Baryon interactions from lattice QCD with physical masses -- S=-2 sector --

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for HAL QCD Collaboration

HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

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Introduction

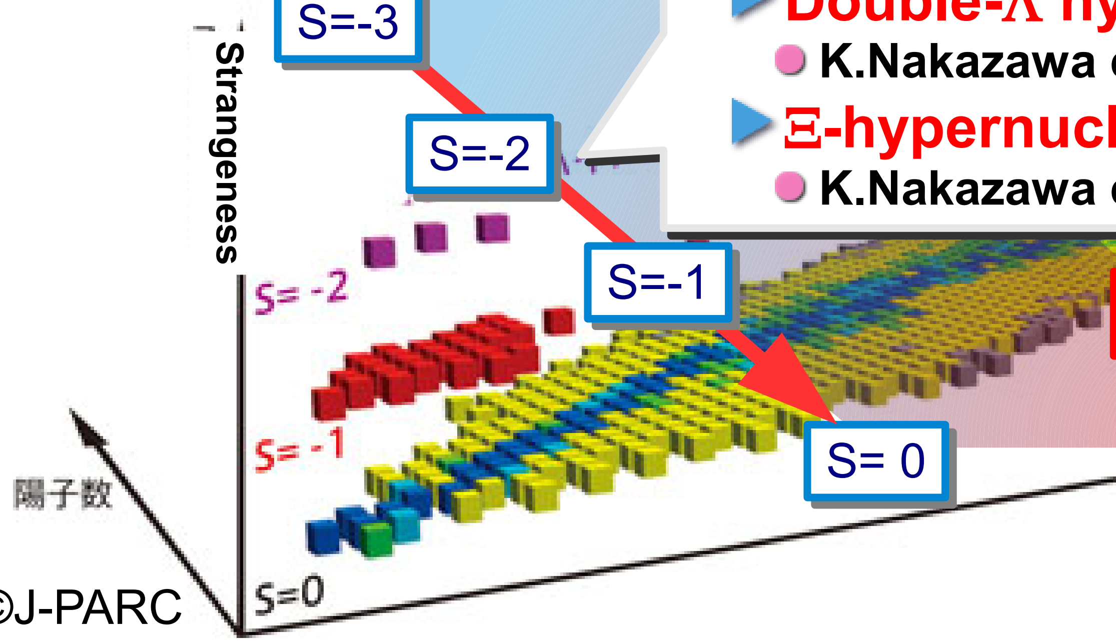
BB interactions are crucial to investigate (hyper-)nuclear structures

Lattice QCD simulation

- **Advantageous for more strange quarks**
- **Signals getting worse as increasing the number of light quarks.**
- **Complementary role to experiment.**

Main topics of S=-2 multi baryon system

- **H-dibaryon**
  - **R.L. Jaffe, PRL 38 (1977) 195**
- **Double-Λ hypernuclei**
  - **K.Nakazawa et al, KEK-E176 Collaboration**
- **Ξ-hypernuclei**
  - **K.Nakazawa et al, KEK-E373 Collaboration**



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Experiment

Baryon-baryon system with S=-2

Spin singlet states

Isospin	BB channels		
I=0	$\Lambda\Lambda$	$N\Xi$	$\Sigma\Sigma$
I=1	$N\Xi$	$\Lambda\Sigma$	——
I=2	$\Sigma\Sigma$	——	——

Spin triplet states

Isospin	BB channels		
I=0	$N\Xi$	——	——
I=1	$N\Xi$	$\Lambda\Sigma$	$\Sigma\Sigma$

Relations between BB channels and SU(3) irreducible representations

$$8 \times 8 = 27 + 8_S + 1 + \overline{10} + 10 + 8_A$$

$J^P=0^+$, I=0

$$\begin{pmatrix} \Lambda\Lambda \\ N\Xi \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{40}} \begin{pmatrix} -\sqrt{5} & -\sqrt{8} & \sqrt{27} \\ \sqrt{20} & \sqrt{8} & \sqrt{12} \\ \sqrt{15} & -\sqrt{24} & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 8 \\ 27 \end{pmatrix}$$

$J^P=0^+$, I=1

$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \end{pmatrix} = \frac{1}{5} \begin{pmatrix} \sqrt{2} & -\sqrt{3} \\ \sqrt{3} & \sqrt{2} \end{pmatrix} \begin{pmatrix} 27 \\ 8 \end{pmatrix}$$

$J^P=0^+$, I=2

$$\Sigma\Sigma \Leftrightarrow 8$$

$J^P=1^+$, I=0

$$N\Xi \Leftrightarrow 8$$

$J^P=1^+$, I=1

$$\begin{pmatrix} N\Xi \\ \Sigma\Lambda \\ \Sigma\Sigma \end{pmatrix} = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{2} & -\sqrt{2} & \sqrt{2} \\ \sqrt{3} & -\sqrt{3} & 0 \\ 1 & 1 & \sqrt{4} \end{pmatrix} \begin{pmatrix} 8 \\ \overline{10} \\ 10 \end{pmatrix}$$

Features of flavor singlet interaction is integrated into the S=-2 $J^P=0^+$, I=0 system.

Keys to understand H-dibaryon

A strongly bound state predicted by Jaffe in 1977 using MIT bag model.

H-dibaryon state is
- **SU(3) flavor singlet [uuddss], strangeness S=-2.**
- **spin and isospin equals to zero, and $J^P = 0^+$**

▶ Strongly attractive interaction is expected in flavor singlet channel.

- **Short range one-gluon exchange contributions**

  Strongly attractive Color Magnetic Interaction

- **Symmetry of two-baryon system (Pauli principle)**

  Flavor singlet channel is free from Pauli blocking effect

	27	8	1	$\underline{10}$	10	8
Pauli	mixed	forbidden	allowed	mixed	forbidden	mixed
CMI	repulsive	repulsive	attractive	repulsive	repulsive	repulsive

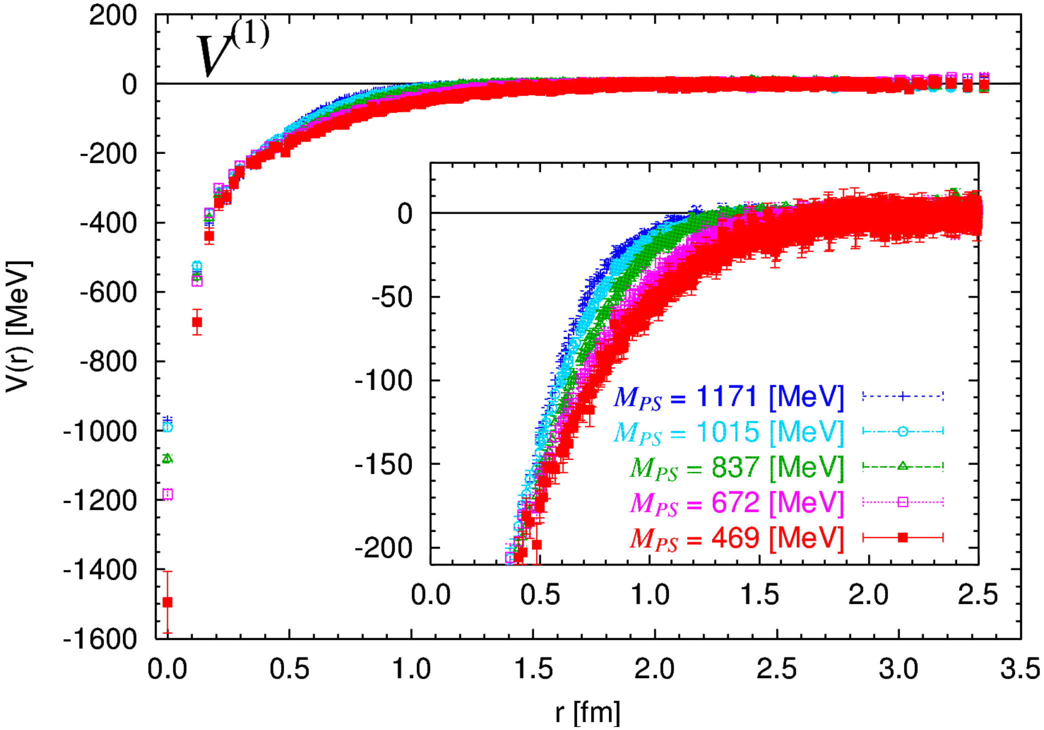
Oka, Shimizu and Yazaki NPA464 (1987)

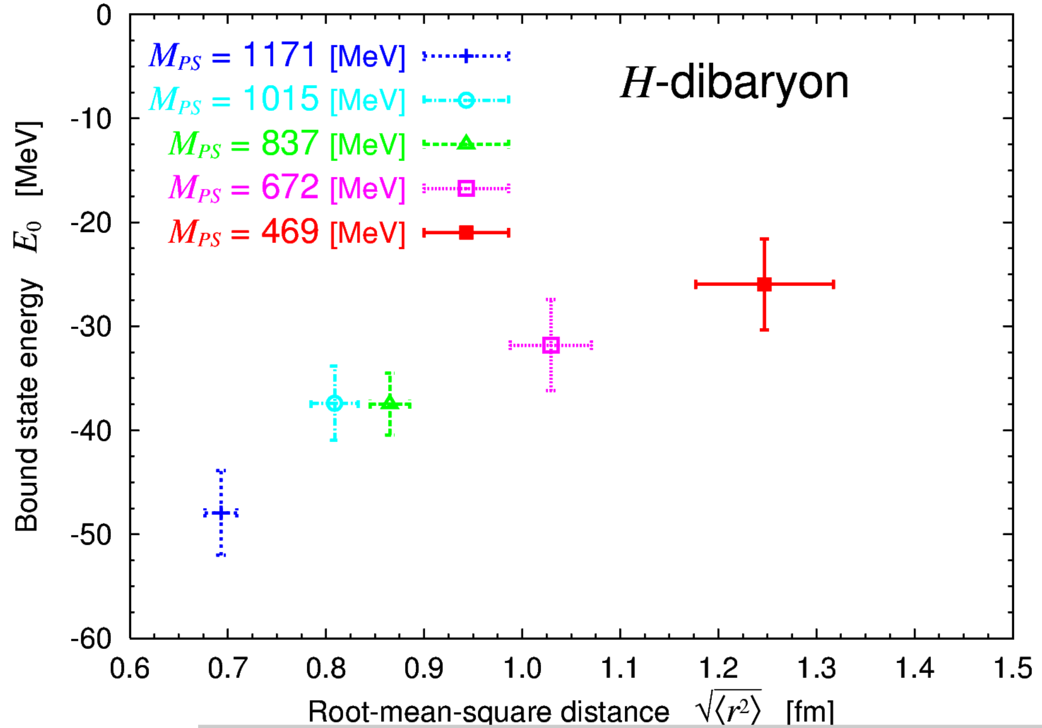
▶ SU(3) breaking effects
- **Threshold separation**
- **Changes of interactions**

⟵ **Non-trivial contributions**

Hunting for H-dibaryon in SU(3) limit

Strongly attractive interaction is expected in flavor singlet channel.

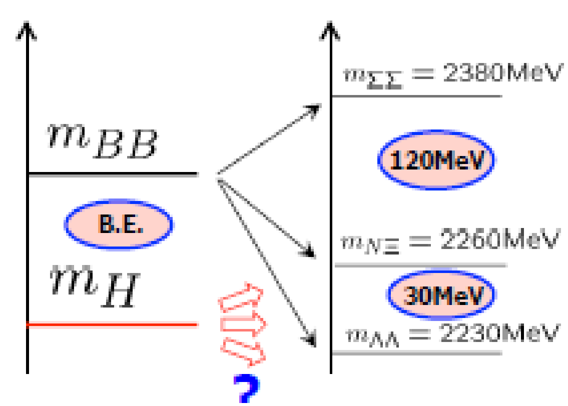




T.Inoue et al[HAL QCD Coll.] NPA881(2012) 28

- **Strongly attractive potential was found in the flavor singlet channel.**
- **Bound state was found in this mass range with SU(3) symmetry.**

What happens at the physical point?

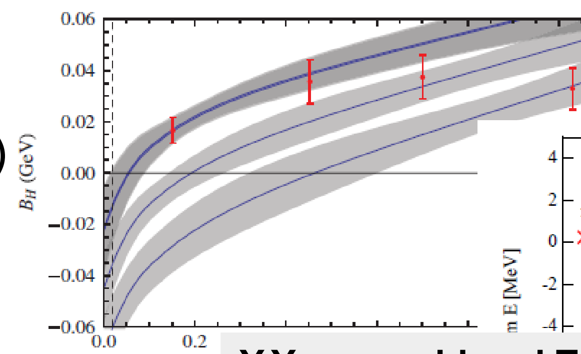


Works on H-dibaryon state

Theoretical status

Several sort of calculations and results
(bag models, NRQM, Quenched LQCD….)

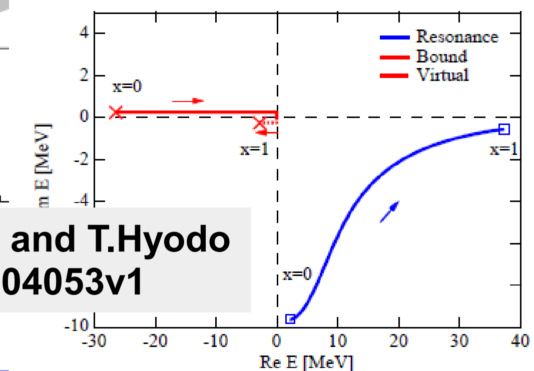
→ There were no conclusive result.



P. E. Shanahan et al
PRL 107(2011) 092004

Chiral extrapolations of recent LQCD data

→ **Unbound or resonance**

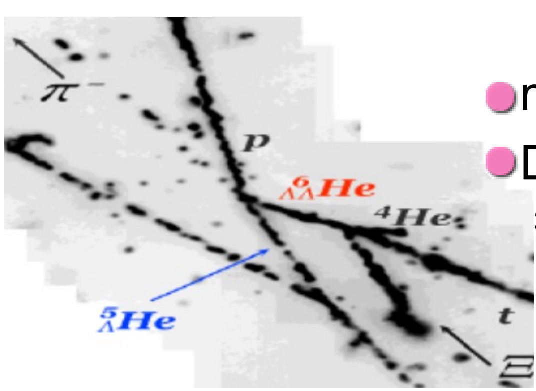


Y.Yamaguchi and T.Hyodo
hep-ph:1607.04053v1

Experimental status

"NAGARA Event"

K.Nakazawa et al KEK-E176 & E373 Collaboration

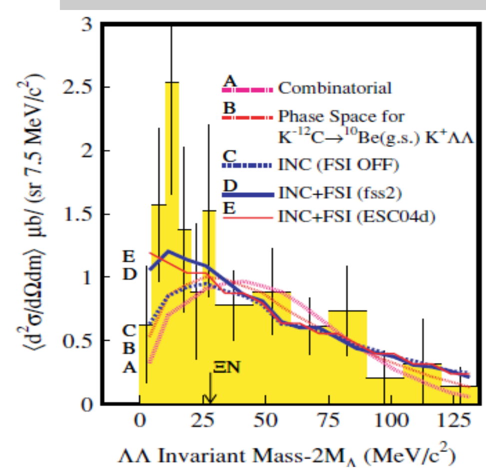


PRL87(2001)212502

- $m_H \geq 2m_\Lambda - 6.9$MeV
- Deeply bound dibaryon state is ruled out

"$^{12}C(K^-,K^+\Lambda\Lambda)$ reaction"

C.J.Yoon et al KEK-PS E522 Collaboration



PRC75(2007)022201(R)

- Significance of enhancements below 30 MeV.

→ **Larger statistics**

HAL QCD method (coupled-channel)

NBS wave function

$$\Psi^{\alpha}(E_i,\vec{r})e^{-E_i t} = \langle 0 | (B_1 B_2)^{\alpha}(\vec{r}) | E_i \rangle$$

$$\Psi^{\beta}(E_i,\vec{r})e^{-E_i t} = \langle 0 | (B_1 B_2)^{\beta}(\vec{r}) | E_i \rangle$$

$$\int dr\, \tilde{\Psi}_{\beta}(E',\vec{r})\Psi^{\gamma}(E,\vec{r}) = \delta(E'-E)\delta_{\beta}^{\gamma}$$

$$R_E^{B_1 B_2}(t,\vec{r}) = \Psi_{B_1 B_2}(\vec{r},E)e^{(-E+m_1+m_2)t}$$

Leading order of velocity expansion and time-derivative method.

Modified coupled-channel Schrödinger equation

$$\begin{pmatrix} \left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\alpha}\right)R^{\alpha}_{E_0}(t,\vec{r}) \\ \left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\beta}\right)R^{\beta}_{E_0}(t,\vec{r}) \end{pmatrix} = \begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})\Delta^{\alpha}{}_{\beta}(t) \\ V^{\beta}{}_{\alpha}(\vec{r})\Delta^{\beta}{}_{\alpha}(t) & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{E_0}(t,\vec{r}) \\ R^{\beta}_{E_0}(t,\vec{r}) \end{pmatrix}$$

$$\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu_\beta}\right)R^{\beta}_{E_1}(t,\vec{r}) \quad \ldots \quad (\vec{r})\Delta^{\alpha}{}_{\beta}(t) \quad V^{\beta}{}_{\beta}(\vec{r}) \quad \begin{pmatrix} R^{\alpha}_{E_1}(t,\vec{r}) \\ R^{\beta}_{E_1}(t,\vec{r}) \end{pmatrix}$$

$$\Delta_{\beta}^{\alpha} = \frac{\exp(-(m_{\alpha_1}+m_{\alpha_2})t)}{\exp(-(m_{\beta_1}+m_{\beta_2})t)}$$

S.Aoki et al [HAL QCD Collab.] Proc. Jpn. Acad., Ser. B, 87 509

K.Sasaki et al [HAL QCD Collab.] PTEP no 11 (2015) 113B01

Considering two different energy eigen states

Potential

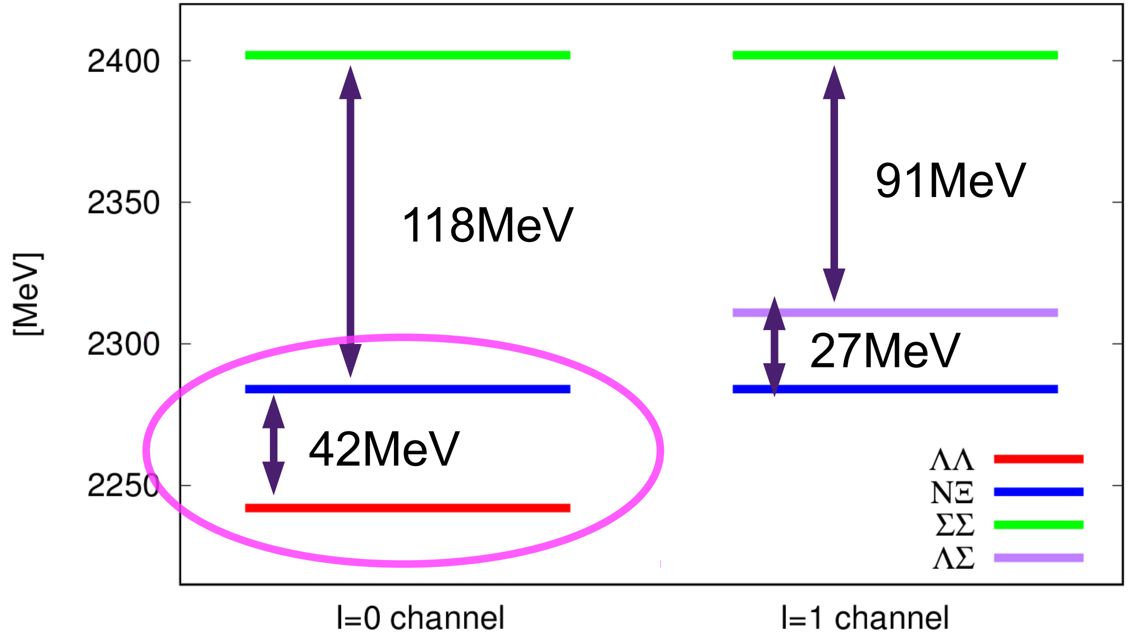
$$\begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r})\Delta^{\alpha}_{\beta} \\ V^{\beta}{}_{\alpha}(\vec{r})\Delta^{\beta}_{\alpha} & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} = \begin{pmatrix} \left(\frac{\nabla^2}{2\mu_\alpha}-\frac{\partial}{\partial t}\right)R^{\alpha}_{E0}(t,\vec{r}) & \left(\frac{\nabla^2}{2\mu_\beta}-\frac{\partial}{\partial t}\right)R^{\alpha}_{E1}(t,\vec{r}) \\ \left(\frac{\nabla^2}{2\mu_\alpha}-\frac{\partial}{\partial t}\right)R^{\beta}_{E0}(t,\vec{r}) & \left(\frac{\nabla^2}{2\mu_\beta}-\frac{\partial}{\partial t}\right)R^{\beta}_{E1}(t,\vec{r}) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{E0}(t,\vec{r}) & R^{\alpha}_{E1}(t,\vec{r}) \\ R^{\beta}_{E0}(t,\vec{r}) & R^{\beta}_{E1}(t,\vec{r}) \end{pmatrix}^{-1}$$

Numerical setup

- 2+1 flavor gauge configurations.
  - Iwasaki gauge action & O(a) improved Wilson quark action
  - a = 0.086 [fm], a^{-1} = 2.300 GeV.
  - 96^3x96 lattice, L = 8.24 [fm].
  - 414 confs x 28 sources x 4 rotations.

- Flat wall source is considered to produce S-wave B-B state.

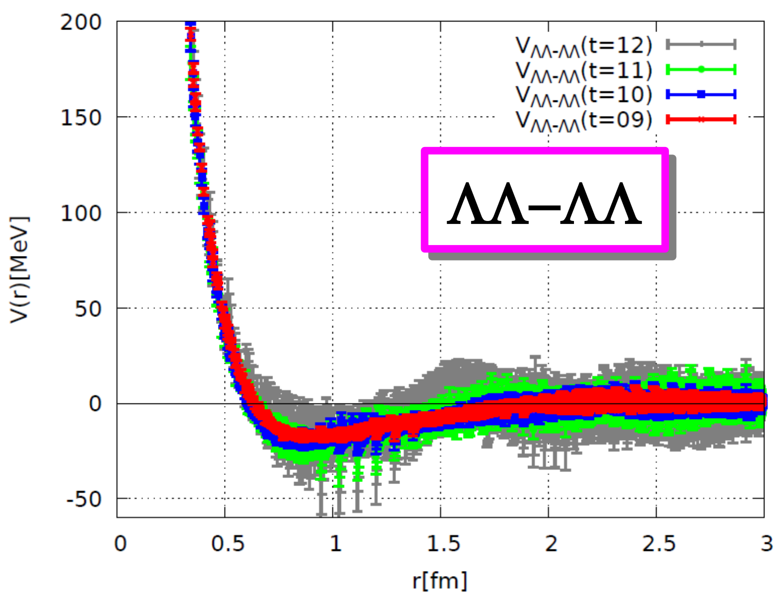
	Mass [MeV]
π	146
K	525
m_π/m_K	0.28
N	956±12
Λ	1121±4
Σ	1201±3
Ξ	1328±3

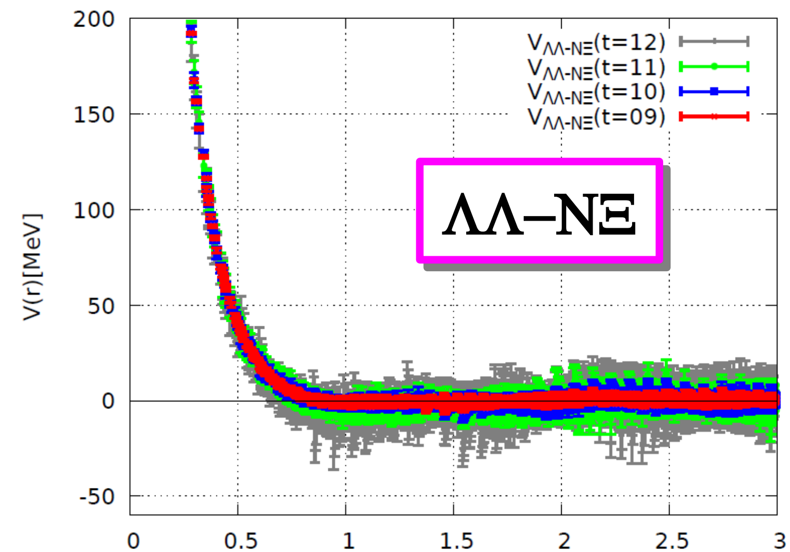


$\Lambda\Lambda$, $N\Xi$ (I=0) $^1S_0$ potential (2ch calc.)

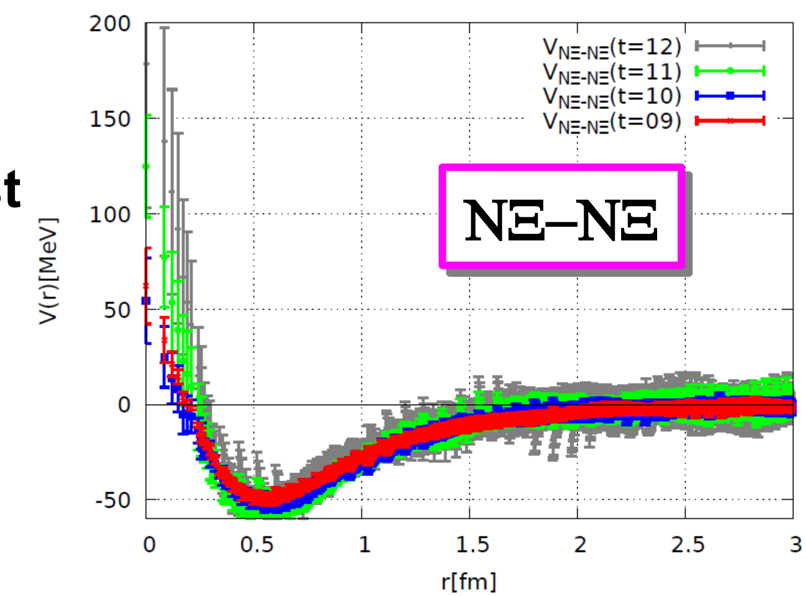
- N_f = 2+1 full QCD with L = 8fm, *m_π = 146 MeV*

Preliminary!





- **Potential calculated by only using $\Lambda\Lambda$ and $N\Xi$ channels.**
- **Long range part of potential is almost stable against the time slice.**
- **Short range part of $N\Xi$ potential changes as time t goes.**
- **$\Lambda\Lambda$–$N\Xi$ transition potential is quite small in r > 0.7fm region**

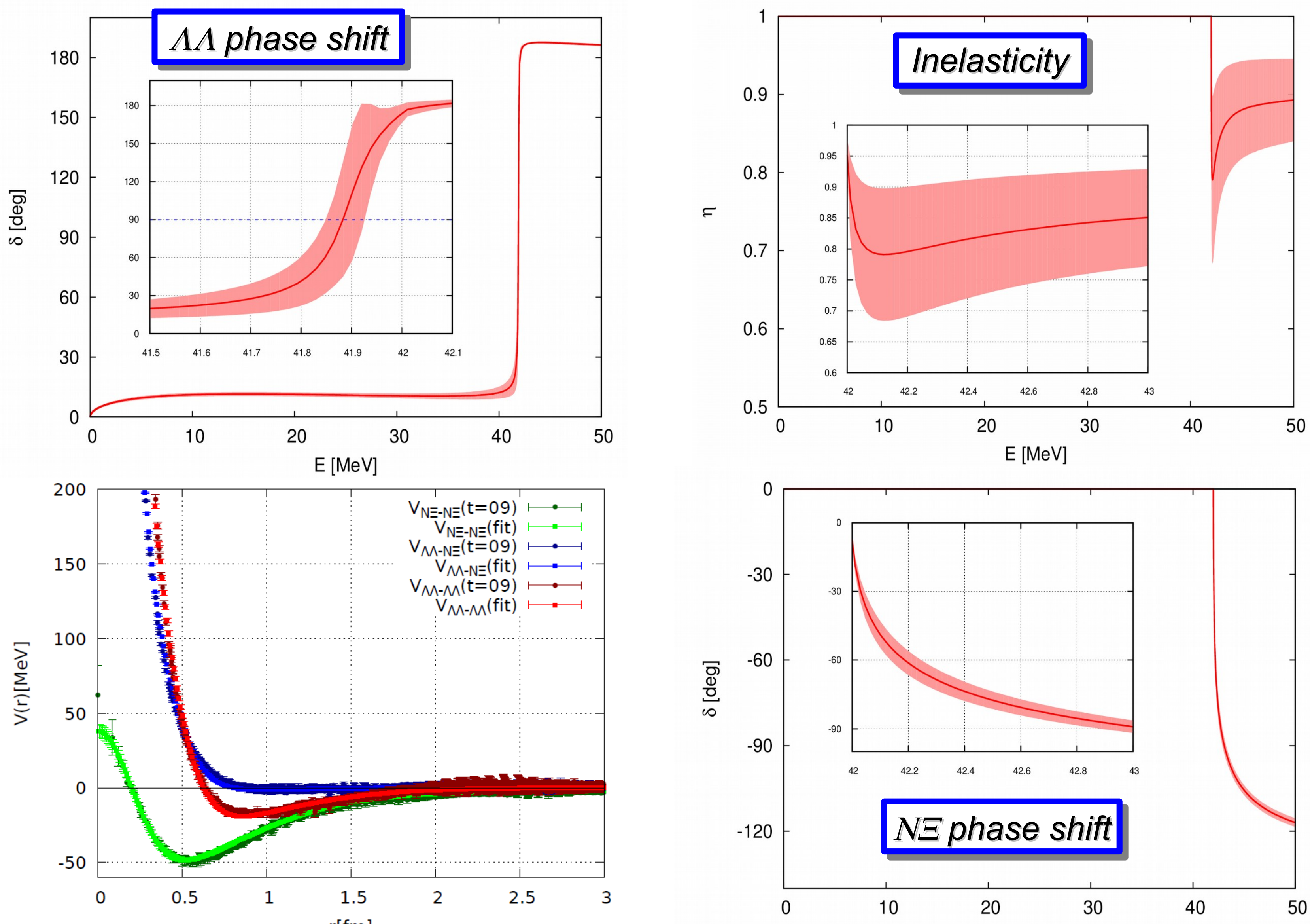


ΛΛ and NΞ phase shift and inelasticity

t=09

▶ N_f = 2+1 full QCD with L = 8fm, m_π = 146 MeV

Preliminary!

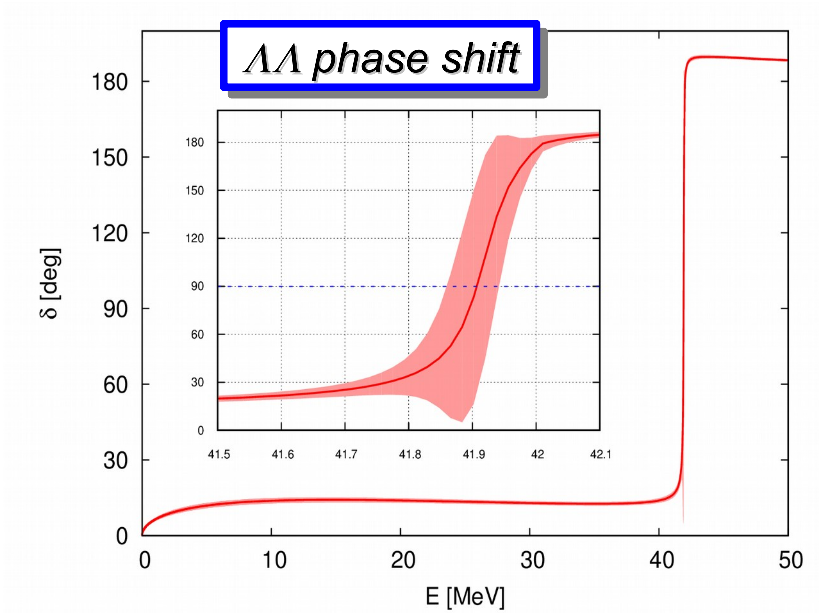


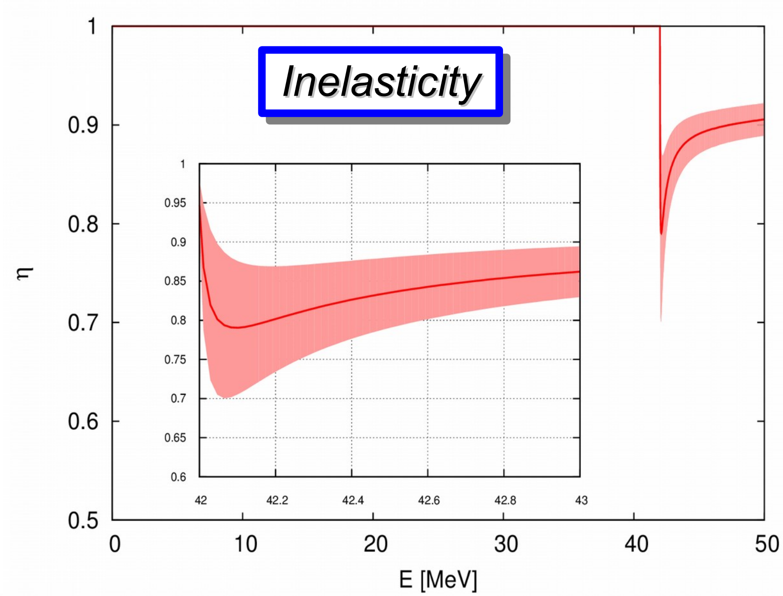
$\Lambda\Lambda$ and $N\Xi$ phase shift and inelasticity

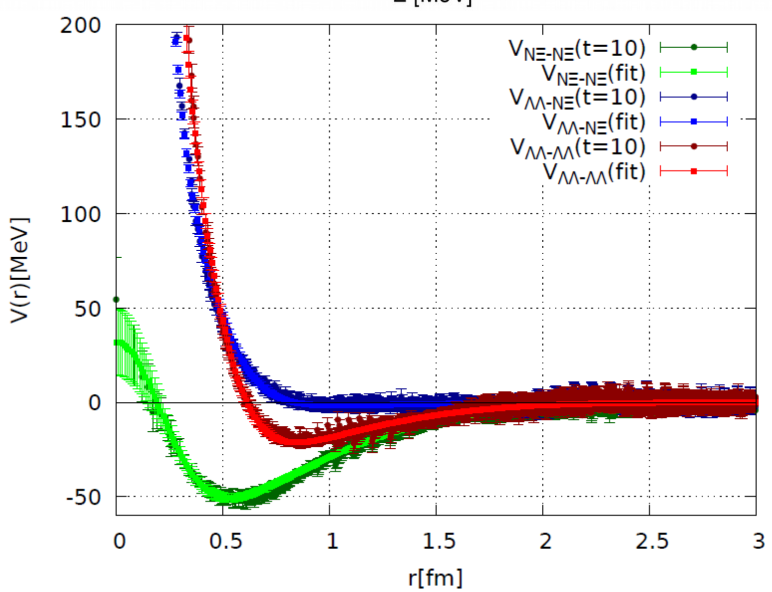
t=10

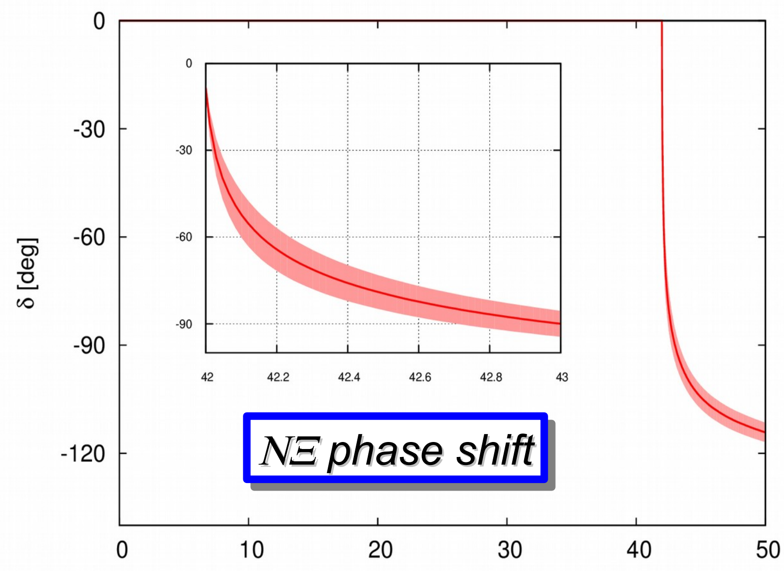
▶ **N_f = 2+1 full QCD with L = 8fm, m_π = 146 MeV**

Preliminary!







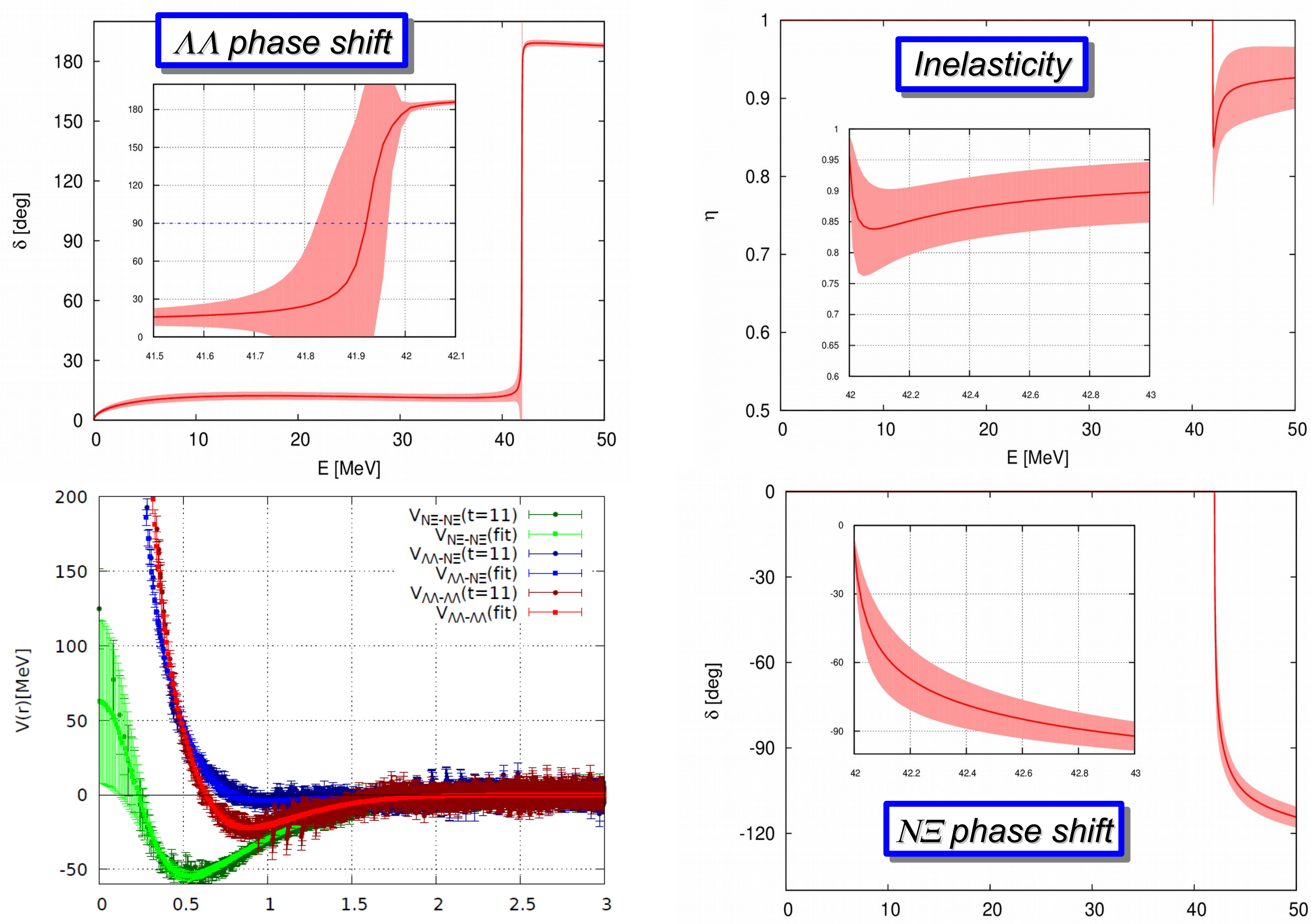


$\Lambda\Lambda$ and $N\Xi$ phase shift and inelasticity

t=11

▶ N_f = 2+1 full QCD with L = 8fm, m_π = 146 MeV

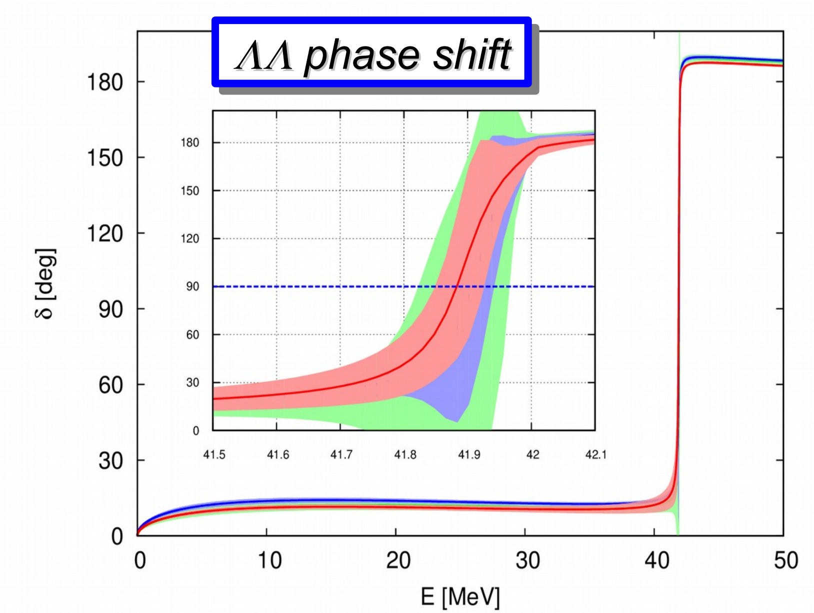
Preliminary!

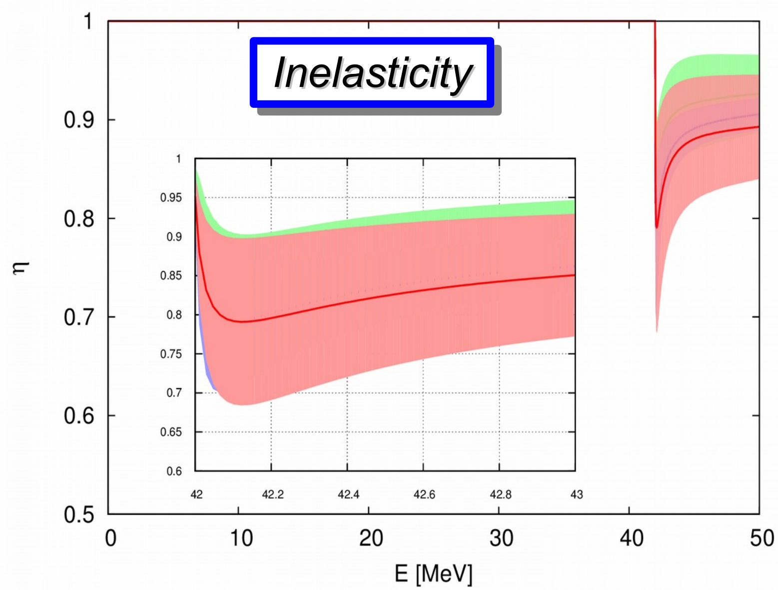


ΛΛ and NΞ phase shift and inelasticity T-dep

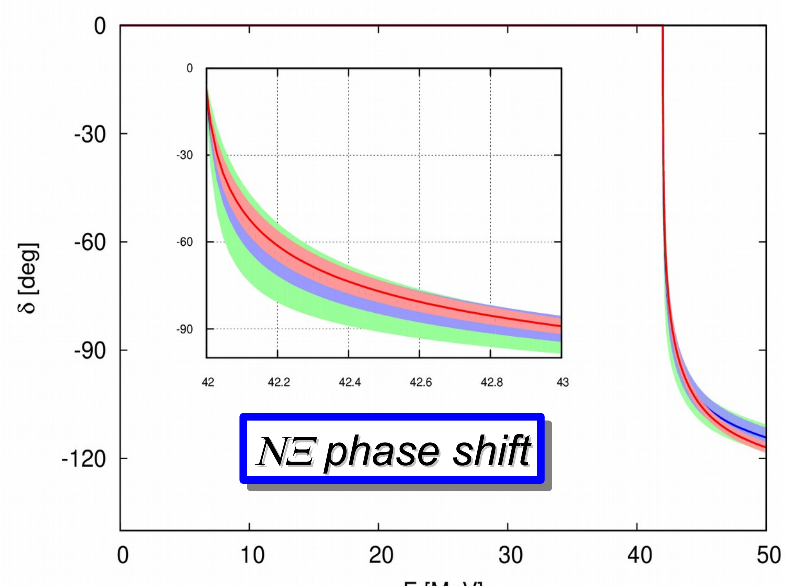
▶ N_f = 2+1 full QCD with L = 8fm, ***m_π = 146 MeV***

Preliminary!





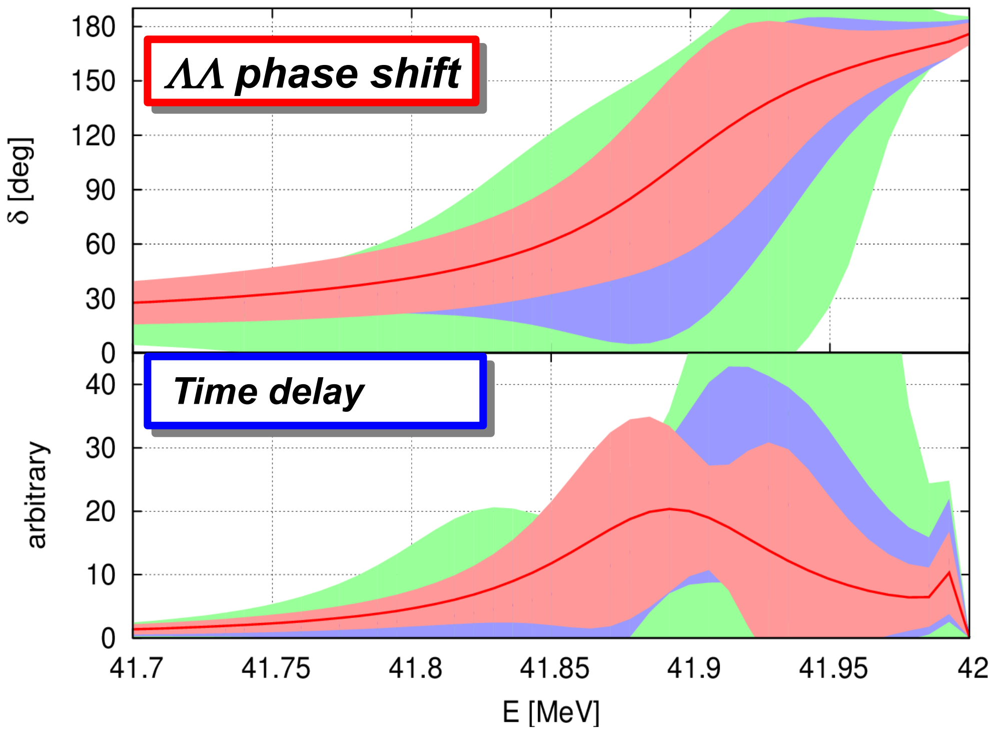
- **ΛΛ and NΞ phase shift is calculated**
  by using 2ch effective potential.
- **A sharp resonance is found**
  just below the NΞ threshold.
- **Inelasticity is small.**



Breit-Wigner mass and width

▶ N_f = 2+1 full QCD with L = 8fm, ***m_π = 146 MeV***

Preliminary!



- **In the vicinity of resonance point,**

$$\delta(E)=\delta_B-\arctan\left(\frac{\Gamma/2}{E-E_r}\right)$$

thus

$$\frac{d\,\delta(E)}{d\,E}=\frac{\Gamma/2}{(E-E_r)^2+(\Gamma/2)^2}$$

- **Fitting the time delay of ΛΛ scattering by the Breit-Wigner type finction,**

Resonance enargy and width

t=09

$$E_R-E_{\Lambda\Lambda}=41.894\pm0.039[MeV]$$
$$\Gamma = 0.099\pm0.059[MeV]$$

t=10

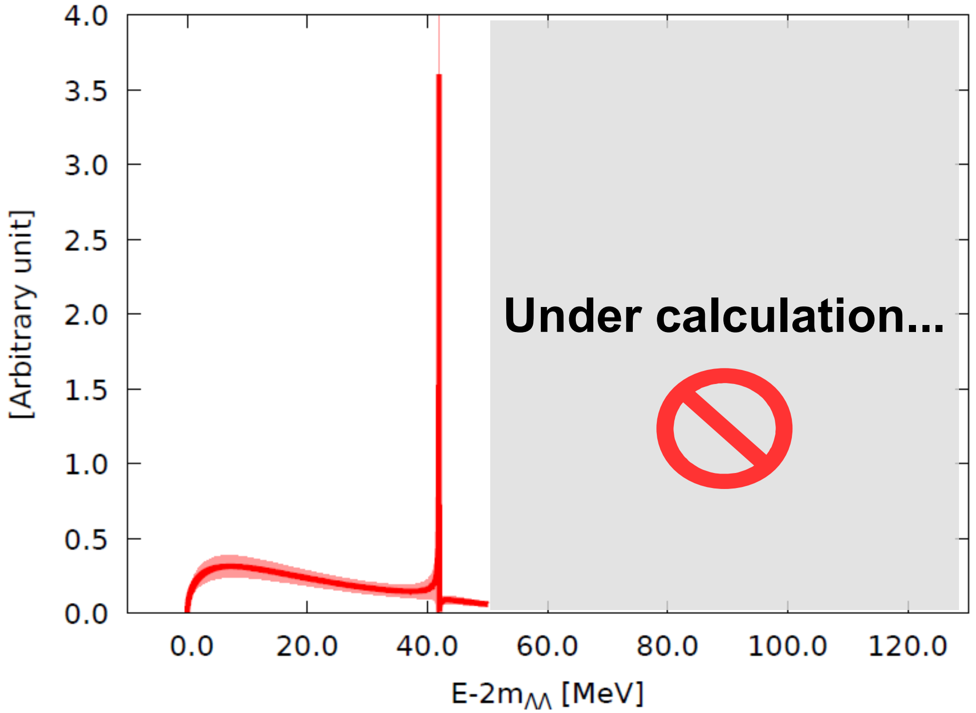
$$E_R-E_{\Lambda\Lambda}=41.917\pm0.056[MeV]$$
$$\Gamma = 0.077\pm0.021[MeV]$$

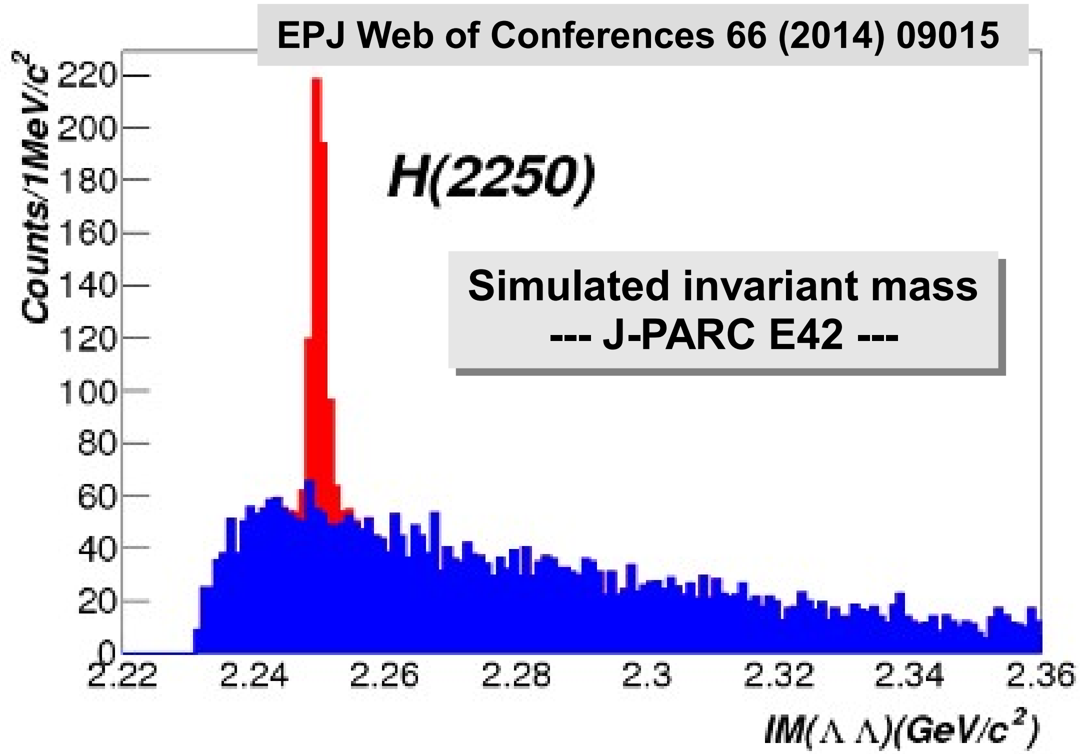
t=11

$$E_R-E_{\Lambda\Lambda}=41.927\pm0.105[MeV]$$
$$\Gamma = 0.050\pm0.053[MeV]$$

Invariant mass spectrum of $\Lambda\Lambda$ channel

- **N_f = 2+1 full QCD with L = 8fm, *m_π = 146 MeV***

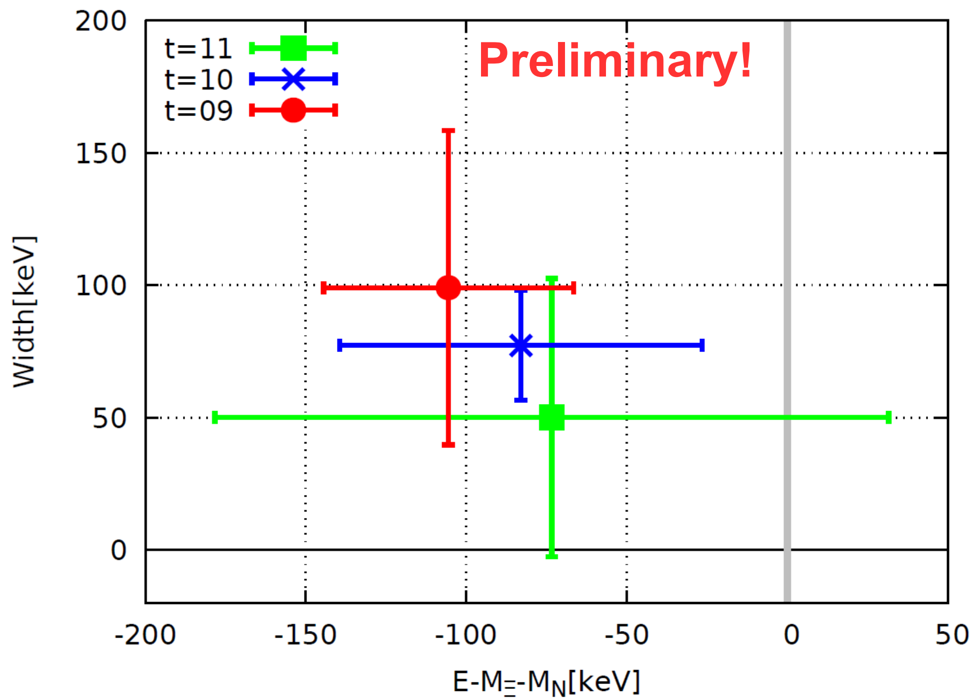




- **Sharp peak below NΞ threshold**
- **Direct comparison with our simulation results and experimental data will be performed in near future?**

Summary

- H-dibaryon state is investigated using 414confs x 28src x 4rot.
- We perform ΛΛ–NΞ coupled channel calculation.
- Sharp resonance is found just below the NΞ threshold.
  - Resonance position and width from Breit-Wigner type fit



- We continue to study it by using higher statistical data.

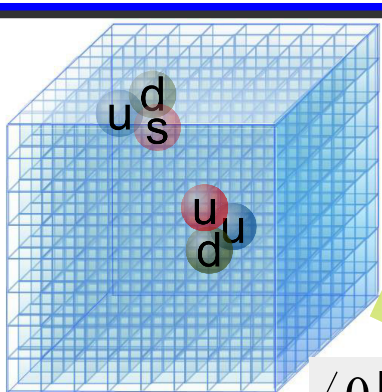
Backup slides

HAL QCD method

Derivation of hadronic interaction from QCD

Start with the fundamental theory,QCD

Lattice QCD simulation



Lüscher's finite volume method

M. Lüscher, NPB354(1991)531

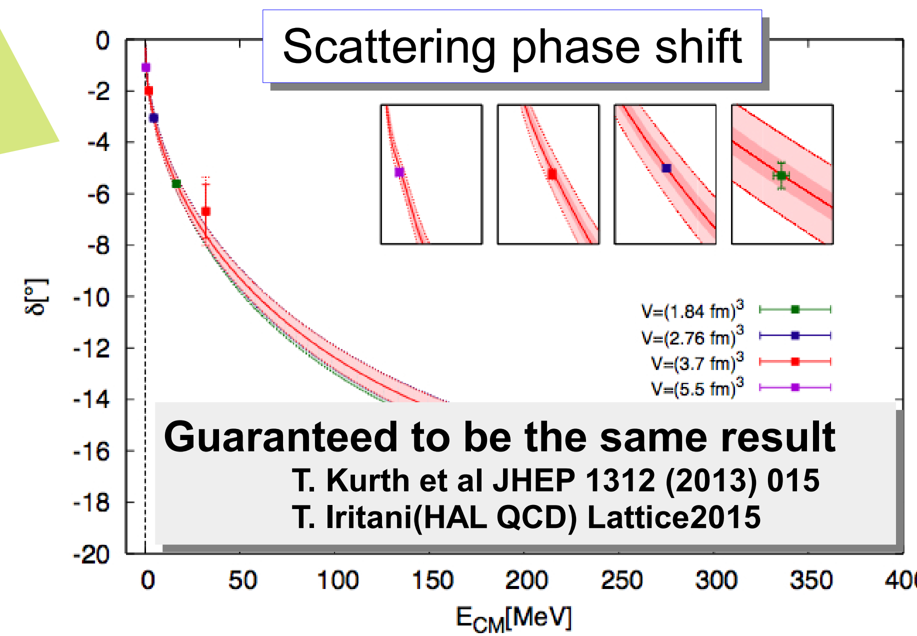
1. Measure the discrete energy spectrum, **E**
2. Put the E into the formula which connects E and **δ**

$$\langle 0|B_1 B_2(t,\vec{r})\bar{B}_2\bar{B}_1(t_0)|0\rangle = A_0 \Psi(\vec{r},E_0)e^{-E_0(t-t_0)}+\cdots$$

HAL QCD method

Ishii, Aoki, Hatsuda, PRL**99** (2007) 022001

1. Measure the NBS wave function, **Ψ**
2. Calculate potential, **V**, through Schrödinger eq.
3. Calculate observables by scattering theory

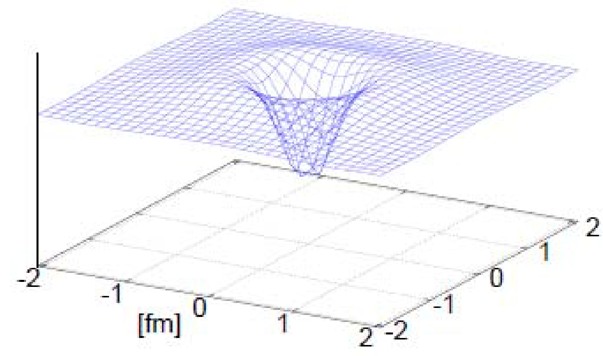


Guaranteed to be the same result
T. Kurth et al JHEP 1312 (2013) 015
T. Iritani(HAL QCD) Lattice2015

Nambu-Bethe-Salpeter wave function

Definition : equal time NBS w.f.

$$\Psi^{\alpha}(E,\vec{r})e^{-Et} = \sum_{\vec{x}} \langle 0 | H_1^{\alpha}(t,\vec{x}+\vec{r}) H_2^{\alpha}(t,\vec{x}) | E \rangle$$



E : Total energy of the system

Local composite interpolating operators

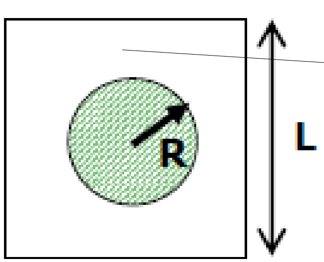
$$B_{\alpha} = \epsilon^{abc}(q_a^T C \gamma_5 q_b) q_{c\alpha} \qquad D_{\mu\alpha} = \epsilon^{abc}(q_a^T C \gamma_{\mu} q_b) q_{c\alpha}$$

$$M = (\bar{q}_a \gamma_5 q_a) \qquad \text{Etc.....}$$

- It satisfies the Helmholtz eq. in asymptotic region : $(p^2 + \nabla^2)\Psi(E,\vec{r}) = 0$
- Using the reduction formula,

C.-J.D.Lin et al.,NPB619 (2001) 467.

$$\Psi^{\alpha}(E,\vec{r}) = \sqrt{Z_{H_1}}\sqrt{Z_{H_2}}\left(e^{i\vec{p}\cdot\vec{r}} + \int \frac{d^3q}{2E_q} \frac{T(q,p)}{4E_p(E_q - E_p - i\epsilon)} e^{i\vec{q}\cdot\vec{r}}\right)$$



$$\Psi(E,\vec{r}) \simeq A \frac{\sin(pr + \delta(E))}{pr}$$

Phase shift is defined as

$$S \equiv e^{i\delta}$$

NBS wave function has a same asymptotic form with quantum mechanics.
(NBS wave function is characterized from phase shift)

Potential in HAL QCD method

We define potentials which satisfy Schrödinger equation

$$\left(p^2+\nabla^2\right)\Psi^\alpha(E,\vec{r}) \equiv \int d^3y\, U^\alpha_\alpha(\vec{x},\vec{y})\,\Psi^\alpha(E,\vec{y})$$

Energy independent potential

$$\left(p^2+\nabla^2\right)\Psi^\alpha(E,\vec{r}) = K^\alpha(E,\vec{r})$$

$$\begin{aligned}K^\alpha(E,\vec{r}) &\equiv \int dE'\, K^\alpha(E',\vec{x}) \int d^3y\, \tilde{\Psi}^\alpha(E',\vec{y})\,\Psi^\alpha(E,\vec{y}) \\ &= \int d^3y \left[\int dE'\, K^\alpha(E',\vec{x})\,\tilde{\Psi}^\alpha(E',\vec{y})\right]\Psi^\alpha(E,\vec{y}) \\ &= \int d^3y\, U^\alpha_\alpha(\vec{x},\vec{y})\,\Psi^\alpha(E,\vec{y})\end{aligned}$$

We can define an energy independent potential but it is fully non-local.

This potential automatically reproduce the scattering phase shift

N. Ishii et al Phys. Lett. B**712**(2012)437

Time-dependent method

Start with the normalized four-point correlator.

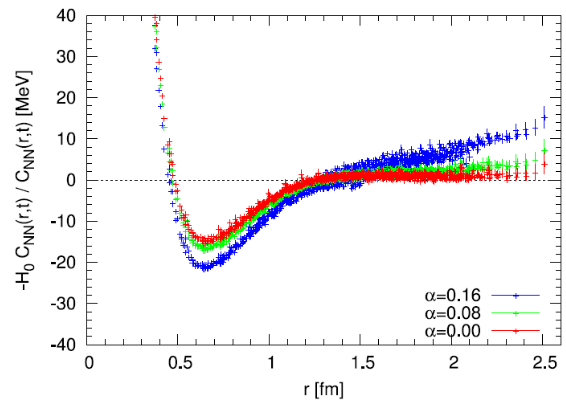
$$R_I^{B_1 B_2}(t,\vec{r}) = F_{B_1 B_2}(t,\vec{r})\, e^{(m_1+m_2)t}$$

$$= A_0 \Psi(\vec{r}, E_0)\, e^{-(E_0 - m_1 - m_2)t} + A_1 \Psi(\vec{r}, E_1)\, e^{-(E_1 - m_1 - m_2)t} + \cdots$$

Each wave functions satisfy
Schrödinger eq. with proper energy

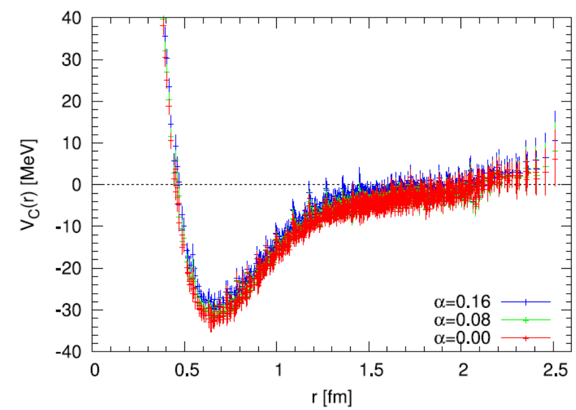
$$\left(\frac{p_0^2}{2\mu} + \frac{\nabla^2}{2\mu}\right)\Psi(\vec{r}, E_0) = \int U(\vec{r}, \vec{r}\,')\Psi(\vec{r}\,', E_0)\, d^3r'$$

$$\left(\frac{p_1^2}{2\mu} + \frac{\nabla^2}{2\mu}\right)\Psi(\vec{r}, E_1) = \int U(\vec{r}, \vec{r}\,')\Psi(\vec{r}\,', E_1)\, d^3r'$$



$$E_n - m_1 - m_2 \approx \frac{p_n^2}{2\mu}$$

$$\left(-\frac{\partial}{\partial t} + \frac{\nabla^2}{2\mu}\right) R_I^{B_1 B_2}(t,\vec{r}) = \int U(\vec{r}, \vec{r}\,')\, R_I^{B_1 B_2}(t,\vec{r})\, d^3r'$$



A single state saturation is not required!!

BB interaction from NBS wave function

$$\left(-\frac{\partial}{\partial t}+\frac{\nabla^2}{2\mu}\right)R_I^{B_1 B_2}(t,\vec{r})=\int U(\vec{r},\vec{r}\,')R_I^{B_1 B_2}(t,\vec{r})d^3 r\,'$$

Derivative (velocity) expansion of U is performed to deal with its nonlocality.

- For the case of oct-oct system,

$$U(\vec{r},\vec{r}\,') = \underbrace{\left[V_C(r)+S_{12}V_T(r)\right]}_{\text{Leading order part}}+\left[\vec{L}\cdot\vec{S}_s V_{LS}(r)+\vec{L}\cdot\vec{S}_a V_{ALS}(r)\right]+O(\nabla^2)$$

- For the case of dec-oct and dec-dec system,

$$U(\vec{r},\vec{r}\,') = \underbrace{\left[V_C(r)+S_{12}V_{T_1}(r)+S_{ii}V_{T_2}(r)+O(Spin\,op^3)\right]}_{\text{Leading order part}}+O(\nabla^2)$$

$$\equiv \left[V_C^{eff}(r)\right]+O(\nabla^2)$$

$$S_{ii}V_{T_2}(r):\quad ((\vec{r}\cdot\vec{S}_1)^2-\frac{\vec{r}^{\,2}}{3}\vec{S}_1^{\,2}+(\vec{r}\cdot\vec{S}_2)^2-\frac{\vec{r}^{\,2}}{3}\vec{S}_2^{\,2})V_{T^2}(r)$$

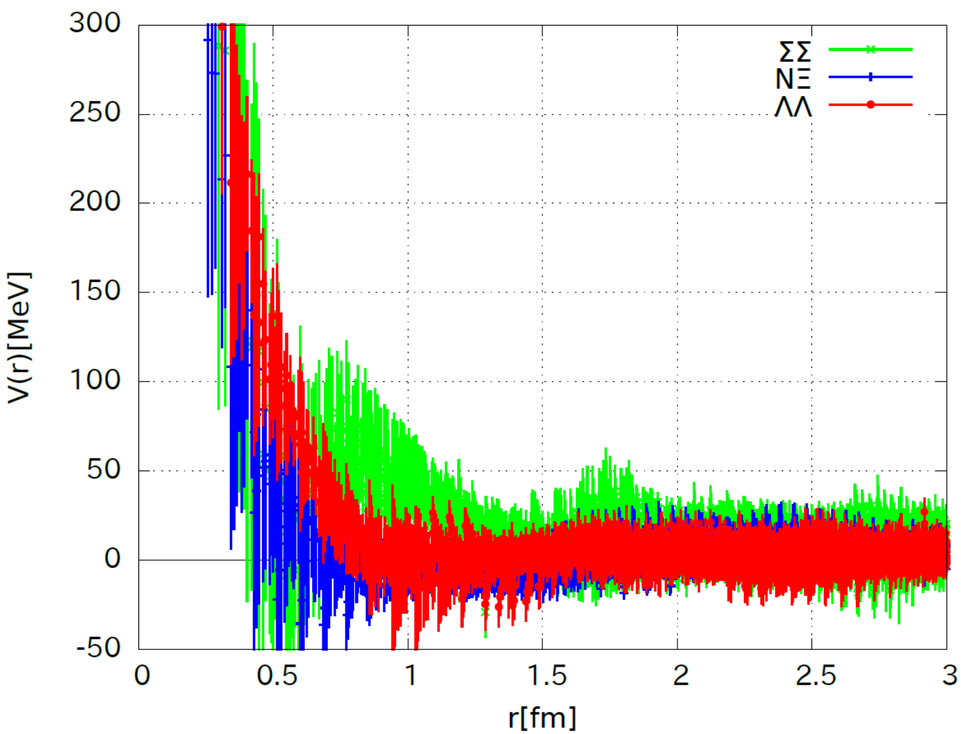
We consider the effective central potential which contains not only the genuine central potential but also tensor parts.

ΛΛ, NΞ, ΣΣ (I=0) $^1S_0$ channel (t=11)

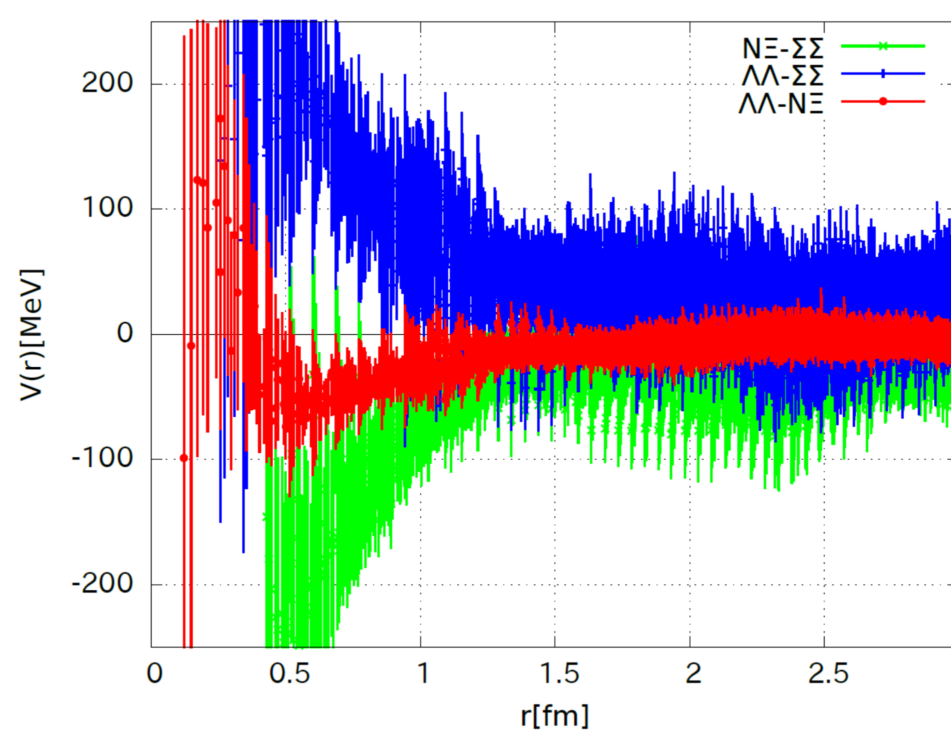
- N_f = 2+1 full QCD with L = 8fm, *m_π = 145 MeV*

Preliminary!

Diagonal elements



Off-diagonal elements



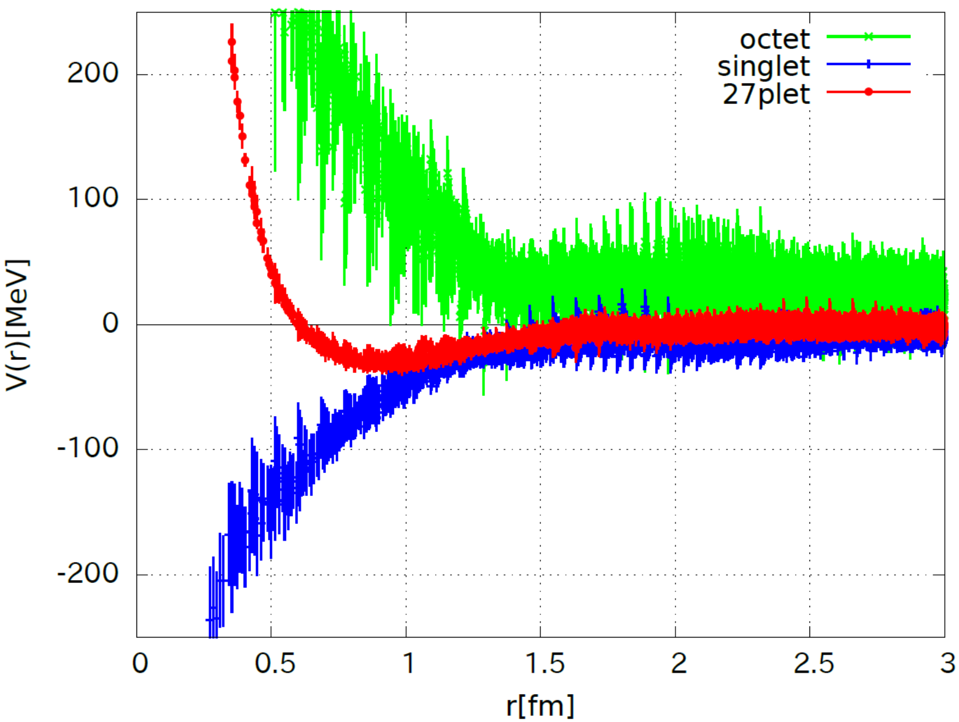
- All diagonal element have a repulsive core. ΣΣ-ΣΣ potential is strongly repulsive.
- The results have relatively large error comparing to the spin triplet cases.
- **Diagonal NΞ potential is more attractive than the ΛΛ potential.**
- We need more statistics to discuss physical observables through this potential.

1, 8, 27plet (I=0) $^1S_0$ channel (t=11)

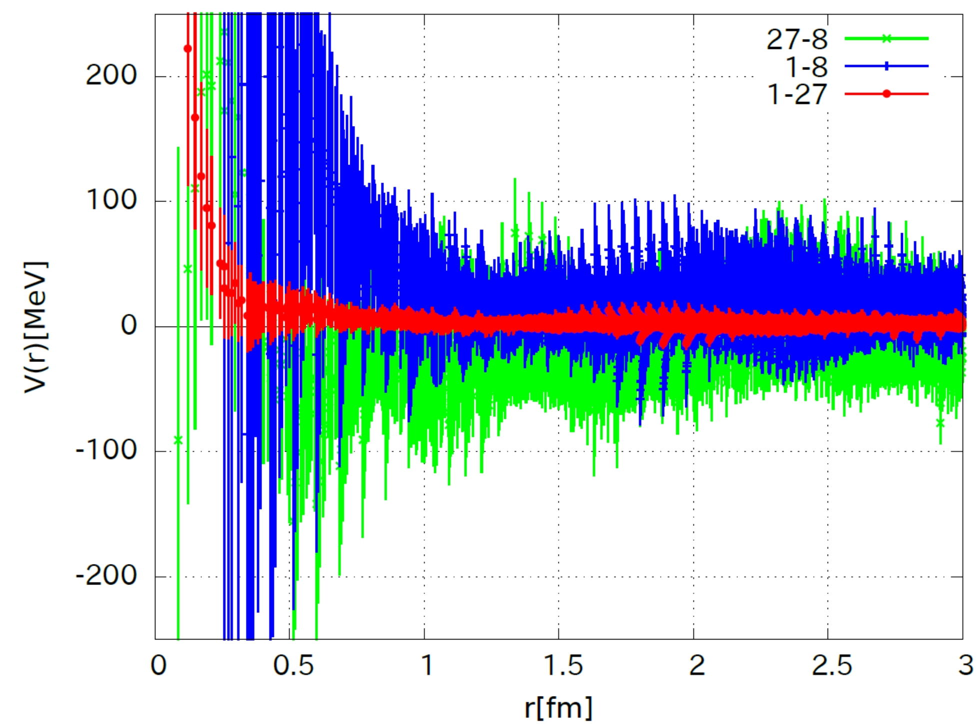
▶ **N_f = 2+1 full QCD with L = 8fm, *m_π = 145 MeV***

Preliminary!

Diagonal elements



Off-diagonal elements



- **Potential of flavor singlet channel does not have a repulsive core**
- Potential of flavor octet channel is strongly repulsive which reflects Pauli effect.
- Off-diagonal potentials are visible only in r<1fm region.

Phase shufts and time delay

T-dep

▶ **N_f = 2+1 full QCD with L = 8fm, *m_π = 145 MeV***

Preliminary!