# Rho meson decay width in SU(2) gauge theories with 2 fundamental flavours.

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# SU(2) with 2 fundamental flavours

See also the talk by V. Drach (Monday) and plenary by C. Pica
See also [1402.0233].
Fundamental representation of SU(2) is pseudo-real → we can construct a flavour multiplet

$$Q = \begin{pmatrix} u_L \\ d_L \\ -i\sigma^2 C \bar{u}_R^T \\ -i\sigma^2 C \bar{d}_R^T \end{pmatrix}$$

which is symmetric under SU(4) flavour group (locally isomorphic to SO(6)).

# SU(2) with 2 fundamental flavours

The mass term can be written as

$$-m(\bar{u}u + \bar{d}d) = \frac{m}{2} Q^T (-i\sigma^2) CEQ + h.c.$$

$$E = \begin{pmatrix} 0 & 1_2 \\ -1_2 & 0 \end{pmatrix}$$

The mass term breaks SU(4) to the subgroup which leaves E invariant, i.e. Sp(4).
Even if the explicit mass term is not present, the symmetry is broken spontaneously by $\bar{u}u + \bar{d}d$ condensate [1109.3513]. In a 2-flavour theory this produces 5 Goldstone bosons ("pions").

# Phenomenology

The model contains a choice of inequivalent vacua. Of particular interest are $\Sigma_B$ and $\Sigma_H$

| | $\Sigma_B$ | $\Sigma_H$ |
|---|---|---|
| EW symmetry | unbroken | broken |
| model | composite Higgs | Technicolor |
| pions | $W^{\pm}$, $Z$, $H$ + 1 extra | $W^{\pm}$, $Z$ + 2 extra |
| Higgs | pion | scalar resonance |

The vacuum can also be a superposition of the two:

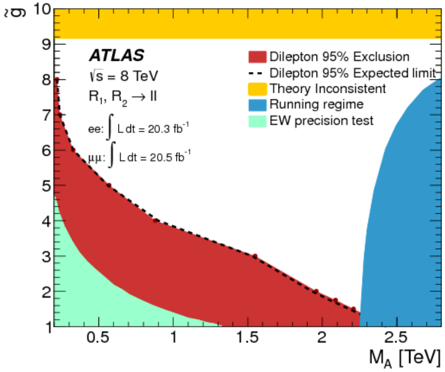
$$\Sigma_0 = \cos\theta \Sigma_B + \sin\theta \Sigma_H$$

**Motivation for this work:**
At high energies, vector boson scattering is equivalent to Goldstone boson scattering. Studying $\pi\pi$ scattering in this model tells us about the resonance structure in vector boson scattering.

We want $m_\rho$ and $g_{\rho\pi\pi}$ defined as

$$\mathcal{L}_{eff} = g_{\rho\pi\pi}\epsilon_{ijk}\rho^{i\mu}\pi^j\partial_\mu\pi^k \quad (1)$$

In QCD $g_{\rho\pi\pi} \approx 6$. This need not be the case for this model.




[1405.4123]

# $\rho \to \pi\pi$ kinematics

- Pions transform in the 5-dimensional representation of Sp(4) ( = fundamental representation of SO(5))
- Two-pion wavefunction can be in one of the following irreducible representations:
  1. Symmetric traceless $\pi^i\pi^j + \pi^j\pi^i - \frac{2}{5}\pi^k\pi^k\delta^{ij}$ - see [1412.4771]
  2. Antisymmetric $\pi^i\pi^j - \pi^j\pi^i$ - this presentation
  3. Trace $\delta^{ij}\pi^k\pi^k$ - in the future
- This is analogous to I=2, 1 and 0 $\pi\pi$ scattering in QCD.
- Rho is a vector (J=1) resonance, so by angular momentum conservation, the two-pion state must be in a p-wave (l=1), which is parity odd.
- Because pions are spin-0 bosons, they can only be combined into an antisymmetric wavefunction if their momenta are different.

# Phase shift

Resonance parameters can be extracted from two-pion phase shift.

Two-pion state can be described in partial-wave basis as $|E, p, l, m\rangle$.

Below inelastic threshold ($4m_\pi$) there is only one state contributing to the S-matrix, which can be written as:

$$S = \langle E, p, l', m' \,|\, E_{CM}, 0, l, m\rangle = \delta(E - E_{CM})\delta(p)\delta_{ll'}\delta_{mm'}e^{2i\delta_l(E_{CM})}$$

This defines the phase shift $\delta_l(E_{CM})$.

Phase shift can be calculated from the energy spectrum using Lüscher's approach.

The strategy is:

Spectrum $\rightarrow$ phase shift $\rightarrow$ $m_\rho$ and $g_{\rho\pi\pi}$

# Moving frames

- In continuum, the angular momentum is conserved (rotational symmetry).
- On the lattice, this symmetry is broken $\rightarrow$ mixing between partial waves.
- Phase shift is extracted in the centre-of-mass frame, so the symmetry depends on the frame of reference we're in:
  1. COM - $O$ ($P = (0, 0, 0)$)
  2. MF1 - $D_{4h}$ ($P = (0, 0, p)$)
  3. MF2 - $D_{2h}$ ($P = (p, p, 0)$)
- $l = 1$ representation reduces to
  1. COM $T_1^-$
  2. MF1 $A_2^- \oplus E^-$
  3. MF2 $B_1^- \oplus B_2^- \oplus B_3^-$

# Moving frames

Phase shift formula depends on the frame and the representation:

| frame | representation | $\tan\delta_1$ |
|---|---|---|
| COM | $T_1^-$ | $\frac{\pi^{3/2}q}{Z_{00}(1;q^2)}$ |
| MF1 | $A_2^-$ | $\frac{\pi^{3/2}q}{Z_{00}(1;q^2)+\frac{2}{\sqrt{5}q^2}Z_{20}}$ |
| MF2 | $B_1^-$ | $\frac{\pi^{3/2}q}{Z_{00}(1;q^2)-\frac{1}{\sqrt{5}q^2}Z_{20}+i\frac{\sqrt{3}}{\sqrt{10}q^2}\left(Z_{22}(1;q^2)-Z_{2(-2)}(1;q^2)\right)}$ |

$$Z_{lm}(s,q^2)=\sum_{n\in\mathbb{Z}^3}\frac{Y_{lm}(n)}{(q^2-n^2)^s}$$

$$\tan\delta_1=\frac{g_{\rho\pi\pi}^2}{6\pi}\frac{p^3}{E_{CM}(m_\rho^2-E_{CM}^2)},\ p=\sqrt{E_{CM}^2/4-m_\pi^2}$$

# GEVP

The correlation functions are given by

$$C_{ij}(t) \equiv \langle 0 \mid O_i^\dagger(t) O_j(0) \mid 0 \rangle = \sum_{n,m} \langle 0 \mid O_i^\dagger \mid n \rangle (e^{-E_n t} \delta_{mn}) \langle m \mid O_j \mid 0 \rangle$$

U and V are square matrices assuming higher-energy states don't contribute.
Then

$$C_{ij}^{-1}(t_0) C_{jk}(t) = V_{in}^{-1} diag \left( e^{-E_n(t-t_0)} \right)_{nm} V_{mj}$$

The spectrum can be extracted from the eigenvalues of $C^{-1}(t_0)C(t)$.

# Interpolating operators

We use singlet representations - $A_2^-$ in MF1 and $B_1^-$ in MF2. We use the following two interpolating operators

$$O_1(t) = \sum_{x,y} \bar{\psi}(x)\gamma^5\psi(x)\bar{\psi}(y)\gamma^5\psi(y)e^{i\mathbf{p}\cdot\mathbf{x}}$$

$$O_2(t) = \sum_{x} \bar{\psi}(x)(\gamma \cdot \hat{p})\psi(x)e^{i\mathbf{p}\cdot\mathbf{x}}$$

- $p = (0,0,1)$ in MF1 and $p = (1,1,0)$ in MF2
- $O_1(t)$ is by itself not in an irreducible representation, projection is done by choosing the contractions the same way as for $\pi_i(p)\pi_j(0) - \pi_j(p)\pi_i(0)$.

# Quark sources

To achieve sources of the form $\sum_x \bar{\psi}(x)\Gamma\psi(x)$ we use propagator sources of the form

$$\psi_\eta(p) = \sum_x \eta(x)\psi(x)e^{ipx}$$

with

$$\langle \eta(x)^\dagger \eta(y) \rangle_\eta = \delta(x-y)$$

then

$$\langle \bar{\psi}_\eta(0)\psi_\eta(p) \rangle_\eta = \sum_x \bar{\psi}(x)\Gamma\psi(x)$$

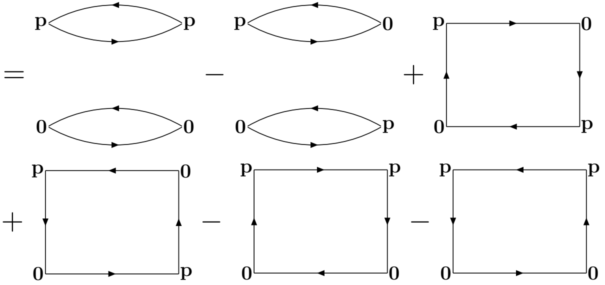
Two-pion sources require separate noise vectors for each pion. We use $\mathbb{Z}_2 \times \mathbb{Z}_2$ noise sources

$$\eta(x) = \mathrm{rand}\,(\pm 1 \pm i)$$

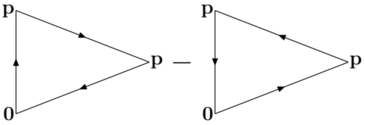
with 3 choices of $\eta$ for each propagator ("hits").

## Contractions

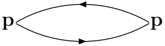
$C_{11}(t) =$




$C_{12}(t) = -C^*_{21}(t) =$



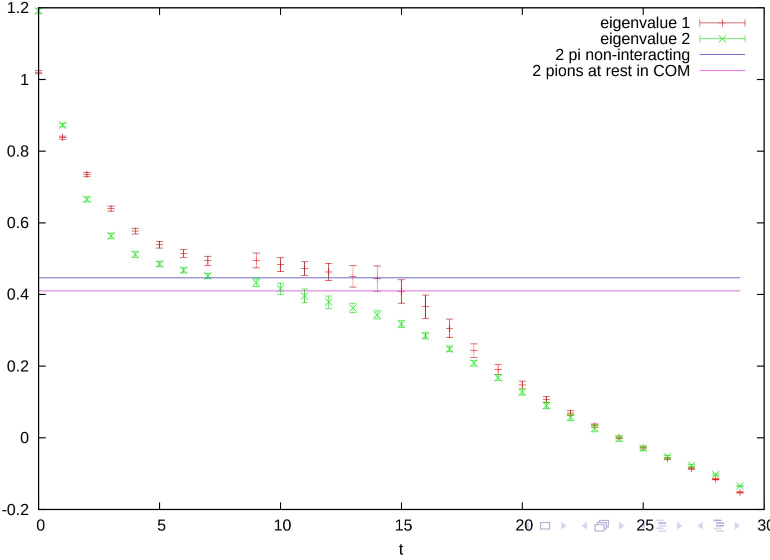

$C_{22}(t) =$



# Ensemble parameters

$$
\begin{aligned}
L^3 T &= 32^4 \\
configs &= 122 \\
\beta &= 2.0 \\
am_0 &= -0.958 \\
af_\pi &= 0.049(3) \\
am_\pi &= 0.18(1) \\
am_\rho &= 0.38(5)
\end{aligned}
$$

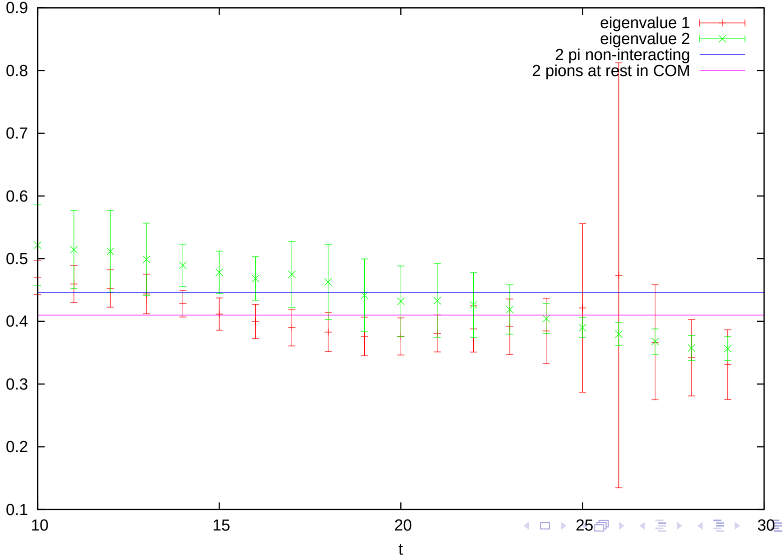
Eigenvalue $\lambda_i(t) = \exp\left(-E_i(t - t_0)\right)$
Effective mass $E_i(t) = -\ln \lambda_i(t)/(t_0 - t)$



# Effective mass plots

Eigenvalue $\lambda_i(t) = \exp\left(-E_i(t - t_0)\right)$

Effective mass $E_i(t) = -\ln \lambda_i(t)/(t_0 - t)$



# Conclusions

First attempt at calculating $\rho$ resonance mass and decay width in non-QCD theory.
Early results - one ensemble with possibly stable rho.
Future work:

- Include centre-of-mass frame to make the fit to $M_R$ and $g_{\rho\pi\pi}$ more reliable
- Several quark masses $\rightarrow$ chiral extrapolation
- Several lattice spacings $\rightarrow$ continuum extrapolation