On the nature of an excited state

Benoît Blossier

CNRS/Laboratoire de Physique Théorique d'Orsay

Lattice 2016, Southampton, 25 – 30 July 2016

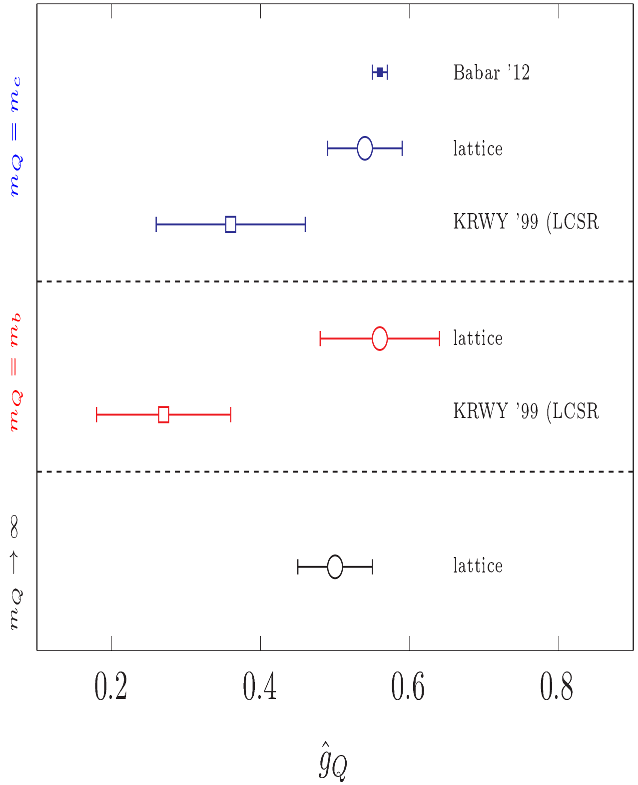
- phenomenological context
- Density distributions of the B meson
- Multihadron states
- Outlook

[B. B. and Antoine Gérardin, arXiv:1604.02891 [hep-lat]]

Phenomenological context

$D^* \to D\pi$: an ideal process to test analytical computations based on the soft pion theorem:

$$\langle D(p')\pi(q)|D^*(p,\epsilon_\lambda)\rangle = g_{D^*D\pi}\, q\cdot\epsilon_\lambda, \quad g_{H^*H\pi} \equiv \frac{2\sqrt{m_H m_{H^*}}\,\hat{g}_Q}{f_\pi}$$



Claim: a negative radial excitation contribution to the hadronic side of LCSR might explain the discrepancy between $g^{\rm exp}_{D^*D\pi}$ and $g^{\rm LCSR}_{D^*D\pi}$ [D. Becirevic *et al*, '03].

Our proposal: check on the lattice that statement in the heavy quark limit.

Transition amplitude under interest, with $q = p' - p$, $\mathcal{A}^\mu = \bar{d}\gamma^\mu\gamma_5 u$,
$T^{mn\,\mu} = \langle B_m(p)|\mathcal{A}^\mu|B_n^*(p',\lambda)\rangle$ and $\epsilon_\perp^\mu(p',\lambda) = \epsilon(p',\lambda)^\mu - \frac{\epsilon(p',\lambda)\cdot q}{q^2} q^\mu$:

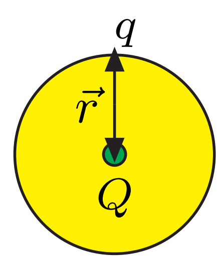
$$\begin{aligned} T^{mn\,\mu} &= 2m_{B_n^*} A_0^{mn}(q^2) \frac{\epsilon(p',\lambda)\cdot q}{q^2} q^\mu + (m_{B_m} + m_{B_n^*}) A_1^{mn}(q^2) \epsilon_\perp^\mu(p',\lambda) \\ &+ A_2^{mn}(q^2) \frac{\epsilon(p',\lambda)\cdot q}{m_{B_m} + m_{B_n^*}} \left[(p+p')^\mu + \frac{m_{B_m}^2 - m_{B_n^*}^2}{q^2} q^\mu \right] \end{aligned}$$

With $\langle B_m(p)|q_\mu \mathcal{A}^\mu|B_n^*(p',\lambda)\rangle = 2\,m_{B_n^*}\, A_0^{mn}(q^2)\; q\cdot\epsilon(p',\lambda)$, PCAC relation, LSZ reduction formula and $\sum_\lambda \epsilon_\mu(k,\lambda)\,\epsilon_\nu^*(k,\lambda) = -g_{\mu\nu} + \frac{k_\mu k_\nu}{m^2}$:

$$g_{H_n^* H_m \pi} = \frac{2\,m_{H_n^*} A_0^{mn}(0)}{f_\pi}\;,\; A_0^{mn}(q^2) = -\sum_\lambda \frac{\langle H_m(p)|q_\mu \mathcal{A}^\mu|H_n^*(p',\lambda)\rangle}{2m_{H_n^*}\, q_i}\, \epsilon_i^*(p',\lambda)$$

Back to the x space: $A_0^{mn}(q^2=0) = -\frac{q_0}{q_i} \int d^3r\, f_{\gamma_0\gamma_5}^{(mn)}(\vec{r})\, e^{i\vec{q}\cdot\vec{r}} + \int d^3r\, f_{\gamma_i\gamma_5}^{(mn)}(\vec{r})\, e^{i\vec{q}\cdot\vec{r}}$

Axial density distributions $f_{\gamma_\mu\gamma_5}^{mn}(r)$ defined
in terms of 2-pt and 3-pt HQET correlation functions



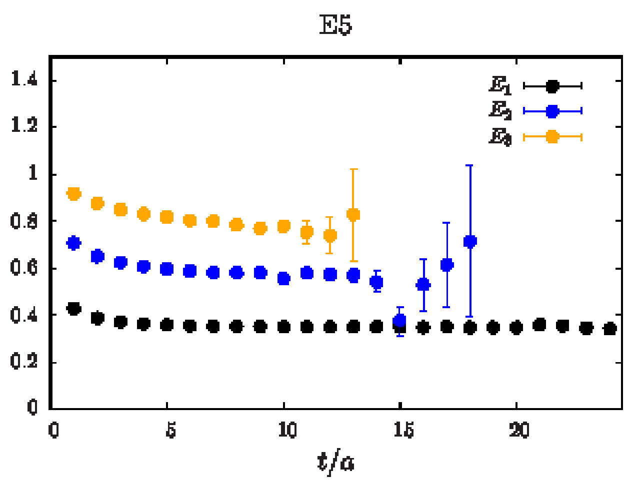
Density distributions of the B meson

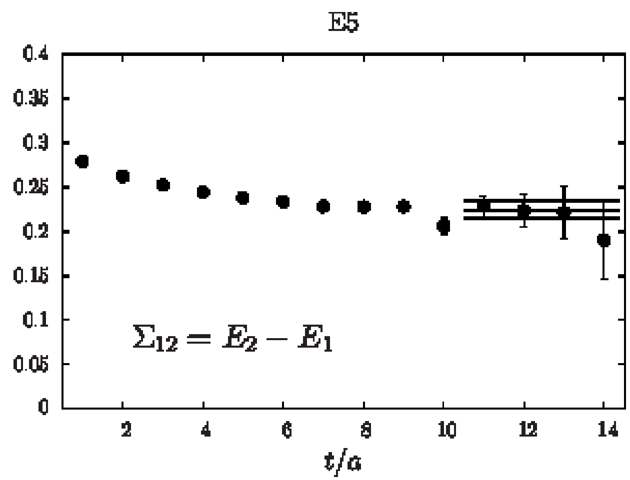
Lattice set-up: $\mathcal{O}(a)$ improved Wilson-Clover (light quark), HYP2 (static quark)

CLS based

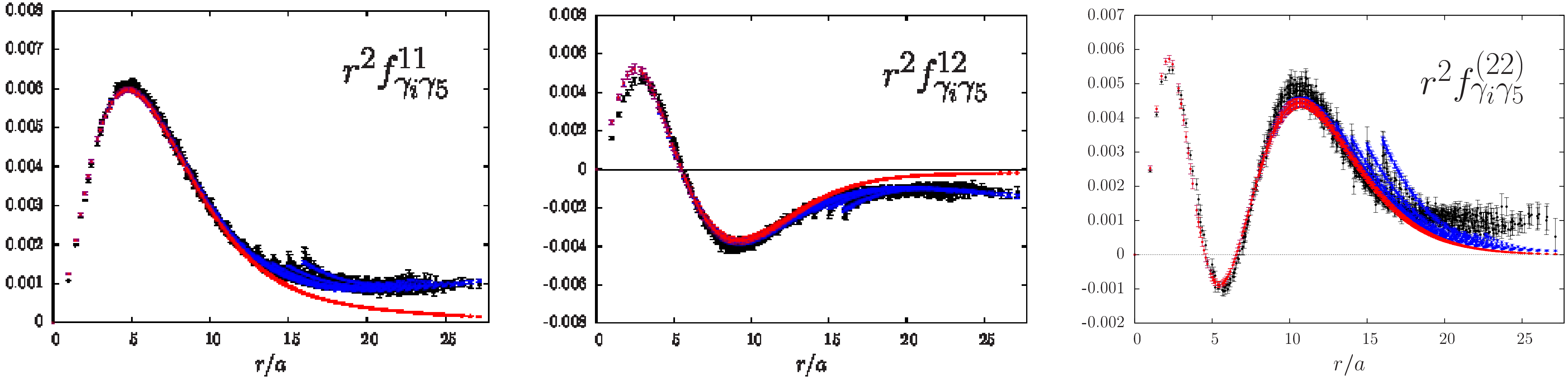
lattice	β	$L^3 \times T$	a[fm]	m_π[MeV]	Lm_π
A5	5.2	$32^3 \times 64$	0.075	330	4
B6		$48^3 \times 96$		280	5.2
D5	5.3	$24^3 \times 48$	0.065	450	3.6
E5		$32^3 \times 64$		440	4.7
F6		$48^3 \times 96$		310	5
N6	5.5	$48^3 \times 96$	0.048	340	4
Q1	6.2885	$24^3 \times 48$	0.06	-	-
Q2	6.2885	$32^3 \times 64$	0.06	-	-

Basis of interpolating fields (4×4 matrix of correlators, Gaussian smearing) large enough to well isolate the ground state and the first excited state.



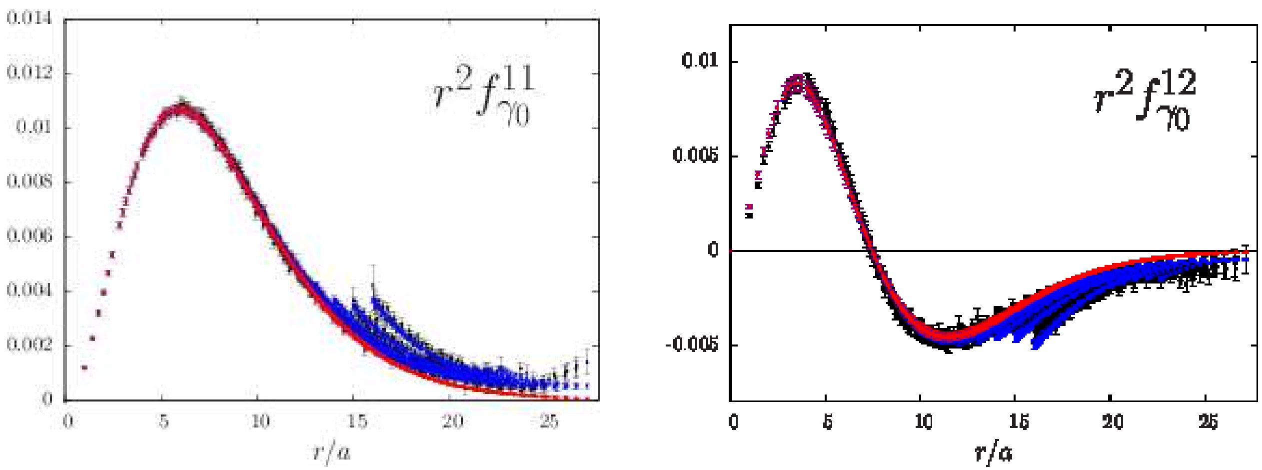


Spatial component of the axial density distributions: systematics from excited states, finite-volume effects and cut-off effects taken into account



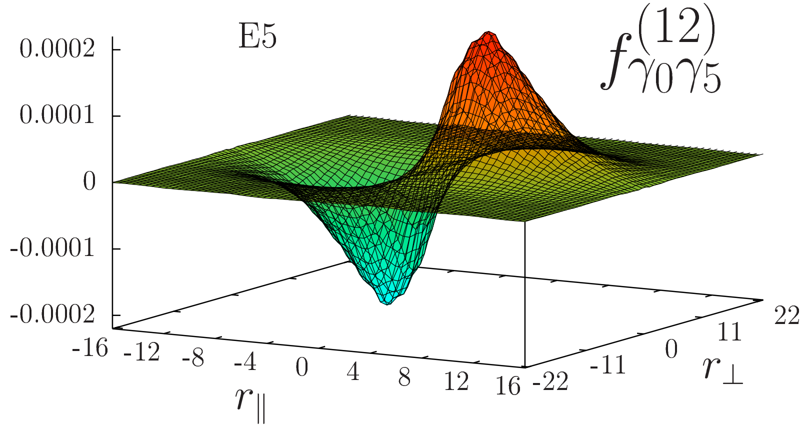
$f^{11}_{\gamma_i\gamma_5}(r)$: positive everywhere; $f^{12}_{\gamma_i\gamma_5}(r)$: there is a node; $f^{22}_{\gamma_i\gamma_5}(r)$: almost positive, negative part interpreted by relativistic effects

Technique employed also for the charge density distribution $f^{mn}_{\gamma_0}(r)$



Including Z_V, $\int dr\, r^2\, f^{11}_{\gamma_0}(r)$ compatible with 1. $\int dr\, r^2 f^{12}_{\gamma_0}(r)$ compatible with 0.

Time component of the axial density distribution also extracted

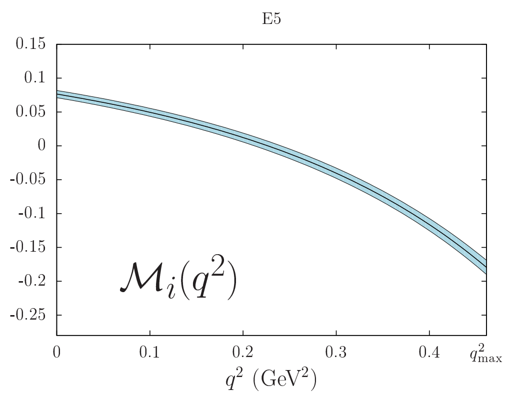


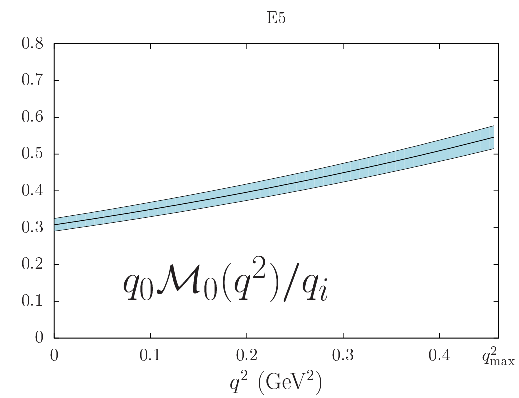
Distribution odd in $r_\parallel$, along the vector meson polarisation

Matrix elements obtained at q after a Fourier transform of the distributions to get $g_{B^{*\prime} B\pi}$

$$\mathcal{M}_i(q^2_{\rm max} - \vec{q}^{\,2}) = 4\pi \int_0^\infty \mathrm{d}r \; r^2 \, \frac{\sin(|\vec{q}|r)}{|\vec{q}|r} f^{(12)}_{\gamma_i\gamma_5}(\vec{r})$$

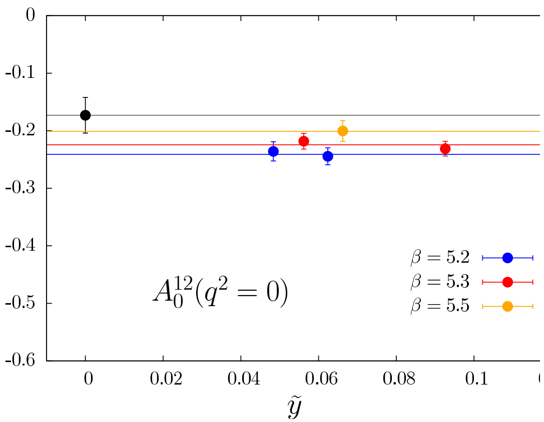
$$\frac{q_0}{q_i}\mathcal{M}_0(q^2_{\rm max} - \vec{q}^{\,2}) = -q_0 4i\pi \int_0^\infty \mathrm{d}r_\parallel \int_0^\infty \mathrm{d}r_\perp \, r_\perp \, f^{(12)}_{\gamma_0\gamma_5}(r_\parallel, r_\perp) \, \frac{\sin(|\vec{q}|\, r_\parallel)}{|\vec{q}|}$$





$$A_0^{12}(q^2) = -\frac{q_0}{q_i}\mathcal{M}_0(q^2_{\rm max} - \vec{q}^{\,2}) + \mathcal{M}_i(q^2_{\rm max} - \vec{q}^{\,2})$$

Lattice results and comparison with quark models (*à la* Bakamjian-Thomas/Godfrey-Isgur, Dirac) [A. Le Yaouanc, orivate communication]



Extrapolation of $A_0^{12}(q^2=0)$ to the physical point:
$A_0^{12}(0,m_\pi^2) = D_0 + D_1 a^2 + D_2 m_\pi^2/(8\pi f_\pi^2)$

Normally, $\mathcal{O}(a)$ effects at $q^2 \neq q^2_{\rm max}$,
not visible in our data.

$$A_0^{12}(0) = -0.173(31)_{\rm stat}(16)_{\rm syst}, \quad g_{B^{*\prime}B\pi} = -15.9(2.8)_{\rm stat}(1.4)_{\rm syst}$$

Quenched result ($m_q = m_s$): $A_0^{12}(0) = -0.143(14)$

	Latt		BT		D	
q^2	$q^2_{\rm max}$	0	$q^2_{\rm max}$	0	$q^2_{\rm max}$	0
$q_0\mathcal{M}_0(q^2)/q_i$	$0.402(54)(27)$	$0.237(27)(28)$	0.252	0.173	0.219	0.164
$\mathcal{M}_i(q^2)$	$-0.172(16)(6)$	$0.064(9)(13)$	-0.103	0.05	-0.223	-0.056

Lattice: $q_0 = 0.701(65)$ GeV
Bakamjian-Thomas with Godfrey-Isgur potential: $q_0 = 0.538$ GeV
Dirac: $q_0 = 0.576$ GeV
global sign of hadronic matrix elements fixed with conventions $f_B > 0$ and $f_{B^{*\prime}} > 0$

Qualitative agreement between lattice and quark models: $q_0\mathcal{M}_0/q_i$ dominates in $A_0^{12}(q^2)$ and explains why $A_0^{12}(q^2=0) < 0$.

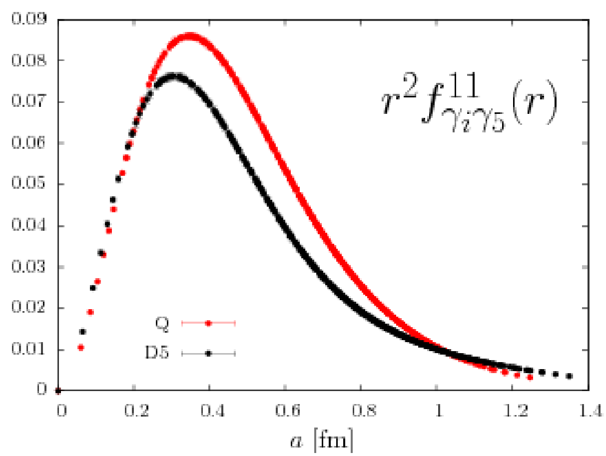
Multihadron states

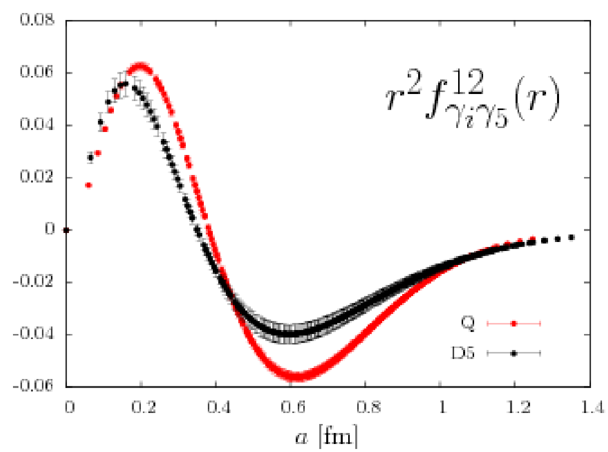
A possible unpleasant systematics of our results is an uncontrolled mixing between radial excitations ($B^{*'}$) and multihadron states ($B_1^*\pi$ in S wave) close to threshold.

$$\delta = m_{B_1^*} - m_B$$

lattice	$a\Sigma_{12}$	$a\delta + am_\pi$
A5	$0.253(7)$	$0.281(4)$
B6	$0.235(8)$	$0.248(4)$
E5	$0.225(10)$	$0.278(6)$
F6	$0.213(11)$	$0.233(3)$
N6	$0.166(9)$	$0.176(3)$

Comparison with quenched data: behaviour of $f^{11}_{\gamma_i\gamma_5}$ and $f^{12}_{\gamma_i\gamma_5}$ similar



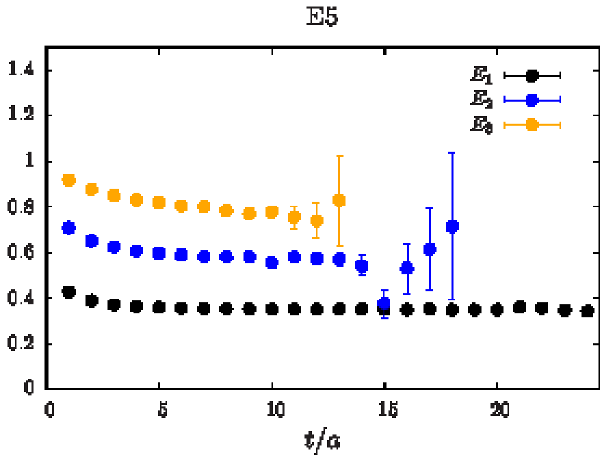


At $\mathrm{N_f} = 2$, position of the node of $f^{12}_{\gamma_i\gamma_5}$ weakly dependent of m_π in the range we have considered

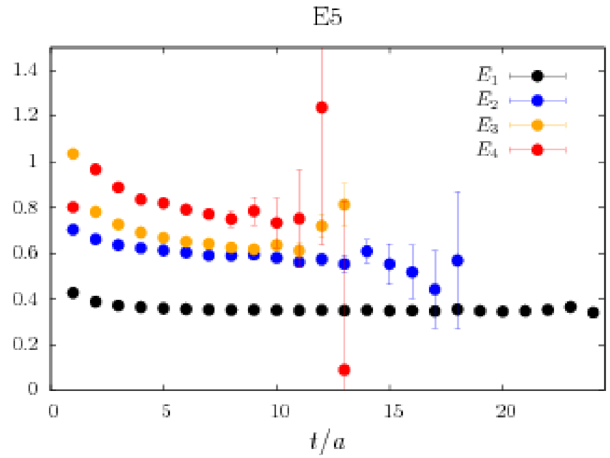
lattice	m_π [MeV]	r_n^{12} [fm]
A5	330	$0.369(13)$
B6	280	$0.374(12)$
E5	440	$0.369(11)$
F6	310	$0.379(20)$
N6	340	$0.365(12)$

Change observed when $\bar{q}\nabla_k h$ is included in addition to $\bar{q}\gamma_k h$ to couple to $B^{*'}$

$\bar{q}\gamma_k h$ only



with $\bar{q}\nabla_k h$



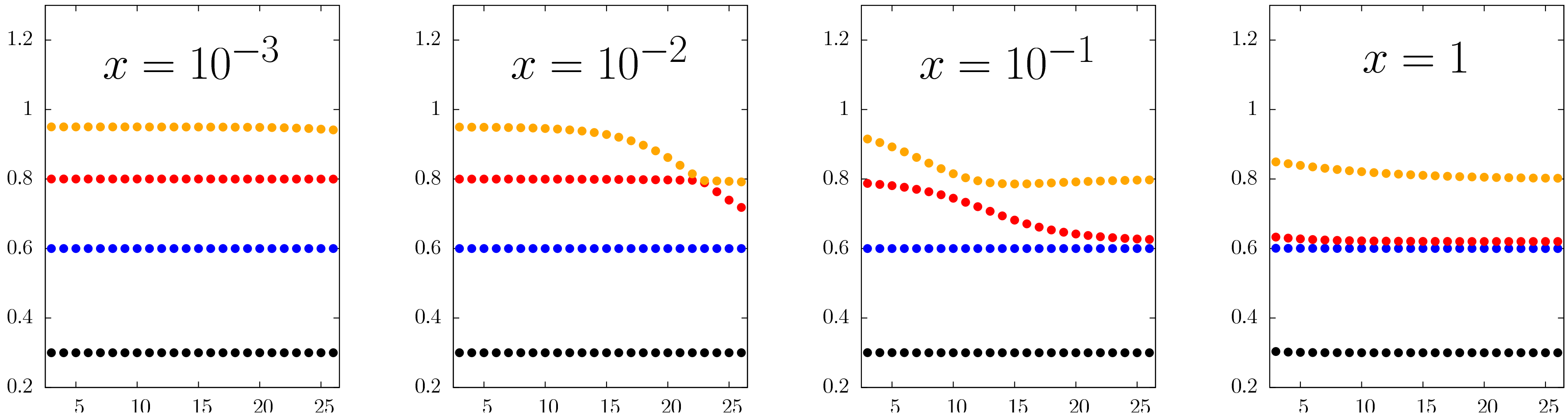
A new state, not seen before, is present in the spectrum close to the first excited state.

A toy model with 5 states in the spectrum to understand this fact:

spectrum
0.3
0.6
0.63
0.8
0.95

Matrix of couplings

$$\begin{bmatrix} 0.6 & 0.25 & x\times 0.4 & 0.1 & 0.5 \\ 0.61 & 0.27 & x\times 0.39 & 0.11 & 0.51 \\ 0.58 & 0.24 & x\times 0.42 & 0.12 & 0.52 \\ 0.57 & 0.25 & x\times 0.41 & 0.1 & 0.49 \\ 0.56 & 0.26 & x\times 0.36 & 0.08 & 0.48 \end{bmatrix}$$



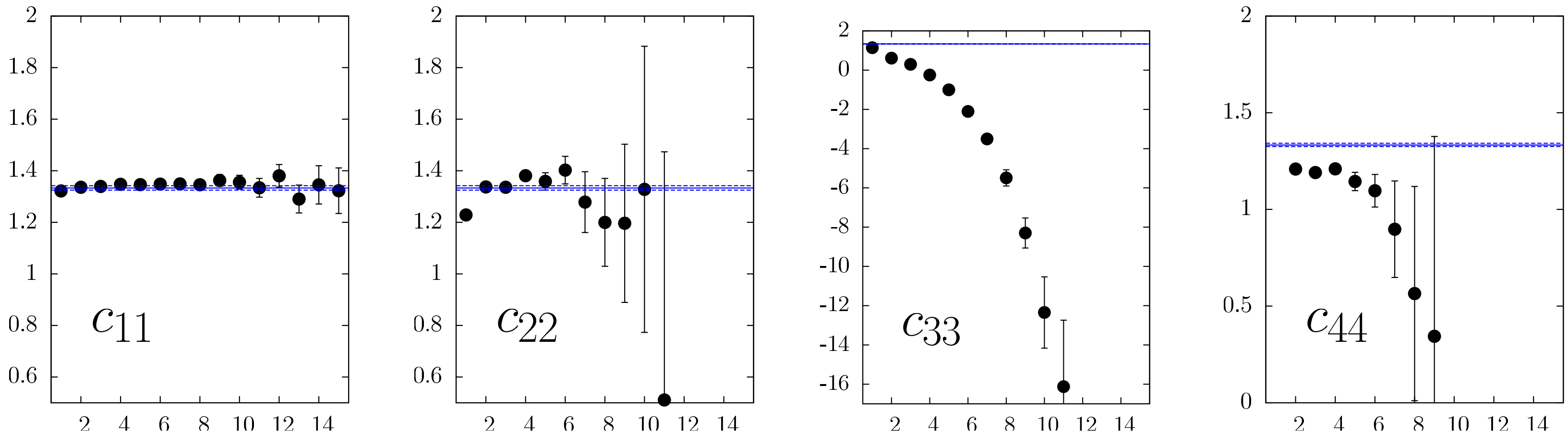
$x \ll 1$: GEVP isolates states 1, 2, 4 and 5; $x \to 1$, GEVP isolates states 1, 2, 3 and 4

A GEVP can "miss" an intermediate state of the spectrum if, by accident, the coupling of the interpolating fields to that state is suppressed.

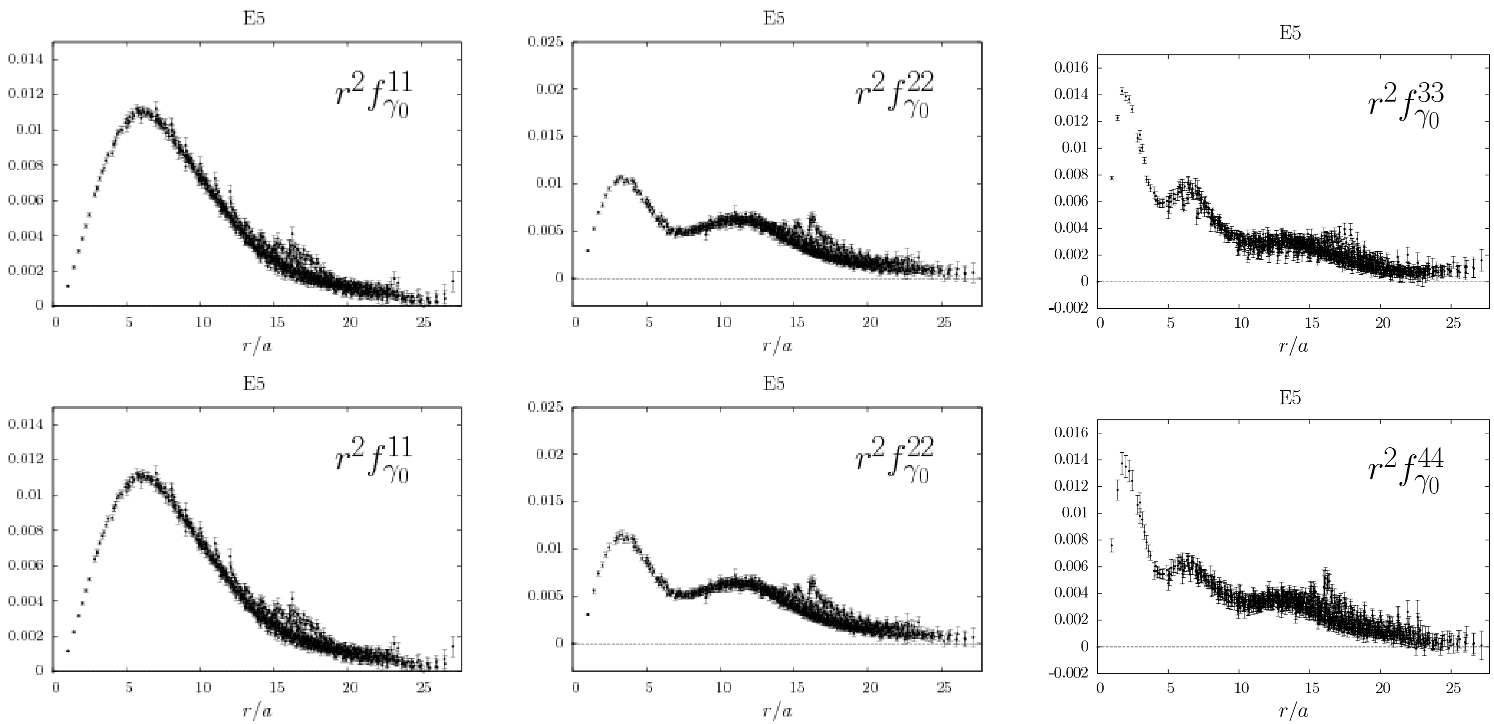
Our claim: using interpolating fields $\bar{q}\gamma_k h$, no chance to couple to multi-hadron states while inserting an operator $\bar{q}\nabla_k h$ may isolate the $B_1^*\pi$ two-particle state.

Clues come from density distributions obtained with that interpolating field.

Conservation of vector charge: not verified in the case of second excited state if the basis of interpolating fields incorporates $\bar{q}\nabla_k h$.



Including or not $\bar{q}\nabla_k h$ does not change the profile of $f^{11}_{\gamma_0}$ nor $f^{22}_{\gamma_0}$: it does in the case of $f^{33}_{\gamma_0}$.



Outlook

- Excited meson states are massively produced in experiments. To exploit fruitfully the numerous data at Super Belle and LHCb, theorists do have to put an important effort in confronting their models predictions with measurements.

- Extract density distributions of the B meson is beneficial to get the form factors at $q^2 = 0$ associated to pionic couplings. Lattice computations allow a detailed comparison with quark models.

- Density distributions may be a check of the absence of any unwanted coupling between a given interpolating field of the B meson and a multihadron state $B\pi$.

First workshop in a series organized in the framework of the Horizon 2020 project RBI-T-Winning.

Topics related to the recent theoretical and experimental developments in flavor physics, both in quark and lepton sectors, with a particular emphasis on scenarios of physics beyond the Standard Model. Lattice QCD will not be let aside.

Please visit the web site https://indico.in2p3.fr/event/13187/