Quark confinement to be caused by Abelian or non-Abelian dual superconductivity in the $SU(3)$ Yang-Mills theory

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outline

1. Introduction
2. Gauge-link decomposition and extract the relevant modes for confinement
3. Lattice data
4. Summary and discussion
Introduction(1)

- Quark confinement follows from the area law of the Wilson loop average \[\text{[Wilson,1974]}\]


- To establish this picture, we must show evidences of the dual version of the superconductivity.

  c.f. center vortex (in the maximal center gage) \[\text{[Greensite ]}\]
Dual superconductivity

Superconductor (condensed matter)

- Condensation of electric charges (Cooper pairs)
- Meissner effect: Abrikosov string (magnetic flux tube) connecting monopole and anti-monopole
- Linear potential between monopoles

Dual superconductor (QCD)

- Condensation of magnetic monopoles
- Dual Meissner effect: formation of a hadron string (chromo-electric flux tube) connecting quark and antiquark
- Linear potential between quarks

Electro-magnetic duality
Extracting relevant mode for confinement

Abelian projection method

Extracting the relevant mode as the diagonal (Abelian) part in the maximal Abelian (MA) gauge. \( U=XY \)

- \( SU(2) \to U(1) \)
- \( SU(3) \to U(1)XU(1) \)

Problems:

The result depends on the gauge fixing of the Yang-Mills theory.

The gauge fixing breaks (global) color symmetry.

Decomposition method

[a new formulation on a lattice]

Extracting the relevant mode \( V \) for quark confinement by solving the defining equation in the gauge independent way (gauge-invariant way).

The Abelian projection method can be reformulated by using the decomposition method in the gauge invariant way.
A new formulation of Yang-Mills theory (on a lattice)  


**Decomposition of SU(N) gauge links**  For SU(N) YM gauge link, there are several possible options of decomposition *discriminated by its stability groups*:

- SU(2) Yang-Mills link variables: unique $U(1) \subset SU(2)$
- SU(3) Yang-Mills link variables: **Two options**

**minimal option**: $U(2) \cong SU(2) \times U(1) \subset SU(3)$

- Minimal case is derived for the Wilson loop, defined for quark in the fundamental representation, which follows from the non-Abelian Stokes’ theorem

**maximal option**: $U(1) \times U(1) \subset SU(3)$

- Maximal case is a gauge invariant version of Abelian projection in the maximal Abelian (MA) gauge. (the maximal torus group)
Dual Superconductivity in SU(3) Yang-Mills

**Abelian Dual superconductivity**
- Abelian projection in MA gauge :: SU(3) $\Rightarrow$ U(1)$\times$U(1) (Maximal torus)
- Perfect Abelian dominance in string tension [Sakumichi-Suganuma]

- Decomposition method
  - **Maximal option** of a new formulation [ours]
  - Cho-Faddev-Niemi-Shavanov decomposition [N Cundy, Y.M. Cho et.al]

**Non-Abelian Dual superconductivity**
- Decomposition method
  - **Minimal option**: (non-Abelian dual superconductivity) based on the U(2) stability sub-group.

  we have showed in the series works
  - V-field dominance, non-Abelian magnetic monopole dominance in string tension
  - chromo-flux tube and dual Meissner effect,
  - confinement/deconfinement phase transition in terms of dual Meissner effect at finite temperature
Dual Superconductivity in SU(3) Yang-Mills (II)

- In the series of workshop, we have studied the minimal option. Because the non-Abelain Stokes theorem shows that Wilson loop of Yang-Mills field in the fundamental representation can be rewritten by using the restricted field $V$ which is decomposed as new variables $(U = \hat{X}V)$.

- Ordinary, Abelian picture (maximal option) has been studied.

- Both can derive dual superconductivity picture such as $V$-field or Abelian dominance in string tension.

Then, following questions come up:

- Whether these two are qualitatively different or not.
- Which picture is a better effective theory for QCD.

Therefore, we investigate the dual Meissner effect in both options at zero and finite temperature.
1. Introduction
2. Gauge-link decomposition and extract the relevant modes for confinement
   - Minimal Option
   - Maximal Option
3. Lattice data
4. Summary and discussion
minimal option: The decomposition of SU(3) link variable

\[
W_C[U] := \text{Tr} \left[ P \prod_{(x,x+\mu) \in C} U_{x,\mu} \right] / \text{Tr}(1)
\]

\[
U_{x,\mu} = X_{x,\mu} V_{x,\mu}
\]

\[
U_{x,\mu} \rightarrow U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega_{x+\mu}^\dagger
\]

\[
V_{x,\mu} \rightarrow V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega_{x+\mu}^\dagger
\]

\[
X_{x,\mu} \rightarrow X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega_{x+\mu}^\dagger
\]

\[
\Omega_x \in G = SU(N)
\]

\[
W_C[V] := \text{Tr} \left[ P \prod_{(x,x+\mu) \in C} V_{x,\mu} \right] / \text{Tr}(1)
\]

\[
W_C[U] = \text{const.} W_C[V]
\]
Introducing a color field $\mathbf{h}_x = \xi(\lambda^8/2)\xi^\dagger \in SU(3)/U(2)$ with $\xi \in SU(3)$, a set of the defining equation of decomposition $U_{x,\mu} = X_{x,\mu}V_{x,\mu}$ is given by

$$D_\mu[V]\mathbf{h}_x = \frac{1}{\epsilon} (V_{x,\mu}\mathbf{h}_{x+\mu} - \mathbf{h}_x V_{x,\mu}) = 0,$$

$$g_x = e^{-2\pi q_x/N} \exp(-a_x^{(0)}\mathbf{h}_x - i \sum_{i=1}^3 a_x^{(i)}u_x^{(i)}) = 1,$$

which correspond to the continuum version of the decomposition, $A_\mu(x) = V_\mu(x) + X_\mu(x)$,

$$D_\mu[V_\mu(x)]\mathbf{h}(x) = 0, \quad \text{tr}(X_\mu(x)\mathbf{h}(x)) = 0.$$

### Exact solution

(N=3)

$$X_{x,\mu} = \hat{L}_{x,\mu}^\dagger (\det \hat{L}_{x,\mu})^{1/N} g_x^{-1} V_{x,\mu} = X_{x,\mu}^\dagger U_x, = g_x \hat{L}_{x,\mu} U_x, (\det \hat{L}_{x,\mu})^{-1/N}$$

$$\hat{L}_{x,\mu} = \left( \sqrt{L_{x,\mu} L_{x,\mu}^\dagger} \right)^{-1} L_{x,\mu}$$

$$L_{x,\mu} = \frac{N^2 - 2N + 2}{N} \mathbf{1} + (N - 2) \sqrt{\frac{2(N - 2)}{N}} (\mathbf{h}_x + U_{x,\mu}\mathbf{h}_{x+\mu}U_{x,\mu}^{-1})$$

$$+ 4(N - 1)\mathbf{h}_x U_{x,\mu} \mathbf{h}_{x+\mu} U_{x,\mu}^{-1}$$

### Continuum limit

$$V_\mu(x) = A_\mu(x) - \frac{2(N - 1)}{N} [\mathbf{h}(x), [\mathbf{h}(x), A_\mu(x)]] - ig^{-1} \frac{2(N - 1)}{N} [\partial_\mu \mathbf{h}(x), \mathbf{h}(x)],$$

$$X_\mu(x) = \frac{2(N - 1)}{N} [\mathbf{h}(x), [\mathbf{h}(x), A_\mu(x)]] + ig^{-1} \frac{2(N - 1)}{N} [\partial_\mu \mathbf{h}(x), \mathbf{h}(x)].$$
**Minimal option: Non-Abelian magnetic monopole**

For Wilson loop in the fundamental representation

From the non-Abelian Stokes theorem and the Hodge decomposition, the magnetic monopole is derived **without using the Abelian projection**

$$W_C[A] = \int [d\mu(\xi)]_\Sigma \exp \left( -ig \int_{S_{C=\partial \Sigma}} dS^{\mu\nu} \sqrt{\frac{N-1}{2N}} \text{tr}(2\mathbf{h}(x) F^{\mu\nu}[V](x)) \right)$$

$$= \int [d\mu(\xi)]_\Sigma \exp \left( ig \sqrt{\frac{N-1}{2N}} (k, \Xi) + ig \sqrt{\frac{N-1}{2N}} (j, N_{\Sigma}) \right)$$

magnetic current $k := \delta^* F = *dF$, \quad $\Xi := \delta^* \Theta_{\Sigma} \Delta^{-1}$

electric current $j := \delta F$, \quad $N_{\Sigma} := \delta \Theta_{\Sigma} \Delta^{-1}$

$\Delta = d\delta + \delta d$, \quad $\Theta_{\Sigma} := \int_{\Sigma} d^2 S^{\mu\nu}(\sigma(x)) \delta D(x - x(\sigma))$

$k$ and $j$ are gauge invariant and conserved currents; $\delta k = \delta j = 0$.

Note that field strength $F[V]$ is described by $V$-field in the minimal option.

The lattice version of magnetic monopole current is defined by using plaquette:

$$\Theta_{\mu \nu}^8 := - \arg \text{Tr} \left[ \left( \frac{1}{3} \mathbf{1} - \frac{2}{\sqrt{3}} \mathbf{h}_x \right) V_{x,\mu} V_{x+\mu,\mu} V_{x+\nu,\mu} V_{x,\nu}^\dagger \right],$$

$$k_\mu = 2\pi n_\mu := \frac{1}{2} \epsilon \epsilon_{\mu \nu \alpha \beta} \partial_\nu \Theta_{\alpha \beta}^8,$$
maximal option: The decomposition of SU(3) link variable

\[ U_{x,\mu} = X_{x,\mu} V_{x,\mu} \]

\[ U_{x,\mu} \rightarrow U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega_x^{\dagger} \]
\[ V_{x,\mu} \rightarrow V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega_x^{\dagger} \]
\[ X_{x,\mu} \rightarrow X'_{x,\mu} = \Omega_x X_{x,\mu} \Omega_x \]

\( \Omega_x \in G = SU(N) \)

Gauge invariant construction of the Abelian projection to maximal torus group U(1) x U(1) in MA gauge.
maximal option: Defining equation for the decomposition

By introducing color fields $n_x^{(3)} = \Theta_x (\lambda^3/2) \Theta^\dagger$, $n_x^{(8)} = \Theta_x (\lambda^8/2) \Theta^\dagger$
\[ \in SU(3)_\omega \times [SU(3)/(U(1) \times U(1))]_\theta, \] a set of the defining equation for the decomposition $U_{x,\mu} = X_{x,\mu} V_{x,\mu}$ is given by
\[ D^\varepsilon_{\mu}[V] n_x^{(k)} = \frac{1}{\varepsilon} (V_{x,\mu} n_{x+\mu}^{(k)} - n_x^{(k)} V_{x,\mu}) = 0, \quad (k = 3, 8) \]
\[ g_x = \exp(2\pi i n/N) \exp(i \sum_{j=3,8} a^{(j)} n_x^{(j)}) = 1 \]

Corresponding to the continuum version of the decomposition $A_\mu(x) = V_\mu(x) + X_\mu(x)$
\[ D_\mu[V_\mu] n_x^{(k)}(x) = 0, \quad tr(n^{(k)}(x) X_\mu(x)) = 0, \quad (k = 3, 8) \]

\[ X_{x,\mu} = \hat{K}_{x,\mu} \det(K_{x,\mu})^{1/3} g_x^{-1}, \quad V_{x,\mu} = g_x \hat{K}_{x,\mu} \det(K_{x,\mu})^{-1/3} \]
where
\[ \hat{K}_{x,\mu} : = \left( \sqrt{K_{x,\mu} K_{x,\mu}^\dagger} \right)^{-1} K_{x,\mu}, \quad \hat{K}_{x,\mu}^\dagger = K_{x,\mu} \left( \sqrt{K_{x,\mu} K_{x,\mu}^\dagger} \right)^{-1} \]
\[ K_{x,\mu} = 1 + 6n_x^{(3)} U_{x,\mu} U_{x,\mu}^\dagger + 6n_x^{(8)} U_{x,\mu} U_{x,\mu}^\dagger \]
Maximal option

- magnetic monopole

We have two kind of magnetic monopoles in the maximal option

- Decomposition in the MA gauge

Decomposition formula is rewritten into Abelian projection in Maximal Abelian gauge

→ Abelian projection in the MA gage
Reduction condition

- The reduction condition is introduced such that the theory in terms of new variables is equipollent to the original Yang-Mills theory.
- We here introduce the reduction condition which is the kinetic term of adjoint gauge-Higgs system.

**Minimal option:**

\[ SU(3)_\omega \times [SU(3)/U(2)]_\theta \rightarrow SU(3)_{\omega-\theta} \]

Determining \( h_x \) to minimize the reduction function for given \( U_{x,\mu} \)

\[
F_{\text{red}}[h_x, U_{x,\mu}] = \sum_{x,\mu} \text{tr}\left\{ (D^\epsilon_{\mu}[U_{x,\mu}]h_x)^\dagger (D^\epsilon_{\mu}[U_{x,\mu}]h_x) \right\}
\]

**Maximal option:**

\[ SU(3)_\omega \times [SU(3)/(U(1) \times U(1))]_\theta \rightarrow SU(3)_{\omega-\theta} \]

Determine \( n^{(3)} \) and \( n^{(8)} \) to minimize the following functional

\[
F_{\text{max}}[n^{(3)}, n^{(8)}; U_{x,\mu}] = \sum_{x,\mu} \text{tr}\left( \| D^\omega_{\mu}[U]n^{(3)}_x \|^2 \right) + \sum_{x,\mu} \text{tr}\left( \| D^\omega_{\mu}[U]n^{(8)}_x \|^2 \right)
\]

\[
n^{(3)}_x = \Theta_x(\lambda^3/2)\Theta_x^\dagger, \quad n^{(8)}_x = \Theta_x(\lambda^8/2)\Theta_x^\dagger
\]

**Reduction condition** for maximal option is rewritten into the gauge fixing of maximal Abelian gauge.
1. Introduction
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3. Lattice data
   Comparison of TWO picture of dual superconductivity: minimal v.s. maximal at zero and nonzero temperature
   - Static potential
   - Chromo flux tube and dual Meissner effect
   - Polyakov loop average :: center symmetry breaking in
   - confinement/deconfinement phase transition in view of the dual superconductor
4. Summary and discussion
DUAL SUPERCONDUCTIVITY AT ZERO TEMPERATURE
String tension: zero temperature

We obtain the restricted field ("Abelian") dominance in the string tension for both the minimal option and the maximal option.

The string tension is almost same with the both options and YM field.

Static potential from Wilson loop average of YM-field and two V-fields in minimal and maximal options

$$\log <W[T=10,R]> \text{ vs } R$$
Measurement of chromo flux:

\[ \rho_W = \frac{\langle \text{tr}(WLUPL^+) \rangle}{\langle \text{tr}(W) \rangle} - \frac{1}{N} \frac{\langle \text{tr}(W) \text{tr}(U_P) \rangle}{\langle \text{tr}(W) \rangle} \]

The field strength by quark and anti quark can be defined as

\[ F_{\mu\nu}(x) = \sqrt{\frac{\beta}{2N}} \rho_W(x) \]

To know the difference between the decomposition, we measure the three types of probes and compare them.

- \[ O^{[\text{YM}]} = L[U]U_PL[U]^{-1} \] :: original YM
- \[ O^{[\text{min}]} = L[V^{[\text{min}]}]V_p^{[\text{min}]}L[V^{[\text{min}]}]^{-1} \] :: V field in minimal option
- \[ O^{[\text{max}]} = L[V^{[\text{max}]}]V_p^{[\text{max}]}L[V^{[\text{max}]}]^{-1} \] :: V field in maximal option

Chromo-electric flux tube

• Chromo flux between quark and antiquark at midpoint
• Chromo-flux tube is observed, only $E_z$ element has non-vanishing values in each.

• Comparison of Chromo flux strength.

24-30 July 2016
Anatomy of chromo flux by color field

• In aximal option, there exists two color fields, n3 and n8.
• Chromo flux can be decomposed into two parts by using the color fields.

⇒ The data shows that decomposed chromo fluxes have almost same amplitude.

\[ Q_w = \frac{\langle tr(L[V]V_P L[V]^\dagger W_C) \rangle - \frac{1}{3}\langle tr(V_P)tr(W_C) \rangle}{\langle tr(W_C) \rangle} \]
\[ = \frac{\langle tr(V_P n_x^{(3)} tr(W_C n_x^{(3)})) \rangle + \langle tr(V_P n_x^{(8)} tr(W_C n_x^{(8)})) \rangle}{\langle tr(W_C) \rangle} \]

\[ Ez = Ez(n3) + Ez(n8) \]

\[ Ez(n8) \quad Ez(n3) \]
Induced magnetic current (monopole)

Yang–Mills equation (Maxell equation) for restricted field $V_\mu$, the magnetic current (monopole) can be calculated as

$$ k = \delta * F[V] = * d F[V], $$

where $F[V]$ is the field strength of $V$, $d$ exterior derivative, $*$ the Hodge dual and $\delta$ the coderivative $\delta := * d^*$, respectively.

Induced magnetic current (monopole) $k$ can be an order parameter of the dual Meissner effect.

$k$ is a order parameter of confinement/deconfinement phase

24-30 July 2016
DUAL SUPERCONDUCTIVITY AT FINITE TEMPERATURE

$L^3 \times T$, $L=24$, $T=6$ fixed lattice size

Temperature is controlled by a parameter $\beta$:

$\beta = 5.8, 5.85, 5.9, 5.925, 5.95, 5.975, 6.0, 6.05, 6.1, 6.15, 6.2, 6.35, 6.3, 6.4, 6.5$
Polyakov loop

Distribution of Polyakov loop values

\[ P_U(x) = \text{tr}\left( \prod_{t=1}^{N_t} U(x_t,4) \right) \] for original Yang-Mills field

\[ P_V(x) = \text{tr}\left( \prod_{t=1}^{N_t} V(x_t,4) \right) \] for restricted field

![Graphs showing the distribution of Polyakov loop values for different values of b, with two options for the V field: minimal and maximal.](image)
Magnitude of Polyakov-loop average is different, but gives the same phase transition temperature ($\beta$).
static potential (correlation function of Plyakov loops)
Measurement of chromo flux at finite temperature

\[ \rho_W = \frac{\langle \text{tr}(WLU_pL^\dagger) \rangle}{\langle \text{tr}(W) \rangle} - \frac{1}{N} \frac{\langle \text{tr}(W)\text{tr}(U_p) \rangle}{\langle \text{tr}(W) \rangle} \]

\[ F_{\mu\nu}(x) = \sqrt{\frac{\beta}{2N}} \rho_W(x) \]

- Using the same operator with that of zero temperature.
- Size of Wilson loop T-direction = Nt
- The source of quark and antiquark are given by Plyakov loops connecting by Wilson line.
- The three types of probes and compare them.

\[ O^{[YM]} = L[U]U_pL[U]^{-1} \quad \text{:: original YM} \]
\[ O^{[\text{min}]} = L[V^{[\text{min}]}]V_p^{[\text{min}]}L[V^{[\text{min}]}]^{-1} \quad \text{:: V field in minimal option} \]
\[ O^{[\text{max}]} = L[V^{[\text{max}]}]V_p^{[\text{max}]}L[V^{[\text{max}]}]^{-1} \quad \text{:: V field in maximal option} \]
Chromo flux in confining phase
Chromo flux in deconfining phase

Yang-Mills beta 6.1

Mineral beta 6.1

Maximal beta 6.1

24-30 July 2016
Lattice 2016, Highfield Campus, University of Southampton
Induced magnetic current (monopole) at finite temperature

Yang–Mills equation (Maxell equation) for restricted field $V_\mu$, the magnetic current (monopole) can be calculated as

$$ k = \delta^* F[V] = *dF[V], $$

where $F[V]$ is the field strength of $V$, $d$ exterior derivative, $*$ the Hodge dual and $\delta$ the coderivative $\delta := *d^*$, respectively.

Minimal option
Summary

- We investigate dual superconductivity applying our new formulation of Yang-Mills theory on the lattice, i.e., in the minimal and maximal options as well as Yang-Mills field at finite temperature.

- In both options we have found that
  - the restricted field (V-field) dominance in the string tension, and the string tension is almost same.
  - In confining phase we directory observe the dual Meissner effects. The induced magnetic (monopole) currents appear around chromo-electro flux tube between a pair of quark and antiquark.
  - In deconfining phase we find no more the dual Meissner effects, i.e., the induced magnetic (monopole) currents become very small or disappears.
  - The Polyakov loop averages, which is the conventional order parameter of confinement/deconfinement phase transition, gives the same critical temperature with both options and the YM field.
outlook

• Determination **type of the dual superconductor** in the maximal option. By using the minimal option, of type I [Phys.Rev. D87 (2013) 054011].

• Investigate the dual Meissner effect phase transition, and determine critical temperature and order of the phase transition.
THANK YOU FOR YOUR ATTENTION