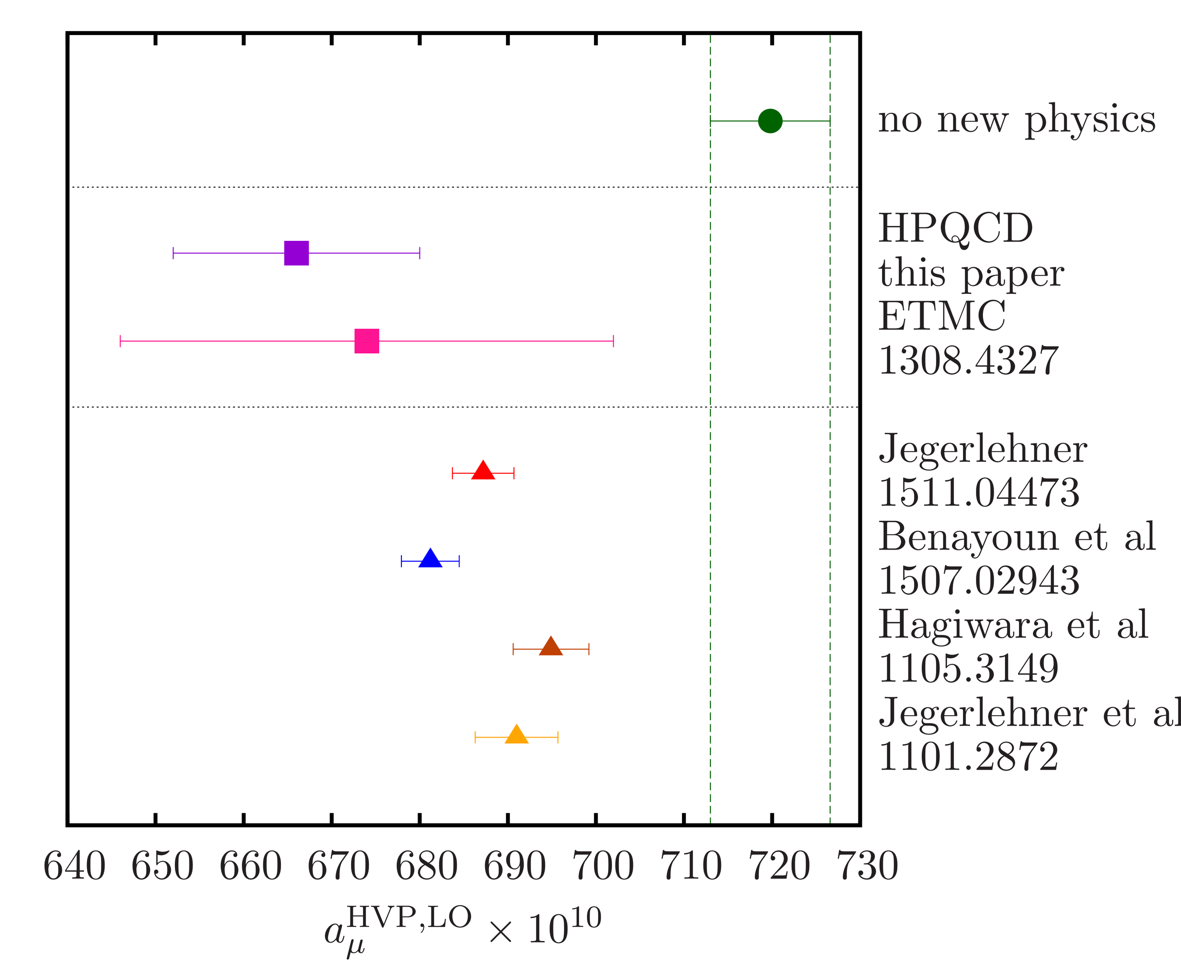


$$a_\mu^{HVP,LO}{}_{\mathrm{conn}} \times 10^{10} = \begin{cases} 598(11) & u/d \\ 53.4(6) & s \\ 14.4(4) & c \\ 0.27(4) & b \end{cases}$$

The total HVP,LO result [1,2,5] is then

$$a_\mu^{\mathrm{HVP,LO}} = 666(6)(12)\times 10^{-10}$$

including a 1.5% systematic uncertainty for quark-line



disconnected diagrams [6]. We compare lattice results to determinations that use the experimental cross-section for e+e- to hadrons above. The green point shows the value expected if there were no new physics (i.e. experiment - [sum of SM values for QED, EW, HVP,HO and Hlbl pieces]).

Conclusions

Lattice QCD calculations of the HVP contribution to a_μ are making fast progress. Our result is the accurate to date and shows a 3σ discrepancy with experiment. Ongoing work with MILC will improve statistics, use finer lattices, add QED and isospin effects and improve analysis of disconnected contributions.

[1] B. Chakraborty et al, HPQCD, 1601.03071.
[2] B. Chakraborty et al, HPQCD, 1403.1778.
[3] T. Blum, hep-lat/0212018
[4] Feng et al, ETMC, 1103.4818
[5] B. Colquhoun et al, HPQCD, 1408.5768.
[6] B. Chakraborty et al, Hadspec/HPQCD, 1512.03270

These calculations used Darwin@Cambridge, a component of the UK STFC's DiRAC facility.