Lattice QCD simulation of the Berry curvature

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Berry Curvature

Berry (1984)

$$H(p)\Phi(p) = E(p)\Phi(p)$$

Berry Curvature

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$$H(p)\Phi(p) = E(p)\Phi(p)$$

Berry connection

$$\tilde{A}_\mu(p) = -i\Phi^\dagger(p)\frac{\partial}{\partial p^\mu}\Phi(p)$$

Berry curvature

$$\tilde{F}_{\mu\nu}(p) = \frac{\partial}{\partial p^\mu}\tilde{A}_\nu(p) - \frac{\partial}{\partial p^\nu}\tilde{A}_\mu(p)$$

Berry Curvature

Berry (1984)

$$H(p)\Phi(p) = E(p)\Phi(p)$$

Berry connection		gauge connection
$\tilde{A}_\mu(p) = -i\Phi^\dagger(p)\dfrac{\partial}{\partial p^\mu}\Phi(p)$	$\Leftrightarrow$	$A_\mu(x)$

Berry curvature		gauge field strength
$\tilde{F}_{\mu\nu}(p) = \dfrac{\partial}{\partial p^\mu}\tilde{A}_\nu(p) - \dfrac{\partial}{\partial p^\nu}\tilde{A}_\mu(p)$	$\Leftrightarrow$	$F_{\mu\nu}(x)$

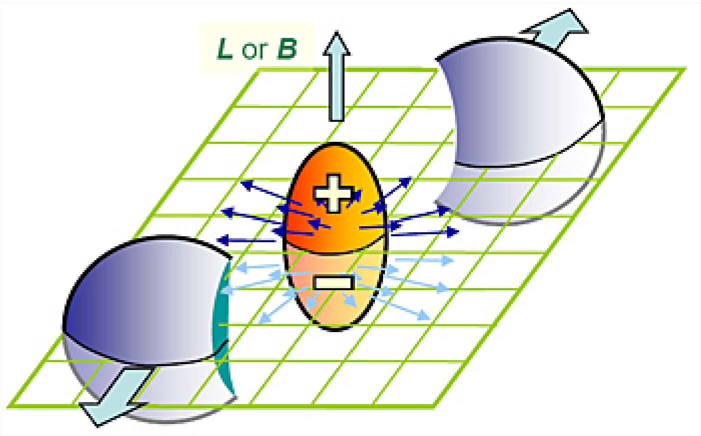
Berry Curvature

https://arxiv.org/abs/0909.1717
https://www.psi.ch/media/insulator-makes-electrons-move-in-an-ordered-way

Berry curvature of fermion spatial momentum

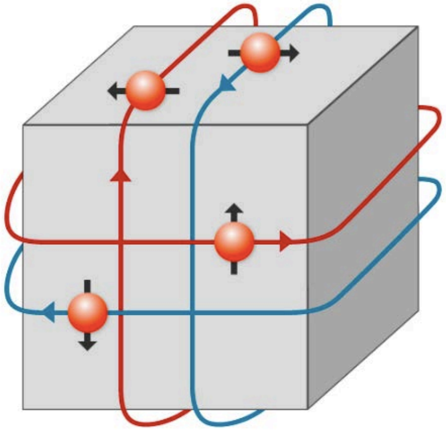
chiral magnetic effect

chiral vortical effect



quantum Hall effect

topological insulators



Formalism

fermion ground state $\Phi(p)$

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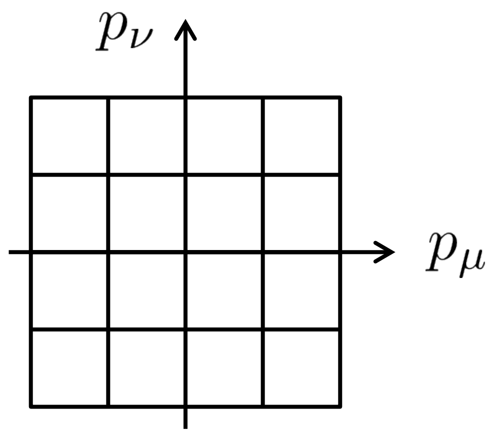
ground state projection

$$\phi(p,\tau) = \sum_{x,x'} e^{ip\cdot(x-x')} \boxed{D^{-1}(x,\tau|x',0)} \phi_{\text{init}}$$

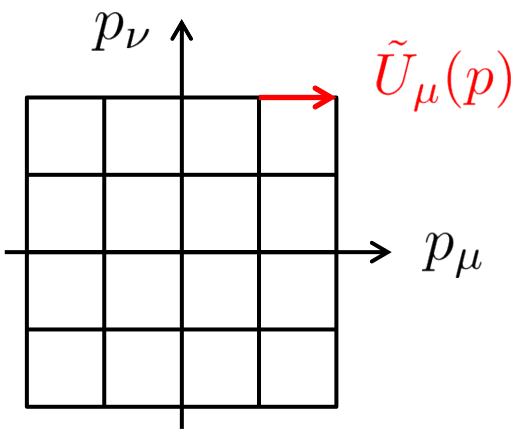
fermion propagator

$$\to \Phi(p) \qquad \text{in } \tau \to \infty$$

Formalism



Formalism



Berry connection

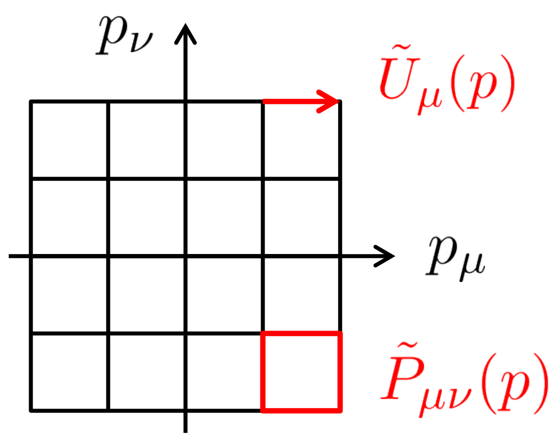
$$\tilde{A}_\mu(p)$$

⇨

Berry link variable

$$\tilde{U}_\mu(p) = \frac{\Phi^\dagger(p)\Phi(p+\tilde{\mu})}{|\Phi^\dagger(p)\Phi(p+\tilde{\mu})|}$$

Formalism



Berry connection

Berry link variable

$$\tilde{A}_\mu(p) \Rightarrow \tilde{U}_\mu(p) = \frac{\Phi^\dagger(p)\Phi(p+\tilde{\mu})}{|\Phi^\dagger(p)\Phi(p+\tilde{\mu})|}$$

Berry curvature

Berry plaquette

$$\tilde{F}_{\mu\nu}(p) \Rightarrow \tilde{P}_{\mu\nu}(p) = \tilde{U}_\mu(p)\tilde{U}_\nu(p+\tilde{\mu})\tilde{U}_\mu^\dagger(p+\tilde{\nu})\tilde{U}_\mu^\dagger(p)$$

Example

(2+1)-dim. Wilson fermion

$$D(x,x') = (ma+3)\delta_{x,x'} - \frac{1}{2}\sum_{\mu=1}^{3}\left[(1-\sigma_\mu)\, U_\mu(x)\delta_{x+\hat{\mu},x'} + (1+\sigma_\mu)\, U_\mu^\dagger(x')\delta_{x-\hat{\mu},x'} \right]$$

Example

(2+1)-dim. Wilson fermion

$$D(x,x') = (ma+3)\delta_{x,x'} - \frac{1}{2}\sum_{\mu=1}^{3}\left[(1-\sigma_\mu)\,U_\mu(x)\delta_{x+\hat{\mu},x'} + (1+\sigma_\mu)\,U_\mu^\dagger(x')\delta_{x-\hat{\mu},x'}\right]$$

⇓

2-dim. U(1) Berry link variables

||

2-dim. U(1) lattice gauge theory

Example

Berry curvature

$$\tilde{F}_{xy}(p) = \mathrm{Im}\ \ln\ \tilde{P}_{xy}(p)$$

⇔

topological charge density

$$F_{xy}(x)$$

1st Chern number

$$N = \frac{1}{2\pi}\sum_{p}\tilde{F}_{xy}(p)$$

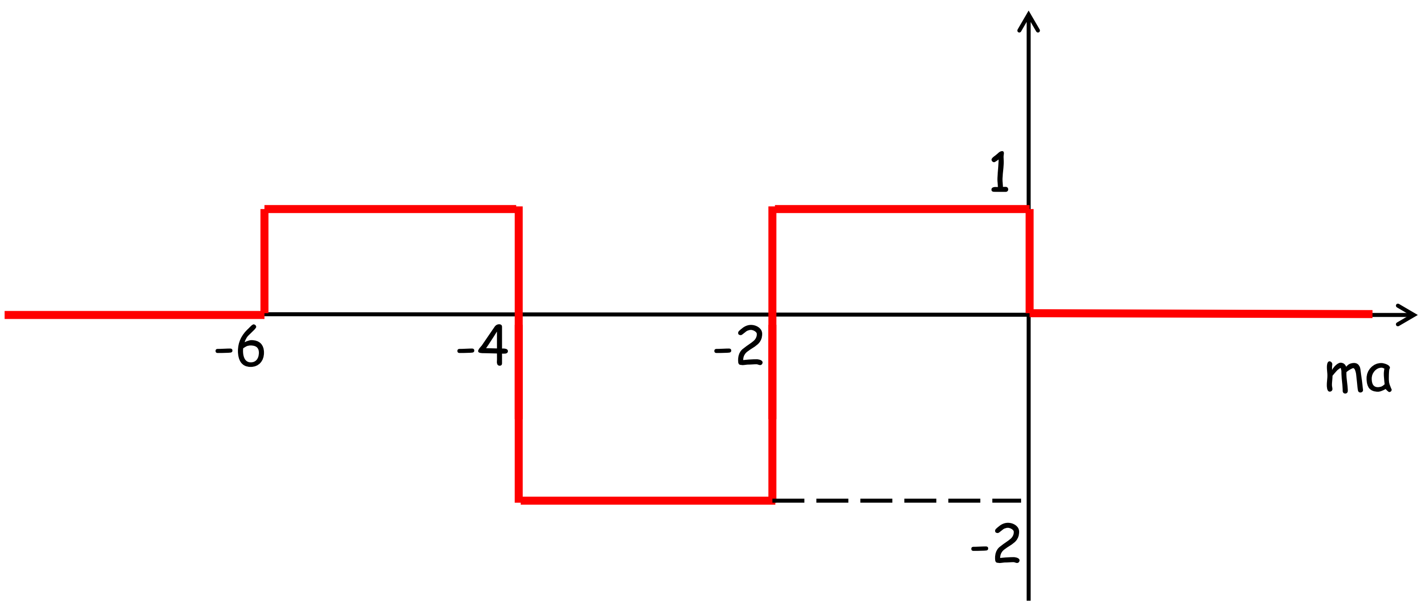
⇔

topological charge

$$Q$$

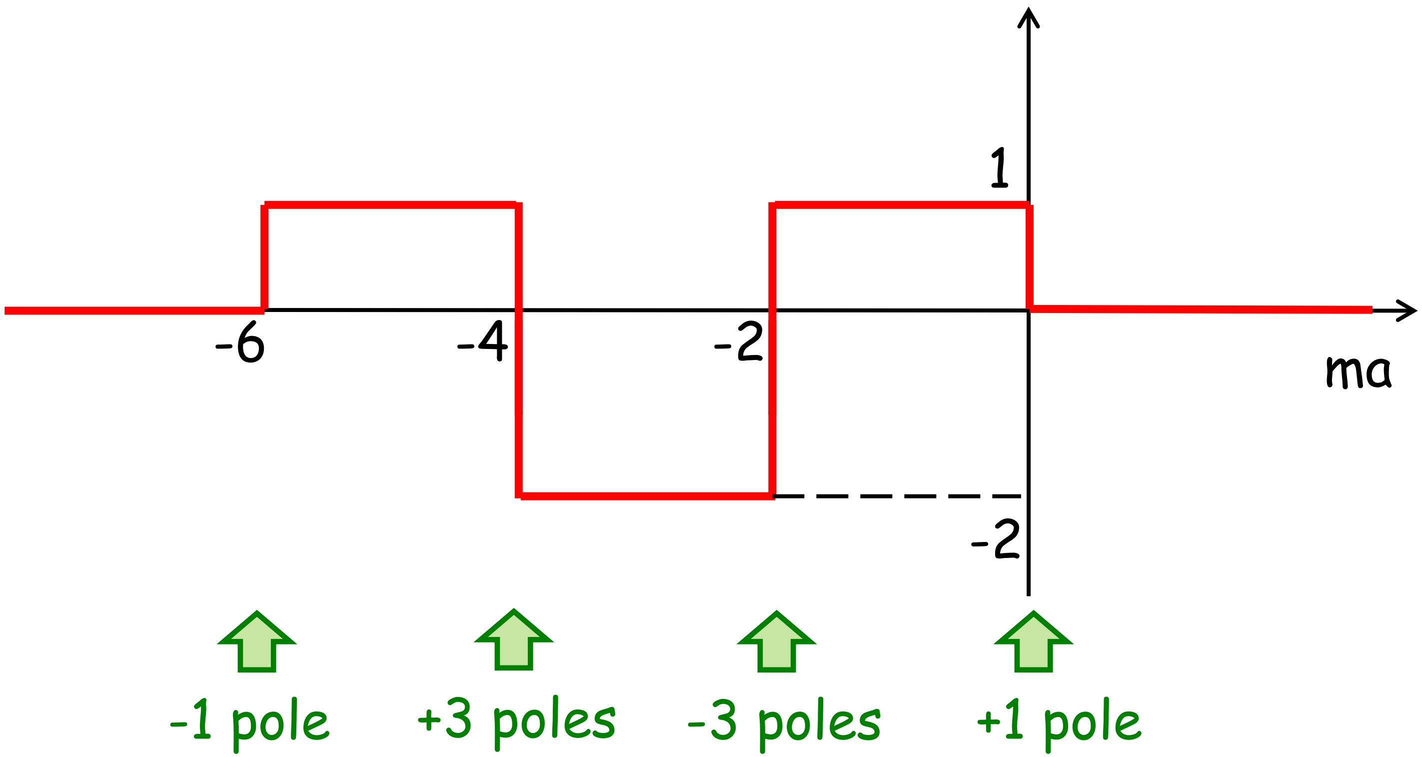
Example

1st Chern number $$N = \frac{1}{2\pi}\sum_p \tilde{F}_{xy}(p)$$



Example

1st Chern number $N = \frac{1}{2\pi} \sum_p \tilde{F}_{xy}(p)$



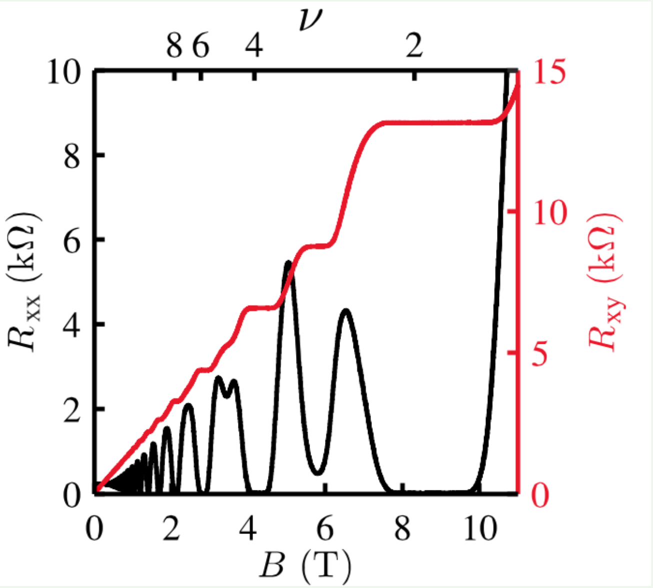
Example

Thouless, Kohmoto, Nightingale, den Nijs (1982)
Suddards, Baumgartner, Henini, Mellor (2011)

1st Chern number
$$N = \frac{1}{2\pi} \sum_p \tilde{F}_{xy}(p)$$

quantum Hall effect

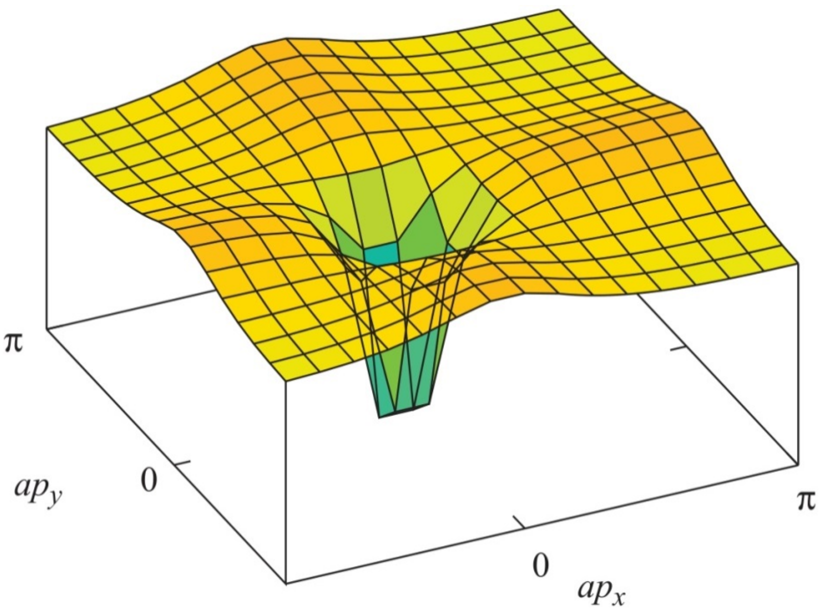
$$R_{xy} = \frac{2\pi}{e^2} \frac{1}{N}$$



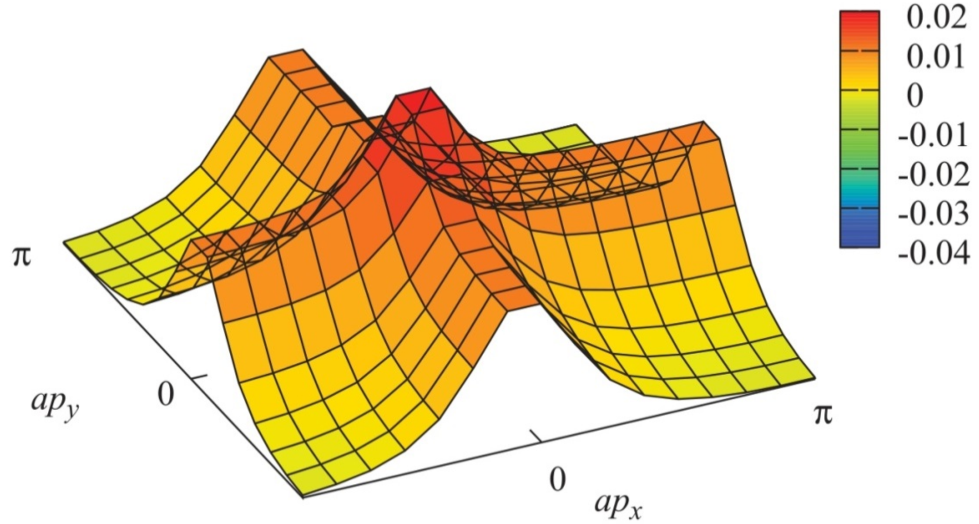
Example

Berry curvature

ma = 0.5



ma = - 0.5



Example

Chern number

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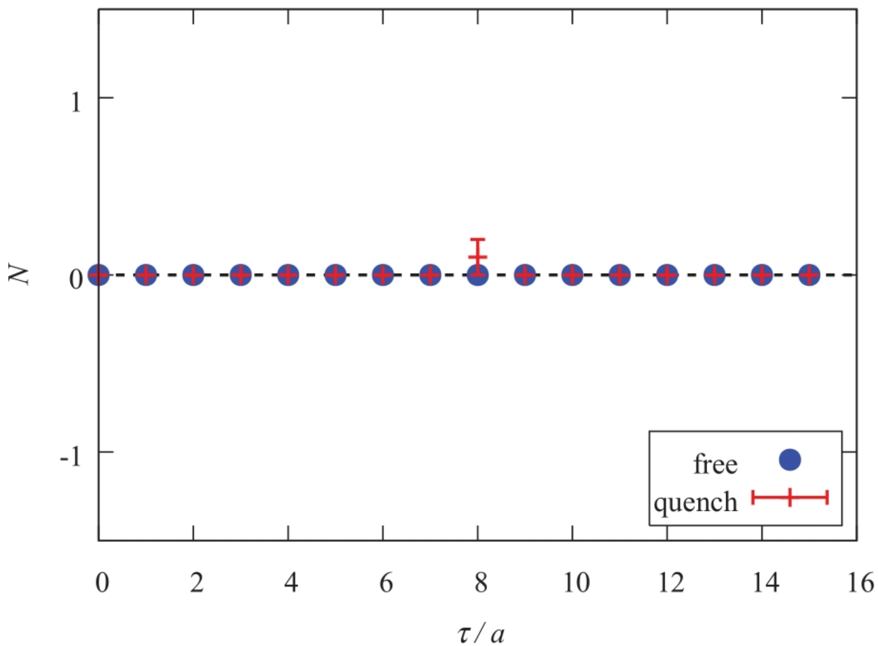

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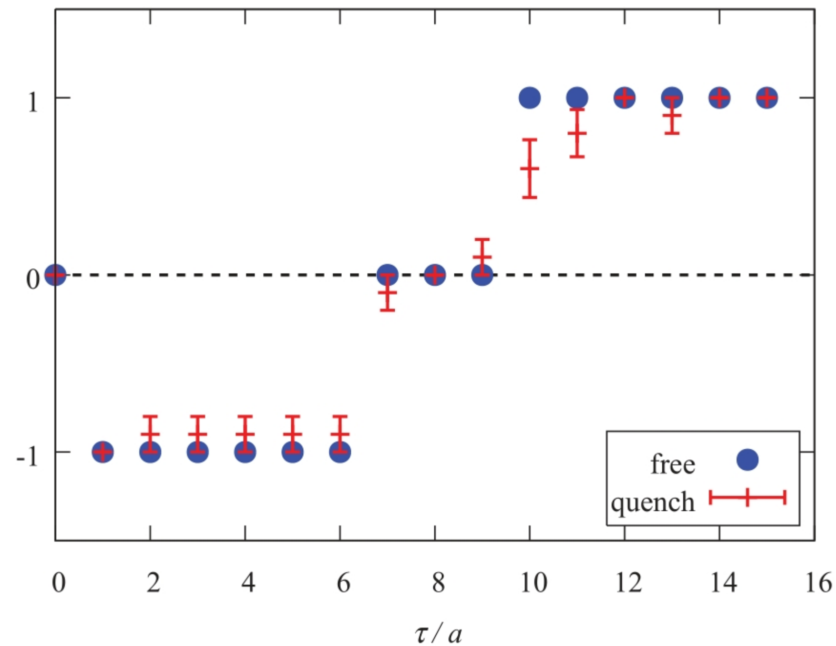

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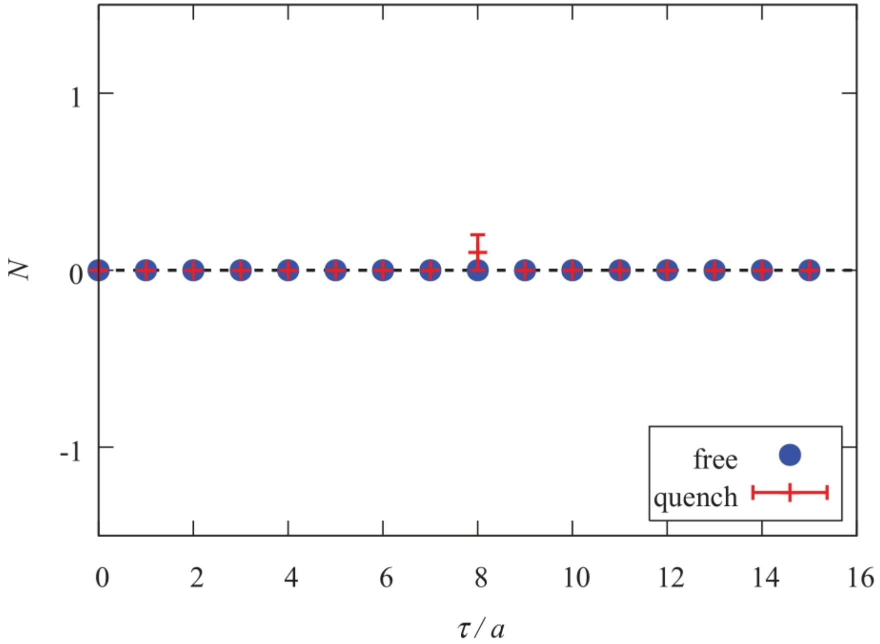

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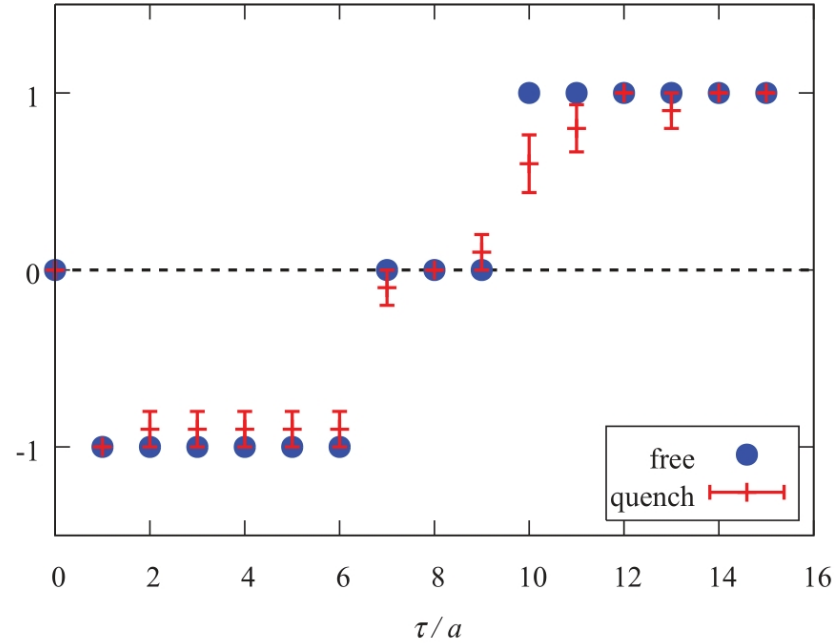

N = 0

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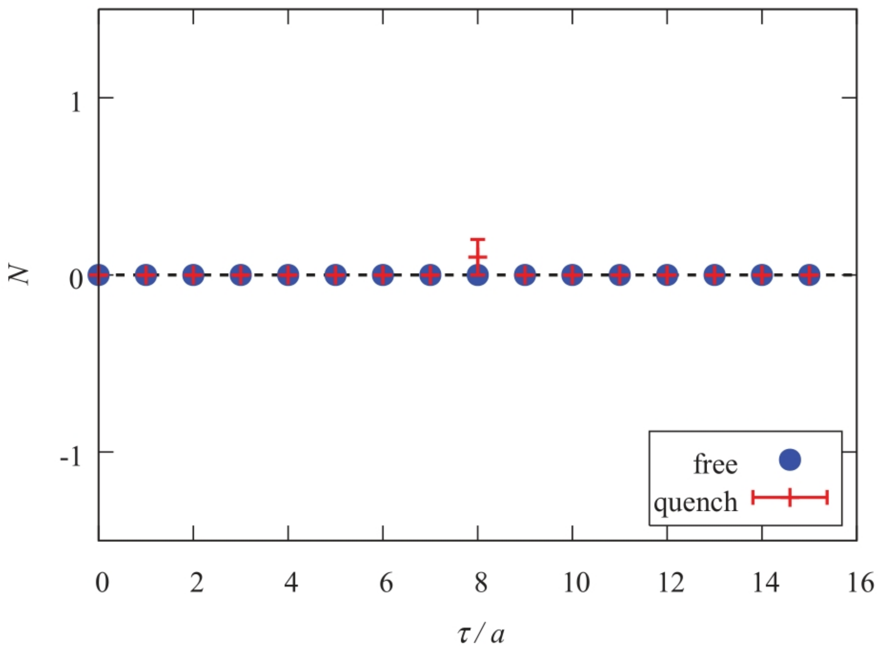

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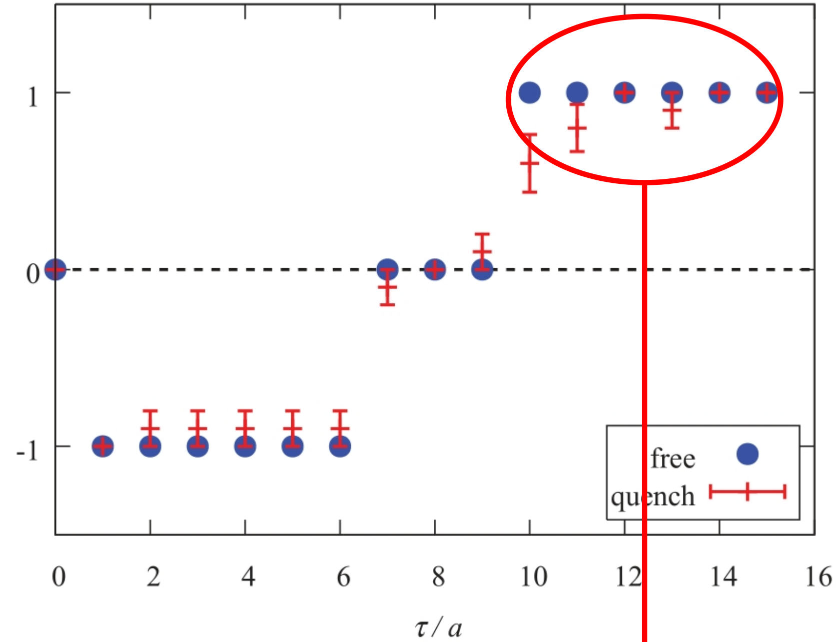

ground state
N = 1

Summary

lattice QCD simulation of the Berry curvature:

- ✓ formulated
- ✓ checked in (2+1)-dim. Wilson fermion
- ✓ applicable to realistic systems