S-duality in Lattice Super-Yang-Mills*

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*** In memory of Tom Kibble**


TRUMP
UNIVERSITY

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- Robert Wells (Cambridge → finance)
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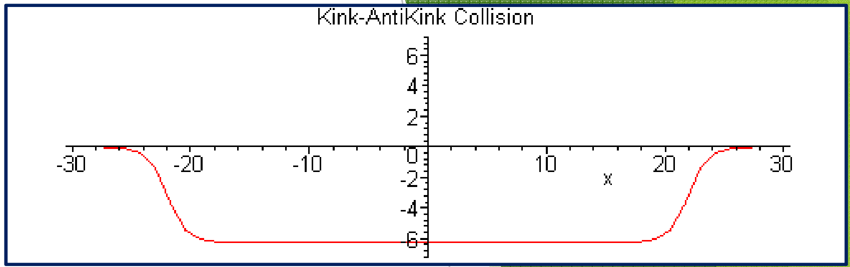


http://homepages.tversu.ru/~s000154/collision/sge_sol_m/images/SGe_solitons41.gif

Kink solitons colliding and annihilating (sine-Gordon model). Dual to fermion -- antifermion collision in Thirring model. The vertical is the value of phi(x), with the convention that the potential is ~ cos(phi). x can be your spatial coordinate (say our x[1]), and the movie is playing out in the time axis (say our x[0])

DUALITIES

Some duality history:

Kramers (Leiden) & Wannier (Texas) [1941]: "Owing to communication difficulties, one of the authors (G. H. W.) is entirely responsible for the printed text."

Complex, nonlinear = diversity of emergent phenomena

- Topological solitons in gauge theories an example: magnetic charge without elementary magnetic charges.
- ‘t Hooft was led to his monopole by considering the dynamics of Nielsen-Olesen vortices in SO(3) gauge theory --- vortex-vortex co-annihilation.

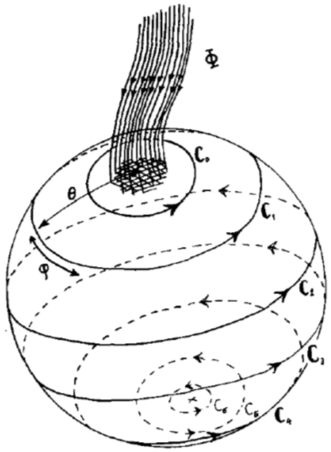
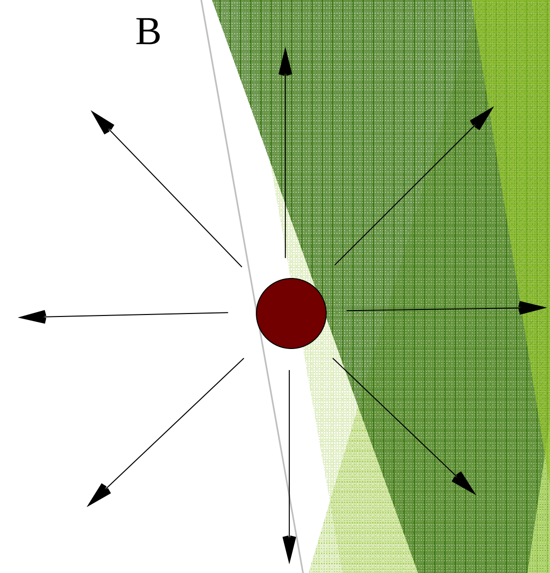


Fig. 1. The contour C on the sphere around the monopole. We deplace it from C_0 to C_1, etc., until it shrinks at the bottom of the sphere. We require that there be no singularity at that point.

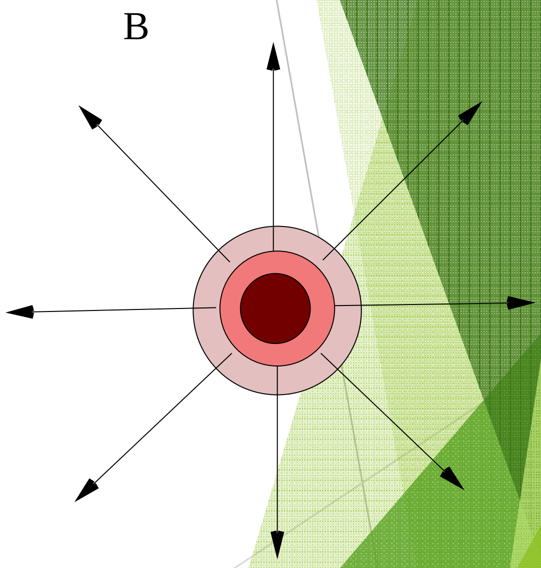
The ‘t Hooft – Polyakov proposal



- We may not have discovered any elementary magnetic monopoles of electromagnetism in nature, but it is quite possible that Yang-Mills theories with adjoint scalars in the Coulomb phase have magnetic monopoles that look like the Dirac monopole at long distance.

A monopole with structure

- It turns out that this is a kind of topological soliton.
- Much is known about the classical theory.
- Somewhat less is known about the quantum theory.
- Attempts have been made on the lattice to study the latter (e.g. Kibble & Co.).
- This proposal is highly relevant to Montonen-Olive duality and S-duality in N=4 super-Yang-Mills
- Of course it is also very important in the Seiberg-Witten theory of N=2 SYM.



- In particular, the ‘t Hooft-Polyakov monopole is supposed to be dual to the W bosons, in the Coulomb phase.

$$U(N) \to U(1)^N$$

- There is some compelling evidence for this in the Montonen-Olive paper, in the context of the 2+1 dimensional Georgi-Glashow model.
- It is an unusual duality: the Lagrangian for the solitons is actually the original Lagrangian, but with some redefinition of the couplings.

What is duality?

- A duality can be thought of as a change of variables in the path integral:

$$\int [d\Phi] e^{-S[\Phi]} = \int [d\Psi] e^{-\tilde{S}[\Psi]}$$

- The amazing thing about the Montonen-Olive duality is that

$$S = \tilde{S}$$

with some appropriate map of the couplings.

S-duality

- Since:

$$m_W \sim g, \quad m_{\text{mono.}} \sim \frac{1}{g}$$

- We expect:

$$\tilde{g} \sim \frac{1}{g}$$

- It is also amusing that we expect elementary field excitations to become solitons in the dual theory.
- Also note the electric-magnetic duality.

- More generally, we have an SL(2,Z) duality

$$\begin{pmatrix} p \\ q \end{pmatrix} \to \begin{pmatrix} p' \\ q' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} ap + bq \\ cp + dq \end{pmatrix} \qquad \tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2}$$

$$\tau \to \tau' = \frac{a\tau + b}{c\tau + d}$$

- Half-BPS solitons

$$M_{p,q} = vg|p + q\tau| = vg\sqrt{\left(p + \frac{\theta}{2\pi}q\right)^2 + \left(\frac{4\pi q}{g^2}\right)^2}$$

- W-boson and ‘t Hooft Polyakov monopole

$$M_{1,0} = gv, \quad M_{0,1} = 4\pi v/g$$

- Under S-duality $\tau \to -1/\tau$

$$S = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

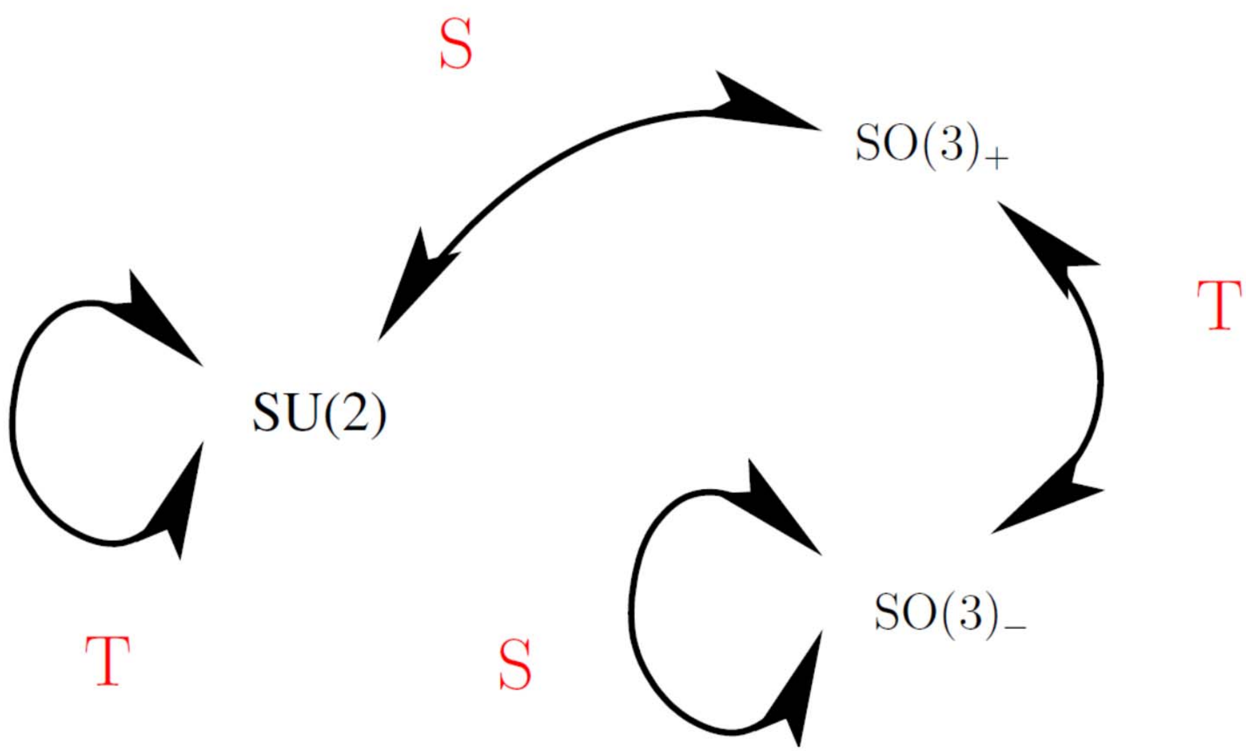
they transform into each other.

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

https://www.youtube.com/watch?v=IcTgDQwBqfc

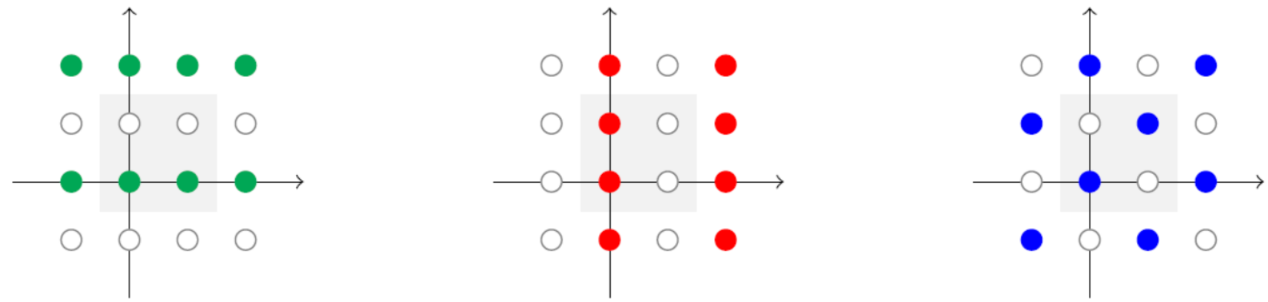
Seiberg et al. recent work…

search: seiberg reading between the lines



N=4 N=2

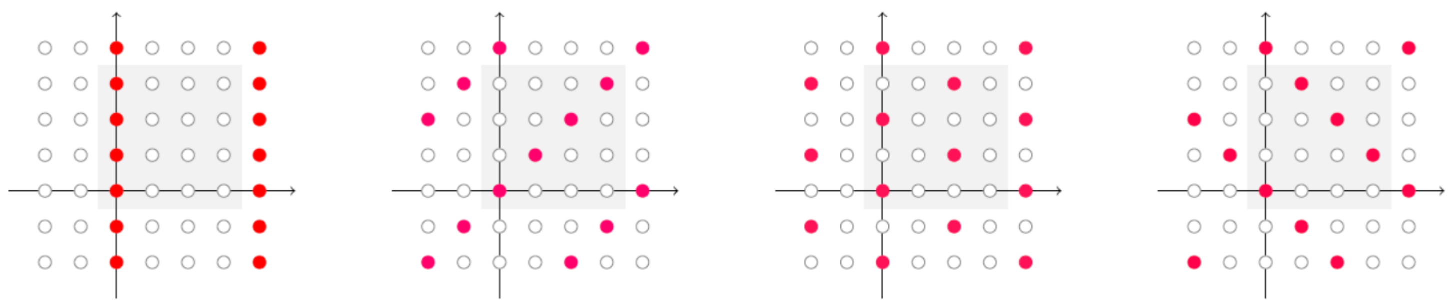
N=1 N=0



$SU(2)$ $SO(3)_+$ $SO(3)_-$

Aharony, Seiberg, Tachikawa 2013

(z_e, z_m)



$(SU(4)/\mathbb{Z}_4)_0$ $(SU(4)/\mathbb{Z}_4)_1$ $(SU(4)/\mathbb{Z}_4)_2$ $(SU(4)/\mathbb{Z}_4)_3$

‘t Hooft rotation

$$\Omega(\theta,\varphi) = \begin{pmatrix} \cos\frac{\theta}{2} e^{i\varphi} & i\sin\frac{\theta}{2} \\ i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} e^{i\varphi} \end{pmatrix}$$

$$R[\Omega] = \begin{pmatrix} \frac{1}{2}(1-\cos\theta) + \frac{1}{2}\cos(2\varphi)(1+\cos\theta) & \frac{1}{2}\sin(2\varphi)(1+\cos\theta) & \sin\varphi\sin\theta \\ -\frac{1}{2}\sin(2\varphi)(1+\cos\theta) & -\frac{1}{2}(1-\cos\theta) + \frac{1}{2}\cos(2\varphi)(1+\cos\theta) & \cos\varphi\sin\theta \\ \sin\varphi\sin\theta & -\cos\varphi\sin\theta & \cos\theta \end{pmatrix}$$

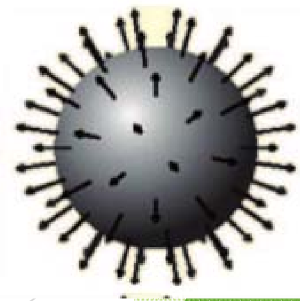
Higgs at infinity

- This gives rise to a topological soliton

$$H(\infty) = R[\Omega]H_0$$

where $H_0 = F = \text{const.}$ is the ordinary vacuum Higgs field.

$$H(\infty) = F(\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$$



Knobs controlling the mass

- Note that in the case

$$F \to 0$$

  the topology disappears and the energy of the monopole (mass) goes to zero (vacuum).

- Also interesting is that at least in the 2+1 dimensional Georgi-Glashow model there is some evidence [Davis et al., hep-lat/0110154] that the monopole mass eventually goes to zero in infinite volume.
- We anticipate that this does not happen in N=4, because of the implications for the W mass.

- So our goal is to study these solitons on the lattice and to measure their mass.
- The way to do that is to impose boundary conditions that lead to the soliton as the leading saddle point solution.
- We can arrange to have either odd numbers of monopoles or even numbers of monopoles.

Following: Davis et al. [hep-lat/0009037, hep-lat/0110154] and Rajantie [hep-lat/0512006]

- At low temperature

$$
\begin{aligned}
Z_{\mathrm{odd}} &= Z_0 e^{-\beta M} + Z_0' e^{-3\beta M} + \cdots \\
Z_{\mathrm{even}} &= Z_0 + Z_0' e^{-2\beta M} + \cdots
\end{aligned}
$$

- So the mass of a single monopole can be obtained from

$$
M = -\lim_{\beta\to\infty} \frac{1}{\beta} \ln \frac{Z_{\mathrm{odd}}}{Z_{\mathrm{even}}}
$$

- In fact, the “mass” is the free energy of the monopole.
- It can be used as an order parameter to distinguish the symmetric (conformal) from the broken (Coulomb) phase.

- Two questions arise:

1. How do we get these even/odd partition functions?
2. How do we calculate a partition function (free energy) with a Monte Carlo calculation?

- The answer to the first question is that we use C-periodic and twisted boundary conditions (in space).

$$Z_{\mathrm{even}} = Z_C, \quad Z_{\mathrm{odd}} = Z_{\mathrm{tw}}$$

- C-periodic is relatively simple:

$$U_\mu(x+N\hat{j}) = U_\mu^*(x) = \sigma_2 U_\mu(x)\sigma_2$$
$$\Phi(x+N\hat{j}) = \Phi^*(x) = -\sigma_2\Phi(x)\sigma_2$$

- Note that this is a symmetry.

$U_\mu \ \Phi$	$U_\mu^* \ \Phi^*$	$U_\mu \ \Phi$
$U_\mu^* \ \Phi^*$	$U_\mu \ \Phi$	$U_\mu^* \ \Phi^*$

- Twisted boundary conditions are a slight generalization:

$$\boldsymbol{U_\mu(x+N\hat{j}) = \sigma_j U_\mu(x)\sigma_j}$$
$$\boldsymbol{\Phi(x+N\hat{j}) = -\sigma_j \Phi(x)\sigma_j}$$

Note that the order does not matter b/c

$$\begin{aligned} \boldsymbol{\sigma_1\sigma_2 U_\mu \sigma_2\sigma_1} &= \boldsymbol{\sigma_2\sigma_1 U_\mu \sigma_1\sigma_2} \\ &= \boldsymbol{\sigma_3 U_\mu \sigma_3} \end{aligned}$$

$\sigma_1\sigma_2 U_\mu \sigma_2\sigma_1$ $-\sigma_1\sigma_2\Phi\sigma_2\sigma_1$	$\sigma_2 U_\mu \sigma_2$ $-\sigma_2\Phi\sigma_2$	$\sigma_1\sigma_2 U_\mu \sigma_2\sigma_1$ $-\sigma_1\sigma_2\Phi\sigma_2\sigma_1$
$\sigma_1 U_\mu \sigma_1$ $-\sigma_1\Phi\sigma_1$	$U_\mu\ \Phi$	$\sigma_1 U_\mu \sigma_1$ $-\sigma_1\Phi\sigma_1$

- These BCs are inspired by the continuum solution

$$\Phi \approx \frac{x_i \sigma_i}{r}, \quad A_i \approx \frac{\epsilon_{ijk} x_j \sigma_k}{r}$$

- As we go from the boundary in the +j direction to the one in the –j direction

$$x_j \to -x_j, \quad \Phi \to -\sigma_j \Phi \sigma_j, \quad A_i \to \sigma_j A_i \sigma_j$$

Actually, although Davis et al. claim this, I find a small problem with the last equation…

- OK, so we know how to get the desired ensembles, but how do we compute the free energy difference?
- Trick:

$$\frac{\partial M}{\partial x} = \frac{1}{\beta}\left(\left\langle\frac{\partial S}{\partial x}\right\rangle_{\mathbf{tw}} - \left\langle\frac{\partial S}{\partial x}\right\rangle_{C}\right)$$

Here x is some parameter that appears in the action.

- There will be a value of the bare scalar mass-squared for which the gauge symmetry is unbroken and the monopole mass vanishes. We use that as a reference point and integrate from there. Finite difference approx.:

$$M(m_{i+1}^2) - M(m_i^2) = -\frac{1}{\beta} \ln \frac{\left\langle \exp\left(-(m_{i+1}^2 - m_i^2) \sum_x \mathrm{Tr}\ \Phi^2\right) \right\rangle_{m_i^2,\mathrm{tw}}}{\left\langle \exp\left(-(m_{i+1}^2 - m_i^2) \sum_x \mathrm{Tr}\ \Phi^2\right) \right\rangle_{m_i^2,C}}$$

How we introduce mass:

$$\Delta S = F \sum_x \mathrm{Tr}\mathcal{U}_a^\dagger \mathcal{U}_a \quad \mathrm{n.s.}a$$

$$\begin{aligned}\mathrm{Tr}\mathcal{U}_a^\dagger \mathcal{U}_a &= \mathrm{Tr}\mathbf{1} + 2i\mathrm{Tr}B_a - 4\mathrm{Tr}B_a^2 + \cdots \\ &= N - \frac{2}{\sqrt{N}} B_a^0 + 4B_a^A B_a^A + \cdots\end{aligned}$$

We can't have F too negative or we'll destabilize the U(1) mode and have a runaway inverse lattice spacing.

$$\mathcal{U}_a = \frac{1}{a} - \frac{1}{\sqrt{N}} B_a^0 + \cdots$$

Linear term → anisotropic lattice

For GG, it actually works…

Rajantie 2005

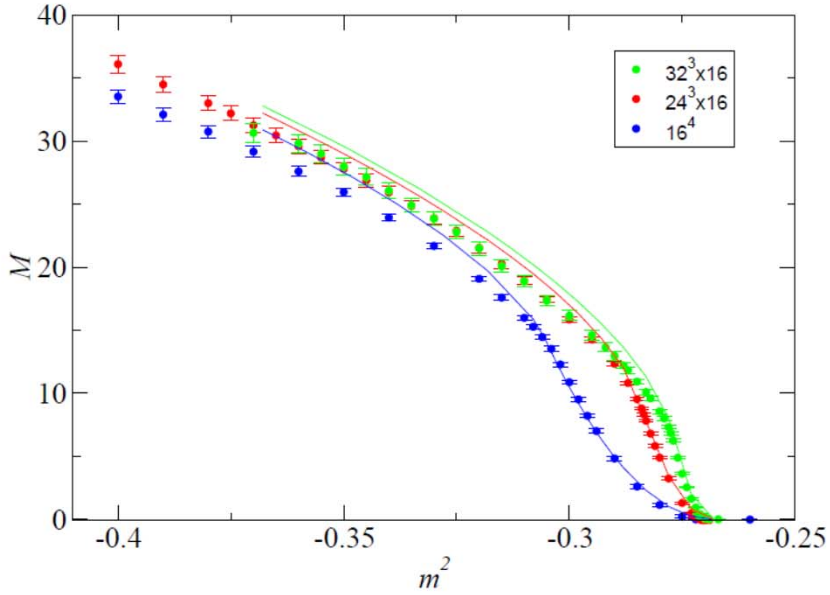
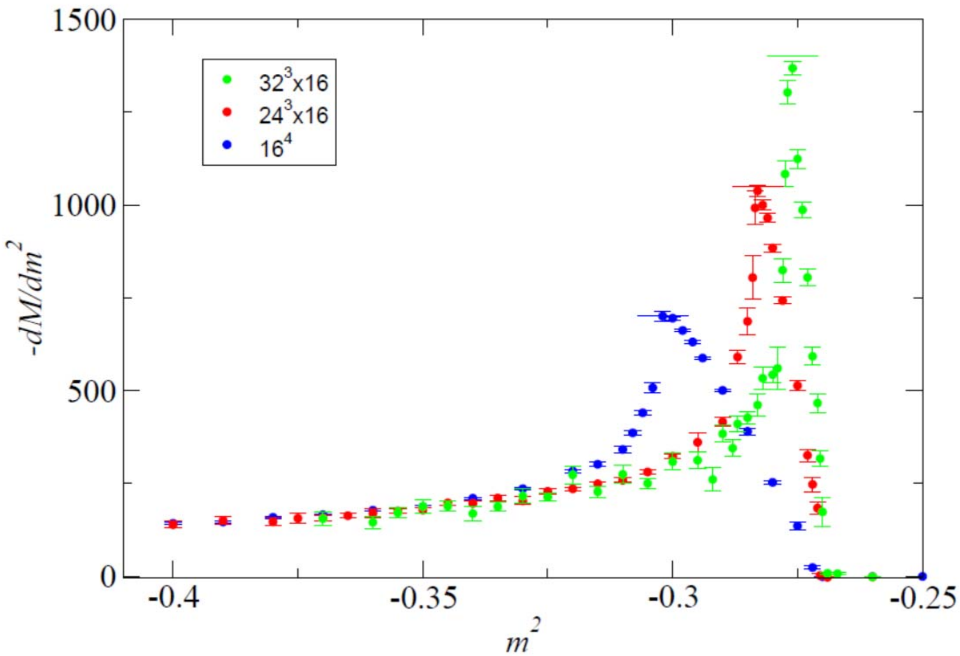


Figure 2: Quantum monopole mass (points) compared with the classical mass (lines).

Note that you can identify the point of symmetry breaking from the slope.

Finding the boundary mass-squared from monopole physics

- We can identify the boundary broken/symmetric from dM:



Rajantie 2005

- It is interesting that one can also measure the renormalized coupling based on a finite volume effect:

$$E(L) \approx M - \frac{10.98}{g^2 L}$$

- This arises from the magnetic Coulomb interaction with periodically extended monopole configurations.
- This is interesting for us because we would like to know

$$g^2 = F(g_0^2)$$

in our lattice theory.

Our BCs --- since our scalars are part of the link variables

- In our lattice theory the twisted BCs look like:

$$\mathcal{U}_a(x + Le_j) = -\sigma_j \mathcal{U}_a^\dagger(x)\sigma_j$$

- This implies:

$$A_a(x + Le_j) = \sigma_j A_a(x)\sigma_j, \quad B_a(x + Le_j) = -\sigma_j B_a(x)\sigma_j$$

- One scalar, A_5, has a funny boundary condition, so we avoid giving it a vev.

W boson mass

- Problem again with Gauss' law on T^3.
- Where does it come from?

$$\mathcal{L} \ni A_0(\mathbf{k}=0)[(\boldsymbol{\nabla}\cdot\boldsymbol{E})_{\mathbf{k}=0} - \rho_{\mathbf{k}=0}]$$

$$A_0(\mathbf{k}=0) = \int_{T^3} d^3x\ A_0$$

$$(\boldsymbol{\nabla}\cdot\boldsymbol{E})_{\mathbf{k}=0} = \int_{T^3} d^3x\ \boldsymbol{\nabla}\cdot\boldsymbol{E}$$

$$\rho_{\mathbf{k}=0} = \int_{T^3} d^3x\ J^0 = Q$$

Becomes δ function constraint in the path integral

$$\frac{\delta\mathcal{L}}{\delta A_0(\mathbf{k}=0)} = 0$$

$\Rightarrow$ Gauss' law

But

$$\int_{T^3} d^3x\ \boldsymbol{\nabla}\cdot\boldsymbol{E} = \oint_{\partial T^3} da\ \boldsymbol{n}\cdot\boldsymbol{E} = 0$$

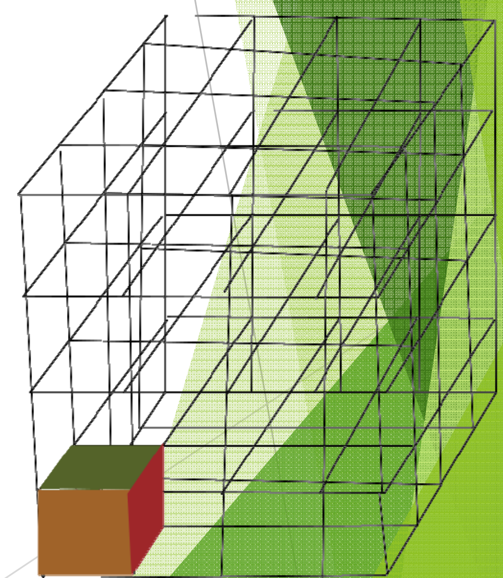
$$\Rightarrow\quad Q = 0$$

- This means that if we attempt the standard spectral approach we will fail:

$$C(t) = \sum_{\mathbf{x}} \langle W^{+}(t, \mathbf{x}) W^{-}(0, \mathbf{0}) \rangle = 0$$

- The solution to this problem is standard: C-periodic BCs

E.g., this comes up when trying to incorporate isospin violating corrections.

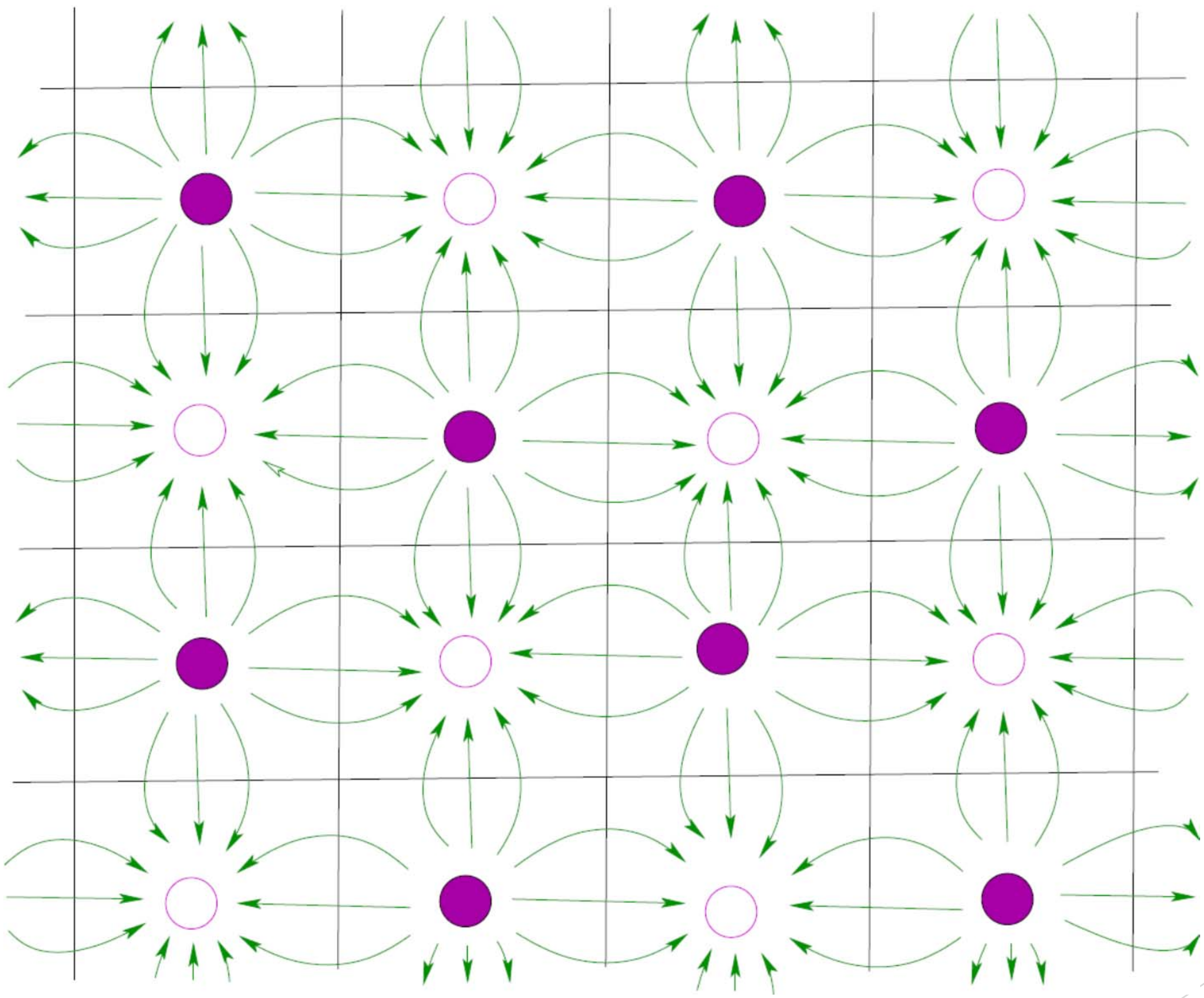


A common solution

- We impose C-periodic BCs

$$A_\mu(x+L_i) = A_\mu^c(x) = -A_\mu(x)$$

- This kills the constant mode
- Another way of looking at it is C-conjugate charges in periodically extended lattice
- This “eats” the flux



Conclusions

- We have a formulation that has the realistic potential of being fine-tuned to recover N=4 SUSY.
- We may see, to some approximation, features of N=4 even without the fine-tuning.
- Measuring W and monopole masses could give us useful information about the relation between the bare and renormalized coupling.
- Based on previous lattice studies, this looks possible.
- We are beginning to study dualities in N=4.
- Not discussed: ‘t Hooft loops and Wilson loops.

Thank you for your attention!

BACKUP SLIDES

Wilson fermion N=4

- Can impose SO(4), gauge invariance

$$
\begin{aligned}
S \;=\; & \int d^4x \ \mathrm{Tr}\Big\{ \frac{1}{2g_r^2} F_{\mu\nu}F_{\mu\nu} + \frac{i}{g_r^2}\overline{\lambda}_i\overline{\sigma}^\mu D_\mu \lambda_i + \frac{1}{g_r^2} D_\mu\phi_m D_\mu\phi_m + m_\phi^2 \phi_m\phi_m \\
& + m_\lambda(\lambda_i\lambda_i + \overline{\lambda}_i\overline{\lambda}_i) + \kappa_1 \phi_m\phi_m\phi_n\phi_n + \kappa_2\phi_m\phi_n\phi_m\phi_n + y_1(\lambda_i[\phi_{ij},\lambda_j] + \overline{\lambda}_i[\phi_{ij},\overline{\lambda}_j]) \\
& + y_2\epsilon_{ijkl}(\lambda_i[\phi_{jk},\lambda_l] + \overline{\lambda}_i[\phi_{jk},\overline{\lambda}_l])\Big\} \\
& + \int d^4x \ \Big\{ \kappa_3(\mathrm{Tr}\phi_m\phi_m)^2 + \kappa_4 \mathrm{Tr}\phi_m\phi_n \mathrm{Tr}\phi_m\phi_n \Big\}
\end{aligned}
$$

- 8-dimensional parameter space to fine-tune in
- Notice rescaling of fields exploited for first three terms---we do that in our twisted theory too.

- There has been significant recent progress on the lattice discretization of N=4 super Yang-Mills (SYM) [Kaplan et al. 05; Catterall et al. 07-present].
- Orbifolding approach; twisting approach; equivalence.
- Other approaches also deserve mention (conjectured equivalence) [Ishii et al. 08; Ishiki et al. 08-09; Hanada et al. 10; Honda et al. 11-13].
- Plane wave matrix model, planar limit equivalence, large N reduction.

Recent work with Catterall

- A necessary ingredient for our previous results on the form of the long distance effective action of the twisted lattice N=4 super Yang-Mills theory is the existence of a real space renormalization group which preserves the lattice structure, both the symmetries and the geometric interpretation of the fields.
- We provide an explicit example of such a blocking scheme.
- We also show that rescaling of the lattice fields greatly reduces the number of fine-tunings.
- In the case that the moduli space is not lifted by nonperturbative effects, there is only a single fine-tuning that would need to be performed.
- It thus becomes comparable to Wilson fermions in terms of fine-tuning.

$S_5, \quad \mathcal{Q}$ shift symmetry, lattice gauge invariance, rescaling, moduli space not lifted

- One motivation for such efforts is that it is highly desirable to test the AdS/CFT correspondence at a finite number of colors Nc , and for moderate values of the 't Hooft coupling

$$\lambda = g^2 N_c \sim 1$$

- Indeed, results in this regime would, in theory, open the way to nonperturbative results for quantum gravity.
- Another reason to study N=4 SYM on the lattice is that the continuum theory is an interacting conformal field theory at all scales, unlike the situation with theories inside the conformal window, which only approach a conformal fixed point in the infrared (IR).
- It is therefore a conformal field theory of a very different character from what is typically studied on the lattice.
- 3+1 analogues of BKT?

- The key new idea which underlies these new lattice constructions is to discretize not the usual theory but a *topologically twisted* cousin.
- In flat space this corresponds merely to an exotic change of variables --- one more suited to discretization.
- In the case of N=4 SYM there are three independent topological twists of the theory and the one that is employed in the lattice work is the Marcus or Geometric-Langlands twist [Marcus 95;Kapustin, Witten 06].

- The resulting lattice action takes the form

$$
\begin{aligned}
S &= \frac{1}{2g^2}(\mathcal{Q}\lambda + S_{\text{closed}}) \\
\lambda &= \sum_x a^4 \text{Tr}(\chi_{ab}\mathcal{F}_{ab} + \eta\overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a - \frac{1}{2}\eta d) \\
S_{\text{closed}} &= -\frac{1}{4}\sum_x a^4 \epsilon_{abcde} \text{Tr}\chi_{de}\overline{\mathcal{D}}_c^{(-)}\chi_{ab}(x)
\end{aligned}
$$

$$\mathcal{Q}S = 0$$

Observes the notion that anything correct should be simple.
Four terms --- will result in four coefficients to fine tune.

- Of course we have to say how the derivatives are implemented:

$$
\begin{aligned}
\mathcal{F}_{ab}(x) &= \mathcal{D}_a^{(+)}\mathcal{U}_b(x) = \mathcal{U}_a(x)\mathcal{U}_b(x+e_a) - \mathcal{U}_b(x)\mathcal{U}_a(x+e_b) \\
\overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a(x) &= \mathcal{U}_a(x)\overline{\mathcal{U}}_a(x) - \overline{\mathcal{U}}_a(x-e_a)\mathcal{U}_a(x-e_a) \\
\epsilon_{abcde}\chi_{de}\overline{\mathcal{D}}_c^{(-)}\chi_{ab}(x) &= \epsilon_{abcde}\chi_{de}(x+e_a+e_b)[\chi_{ab}(x)\overline{\mathcal{U}}_c(x-e_c) \\
&\quad -\overline{\mathcal{U}}_c(x-e_c+e_a+e_b)\chi_{ab}(x-e_c)]
\end{aligned}
$$

$$\mathcal{U}_a = \frac{1}{a} + \mathcal{A}_a \qquad\qquad e_a \leftrightarrow A_4^*$$

$$gl(N,\mathbb{C}) \qquad \mathcal{A}_a = A_a + iB_a$$

- The supersymmetry transformation is:

$$\mathcal{Q}\mathcal{U}_a = \psi_a, \quad \mathcal{Q}\psi_a = 0, \quad \mathcal{Q}\overline{\mathcal{U}}_a = 0$$
$$\mathcal{Q}\chi_{ab}(x) = -\overline{\mathcal{F}}_{ab}(x) \equiv \overline{\mathcal{U}}_b(x+e_a)\overline{\mathcal{U}}_a(x) - \overline{\mathcal{U}}_a(x+e_b)\overline{\mathcal{U}}_b(x)$$
$$\mathcal{Q}\eta = d, \quad \mathcal{Q}d = 0$$

Bianchi identity for Q closed term

$$\epsilon_{mncde}\overline{\mathcal{D}}_c^{(-)}\overline{\mathcal{F}}_{mn}(x+e_c)=0$$

$$\epsilon_{mncde}\overline{\mathcal{D}}_c^{(-)}\overline{\mathcal{F}}_{mn}(x+e_c)=\epsilon_{mncde}[\overline{\mathcal{F}}_{mn}(x+e_c)\overline{\mathcal{U}}_c(x)-\overline{\mathcal{U}}_c(x+e_m+e_n)\overline{\mathcal{F}}_{mn}(x)]$$

$$\overline{\mathcal{F}}_{mn}(x)=\overline{\mathcal{U}}_n(x+e_m)\overline{\mathcal{U}}_m(x)-\overline{\mathcal{U}}_m(x+e_n)\overline{\mathcal{U}}_n(x)$$

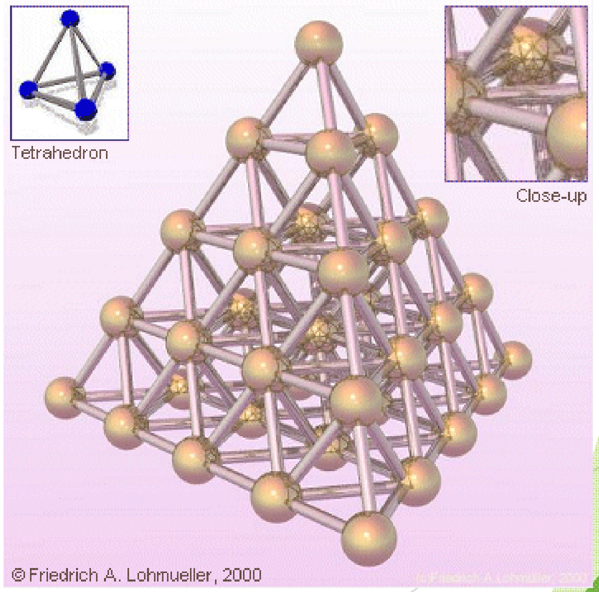
then algebra

Primitive vectors

A_4^* lattice

S_5 point group symmetry

$$
\begin{aligned}
e_1 &= \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right)\\
e_2 &= \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right)\\
e_3 &= \left(0, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right)\\
e_4 &= \left(0, 0, -\frac{3}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right)\\
e_5 &= \left(0, 0, 0, -\frac{4}{\sqrt{20}}\right)
\end{aligned}
$$



A_3^* lattice

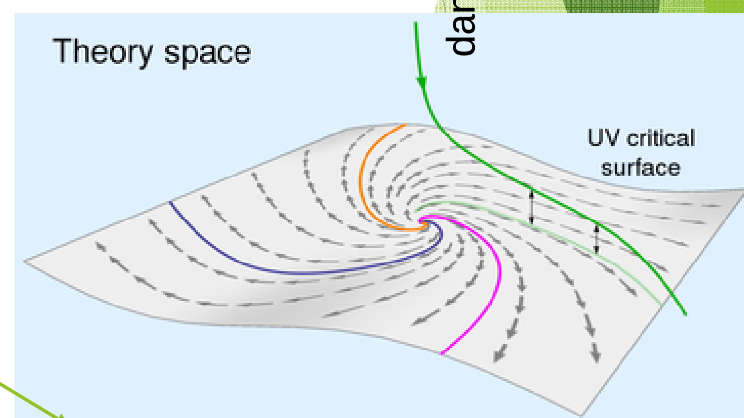
Long distance effective theory allowed by lattice symmetries

- In work from 2011 we showed that the most general renormalizable action consistent with the lattice symmetries is

$$\mathcal{Q}\mathrm{Tr}\{\alpha_1\chi_{ab}\mathcal{F}_{ab}+\alpha_2\eta[\overline{\mathcal{D}}_a,\mathcal{D}_a]-\frac{\alpha_3}{2}\eta d\}-\frac{\alpha_4}{4}\epsilon_{abcde}\mathrm{Tr}\chi_{de}\overline{\mathcal{D}}_c\chi_{ab}$$

$$\underbrace{+\beta\mathcal{Q}\{\mathrm{Tr}\eta\mathcal{U}_a\overline{\mathcal{U}}_a-\frac{1}{N}\mathrm{Tr}\eta\mathrm{Tr}\mathcal{U}_a\overline{\mathcal{U}}_a\}}_{\text{new}}$$

dangerous irrelevant operator



marginally relevant

- In 2013 we studied discrete subgroup of R symmetry in twisted formulation.

$$R_a, \quad R_{ab}$$

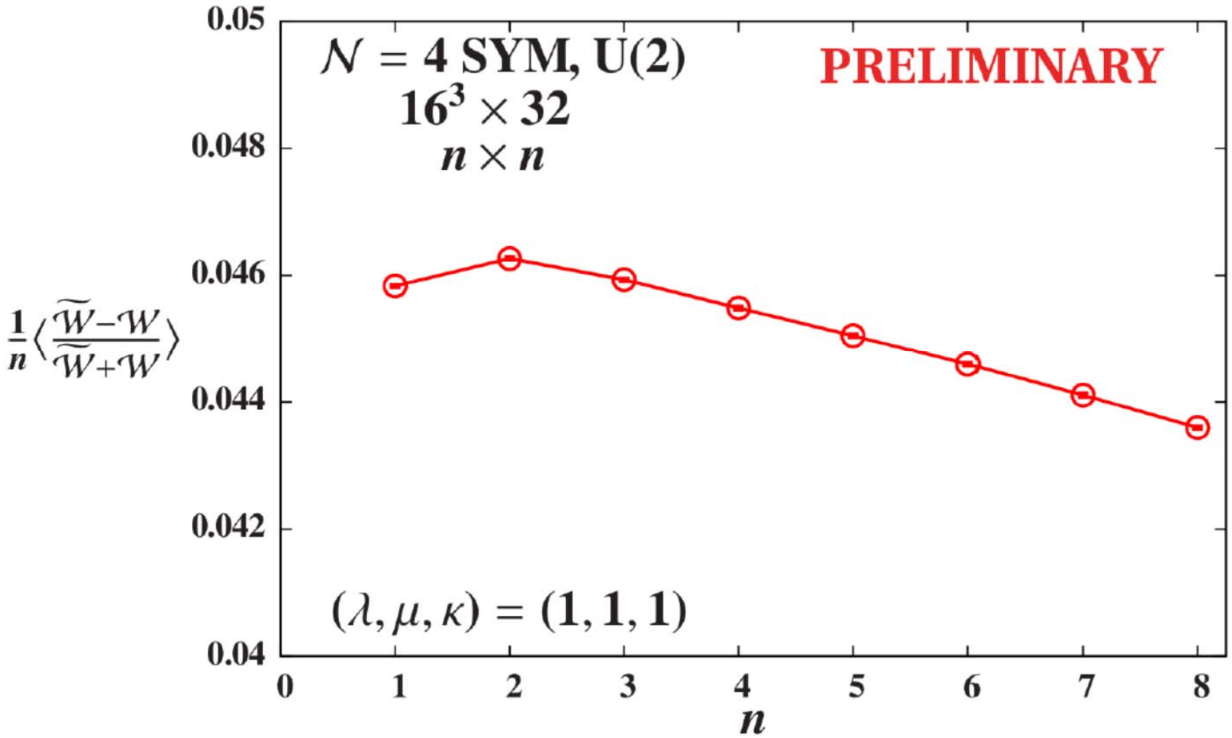
- This allowed us to write down the other 15 supercharges

$$\mathcal{Q}_a = R_a \mathcal{Q}, \quad \mathcal{Q}_{ab} = R_{ab} \mathcal{Q}$$

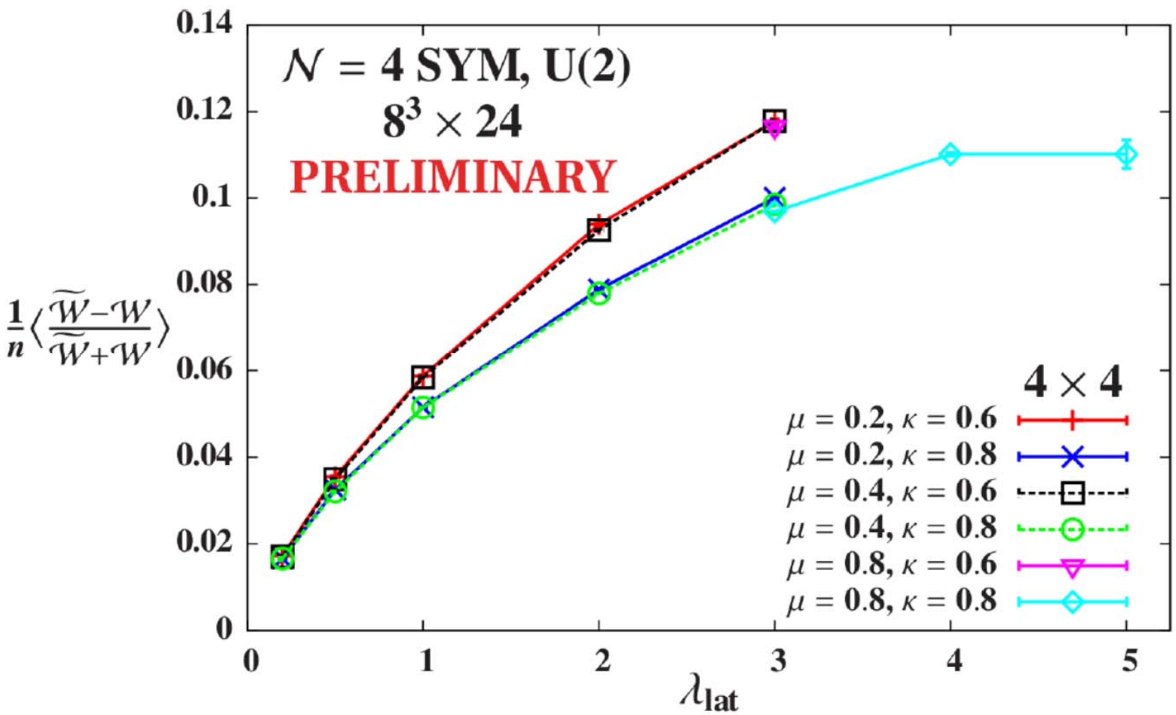
- More importantly, imposing any one of these 15 R symmetries forces

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4, \quad \beta = 0$$

- Hence, together with exact Q, restoration of discrete R symmetry guarantees N=4 SUSY continuum limit.
- We measured the difference between the transformed plaquette and the original plaquette.



0.05
0.048
0.046
0.044
0.042
0.04
0
1
2
3
4
5
6
7
8
n
$\frac{1}{n}\langle\frac{\widetilde{W}-W}{\widetilde{W}+W}\rangle$
$\mathcal{N}$ = 4 SYM, U(2)
$16^3 \times 32$
$n \times n$
PRELIMINARY
$(\lambda, \mu, \kappa) = (1, 1, 1)$



$\mathcal{N}$ = 4 SYM, U(2)
$8^3 \times 24$
PRELIMINARY
$\frac{1}{n}\langle\frac{\widetilde{W}-W}{\widetilde{W}+W}\rangle$
$\lambda_{\rm lat}$
4 × 4
μ = 0.2, κ = 0.6
μ = 0.2, κ = 0.8
μ = 0.4, κ = 0.6
μ = 0.4, κ = 0.8
μ = 0.8, κ = 0.6
μ = 0.8, κ = 0.8
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0
1
2
3
4
5

- In [Catterall, Giedt 14] we obtained a RG blocking transformation that preserves the lattice symmetries:

$$
\begin{aligned}
\mathcal{U}'_a(x) &= \xi\mathcal{U}_a(x)\mathcal{U}_a(x+e_a), \quad \overline{\mathcal{U}}'_a(x) = \xi\overline{\mathcal{U}}_a(x+e_a)\overline{\mathcal{U}}_a(x) \\
d'(x) &= \xi d(x), \quad \eta'(x) = \xi\eta(x) \\
\psi'_a(x) &= \xi[\psi_a(x)\mathcal{U}_a(x+e_a) + \mathcal{U}_a(x)\psi_a(x+e_a)] \\
\chi'_{ab}(x) &= \frac{\xi}{2}[\overline{\mathcal{U}}_a(x+e_a+2e_b)\overline{\mathcal{U}}_b(x+e_a+e_b)\chi_{ab}(x) + \overline{\mathcal{U}}_b(x+2e_a+e_b)\overline{\mathcal{U}}_a(x+e_a+e_b)\chi_{ab}(x)] \\
&+\xi[\overline{\mathcal{U}}_a(x+e_a+2e_b)\chi_{ab}(x+e_b)\overline{\mathcal{U}}_b(x) + \overline{\mathcal{U}}_b(x+2e_a+e_b)\chi_{ab}(x+e_a)\overline{\mathcal{U}}_a(x)] \\
&+\frac{\xi}{2}[\chi_{ab}(x+e_a+e_b)\overline{\mathcal{U}}_a(x+e_b)\overline{\mathcal{U}}_b(x) + \chi_{ab}(x+e_a+e_b)\overline{\mathcal{U}}_b(x+e_a)\overline{\mathcal{U}}_a(x)]
\end{aligned}
$$

$$\Lambda = \{a\textstyle\sum_{\mu=1}^4 n_\mu e_\mu | n \in \mathbb{Z}^4\} \qquad\qquad \Lambda' = \{2a\textstyle\sum_{\mu=1}^4 n_\mu e_\mu | n \in \mathbb{Z}^4\}$$

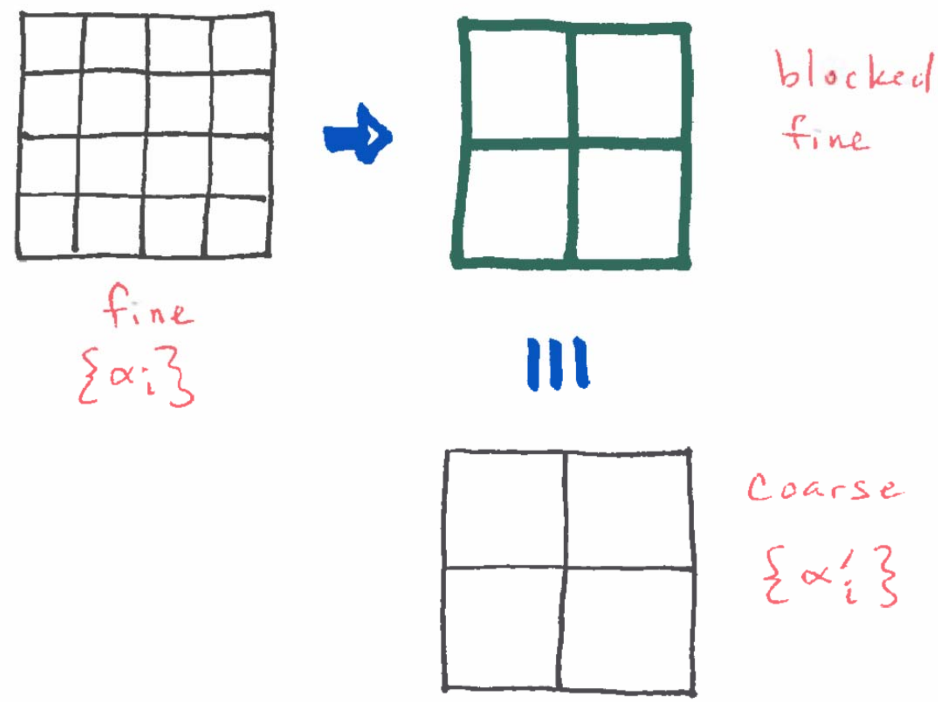
fine lattice

coarse lattice

- Shows that the 2011 analysis of renormalization makes sense: i.e., we really can impose the lattice symmetries on the long distance effective action.
- It also opens up the possibility of MCRG analysis of flow of couplings.

$$W(1,1)_{\text{b.f.}} = \xi^4 W(2,2)_{\text{fine}} \equiv W(1,1)_{\text{coarse}}$$

$$\Rightarrow \quad \xi^4 = \frac{W(1,1)_{\text{coarse}}}{W(2,2)_{\text{fine}}}$$



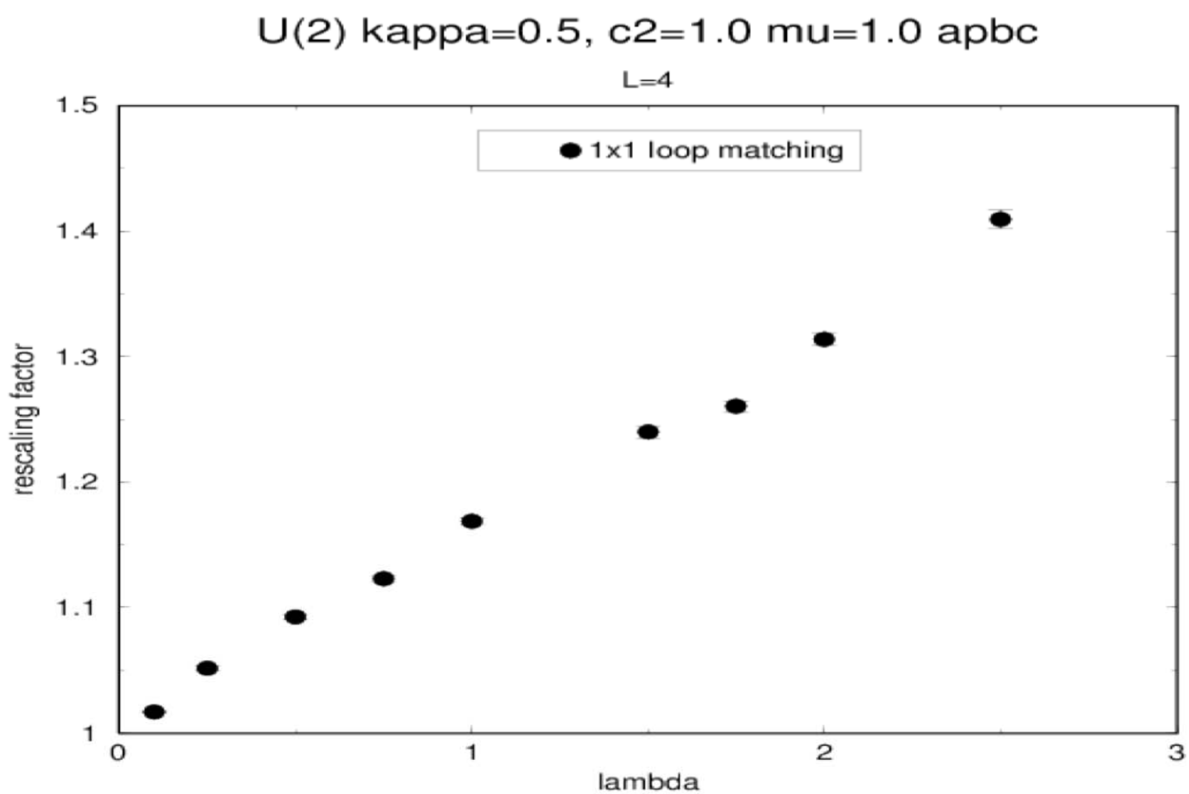
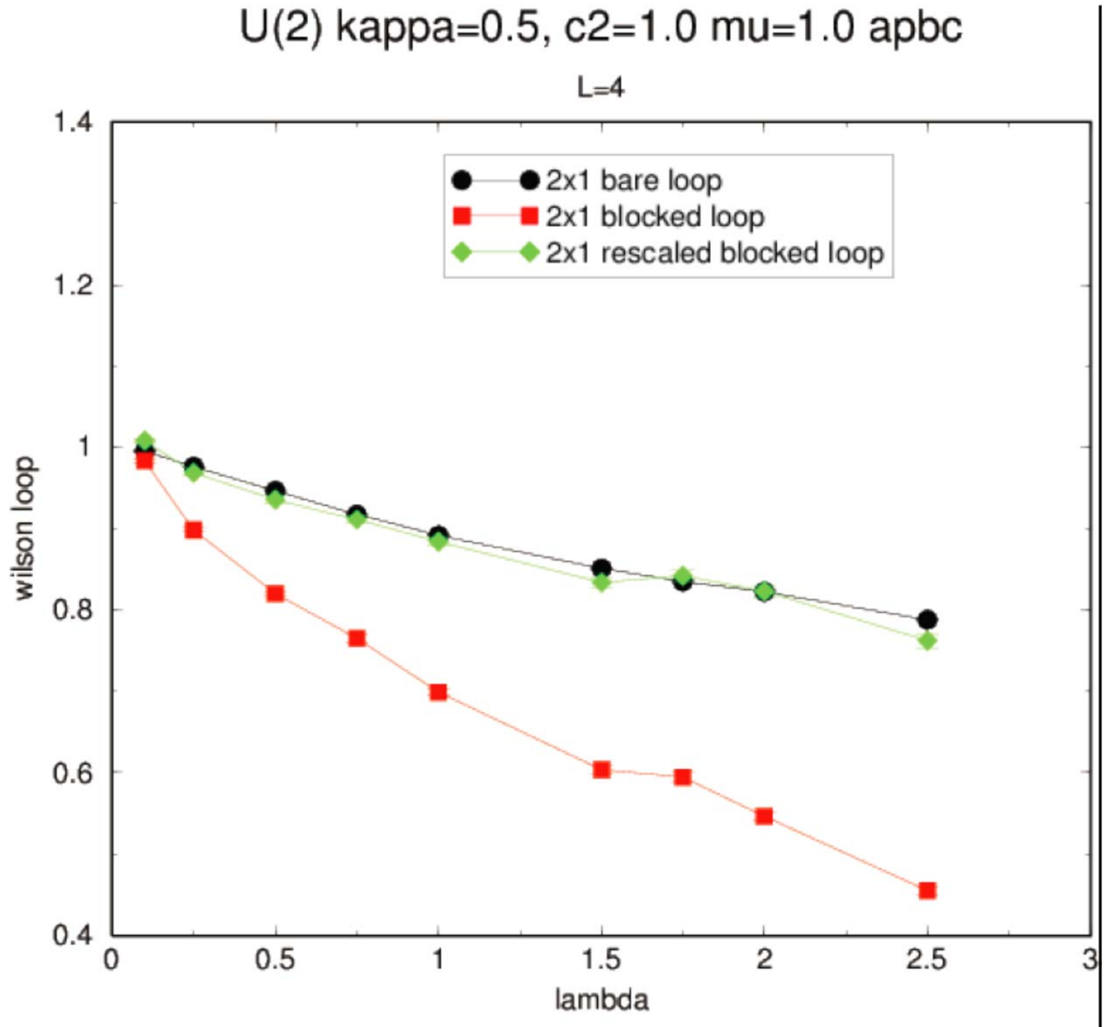


Figure 1. Determination of the scaling parameter ξ. Plotted on the vertical axis is ξ^4, the rescaling factor needed to match the 1×1 Wilson loop measured on the blocked lattices to its value on the coarse lattice.



U(2) kappa=0.5, c2=1.0 mu=1.0 apbc
L=4
2x1 bare loop
2x1 blocked loop
2x1 rescaled blocked loop
wilson loop
lambda
1.4
1.2
1
0.8
0.6
0.4
0
0.5
1
1.5
2
2.5
3

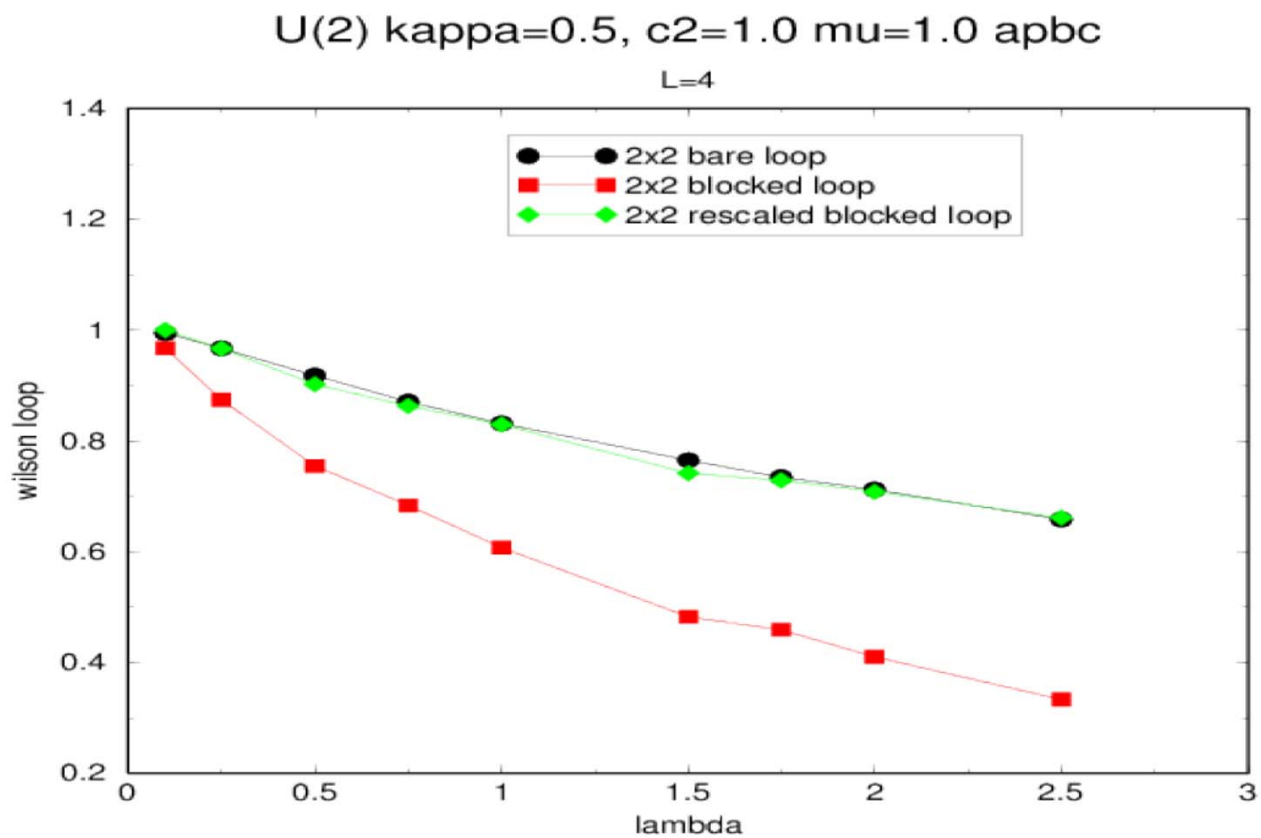


Figure 3. A comparison of $W(2,2)_{b.f.}$ and $W(2,2)_{\text{coarse}}$ with the rescaling factor taken into account.

- Amazingly, we did not have to adjust the couplings

$$\alpha_i = \alpha_i' \quad \forall \quad i$$

- This is perhaps evidence that the couplings are approximately not running; i.e., a nearly conformal behavior.
- It is interesting that this is in spite of the R-symmetry violation that we measured.

- Given that the violation of SUSY on small scales is modest, perhaps we can see some of the interesting features of N=4 SUSY?
- We are beginning to focus on S-duality.
- Identifying the self-dual point would give us valuable information on the map between lattice coupling and continuum coupling.

$$g_{\text{cont.,SD}} = 4\pi, \quad g_{\text{lat.,SD}} = ?$$