

Lattice 2016

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Quark confinement to be caused by Abelian or non-Abelian dual superconductivity in the $SU(3)$ Yang-Mills theory

Akihiro Shibata (KEK)

In collaboration with:

Kei-Ichi. Knodo (Chiba U), Seikou Kato (Oyama NTC),

Toru Shinohara (Chiba U.)

outline

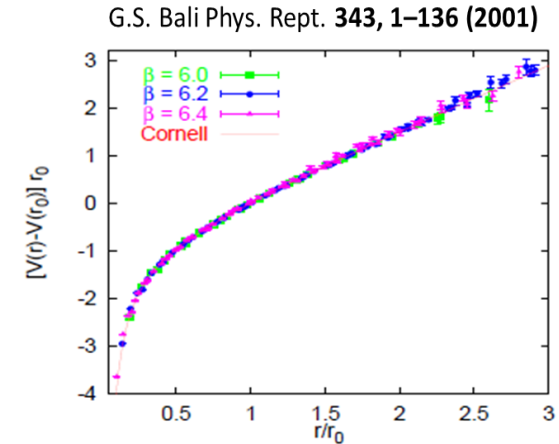
1. Introduction
2. Gauge-link decomposition and extract the relevant modes for confinement
3. Lattice data
4. Summary and discussion

Introduction(1)

- ❑ Quark confinement follows from the area law of the Wilson loop average [Wilson,1974]
- ❑ **Dual superconductivity** is promising mechanism. [Y.Nambu (1974). G.'t Hooft, (1975). S.Mandelstam(1976), A.M. Polyakov (1975)]

❑ To establish this picture, we must show evidences of the dual version of the superconductivity.

c.f. center vortex (in the maximal center gage) [Greensite]



Dual superconductivity

Superconductor (condensed matter)

- Condensation of electric charges (Cooper pairs)
- Meissner effect: Abrikosov string (magnetic flux tube) connecting monopole and anti-monopole
- Linear potential between monopoles

Dual superconductor (QCD)

- Condensation of magnetic monopoles
- Dual Meissner effect: formation of a hadron string (chromo-electric flux tube) connecting quark and antiquark
- Linear potential between quarks



Extracting relevant mode for confinement

Abelian projection method

Extracting the relevant mode as the diagonal (Abelian) part in the maximal Abelian (MA) gauge. $U=XV$

- $SU(2) \rightarrow U(1)$
- $SU(3) \rightarrow U(1)XU(1)$

Problems:

The result depends on the gauge fixing of the Yang-Mills theory.

The gauge fixing breaks (global) color symmetry.

Decomposition method

[a new formulation on a lattice]

Extracting the relevant mode V for quark confinement by solving the defining equation in the gauge independent way (gauge-invariant way).

→ The Abelian projection method can be reformulated by using the decomposition method in the gauge invariant way.

A new formulation of Yang-Mills theory (on a lattice)

[Phys.Rept. 579 (2015) 1-226]

Decomposition of SU(N) gauge links For SU(N) YM gauge link, there are several possible options of decomposition *discriminated by its stability groups*:

- SU(2) Yang-Mills link variables: unique $U(1) \subset SU(2)$
- SU(3) Yang-Mills link variables: Two options

minimal option : $U(2) \cong SU(2) \times U(1) \subset SU(3)$

Minimal case is derived for the Wilson loop, defined for quark in the fundamental representation, which follows from the **non-Abelian Stokes' theorem**

maximal option : $U(1) \times U(1) \subset SU(3)$

Maximal case is a gauge invariant version of Abelian projection in the maximal Abelian (MA) gauge. (the maximal torus group)

Dual Superconductivity in SU(3) Yang-Mills

Abelian Dual superconductivity

□ Abelian projection in MA gauge ::
SU(3) \rightarrow U(1) \times U(1) (Maximal torus)

• Perfect Abelian dominance in string tension [Sakumichi-Suganuma]

□ Decomposition method

• **Maximal option** of a new formulation [ours]

• Cho-Faddev-Niemi-Shavanov decomposition [N Cundy, Y.M. Cho et.al]

Non-Abelian Dual superconductivity

□ Decomposition method

• **Minimal option:** (non-Abelian dual superconductivity) based on the U(2) stability sub-group.

we have showed in the series works

✓ V-field dominance, non-Abelian magnetic monopole dominance in string tension

✓ chromo-flux tube and dual Meissner effect,

✓ confinement/deconfinement phase transition in terms of dual Meissner effect at finite temperature

Dual Superconductivity in SU(3) Yang-Mills (II)

- In the series of workshop, we have studied [the minimal option](#).

Because the non-Abelian Stokes theorem shows that *Wilson loop of Yang-Mills field in the fundamental representation can be rewritten by using the restricted field V which is decomposed as new variables ($U = XV$)*

- Ordinary, Abelian picture ([maximal option](#)) has been studied.
- Both can derive dual superconductivity picture such as V-field or Abelian dominance in string tension.

Then, following questions come up:

- Whether these two are qualitatively different or not.
- Which picture is a better effective theory for QCD

- **Therefore, we investigate the dual Meissner effect in both options at zero and finite temperature**

outline

1. Introduction
2. Gauge-link decomposition and extract the relevant modes for confinement
 - Minimal Option
 - Maximal Option
3. Lattice data
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minimal option: The decomposition of SU(3) link variable

$$W_C[U] := \text{Tr} \left[P \prod_{\langle x, x+\mu \rangle \in C} U_{x,\mu} \right] / \text{Tr}(\mathbf{1})$$

$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

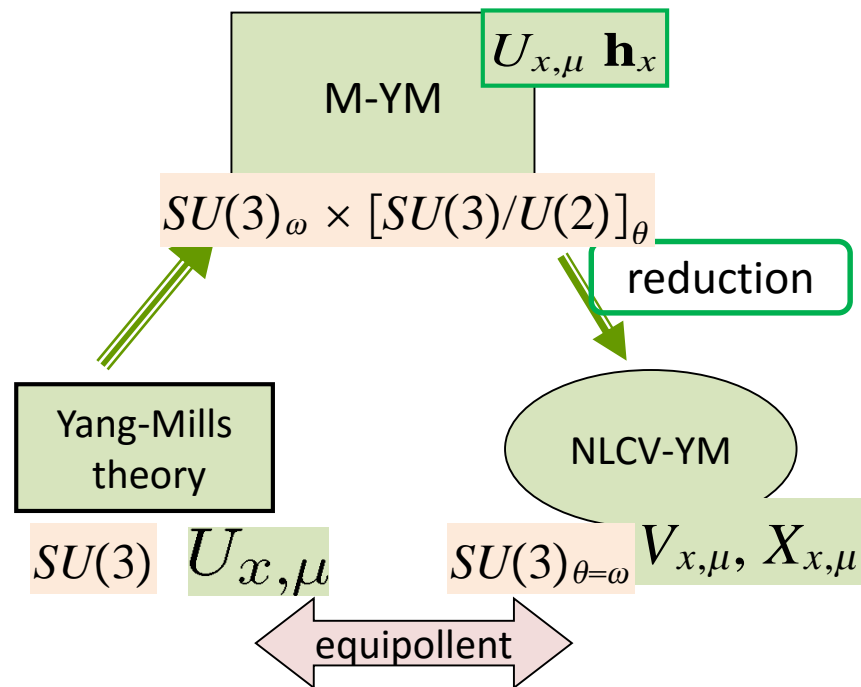
$$U_{x,\mu} \rightarrow U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega_{x+\mu}^\dagger$$

$$V_{x,\mu} \rightarrow V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega_{x+\mu}^\dagger$$

$$X_{x,\mu} \rightarrow X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega_x^\dagger$$

$$\Omega_x \in G = SU(N)$$

$$W_C[V] := \text{Tr} \left[P \prod_{\langle x, x+\mu \rangle \in C} V_{x,\mu} \right] / \text{Tr}(\mathbf{1})$$



$$W_C[U] = \text{const.} W_C[V] \quad !!$$

Minimal option: Defining equation for the decomposition

Introducing a color field $\mathbf{h}_x = \xi(\lambda^8/2)\xi^\dagger \in SU(3)/U(2)$ with $\xi \in SU(3)$, a set of the defining equation of decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D_\mu^\epsilon[V]\mathbf{h}_x = \frac{1}{\epsilon}(V_{x,\mu}\mathbf{h}_{x+\mu} - \mathbf{h}_x V_{x,\mu}) = 0,$$

$$g_x = e^{-2\pi q_x/N} \exp(-a_x^{(0)}\mathbf{h}_x - i \sum_{i=1}^3 a_x^{(i)} u_x^{(i)}) = 1,$$

which correspond to the continuum version of the decomposition, $\mathcal{A}_\mu(x) = \mathcal{V}_\mu(x) + \mathcal{X}_\mu(x)$,

$$D_\mu[\mathcal{V}_\mu(x)]\mathbf{h}(x) = 0, \quad \text{tr}(\mathcal{X}_\mu(x)\mathbf{h}(x)) = 0.$$

Exact solution
(N=3)

$$X_{x,\mu} = \hat{L}_{x,\mu}^\dagger (\det \hat{L}_{x,\mu})^{1/N} g_x^{-1} \quad V_{x,\mu} = X_{x,\mu}^\dagger U_x = g_x \hat{L}_{x,\mu} U_x (\det \hat{L}_{x,\mu})^{-1/N}$$

$$\hat{L}_{x,\mu} = \left(\sqrt{L_{x,\mu} L_{x,\mu}^\dagger} \right)^{-1} L_{x,\mu}$$

$$L_{x,\mu} = \frac{N^2 - 2N + 2}{N} \mathbf{1} + (N - 2) \sqrt{\frac{2(N - 2)}{N}} (\mathbf{h}_x + U_{x,\mu} \mathbf{h}_{x+\mu} U_{x,\mu}^{-1}) \\ + 4(N - 1) \mathbf{h}_x U_{x,\mu} \mathbf{h}_{x+\mu} U_{x,\mu}^{-1}$$

continuum limit

$$\mathbf{V}_\mu(x) = \mathbf{A}_\mu(x) - \frac{2(N-1)}{N} [\mathbf{h}(x), [\mathbf{h}(x), \mathbf{A}_\mu(x)]] - ig^{-1} \frac{2(N-1)}{N} [\partial_\mu \mathbf{h}(x), \mathbf{h}(x)],$$

$$\mathbf{X}_\mu(x) = \frac{2(N-1)}{N} [\mathbf{h}(x), [\mathbf{h}(x), \mathbf{A}_\mu(x)]] + ig^{-1} \frac{2(N-1)}{N} [\partial_\mu \mathbf{h}(x), \mathbf{h}(x)].$$

Minimal option: Non-Abelian magnetic monopole

For Wilson loop in the fundamental representation

From the non-Abelian Stokes theorem and the Hodge decomposition, the magnetic monopole is derived **without using the Abelian projection**

$$\begin{aligned}
 W_C[\mathcal{A}] &= \int [d\mu(\xi)]_\Sigma \exp\left(-ig \int_{S:C=\partial\Sigma} dS^{\mu\nu} \sqrt{\frac{N-1}{2N}} \operatorname{tr}(2\mathbf{h}(x)\mathcal{F}_{\mu\nu}[\mathcal{V}](x))\right) \\
 &= \int [d\mu(\xi)]_\Sigma \exp\left(ig \sqrt{\frac{N-1}{2N}} (k, \Xi_\Sigma) + ig \sqrt{\frac{N-1}{2N}} (j, N_\Sigma)\right)
 \end{aligned}$$

magnetic current $k := \delta^* F = *dF$, $\Xi_\Sigma := \delta^* \Theta_\Sigma \Delta^{-1}$

electric current $j := \delta F$, $N_\Sigma := \delta \Theta_\Sigma \Delta^{-1}$

$$\Delta = d\delta + \delta d, \quad \Theta_\Sigma := \int_\Sigma d^2 S^{\mu\nu}(\sigma(x)) \delta^D(x - x(\sigma))$$

k and j are gauge invariant and conserved currents; $\delta k = \delta j = 0$.

K.-I. Kondo
PRD77
085929(2008)

Note that field strength $F[V]$ is described by V-field in the minimal option.

The lattice version of magnetic monopole current is defined by using plaquette:

$$\Theta_{\mu\nu}^8 := -\arg \operatorname{Tr} \left[\left(\frac{1}{3} \mathbf{1} - \frac{2}{\sqrt{3}} \mathbf{h}_x \right) V_{x,\mu} V_{x+\mu,\mu} V_{x+\nu,\mu}^\dagger V_{x,\nu}^\dagger \right],$$

$$k_\mu = 2\pi n_\mu := \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \partial_\nu \Theta_{\alpha\beta}^8,$$

maximal option: The decomposition of SU(3) link variable

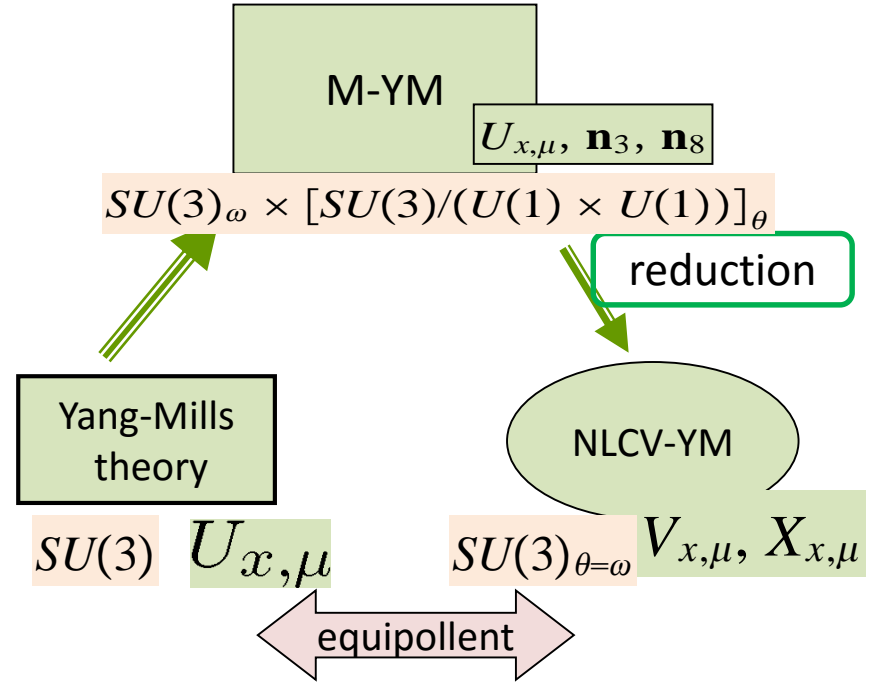
$$U_{x,\mu} = X_{x,\mu} V_{x,\mu}$$

$$U_{x,\mu} \rightarrow U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega_{x+\mu}^\dagger$$

$$V_{x,\mu} \rightarrow V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega_{x+\mu}^\dagger$$

$$X_{x,\mu} \rightarrow X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega_x^\dagger$$

$$\Omega_x \in G = SU(N)$$



Gauge invariant construction of the Abelian projection to maximal torus group $U(1) \times U(1)$ in MA gauge.

maximal option: Defining equation for the decomposition

By introducing color fields $\mathbf{n}_x^{(3)} = \Theta_x(\lambda^3/2)\Theta^\dagger$, $\mathbf{n}_x^{(8)} = \Theta_x(\lambda^8/2)\Theta^\dagger$
 $\in SU(3)_\omega \times [SU(3)/(U(1) \times U(1))]_\theta$, a set of the defining equation for the
 decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D_\mu^\varepsilon[V]n_x^{(k)} = \frac{1}{\varepsilon}(V_{x,\mu}n_{x+\mu}^{(k)} - n_x^{(k)}V_{x,\mu}) = 0, \quad (k = 3, 8)$$

$$g_x = \exp(2\pi i n/N) \exp(i \sum_{j=3,8} a^{(j)} n_x^{(j)}) = 1$$

Corresponding to the continuum version of the decomposition $\mathcal{A}_\mu(x) = V_\mu(x) + \mathcal{X}_\mu(x)$

$$D_\mu[V_\mu]\mathbf{n}^{(k)}(x) = 0, \quad \text{tr}(\mathbf{n}^{(k)}(x)\mathcal{X}_\mu(x)) = 0, \quad (k = 3, 8)$$

$$X_{x,\mu} = \hat{K}_{x,\mu}^\dagger \det(K_{x,\mu})^{1/3} g_x^{-1}, \quad V_{x,\mu} = g_x \hat{K}_{x,\mu} \det(K_{x,\mu})^{-1/3}$$

where

$$\hat{K}_{x,\mu} := \left(\sqrt{K_{x,\mu} K_{x,\mu}^\dagger} \right)^{-1} K_{x,\mu}, \quad \hat{K}_{x,\mu}^\dagger = K_{x,\mu}^\dagger \left(\sqrt{K_{x,\mu} K_{x,\mu}^\dagger} \right)^{-1}$$

$$K_{x,\mu} = 1 + 6\mathbf{n}_x^{(3)} U_{x,\mu} \mathbf{n}_{x+\mu}^{(3)} U_{x,\mu}^\dagger + 6\mathbf{n}_x^{(8)} U_{x,\mu} \mathbf{n}_{x+\mu}^{(8)} U_{x,\mu}^\dagger$$

Maximal option

□ magnetic monopole

We have two kind of magnetic monopoles in the maximal option

□ Decomposition in the MA gauge

Decomposition formula is rewritten into Abelian projection in Maximal Abelian gauge

→ Abelian projection in the MA gage

$$k_\mu^{(j)} := \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} \partial_\nu \Theta_{\alpha\beta}^{(j)}$$

$$\Theta_{\alpha\beta}^{(1)} = \arg \left[\left(\frac{1}{3} \mathbf{1} + \mathbf{n}_x + \frac{1}{\sqrt{3}} \mathbf{m}_x \right) V_{x,\alpha} V_{x+\alpha,\beta} V_{x+\beta,\alpha}^\dagger V_{x,\beta}^\dagger \right]$$

$$\Theta_{\alpha\beta}^{(2)} = \arg \left[\left(\frac{1}{3} \mathbf{1} - \frac{2}{\sqrt{3}} \mathbf{m}_x \right) V_{x,\alpha} V_{x+\alpha,\beta} V_{x+\beta,\alpha}^\dagger V_{x,\beta}^\dagger \right]$$

$$\mathbf{n}_x^{(3)} = \Theta_x (\lambda^3/2) \Theta_x^\dagger, \quad \mathbf{n}_x^{(8)} = \Theta_x (\lambda^8/2) \Theta_x^\dagger, \quad \Theta U_{x,\mu} = \Theta_x^\dagger U_{x,\mu} \Theta_{x+\mu}$$

$$K_{x,\mu} = \left(U_{x,\mu} + 6\mathbf{n}_x^{(3)} U_{x,\mu} \mathbf{n}_{x+\mu}^{(3)} + 6\mathbf{n}_x^{(8)} U_{x,\mu} \mathbf{n}_{x+\mu}^{(8)} \right) U_{x,\mu}^\dagger$$

$$= \Theta_x \left[\Theta U_{x,\mu}^\dagger + 6 \frac{\lambda^3}{2} \Theta U_{x,\mu}^\dagger \frac{\lambda^3}{2} + 6 \frac{\lambda^8}{2} \Theta U_{x,\mu}^\dagger \frac{\lambda^8}{2} \right] \Theta_{x+\mu}^\dagger U_{x,\mu}^\dagger$$

$$= 3\Theta_x \begin{bmatrix} \Theta u_{x,\mu}^{11} & 0 & 0 \\ 0 & \Theta u_{x,\mu}^{22} & 0 \\ 0 & 0 & \Theta u_{x,\mu}^{33} \end{bmatrix} \Theta_{x+\mu}^\dagger U_{x,\mu}^\dagger$$

$$V = \text{diag} \left(\frac{\Theta u_{x,\mu}^{11}}{|\Theta u_{x,\mu}^{11}|}, \frac{\Theta u_{x,\mu}^{22}}{|\Theta u_{x,\mu}^{22}|}, \frac{\Theta u_{x,\mu}^{33}}{|\Theta u_{x,\mu}^{33}|} \right)$$

Reduction condition

- The reduction condition is introduced such that the theory in terms of new variables is equipollent to the original Yang-Mills theory
- We here introduce the reduction condition which is **the kinetic term of adjoint gauge-Higgs system**.

Minimal option:

$$SU(3)_{\omega} \times [SU(3)/U(2)]_{\theta} \rightarrow SU(3)_{\omega-\theta}$$

Determining \mathbf{h}_x to minimize the reduction function for given $U_{x,\mu}$

$$F_{\text{red}}[\mathbf{h}_x, U_{x,\mu}] = \sum_{x,\mu} \text{tr} \left\{ (D_{\mu}^{\epsilon}[U_{x,\mu}]\mathbf{h}_x)^{\dagger} (D_{\mu}^{\epsilon}[U_{x,\mu}]\mathbf{h}_x) \right\}$$

Maximal option:

$$SU(3)_{\omega} \times [SU(3)/(U(1) \times U(1))]_{\theta} \rightarrow SU(3)_{\omega=\theta}$$

Determine $\mathbf{n}^{(3)}$ and $\mathbf{n}^{(8)}$ to minimize the following functional

$$F_{\text{max}}[\mathbf{n}^{(3)}, \mathbf{n}^{(8)}; U_{x,\mu}] = \sum_{x,\mu} \text{tr} \left(\| D_{\mu}[U]\mathbf{n}_x^{(3)} \|^2 \right) + \sum_{x,\mu} \text{tr} \left(\| D_{\mu}[U]\mathbf{n}_x^{(8)} \|^2 \right)$$
$$\mathbf{n}_x^{(3)} = \Theta_x(\lambda^3/2)\Theta_x^{\dagger}, \quad \mathbf{n}_x^{(8)} = \Theta_x(\lambda^8/2)\Theta_x^{\dagger}$$

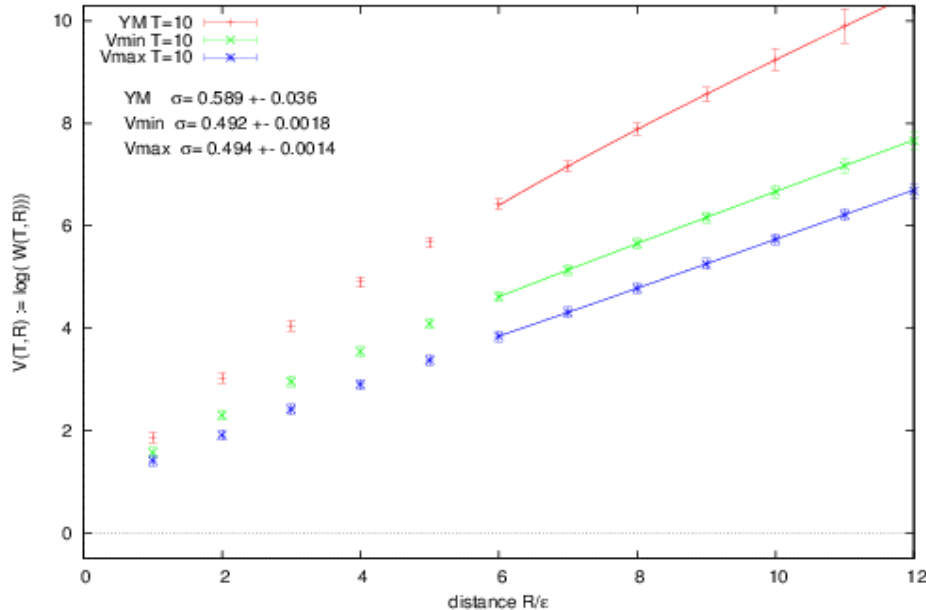
Reduction condition for maximal option is rewritten into the gauge fixing of maximal Abelian gauge

outline

1. Introduction
2. Gauge-link decomposition and extract the relevant modes for confinement
3. Lattice data
Comparison of TWO picture of dual superconductivity: minimal v.s. maximal at zero and nonzero temperature
 - Static potential
 - Chromo flux tube and dual Meissner effect
 - Polyakov loop average :: center symmetry breaking in
 - confinement/deconfinement phase transition in view of the dual super conductor
4. Summary and discussion

DUAL SUPERCONDUCTIVITY AT ZERO TEMPERATURE

String tension: zero temperature



Static potential from Wilson loop average of YM-field and two V-fields in minimal and maximal options

$\log \langle W[T=10,R] \rangle$ vs R

- We obtain **the restricted field (“Abelian”) dominance in the string tension** for **both** the minimal option and the maximal option.
- The string tension is almost same with the both options and YM field

Measurement of chromo flux:

$$\rho_W = \frac{\langle \text{tr}(WLU_pL^\dagger) \rangle}{\langle \text{tr}(W) \rangle} - \frac{1}{N} \frac{\langle \text{tr}(W)\text{tr}(U_p) \rangle}{\langle \text{tr}(W) \rangle}$$

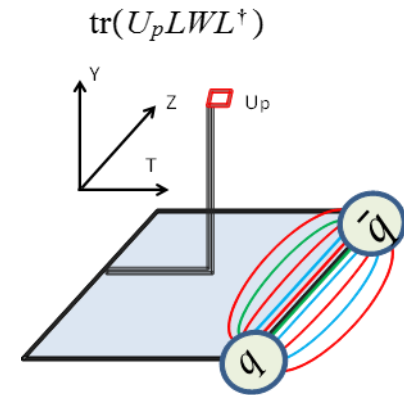
The field strength by quark and anti quark can be defined as

$$F_{\mu\nu}(x) = \sqrt{\frac{\beta}{2N}} \rho_W(x)$$

To know **the difference between the decomposition**,
we measure the three types of probes and compare them.

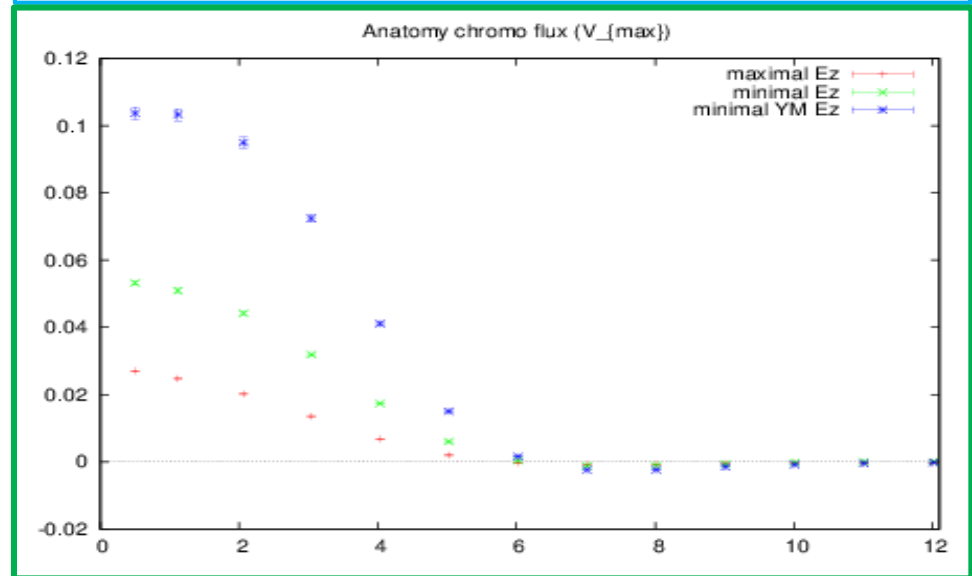
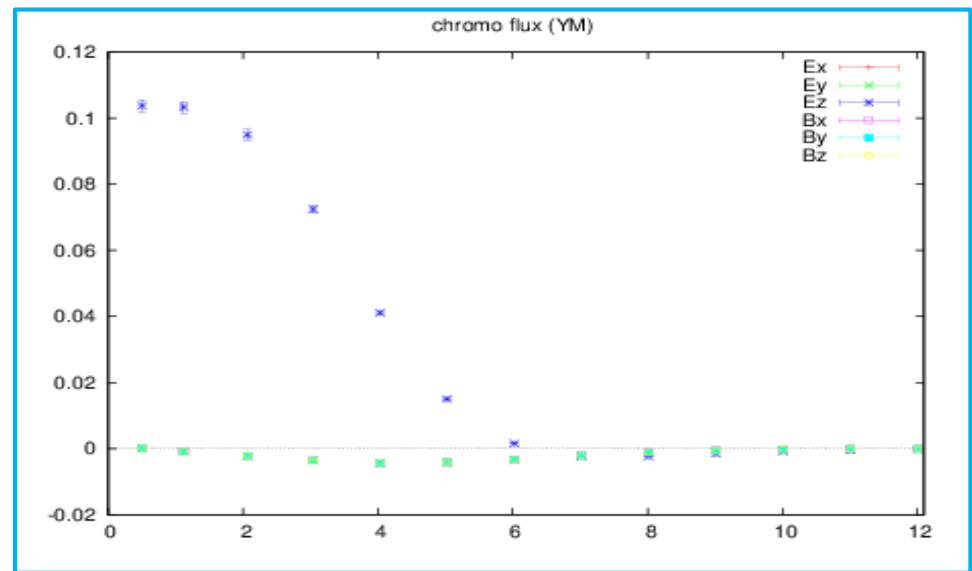
$$\begin{aligned} O^{[YM]} &= L[U]U_pL[U]^{-1} && :: \text{original YM} \\ O^{[nin]} &= L[V^{[\min]}]V_p^{[\min]}L[V^{[\min]}]^{-1} && :: V \text{ field in minimal option} \\ O^{[max]} &= L[V^{[\max]}]V_p^{[\max]}L[V^{[\max]}]^{-1} && :: V \text{ field in maximal option} \end{aligned}$$

Proposed by Adriano Di Giacomo et.al.
[Phys.Lett.B236:199,1990]
[Nucl.Phys.B347:441-460,1990]



Chromo-electric flux tube

- Chromo flux between quark and antiquark at midpoint
- Chromo-flux tube is observed, only E_z element has non-vanishing values in each.
- Comparison of Chromo flux strength.

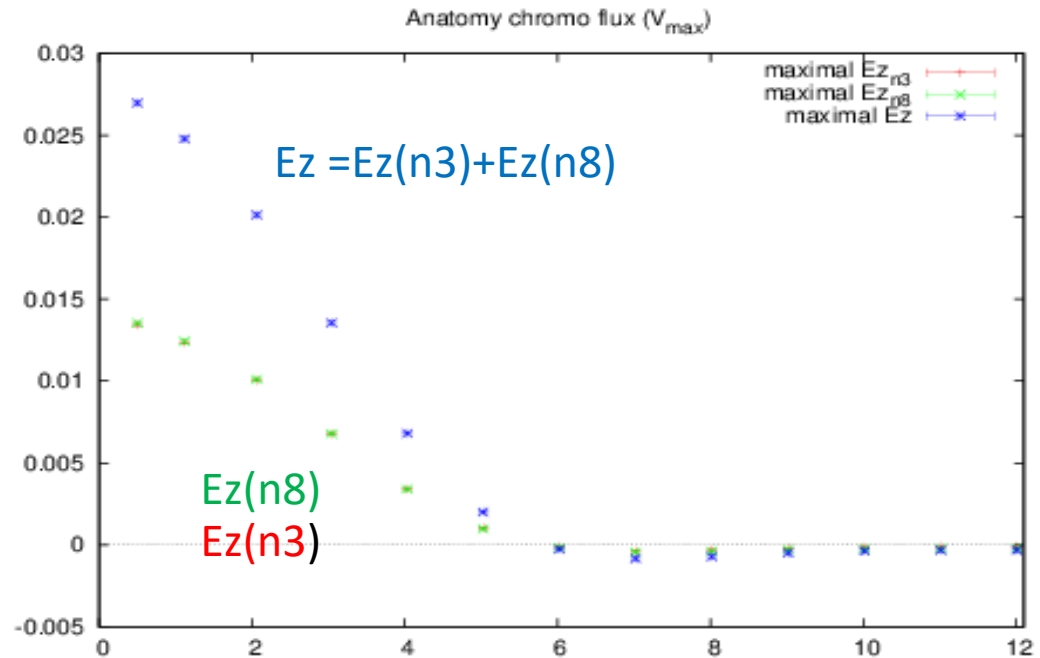


Anatomy of chromo flux by color field

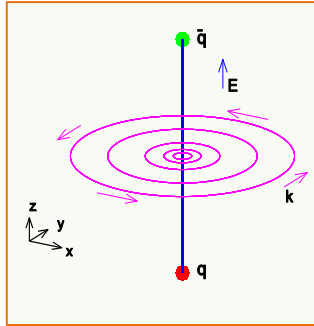
- In aximal option, there exists two color fields, n3 and n8.
- Chromo flux can be decomposed into two parts by using the color fields.

→ The data shows that decomposed chromo fluxes have almost same amplitude.

$$\begin{aligned}
 \rho_W &= \frac{\langle \text{tr}(L[V]V_P L[V]^\dagger W_C) \rangle - \frac{1}{3} \langle \text{tr}(V_P) \text{tr}(W_C) \rangle}{\langle \text{tr}(W_C) \rangle} \\
 &= \frac{\langle \text{tr}(V_P \mathbf{n}_x^{(3)}) \text{tr}(W_C \mathbf{n}_x^{(3)}) \rangle + \langle \text{tr}(V_P \mathbf{n}_x^{(8)}) \text{tr}(W_C \mathbf{n}_x^{(8)}) \rangle}{\langle \text{tr}(W_C) \rangle}
 \end{aligned}$$



Induced magnetic current (monopole)



Yang–Mills equation (Maxell equation) fo rrestricted field V_μ , the magnetic current (monopole) can be calculated as

$$k = \delta^*F[V] = {}^*dF[V],$$

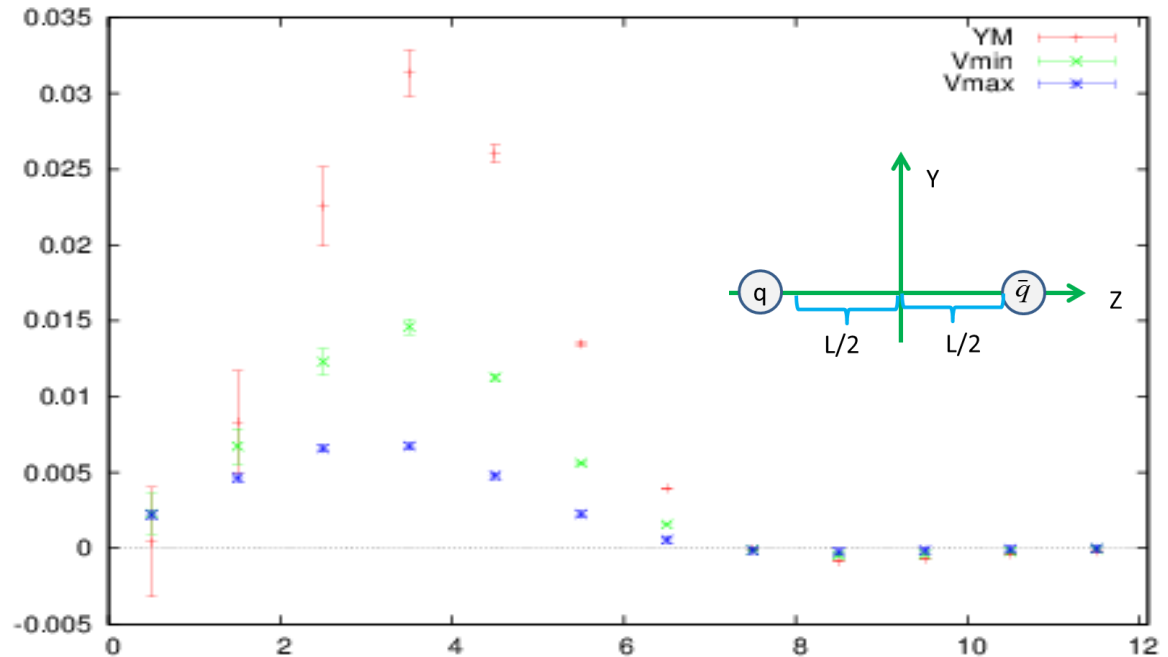
where $F[V]$ is the field strength of V , d exterior derivative, $*$ the Hodge dual and δ the coderivative $\delta := {}^*d^*$, respectively.

Induced magnetic current (monopole) k can be a order parameter of the dual Meissner effect.



k is a *order parameter* of confinement/deconfinement phase

comparison magnetic monopole current



DUAL SUPERCONDUCTIVITY AT FINITE TEMPERATURE

$L^3 \times T$, $L=24, T=6$ fixed lattice size

Temperature is controlled by a parameter β :

$\beta = 5.8, 5.85, 5.9,$

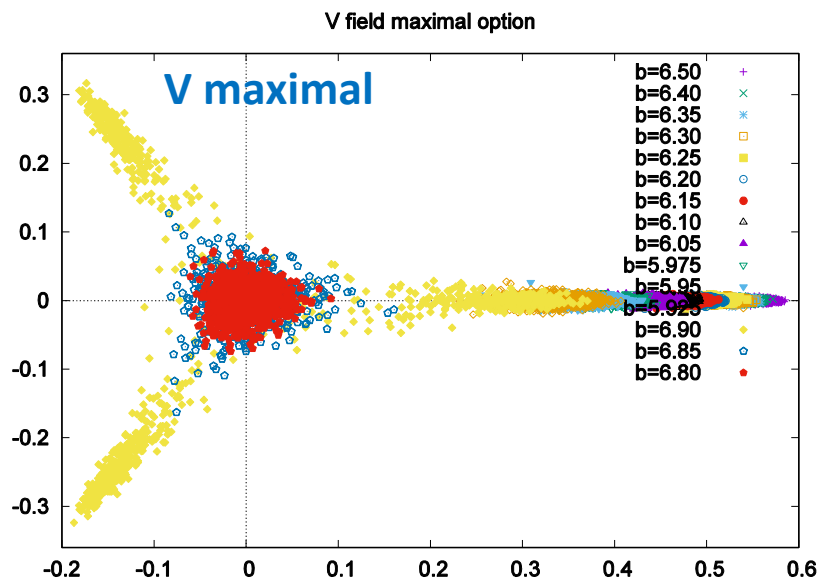
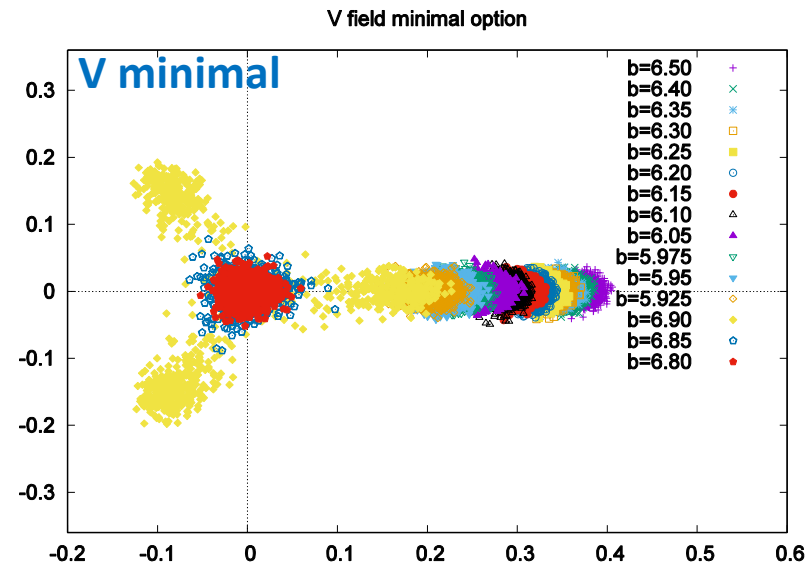
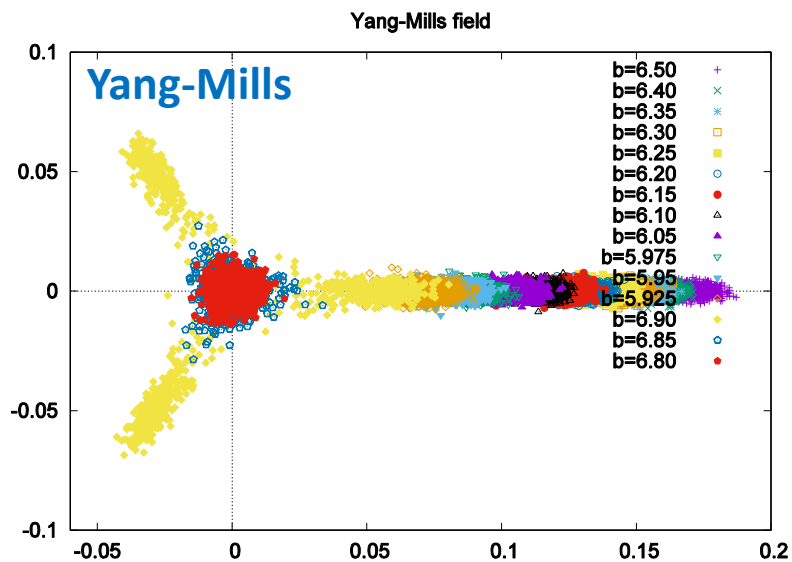
5.925, 5.95, 5.975, 6.0, 6.05, 6.1, 6.15, 6.2, 6.35, 6.3, 6.4 6.5

Polyakov loop

Distribution of Polyakov loop values

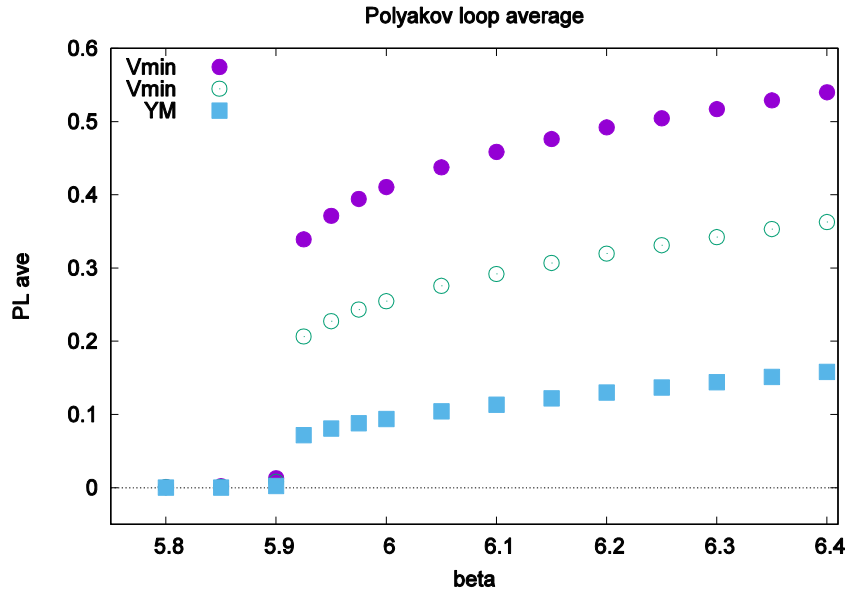
$P_U(x) = \text{tr}\left(\prod_{t=1}^{N_t} U_{(x,t),4}\right)$ for original Yang-Mills field

$P_V(x) = \text{tr}\left(\prod_{t=1}^{N_t} V_{(x,t),4}\right)$ for restricted field

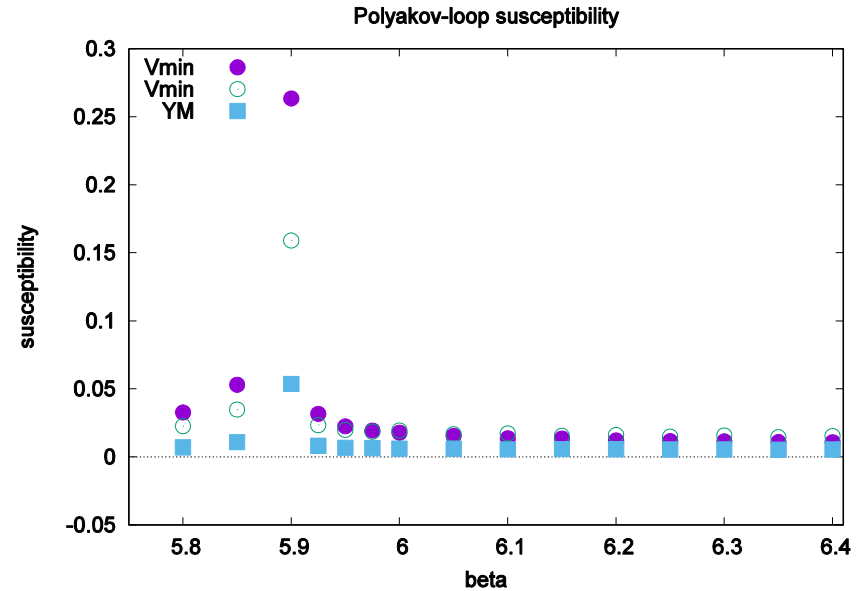


Polyakov loop average and center symmetry

Polyakov loop average

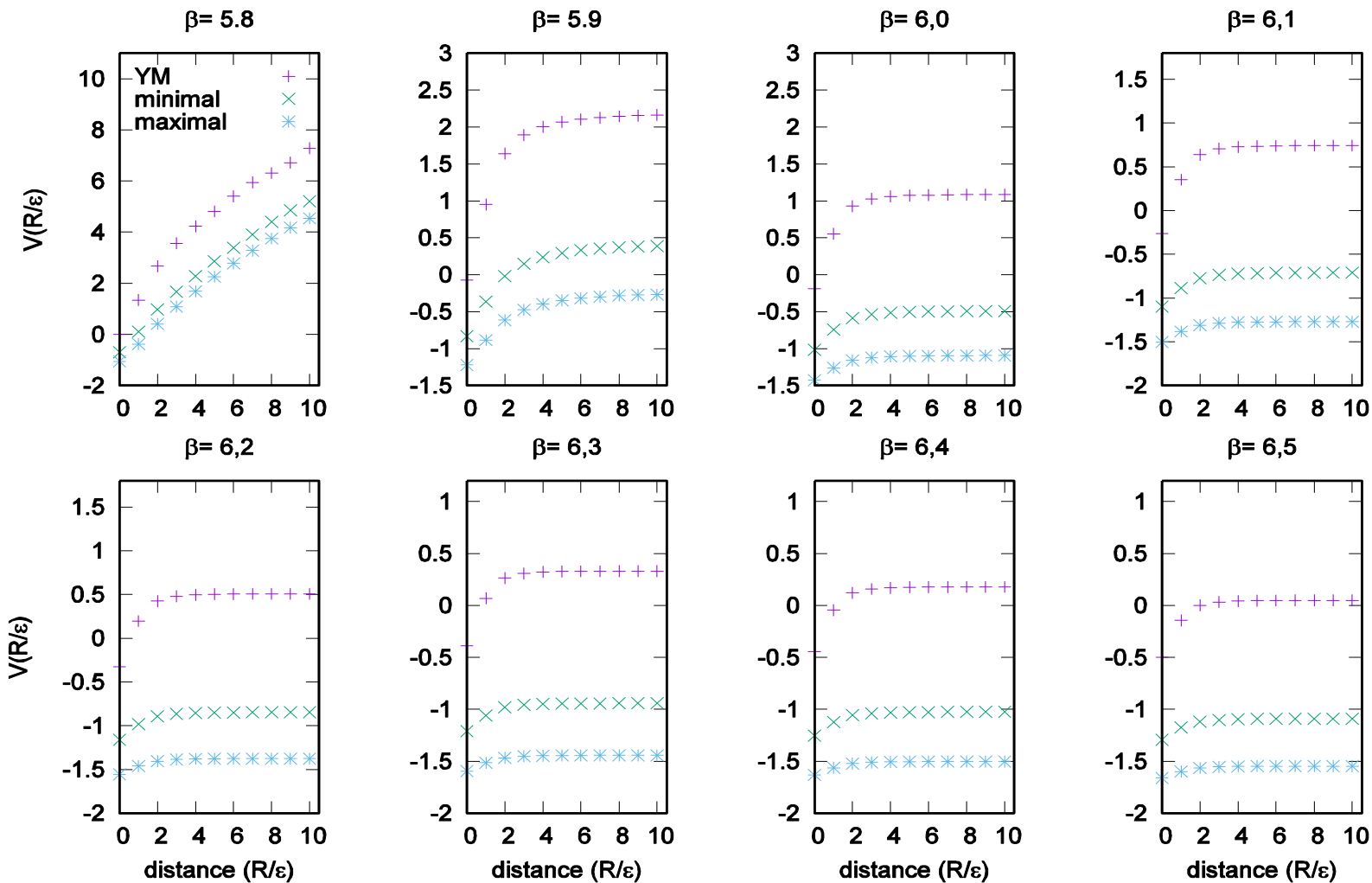


Polyakov loop susceptibility

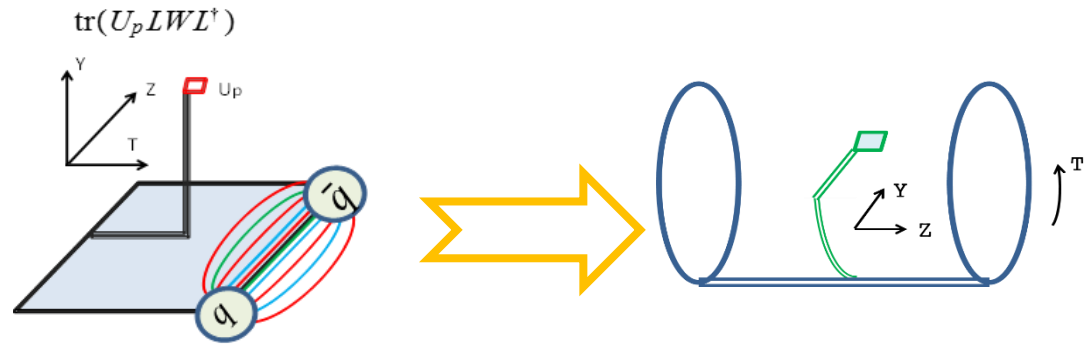


Magnitude of Polyakov-loop average is different, but gives the same phase transition temperature (β).

static potential (correlation function of Polyakov loops)



Measurement of chromo flux at finite temperature



$$\rho_W = \frac{\langle \text{tr}(W L U_p L^\dagger) \rangle}{\langle \text{tr}(W) \rangle} - \frac{1}{N} \frac{\langle \text{tr}(W) \text{tr}(U_p) \rangle}{\langle \text{tr}(W) \rangle}$$

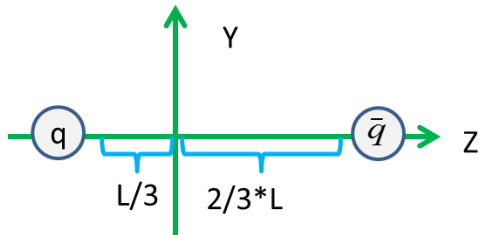
$$F_{\mu\nu}(x) = \sqrt{\frac{\beta}{2N}} \rho_W(x)$$

❑ Using the same operator with that of zero temperature.

❑ Size of Wilson loop T-direction = Nt

➔ The source of quark and antiquark are given by **Plyakov loops** connecting by Wilson line.

❑ The three types of probes and compare them.

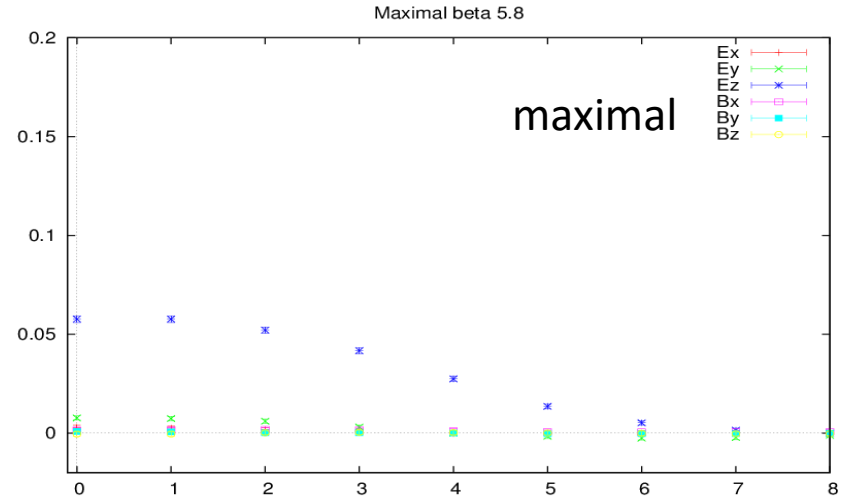
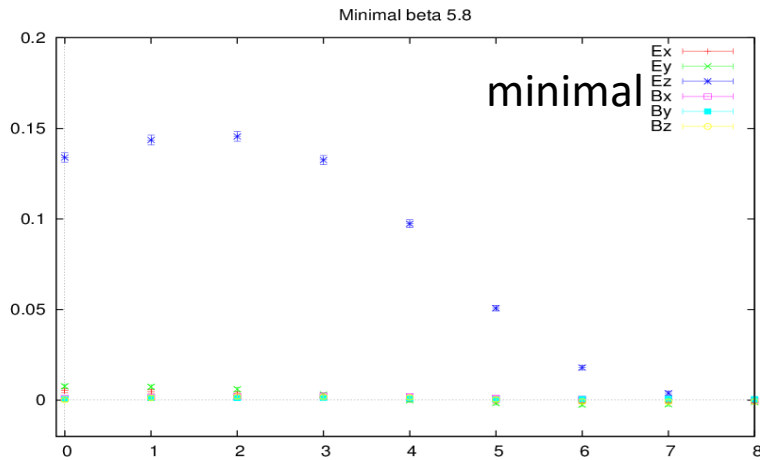
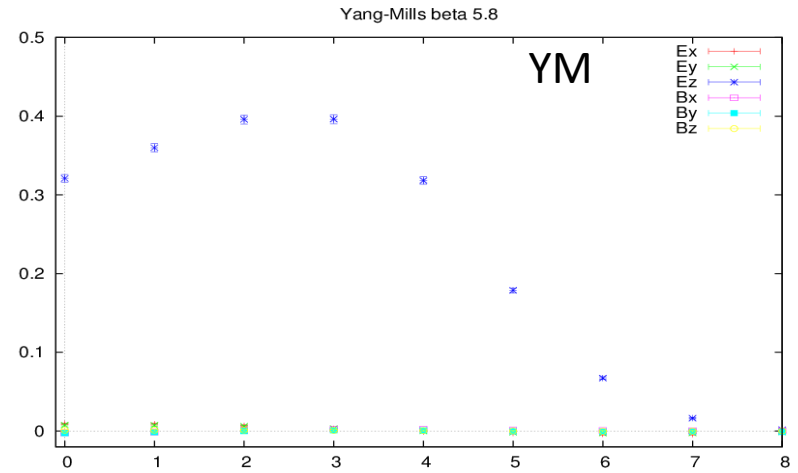


$$O^{[YM]} = L[U] U_p L[U]^{-1} \quad :: \text{original YM}$$

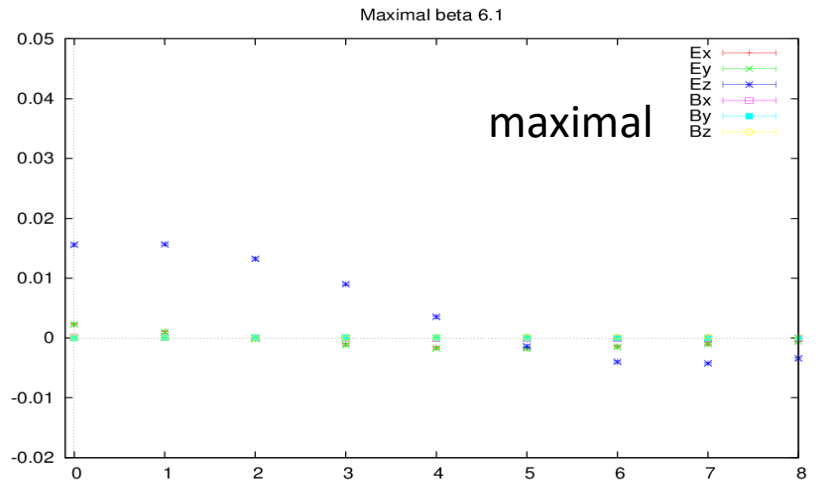
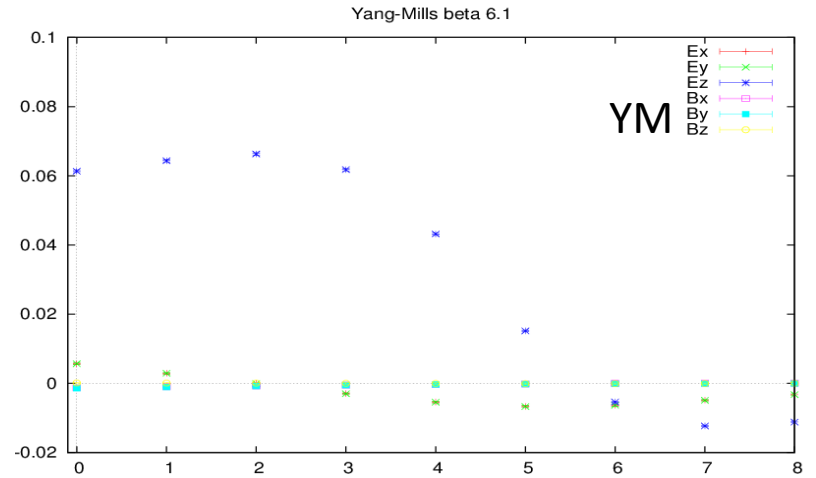
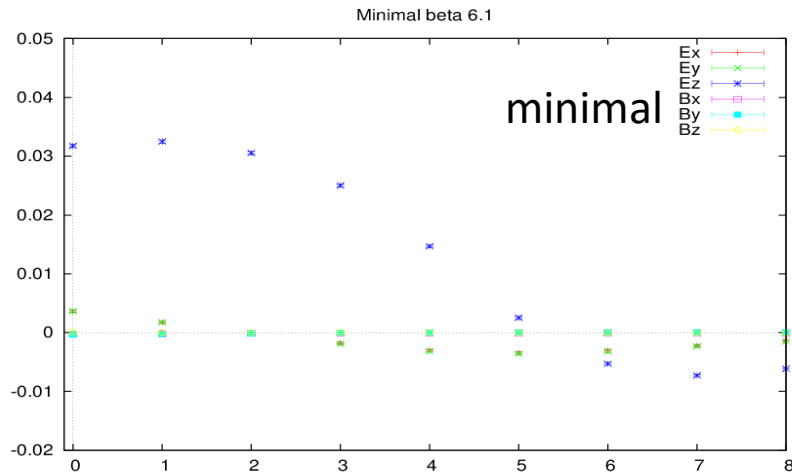
$$O^{[min]} = L[V^{[min]}] V_p^{[min]} L[V^{[min]}]^{-1} \quad :: V \text{ field in minimal option}$$

$$O^{[max]} = L[V^{[max]}] V_p^{[max]} L[V^{[max]}]^{-1} \quad :: V \text{ field in maximal option}$$

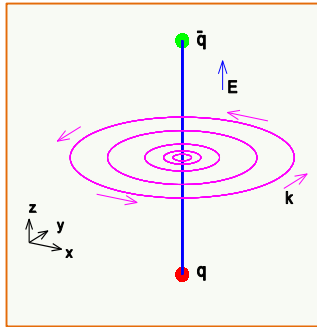
Chromo flux in confining phase



Chromo flux in deconfining phase



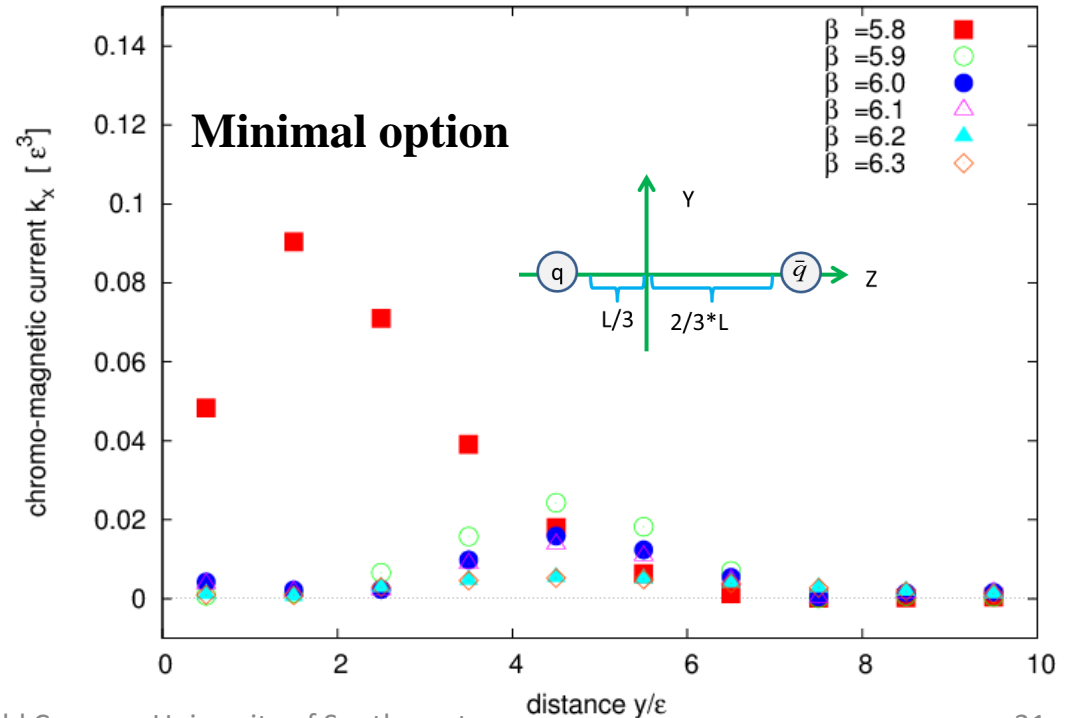
Induced magnetic current (monopole) at finite temperature



Yang–Mills equation (Maxell equation) fo rrestricted field V_μ , the magnetic current (monopole) can be calculated as

$$k = \delta^*F[V] = *dF[V],$$

where $F[V]$ is the field strength of V , d exterior derivative, $*$ the Hodge dual and δ the coderivative $\delta := *d^*$, respectively.



Summary

- We investigate dual superconductivity applying our new formulation of Yang-Mills theory on the lattice, i.e., in the minimal and maximal options as well as Yang-Mills field at finite temperature.
- In both options we have found that
 - the restricted field (**V-field**) **dominance** in the string tension, and **the string tension is almost same**.
 - **In confining phase we directly observe the dual Meissner effects**. The induced magnetic (monopole) currents appear around chromo-electro flux tube between a pair of quark and antiquark.
 - **In deconfining phase we find no more the dual Meissner effects**, i.e., the induced magnetic (monopole) currents become very small or disappears.
 - **The Polyakov loop averages**, which is the conventional order parameter of confinement/deconfinement phase transition, gives the same critical temperature with both options and the YM field.

outlook

- Determination **type of the dual superconductor** in the maximal option.

By using the minimal option, of type I [[Phys.Rev. D87 \(2013\) 054011](#)].

- Investigate the dual Meissner effect phase transition, and determine critical temperature and order of the phase transition.

THANK YOU FOR YOUR ATTENTION