LATTICE 2016

Hypercubic effects in semileptonic $D \to \pi$ decays on the lattice

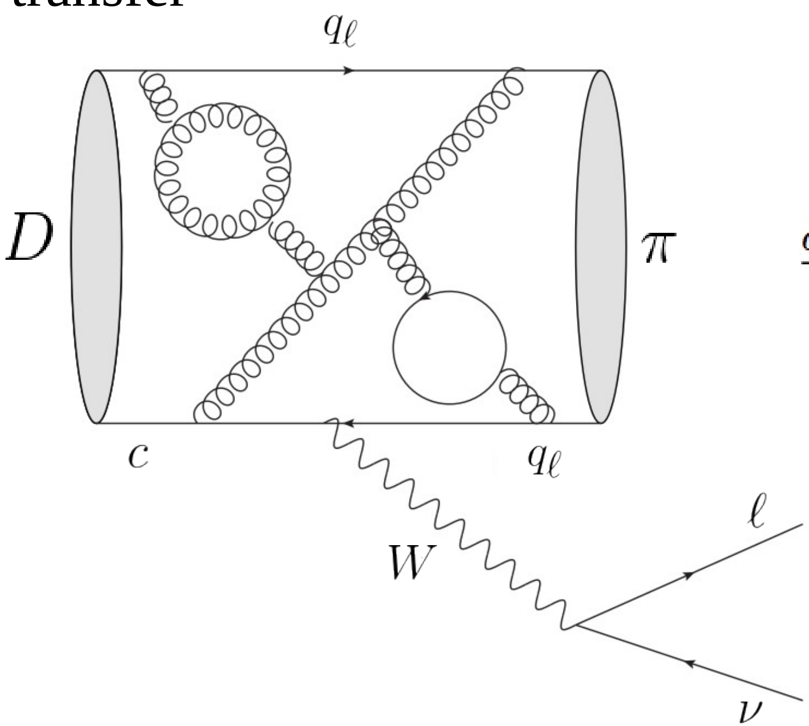
Giorgio Salerno (ETM Collaboration)

Università di Roma Tre, INFN Roma Tre

V. Lubicz, L. Riggio, G. Salerno, S. Simula, C. Tarantino

Motivation

The hadronic contribution to the D → π semileptonic decay rate is regulated by the vector and scalar form factors f_0 and f_+ which are function of the squared 4-momentum transfer



Decay rate in the SM:

$$\frac{d\Gamma(D\to P\ell\nu)}{dq^2} = \frac{G_F^2|V_{cx}|^2}{24\pi^3}\frac{(q^2-m_\ell^2)^2\sqrt{E_P^2-m_P^2}}{q^4 m_D^2}\times$$
$$\times\left[\left(1+\frac{m_\ell^2}{2q^2}\right)m_D^2(E_P^2-m_P^2)|f_+(q^2)|^2+\frac{3m_\ell^2}{8q^2}(m_D^2-m_P^2)^2|f_0(q^2)|^2\right]$$

Typical lattice calculations focus on the value of the form factors at $q^2 = 0$, which allow us to determine the CKM matrix elements V_{cd}

It is however interesting to compute these form factors on the lattice in all the physical q^2 range and to compare them with experimental measurements

Simulation Details

Something on the action:

- Wilson Twisted Mass action at maximal twist with Nf=2+1+1 sea quarks
- Osterwalder-Seiler valence quark action
- Iwasaki gluon action

Simulation Details

Details of the ensembles used in this Nf =2+1+1 analysis

The valence light quark mass is put equal to the sea quark mass

ensemble	β	V/a^4	$a\mu_{sea} = a\mu_\ell$	N_{cfg}	$a\mu_s$	$a\mu_c$
$A30.32$	1.90	$32^3 \times 64$	0.0030	150	0.0180,	0.21256, 0.25000,
$A40.32$			0.0040	90	0.0220,	0.29404, 0.34583
$A50.32$			0.0050	150	0.0260	
$A40.24$	1.90	$24^3 \times 48$	0.0040	150		
$A60.24$			0.0060	150		
$A80.24$			0.0080	150		
$A100.24$			0.0100	150		
$B25.32$	1.95	$32^3 \times 64$	0.0025	150	0.0155,	0.18705, 0.22000,
$B35.32$			0.0035	150	0.0190,	0.25875, 0.30433
$B55.32$			0.0055	150	0.0225	
$B75.32$			0.0075	75		
$B85.24$	1.95	$24^3 \times 48$	0.0085	150		
$D15.48$	2.10	$48^3 \times 96$	0.0015	60	0.0123,	0.14454, 0.17000,
$D20.48$			0.0020	90	0.0150,	0.19995, 0.23517
$D30.48$			0.0030	90	0.0177	

Range of the simulated pion masses

β	L(fm)	M_D(MeV)	M_π(MeV)	$M_\pi L$
1.90	2.84	2008	245	3.53
		2005	282	4.06
		2021	314	4.53
1.90	2.13	2020	282	3.05
		2005	344	3.71
		2038	396	4.27
		2040	443	4.78
1.95	2.61	1941	239	3.16
		1938	281	3.72
		1965	350	4.64
		1970	408	5.41
1.95	1.96	1961	435	4.32
2.10	2.97	1929	211	3.19
		1936	243	3.66
		1933	296	4.46

Three different values of the lattice spacing: 0.06 *fm* ÷ 0.09 *fm*

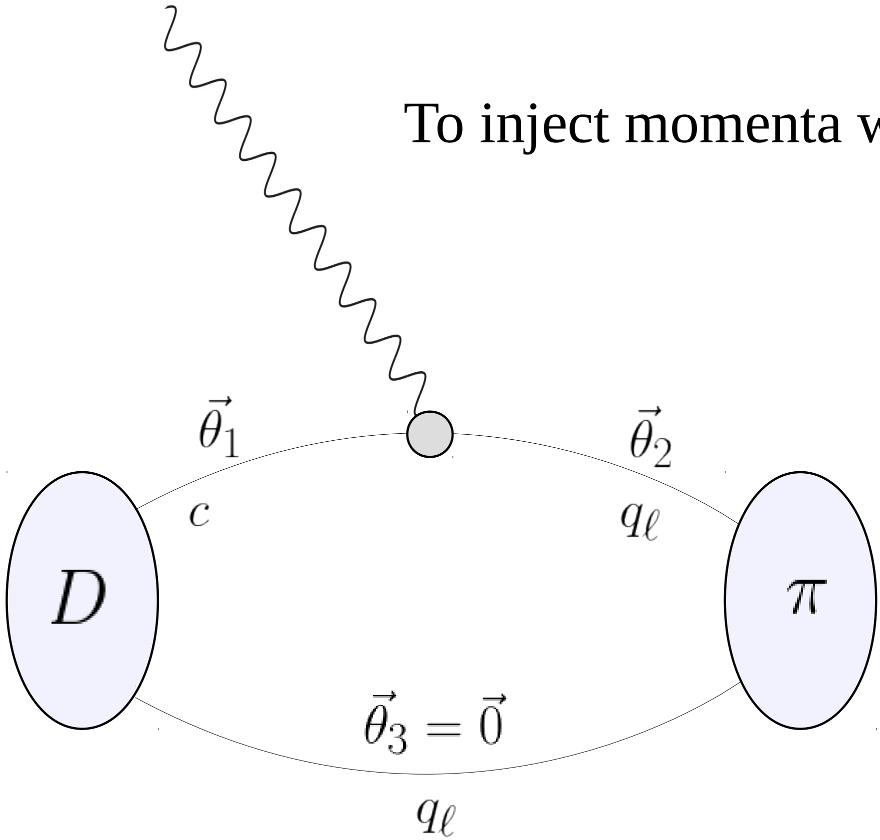
Different volumes: 2 *fm* ÷ 3 *fm*

Pion masses in range 210 ÷ 440 MeV

Lattice Spacings	
a($\beta = 1.90$)	0.0885(36)fm
a($\beta = 1.95$)	0.0815(30)fm
a($\beta = 2.10$)	0.0619(18)fm

The four values of the bare charm mass are used to interpolate to m_c^{phys}

To inject momenta we used non-periodic boundary conditions



$$\vec{p}_D = \frac{2\pi}{L}\vec{\theta}_1$$

$$\vec{p}_\pi = \frac{2\pi}{L}\vec{\theta}_2$$

$$\vec{\theta} = \theta(1,1,1)$$

Both the D and the π mesons can have non-zero momentum

β	V/a^4	θ
1.90	$32^3 \times 64$	0.0, ±0.400, ±0.933, ±1.733
	$24^3 \times 48$	0.0, ±0.300, ±0.700, ±1.300
1.95	$32^3 \times 64$	0.0, ±0.366, ±0.854, ±1.588
	$24^3 \times 48$	0.0, ±0.275, ±0.641, ±1.191
2.10	$48^3 \times 96$	0.0, ±0.424, ±0.986, ±1.832

General Strategy

- Extract for each ensemble and for different values of q^2 the vector and scalar matrix elements V_0 ,V_i and S studying the time dependence of ratios of 3-points and 2-points correlation functions (smeared interpolating fields are adopted)

$$C_2(t) \xrightarrow{t \gg a} \frac{Z(p)}{2E_0}\left(e^{-E_0 t} + e^{-E_0(T-t)}\right)$$

$$\widetilde{C}_2 = \frac{1}{2}\left[C_2(t_s) + \sqrt{C_2(t_s)^2 - C_2(T/2)^2}\right] = \frac{Z(p)}{2E_0}\, e^{-E_0 t_s}$$

$$C_\mu^{D\pi}(t,\, t_s,\, \vec{p}_D,\, \vec{p}_\pi) \xrightarrow{t \gg a \quad (t_s - t) \gg a} Z_V \frac{\sqrt{Z_D(p_D) Z_\pi(p_\pi)}}{4 E_D E_\pi} \langle \pi(p_\pi) | V_\mu | D(p_D) \rangle_0 \; e^{-E_D t - E_\pi (t_s - t)}$$

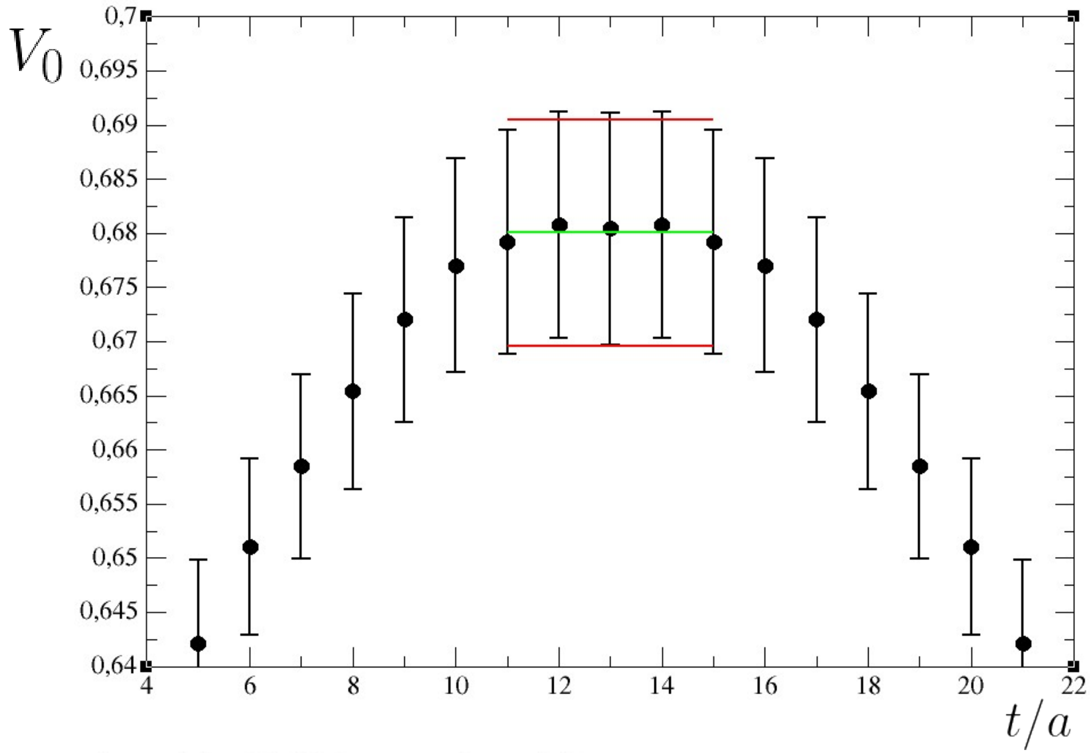
$$R_\mu(t,\, \vec{p}_D,\, \vec{p}_\pi) = \frac{C_\mu^{D\pi}(t,\, t_s,\, \vec{p}_D,\, \vec{p}_\pi)\; C_\mu^{\pi D}(t,\, t_s,\, \vec{p}_\pi,\, \vec{p}_D)}{C_\mu^{\pi\pi}(t,\, t_s,\, \vec{p}_\pi,\, \vec{p}_\pi)\; C_\mu^{DD}(t,\, t_s,\, \vec{p}_D,\, \vec{p}_D)} \xrightarrow{t \gg a \quad (t_s - t) \gg a} \frac{1}{4} \left| \langle \pi(p_\pi) | V_\mu | D(p_D) \rangle \right|^2 (p_D)_\mu (p_\pi)_\mu$$

$$R_S(t,\, \vec{p}_D,\, \vec{p}_\pi) = \frac{C_S^{D\pi}(t,\, t_s,\, \vec{p}_D,\, \vec{p}_\pi)\; C_S^{\pi D}(t,\, t_s,\, \vec{p}_\pi,\, \vec{p}_D)}{\widetilde{C}_2^D\, \widetilde{C}_2^\pi} \xrightarrow{t \gg a \quad (t_s - t) \gg a} \frac{1}{4} \left| \langle \pi(p_\pi) | S | D(p_D) \rangle \right|^2 p_D^0\, p_\pi^0$$

General Strategy

- Extract for each ensemble and for different values of q^2 the vector and scalar matrix elements V_0 ,V_i and S studying the time dependence of ratios of 3-points and 2-points correlation functions (smeared interpolating fields are adopted)

Ensemble D20.48
$M_\pi = 243\,\mathrm{MeV}$
$p_D = p_\pi = 151\,\mathrm{MeV}$



$$R_\mu(t,\,\vec{p}_D,\,\vec{p}_\pi) = \frac{C_\mu^{D\pi}\,(t,\,t_s,\,\vec{p}_D,\,\vec{p}_\pi)\;C_\mu^{\pi D}\,(t,\,t_s,\,\vec{p}_\pi,\,\vec{p}_D)}{C_\mu^{\pi\pi}\,(t,\,t_s,\,\vec{p}_\pi,\,\vec{p}_\pi)\;C_\mu^{DD}\,(t,\,t_s,\,\vec{p}_D,\,\vec{p}_D)} \xrightarrow{t\gg a \quad (t_s-t)\gg a} \frac{1}{4}\,|\,\langle\pi(p_\pi)|V_\mu|D(p_D)\rangle\,|^2\;(p_D)_\mu(p_\pi)_\mu$$

$$R_S(t,\,\vec{p}_D,\,\vec{p}_\pi) = \frac{C_S^{D\pi}\,(t,\,t_s,\,\vec{p}_D,\,\vec{p}_\pi)\;C_\mu^{\pi D}\,(t,\,t_s,\,\vec{p}_\pi,\,\vec{p}_D)}{\widetilde{C}_2^D\,\widetilde{C}_2^\pi} \xrightarrow{t\gg a \quad (t_s-t)\gg a} \frac{1}{4}\,|\,\langle\pi(p_\pi)|S|D(p_D)\rangle\,|^2\;p_D^0\,p_\pi^0$$

General Strategy

- Extract for each ensemble and for different values of q^2 the vector and scalar matrix elements V_0 ,V_i and S studying the time dependence of ratios of 3-points and 2-points correlation functions (smeared interpolating fields are adopted)

- Combine for each ensemble the informations coming from the vector and scalar matrix elements in order to extract the scalar and vector form factors f_0 and f_+

Extraction of the form factors

The two semileptonic form factors f_0 and f_+ can be determined from the matrix element of the vector current

$$f_+(q^2) = \frac{(E_D - E_\pi)\,\langle V_i \rangle - (p_{D\,i} - p_{\pi\,i})\,\langle V_0 \rangle}{2E_D\,p_{\pi\,i} - 2E_\pi\,p_{D\,i}}$$

$$f_-(q^2) = \frac{(p_{D\,i} + p_{\pi\,i})\,\langle V_0 \rangle - (E_D + E_\pi)\,\langle V_i \rangle}{2E_D\,p_{\pi\,i} - 2E_\pi\,p_{D\,i}}$$

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{M_D^2 - M_\pi^2}\,f_-(q^2)$$

An alternative way to determine f_0 is to use the scalar density

$$f_0(q^2) = \frac{m_s - m_l}{M_D^2 - M_\pi^2}\,\langle \pi(p_\pi)|S|D(p_D)\rangle$$

Therefore using both matrix elements we over-constrain f_0 and f_+ and we get our determination of the form factors on each ensemble for the different values of q^2 from a combined fit

General Strategy

- Extract for each ensemble and for different values of q^2 the vector and scalar matrix elements V_0 ,V_i and S studying the time dependence of ratios of 3-points and 2-points correlation functions (smeared interpolating fields are adopted)

- Combine for each ensemble the informations coming from the vector and scalar matrix elements in order to extract the scalar and vector form factors f_0 and f_+

- Lorentz symmetry breaking effects in the behavior of the scalar and vector form factors have been observed and interpreted as due to hypercubic effects (LAT'15 arXiv:1511.04877)

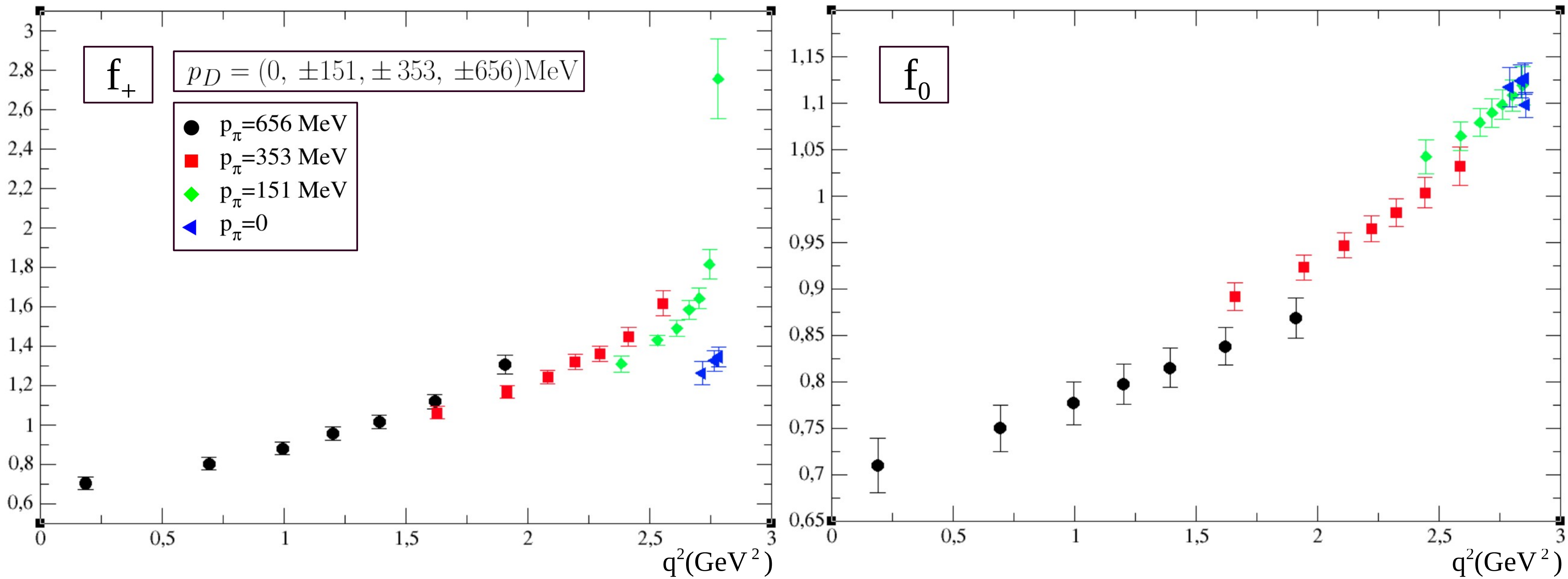
- In this talk we present our new investigation of such hypercubic effects and their removal

General Strategy

Lorentz symmetry breaking effects

Ensemble A50.32
$M_\pi = 314\,\mathrm{MeV}$

Ensemble A40.32
$M_\pi = 282\,\mathrm{MeV}$



Clear evidence of hypercubic effects which affect V_0, V_i and S :

- ✦ the extracted values of f_0 and f_+ are not uniquely determined as functions of q^2
- ✦ need to estimate and get rid of such hypercubic effects

General Strategy

- The determination of the hypercubic effects and the estimate of f_0 and f_+ as functions of q^2 has been achieved performing a global fit which combine together V_0, V_i and S of all our ensembles, studying simultaneously the q^2, m_l and a^2 dependence under the following assumptions:
    - ✦ modified z expansion for the form factors
    - ✦ $\mathcal{O}(a^2)$ breaking of the Lorentz invariance for the vector current
    - ✦ $\mathcal{O}(a^2)$ breaking of the Ward-Takahashi identity relating the 4-divergence of the vector current to the scalar current

Goal:

From the global fit we get directly the value of the vector form factor at zero-momentum transfer, $f_+(0)$, and the behavior of f_0 and f_+ as functions of q^2

Global fit

Modified z expansion

BCL
Phys.Rev. D79 (2009) 013008

$$f_+(q^2) = \frac{f_+(0) + a_+(a^2)\,(z - z_0)\left(1 + \frac{z+z_0}{2}\right)}{1 - \frac{q^2}{M_V^2} K_{FSE}^+}$$

$$f_0(q^2) = \frac{f_+(0) + a_0(a^2)\,(z - z_0)\left(1 + \frac{z+z_0}{2}\right)}{1 - \frac{q^2}{M_S^2} K_{FSE}^0}$$

$$t_+ = (M_D + M_\pi)^2$$

$$t_0 = (M_D + M_\pi)\left(\sqrt{M_D} + \sqrt{M_\pi}\right)^2$$

$$z = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$z_0 = z\,(q^2 = 0)$$

$a_+(a^2)$ and $a_0(a^2)$ have a polynomial dependence on the lattice spacing squared

Hard Pion SU(2) ChPT
Bijnens & Jemos
NPB 840 (2010) 54

$$f_+(0) = b_0\left[1 - \frac{3}{4}\,(1 + 3\,g^2)\,\xi_l \log \xi_l + b_1\,\xi_l + b_2\,a^2\right]$$

$$\xi_l = \frac{2B\,m_l}{16\pi^2 f^2}$$

$b_0, b_1\,, b_2$ free parameters

We add a FSE correction, adopting the following phenomenological form:

$$K_{FSE}^{+(0)} = 1 + C_{FSE}^{+(0)} \frac{e^{-M_\pi L}}{(M_\pi L)^{\alpha_{eff}}}$$

$C_{FSE}^{+(0)}$ are parameters left free to move in the fit

$$\alpha_{eff} = 0,\ 0.5,\ 1,\ 1.5$$

Global fit

Hypercubic effects: breaking of the Lorentz invariance

We considered an $O(a^2)$ breaking of the Lorentz invariance introducing an hypercubic effect in the vector matrix elements that gives rise to a new set of hypercubic form factors H_i:

$$\langle \pi(p_\pi)|V_\mu|D(p_D)\rangle = \langle V_\mu^L\rangle + \langle V_\mu^{hyp}\rangle$$

$$\langle V_\mu^{hyp}\rangle = a^2\left[\, q_\mu^3\, H_1 + q_\mu^2\, P_\mu\, H_2 + q_\mu\, P_\mu^2\, H_3 + P_\mu^3\, H_4\right]$$

Lorentz decomposition of the vector matrix elements

$$\langle V_\mu^L\rangle = P_\mu\, f_+(q^2) + q_\mu\, f_-(q^2)$$

$$q_\mu = (p_D - p_\pi)_\mu$$

$$P_\mu = (p_D + p_\pi)_\mu$$

The hypercubic form factors are in general function of q^2 and the light quark mass m_l

$$H_i = H_i\left(q^2, m_l\right)$$

We adopted for H_i a polynomial expression in the z variable

$$H_i\,(z) = c_0^i + c_1^i\,(z - z_0) + c_2^i\,(z - z_0)^2$$

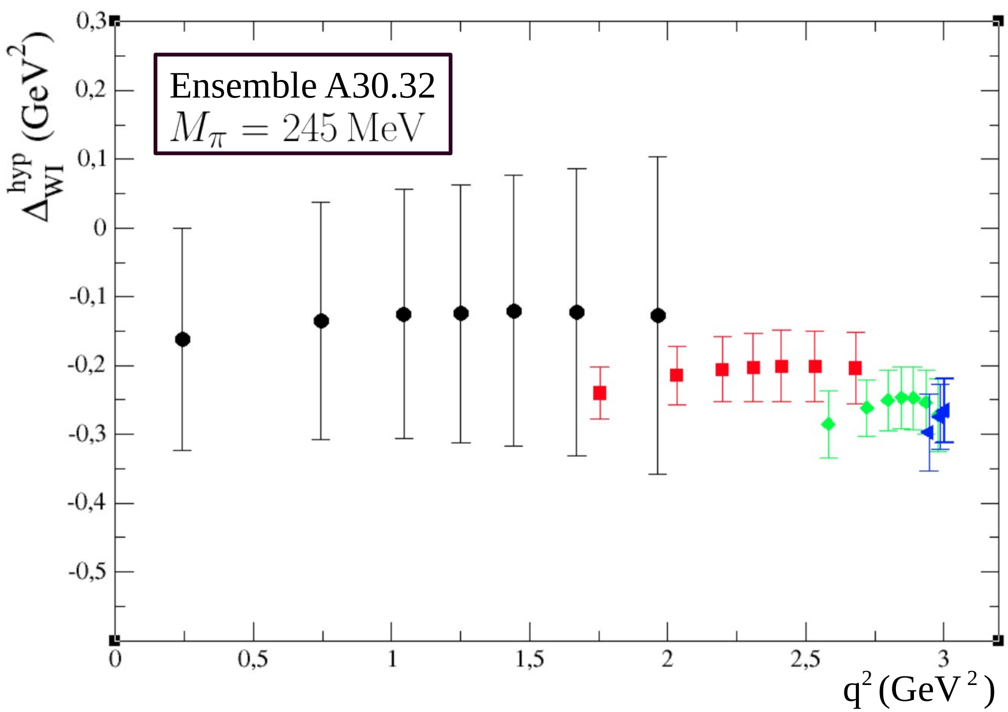
c_0^i, c_1^i and c_2^i are free parameters

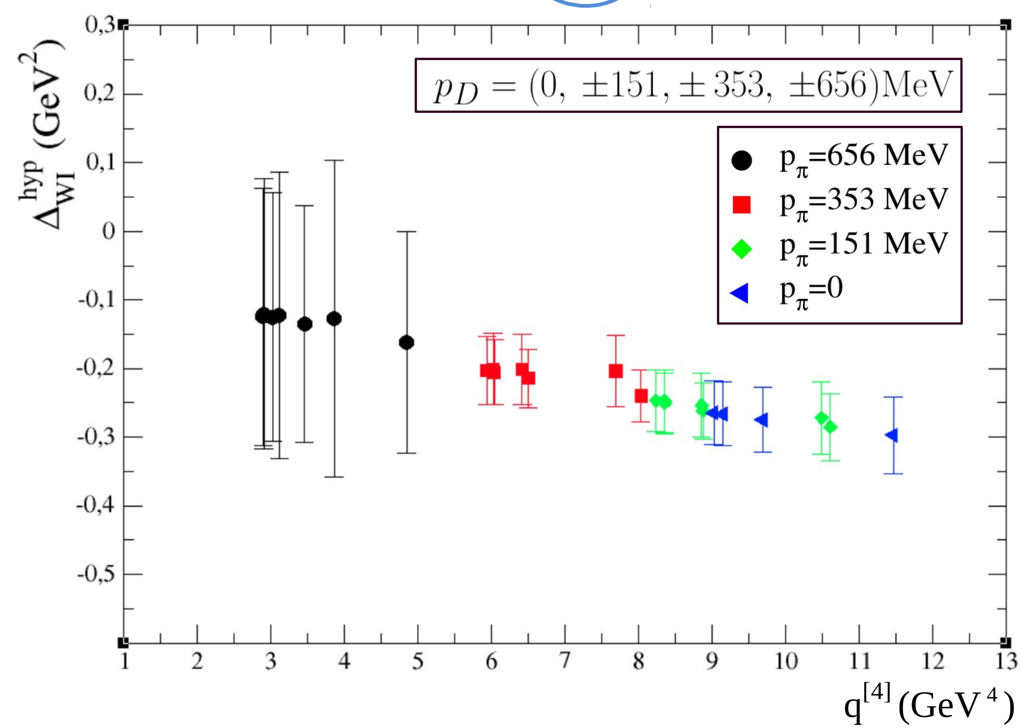
Global fit

Hypercubic effects: breaking of the Ward-Takahashi identity

We also considered the hypercubic effects responsible for the breaking of the Ward-Takahashi identity relating the 4-divergence of the vector current to the scalar current

$$q^\mu \langle \pi(p_\pi)|V_\mu|D(p_D)\rangle = (m_c - m_l)\langle \pi(p_\pi)|S|D(p_D)\rangle + \Delta_{WI}^{hyp}$$





We supposed a simple form for the WI violating term, depending only from $q^{[4]}$ and one hypercubic form factor

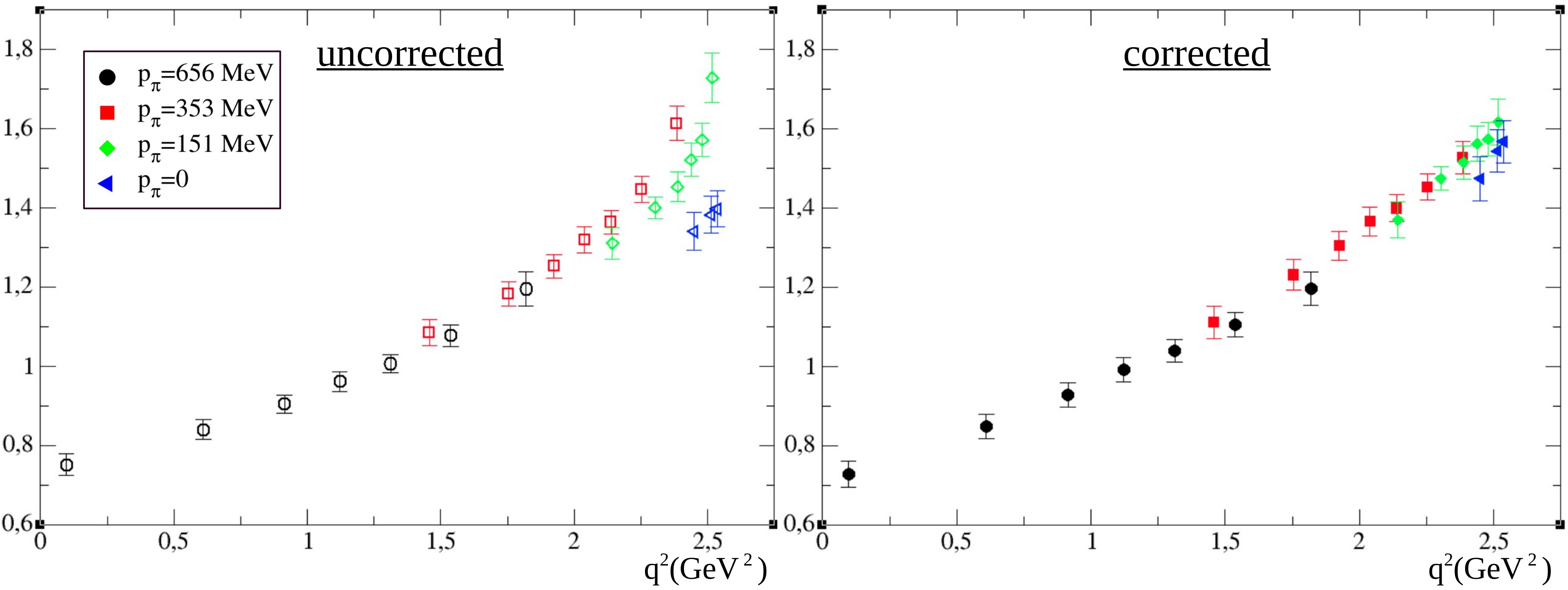
$$q^{[4]} = \sum_\mu q_\mu^4 \qquad \Delta_{WI}^{hyp} = a^2 H_5\, q^{[4]} \qquad H_5(m_l) = c_0^5 + c_1^5 m_l$$

Check of the disappearance of the hypercubic effects in the vector form factor

$$f_+(q^2)$$

Ensemble A80.24
$M_\pi = 396\ \mathrm{MeV}$

$p_D = (0, \pm 151, \pm 353, \pm 656)\mathrm{MeV}$



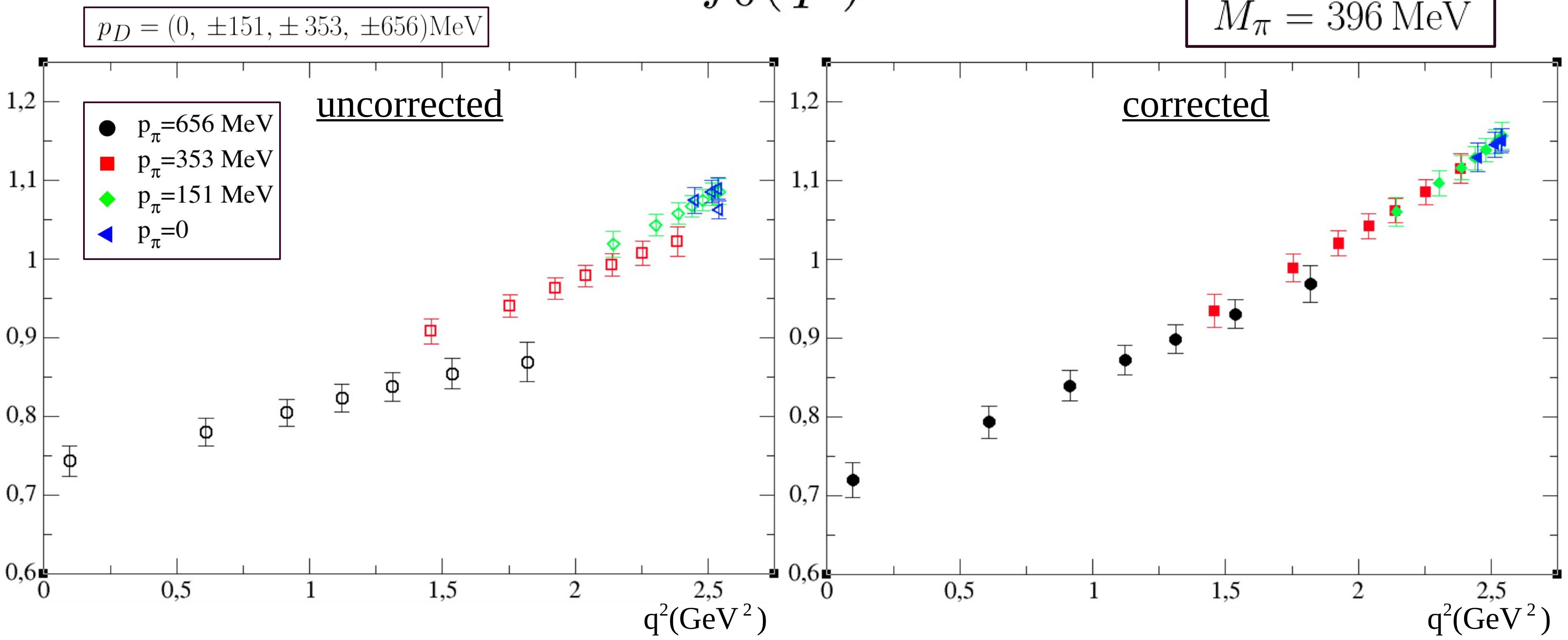
After subtracting the hypercubic effects determined with the global fit the dependence of the scalar and vector form factor on q^2 is restored

Results

Check of the disappearance of the hypercubic effects in the scalar form factor

$$f_0(q^2)$$

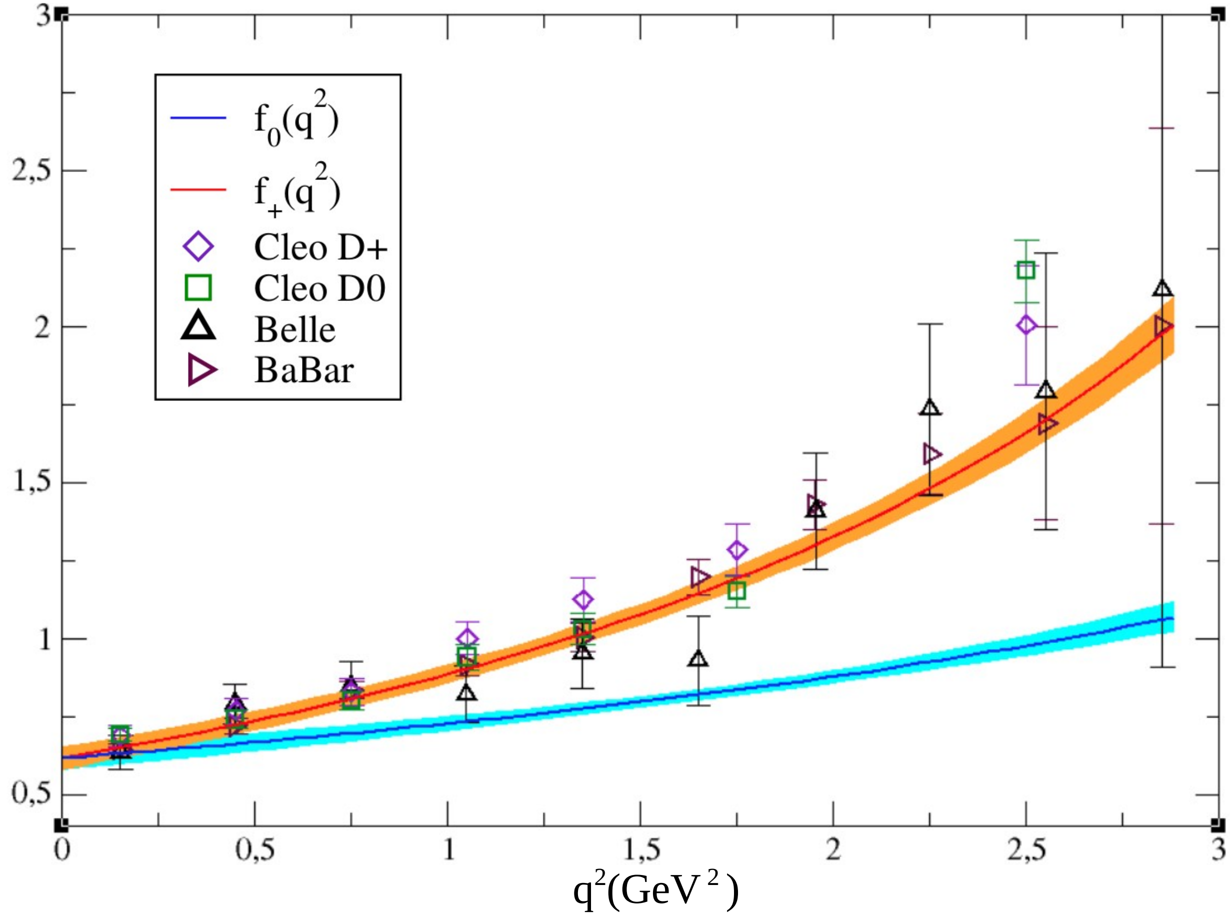
Ensemble A80.24
$M_\pi = 396\ \mathrm{MeV}$



After subtracting the hypercubic effects determined with the global fit the dependence of the scalar and vector form factor on q^2 is restored

The form factors $f_0(q^2)$ and $f_+(q^2)$ at the physical point from the global fit

$$\chi^2/d.o.f. \approx 1 \quad d.o.f. \approx 1100$$



Our estimate of the vector form factor at zero-momentum transfer is:

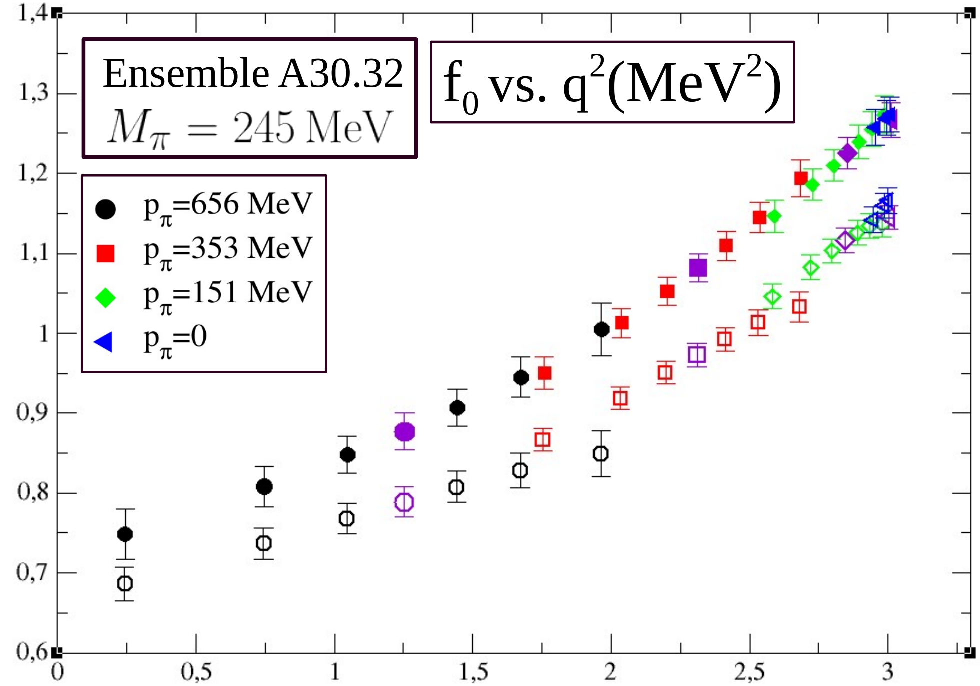
$$f_+^{D\to\pi}(0) = 0.631\,(37)_{\text{stat}}\,(14)_{\text{Chiral}}\,(08)_{\text{Disc}} = 0.631\,(40)$$

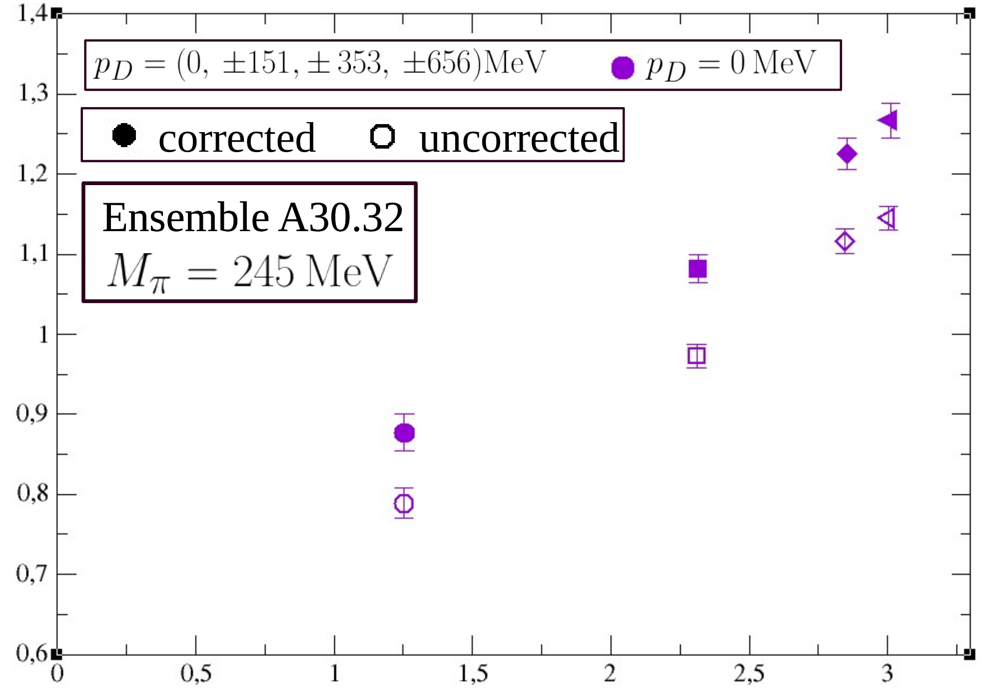
FLAG average

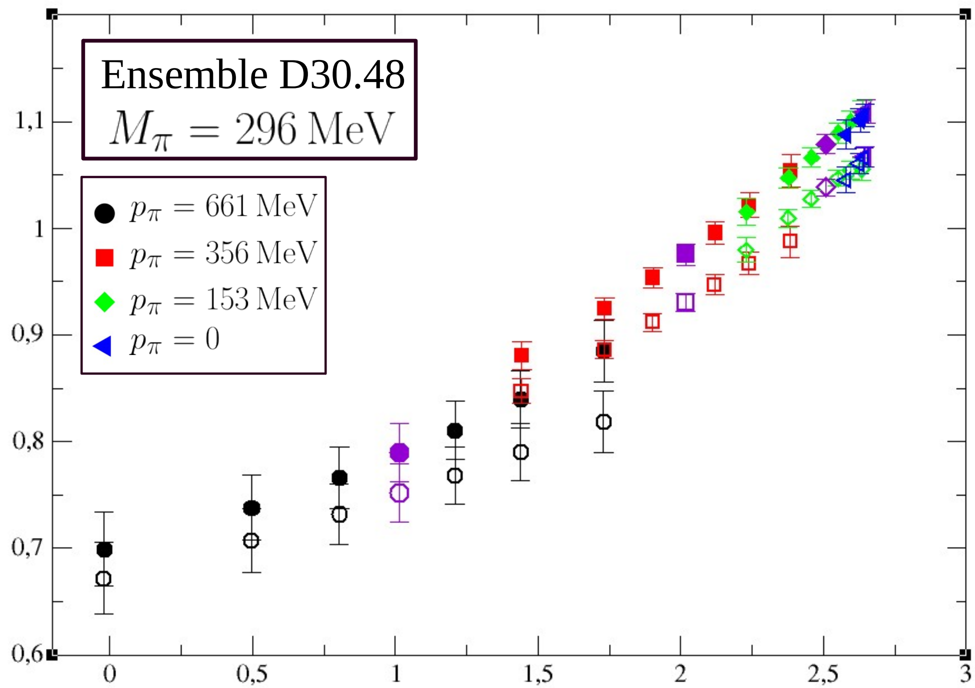
$$f_+(0)^{(D\to\pi)} = 0.666(29)$$

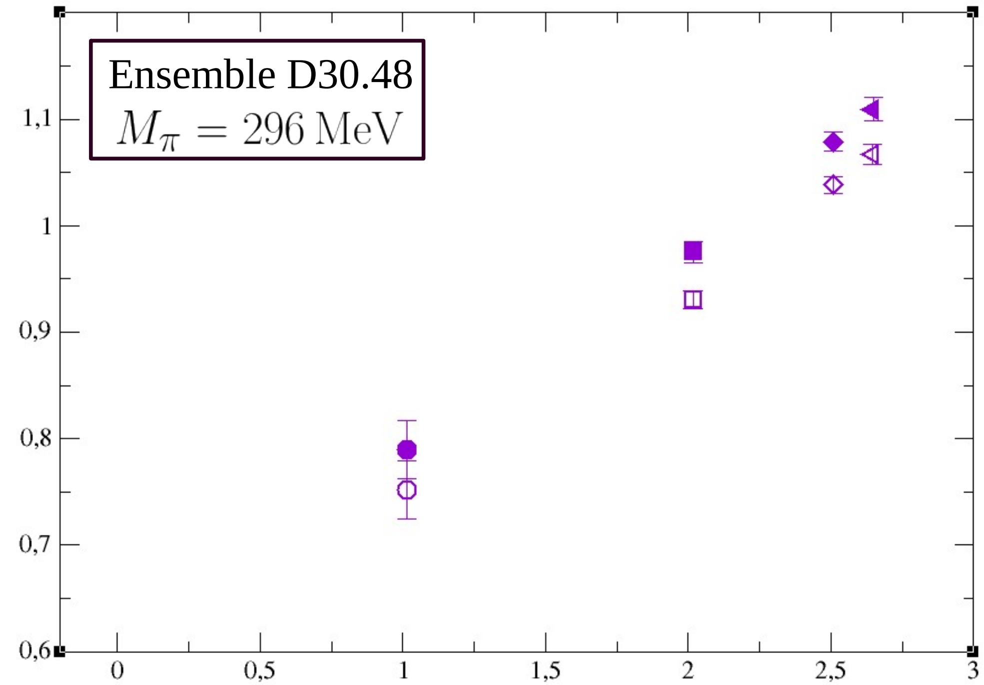
arXiv: 1607.00299

Results


Ensemble A30.32
$M_\pi = 245$ MeV
f_0 vs. q^2(MeV^2)
p_π=656 MeV
p_π=353 MeV
p_π=151 MeV
p_π=0


$p_D = (0, \pm151, \pm353, \pm656)$MeV
$p_D = 0$ MeV
corrected
uncorrected
Ensemble A30.32
$M_\pi = 245$ MeV


Ensemble D30.48
$M_\pi = 296$ MeV
$p_\pi = 661$ MeV
$p_\pi = 356$ MeV
$p_\pi = 153$ MeV
$p_\pi = 0$


Ensemble D30.48
$M_\pi = 296$ MeV

Conclusions & Outlooks

We have presented the momentum dependence of the form factors $f_0(q^2)$ and $f_+(q^2)$ for the semileptonic decay $D \rightarrow \pi$ using ETMC gauge ensembles with Nf=2+1+1 dynamical quarks

The form factors have been determined using both the vector and the scalar currents and adopting different kinematical conditions in which both the D and the π mesons have non-zero momentum

Lorentz symmetry breaking due to hypercubic effects is clearly observed and included in the decomposition of the current matrix elements in terms of form factors

Our preliminary result for $f_+(0)$ at the physical point is $f_+^{D\rightarrow\pi}(0) = 0.631(40)$

<u>To do list:</u>

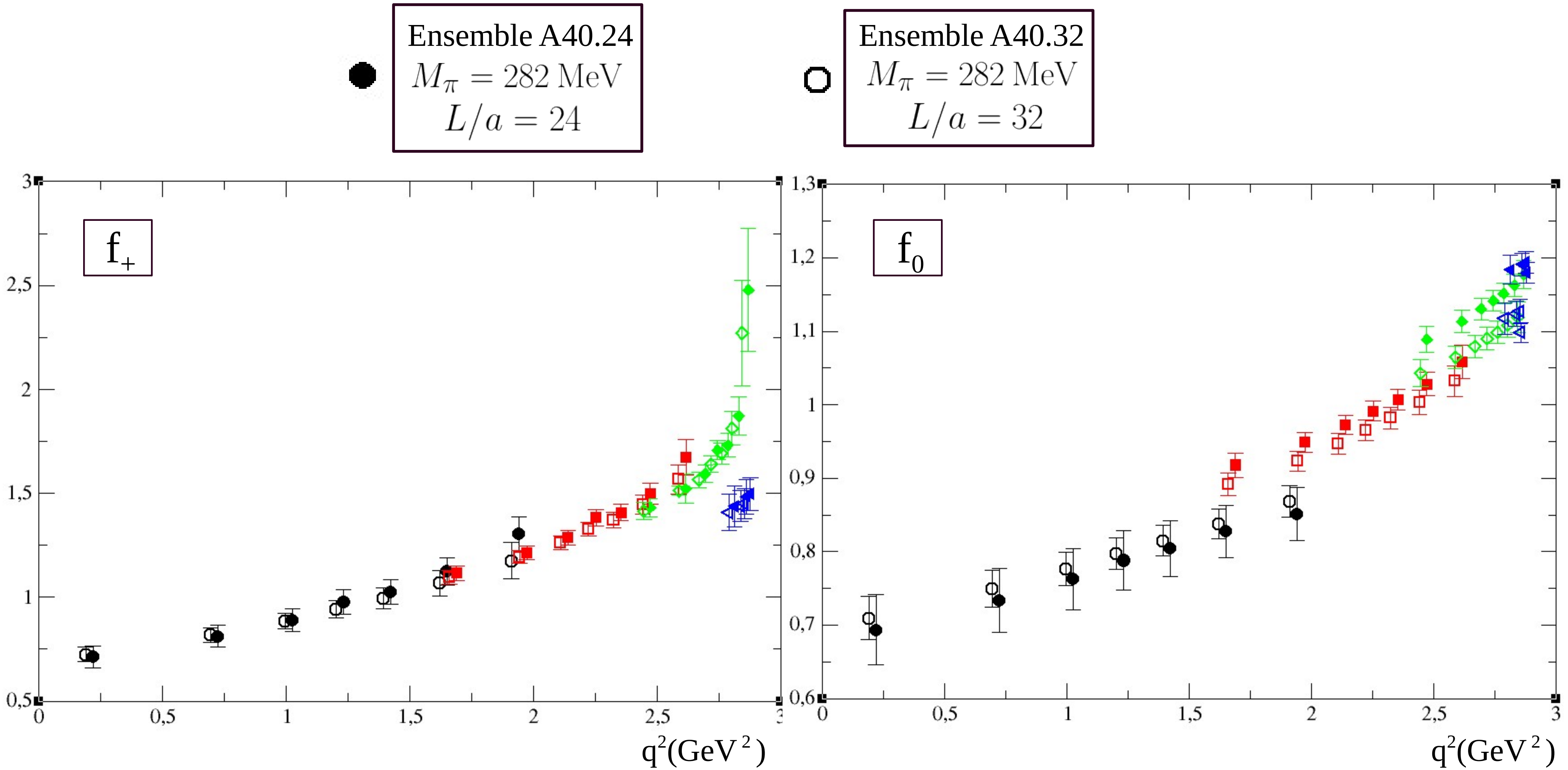
Improve the statistics

Extension to $D \rightarrow K$ semileptonic decays

Check of hypercubic effects in other semileptonic decays (K_{l3} , …)

Thank you for the attention

Thank you for the attention


Ensemble A40.24
$M_\pi = 282$ MeV
$L/a = 24$
Ensemble A40.32
$M_\pi = 282$ MeV
$L/a = 32$
f_+
f_0
$q^2(\mathrm{GeV}^2)$
$q^2(\mathrm{GeV}^2)$