

Quark Mass Dependence of the QCD Critical End Point in the Strong Coupling Limit

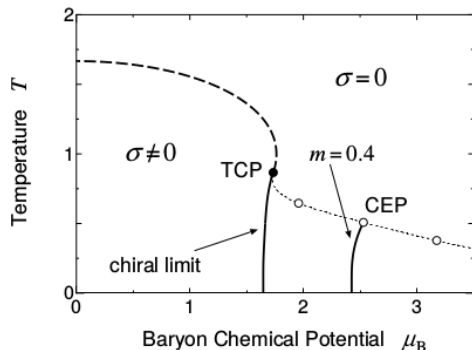
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Introduction

- ▶ Strong coupling lattice QCD in the dual representation allows to study the full μ - T phase diagram, due to the mildness of the sign problem.
- ▶ This has been done in the chiral limit. [Ph. de Forcrand, M. Fromm, PRL, 2009]
- ▶ Here we extend the phase diagram to **finite quark masses**.



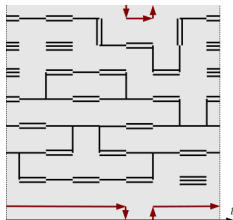
- ▶ No Monte Carlo result yet, only mean field [Nishida,PRD,2004] on phase diagram at $m_q > 0$.
- ▶ We present our results on the quark mass dependence of the QCD critical end point and 1st order line obtained by Monte Carlo via the worm algorithm.

Quark Mass dependence in Dual Representation

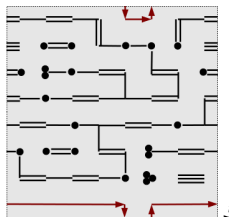
- ▶ Dual Representation - presented by Wolfgang Unger.
- ▶ The partition function after Grassmann integration,

$$Z = \sum_{k,n,\ell} \underbrace{\prod_{b=(x,\hat{\mu})} \frac{(N_c - k_b)!}{N_c! k_b!} \gamma^{2k_b \delta_{0,\hat{\mu}}}}_{\text{meson hoppings}} \underbrace{\prod_x \frac{N_c!}{n_x!} (2am_q)^{n_x}}_{\text{chiral condensate}} \underbrace{\prod_{\ell} w(\ell, \mu)}_{\text{baryon hoppings}} \quad (1)$$

- ▶ $w(\ell, \mu) = \frac{1}{\prod_{x \in \ell} a_t} \sigma(\ell) \gamma^{3N_0} \exp(3N_t r_{\ell} a_t \mu)$
- ▶ γ : anisotropy, $\gamma^2 = a/a_t$. (see Helvio's talk)
- ▶ $\sigma(\ell) = (-1)^{r_{\ell} + N - (\ell) + 1}$



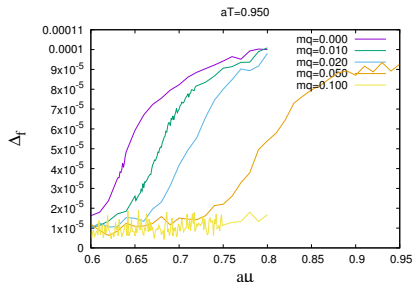
(a) monomers absent at zero m_q



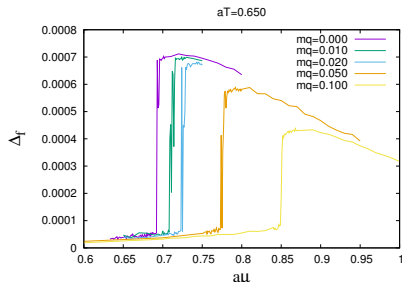
(b) Finite m_q

Mildness of Sign Problem at Finite Quark Masses

- ▶ $\langle sign \rangle = \exp(-L^3 Nt \Delta f)$
- ▶ Δf is the difference between full and sign-quenched free energy density



(c) $0.95 = aT > aT_{tric}$



(d) $0.65 = aT < aT_{tric}$

- ▶ We use sign reweighting method, $\langle O \rangle = \frac{\langle sign O \rangle}{\langle sign \rangle}$

Our Simulation Strategy

- ▶ Aim: Obtain trajectory of chiral/Nuclear CEP as function of m_q .
- ▶ Starting point: Tricritical point in chiral limit. $(aT_{\text{tric}}, a\mu_{\text{tric}}) = (0.825(25), 0.6672(2))$

$$\gamma/\nu = \begin{cases} 1.3177/0.67155 & = \text{2nd order O}(2) \\ 2 & = \text{tricritical} \\ 3 & = \text{1st order} \end{cases}$$

- ▶ Our observables are $\langle \bar{\psi}\psi \rangle$, χ , $\langle n_B \rangle$, χ_B .
- ▶ No Binder Cumulant via Worm algorithm yet.
- ▶ Chiral transition has 2nd order phase transition line and tricritical point and 1st order line in the chiral limit.
- ▶ We scan the parameter space along the μ direction for various temperatures in the range of $0.650 \leq aT \leq 1.000$.

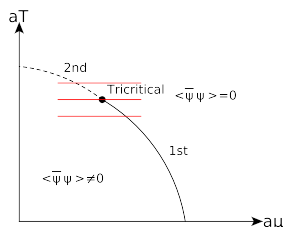
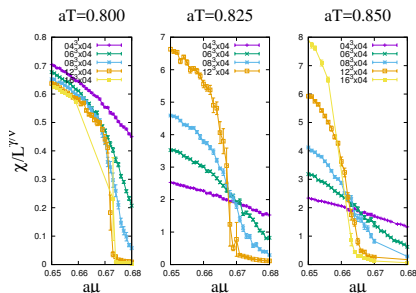


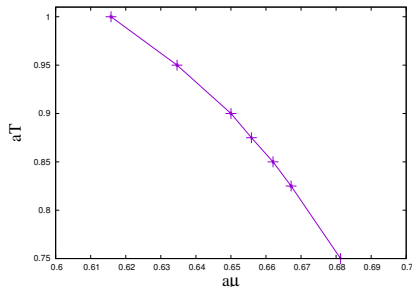
Figure: Strong coupling phase diagram at zero quark mass for chiral transition

Chiral Transition at Zero Quark Mass

- ▶ Our chiral observables are chiral condensate $\langle \bar{\psi}\psi \rangle$ and chiral susceptibility χ .
- ▶ $\langle \bar{\psi}\psi \rangle = \frac{1}{2m_q V} \sum_x \langle N_M(x) \rangle$, where N_M is the number of monomers. $\langle N_M(x) \rangle = 0$ in the chiral limit.
- ▶ $\chi = \frac{1}{V} \sum_{x_1, x_2} G(x_1, x_2)$, where $G(x_1, x_2)$ is 2pt function, sampled by the worm algorithm.
- ▶ We obtain the phase boundary from the crossing points.
- ▶ $\gamma/\nu = 1$ st ($aT = 0.8$), $\gamma/\nu = \text{tric}$ ($aT = 0.825$), $\gamma/\nu = 2$ nd ($aT = 0.85$)



(a) Chiral Susceptibility



(b) Phase Boundary at Zero Quark Mass

Nuclear Transition at Zero Quark Mass

- ▶ Nuclear transition has 2nd order end point and 1st order line.
- ▶ Our nuclear observables are baryon density $\langle n_B \rangle$ and baryon susceptibility χ_B .
- ▶ We can find the end point temperature from baryon susceptibility.

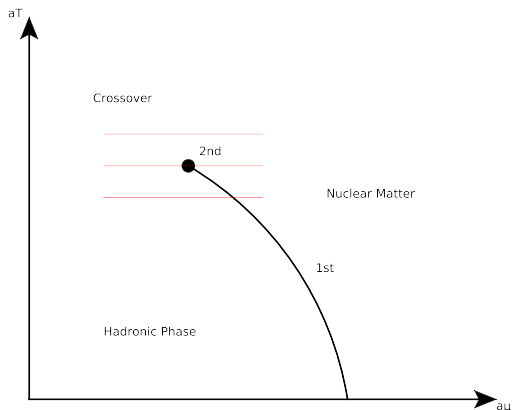
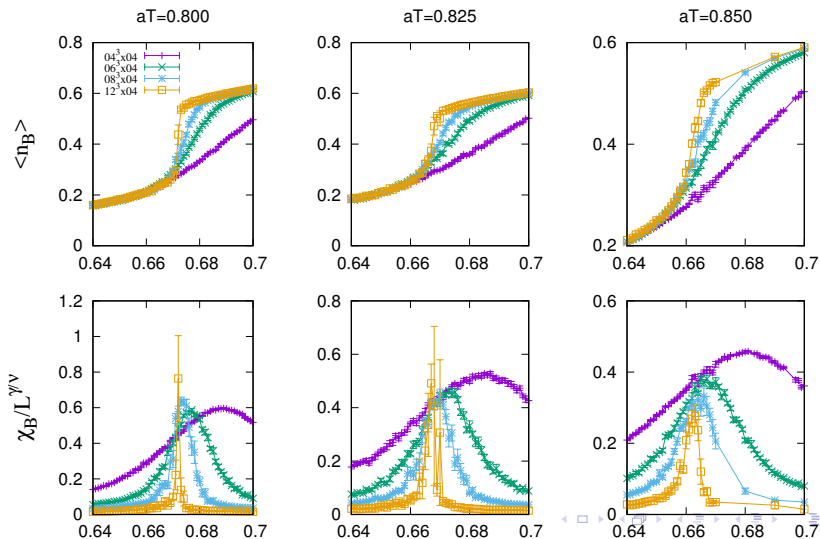


Figure: Strong coupling phase diagram for nuclear transition

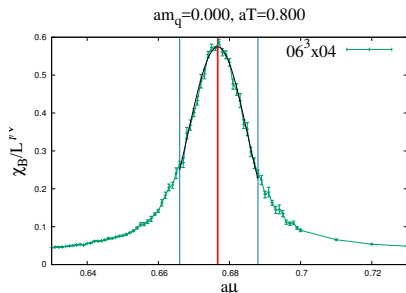
Baryon Density and Susceptibility at Zero Quark Mass

- ▶ We rescale the baryon susceptibility using 2nd-critical exponent.
- ▶ Then the peak heights of susceptibility for different lattice sizes will be overlapped at the critical endpoint temperature.

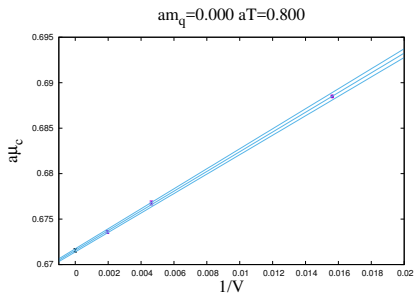


Analysis Method

- ▶ To find the critical chemical potential $a\mu_c$, we try the Gaussian fit to the χ_B .
- ▶ We extrapolate to the thermodynamic limit.
- ▶ This extrapolation is very well controlled.

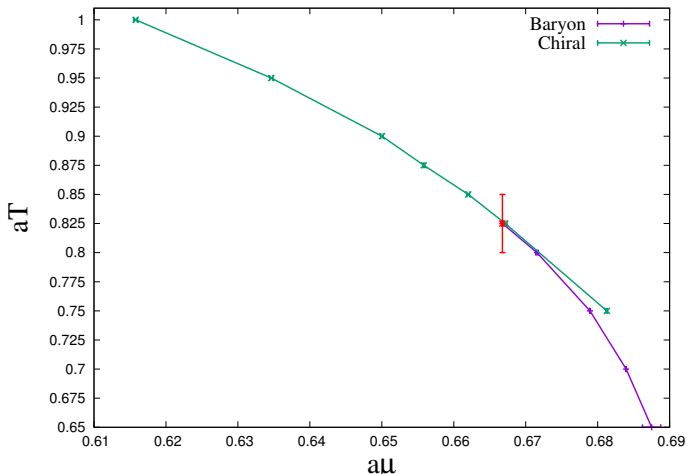


(a) Gaussian fit



(b) Extrapolation to Thermodynamic limit

QCD Phase Diagram in Chiral Limit



- ▶ Red is end point of nuclear transition.
- ▶ Purple is 1st order phase transition line obtained from baryon susceptibility.
- ▶ Green is obtained from chiral susceptibility.
- ▶ In the strong coupling limit, the tricritical point of chiral transition and the end point of nuclear transition are same.

Finite Quark Mass

- ▶ No 2nd order line but 2nd order end point at finite quark mass.
- ▶ The number of monomers is not zero, $\langle N_M \rangle \neq 0$.
- ▶ Chiral susceptibility has peak. $\chi = \frac{1}{(2m_q)^2 V} (\langle N_M^2 \rangle - \langle N_M \rangle^2)$
- ▶ We simulate on 4 quark masses (0.01, 0.02, 0.05, 0.10).

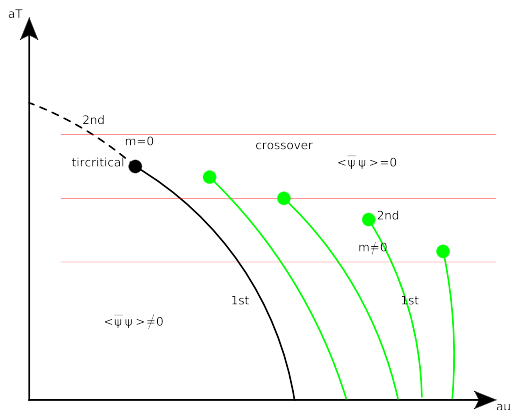
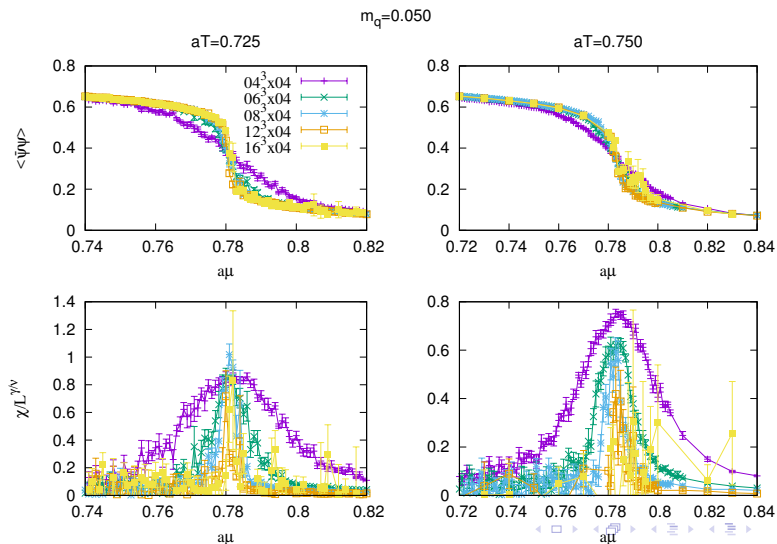


Figure: Strong coupling phase diagram at finite quark mass

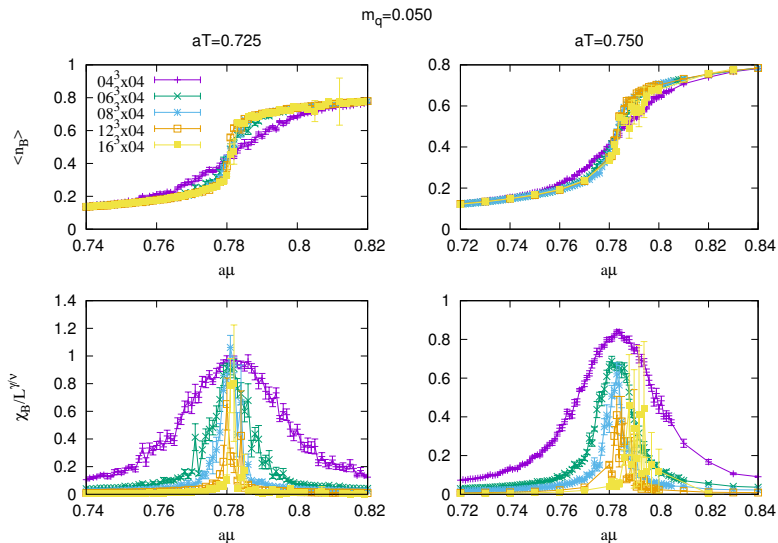
Chiral Observables at Finite Quark Mass

- ▶ We rescale the data using Z_2 -critical exponent ($L^{-\gamma_{Z_2}/\nu_{Z_2}}$)
- ▶ Then the peak heights of susceptibility for different lattice sizes will be overlapped at the critical endpoint temperature.



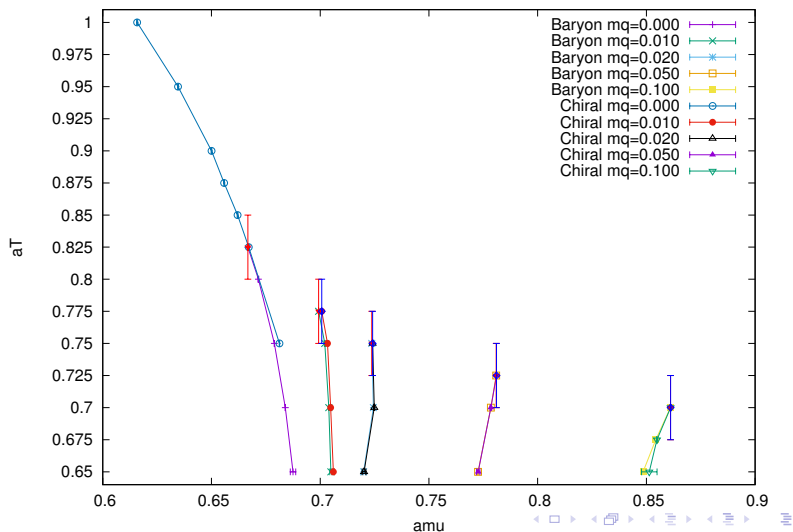
Baryon Observables at Finite Quark Mass

- From these plots, we can determine the temperature of end point.



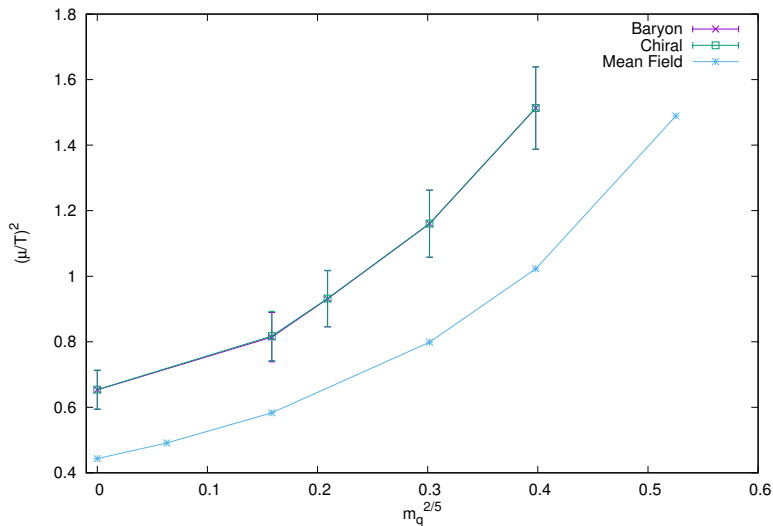
Quark Mass dependence of the QCD Critical End Point

- ▶ We obtain the critical end points for various quark masses 0.00, 0.01, 0.02, 0.05, 0.1.
- ▶ We adopt $\gamma^2 = \frac{a}{a_t}$, $aT = \frac{\gamma^2}{N_t}$. (see Helvio's talk)



Comparison with mean field theory

- ▶ We compare our results with mean field theory. [Nishida, PRD, 2004]



Summary and Future Work

- ▶ We obtain the critical end points for various quark masses in the strong coupling limit using Monte Carlo via worm algorithm and compare with mean field theory.
- ▶ We plan to measure the Binder cumulant using parallel worm algorithm.
 - ▶ To measure the Binder cumulant, we will use 2 worms.
 - ▶ The first worm is frozen until the second worm is closed.
 - ▶ We measure Z_4 while two worms are open.
 - ▶ Z_2 is measured from the evolution of first worm (second worm is closed).
 - ▶ Not yet applicable to $SU(3)$.
- ▶ We will extend to finite quark masses and finite β .