Determination of topological charge following several definitions

and preliminary results of χ_t in $N_f = 1 + 2$

Julien Frison, Ryuichiro Kitano, Nori Yamada
KEK

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The strong CP problem today

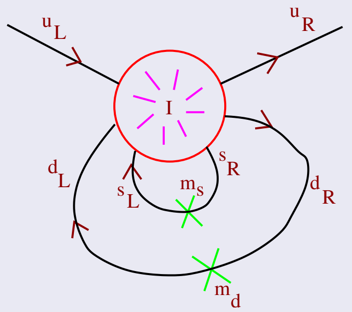
- Why is there no $\theta F\tilde{F}$ term in the Lagrangian?
- Trivial solution: $m_u e^{i\theta} = 0$
- Other popular solution: Peccei-Quinn mechanism (axion)

$m_u = 0$ solution

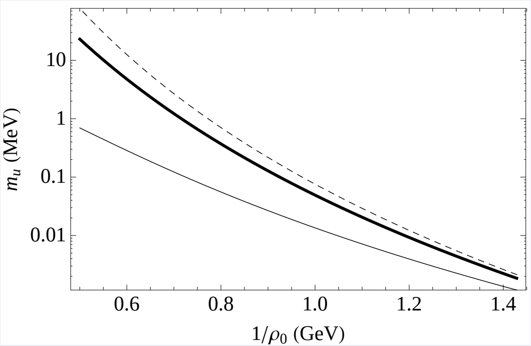
- New lattice computations make $m_u^{\overline{\rm MS}} = 0$ very unlikely
- Is $m_u = 0$ physically defined without massless pion?
- Is perturbative $\overline{\rm MS}$ really what we need?
- Non-perturbative contributions make this solution ill-defined
- What latticists should really check is whether $\chi_t^{\rm physical} = 0$

Instanton contribution to the mass

't Hooft vertex [Creutz:0711.2640]



Instanton computation [Dine:1410.8505]



Topology on the lattice

- On the lattice, Q is ill-defined too!
- Only defined on smooth configurations
- How arbitrary are the definitions? Are some better than others?
- Bosonic versus fermionic definitions
- Does continuum limit trivially remove ambiguity? Even with Wilson fermions? On Q or on $\langle Q^2 \rangle$?

Topology ambiguity or mass ambiguity?

- Mass and topology are related through Ward identities
- Earlier works have tried to make both definitions compatible [Bochicchio'84-85-86]
- In general, arbitrary definitions will break singlet Ward identities at finite lattice spacing, and $\chi_t(m_u = 0) = 0$ is not guaranteed.
- In $N_\mathrm{f} = 2 + 1$, $\chi_t(m_u = 0) = 0$ has been empirically checked, agreeing with ChPT prediction $\chi_t^{-1} \propto \sum m^{-1}$
- What in $N_\mathrm{f} = 1 + \mathrm{smthg}$? "SU(1) ChPT" makes no sense.

Objective

We want to determine χ_t at $m_u^{PCAC} = 0$

- In $N_f = 1 + 2$, where $m_d = m_s^{\text{physical}}$ so that the 't Hooft vertex effect is amplified
- Only m_u will be taken close to zero
- We use Wilson-like fermions to study the worst scenario
- We choose parameters similar to BMW HEX2 $N_f = 2 + 1$ ensembles

Ensembles

$N_{\mathrm{f}} = 2 + 1$ Ensemble (cross-check)

$\beta = 3.31$ Lüscher-Weisz w/ HEX2 Clover ($a \sim 0.116\ \mathrm{fm}$), $m_{ud}^{\mathrm{bare}} = -0.07$, $m_s^{\mathrm{bare}} = -0.04$, $16^3 \times 32$

$N_{\mathrm{f}} = 1 + 2$ Ensembles

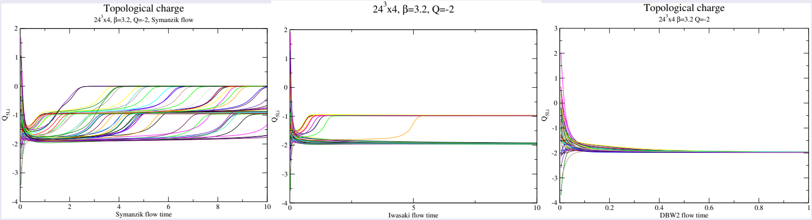
$m_u^{\mathrm{bare}} = -0.07, -0.093, -0.09756$, $m_{ds}^{\mathrm{bare}} = -0.04$, $16^3 \times 32$
A larger volume and a finer lattice are both being generated

Other Ensembles

Many quenched ensembles have been used for tests, either generated for this project or for another project

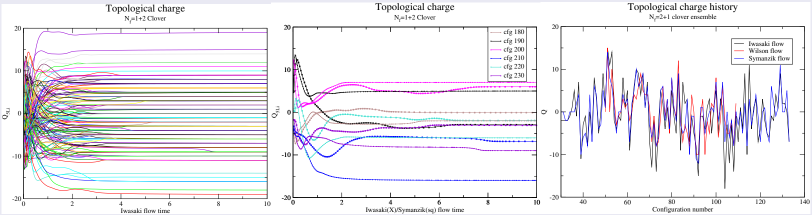
Gradient flow at large flow time

Fixed fermionic topology (finite temperature)



Remark: c_1 increases both stability and convergence speed ($n_c = (3 - 15c_1)\tau$ [Alexandrou:1509.04259])

Main ensembles

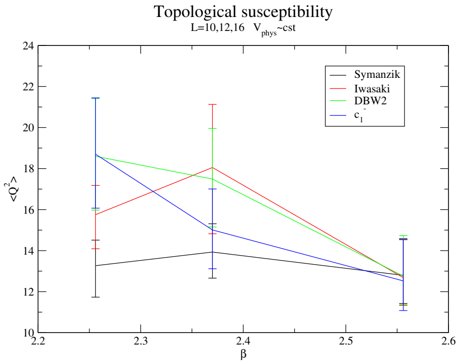


Continuum limit and universality

$\beta = \mathbf{2.256}$	Sym	Iwa	DBW2	c_1^{-}
Sym	X	0.922	0.914	0.908
Iwa	X	X	0.961	0.948
DBW2	X	X	X	0.984
c_1^{-}	X	X	X	X

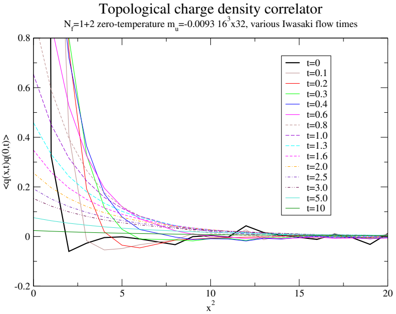
$\beta = \mathbf{2.37}$	Sym	Iwa	DBW2	c_1^{-}
Sym	X	0.985	0.969	0.954
Iwa	X	X	0.989	0.976
DBW2	X	X	X	0.989
c_1^{-}	X	X	X	X

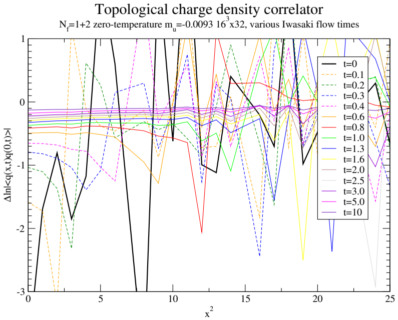
$\beta = \mathbf{2.556}$	Sym	Iwa	DBW2	c_1^{-}
Sym	X	0.981	0.974	0.977
Iwa	X	X	0.994	0.991
DBW2	X	X	X	0.998
c_1^{-}	X	X	X	X



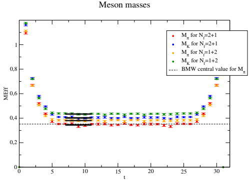
- Quenched ensembles at fixed physical volume
- Strong correlations at finest ensemble
- Nevertheless individual Q values almost never agree/plateau
- The closer the c_1 the stronger the correlation

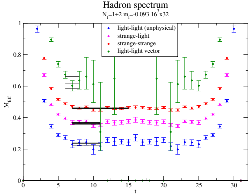
Topological Charge Density Correlator

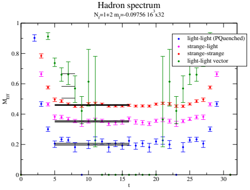


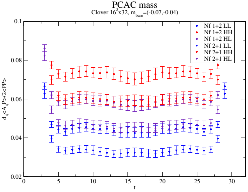


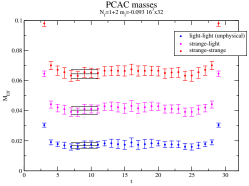
Spectrum and PCAC masses

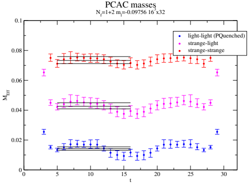




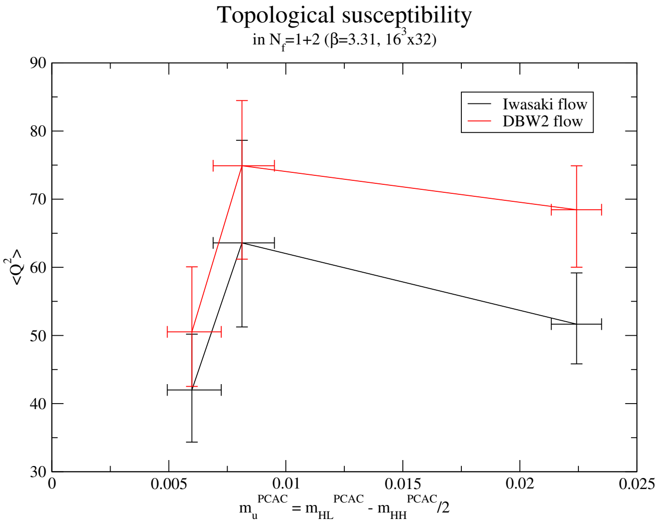








(m_u, χ_t) plot



Topological susceptibility
in N_f=1+2 (β=3.31, 16^3x32)
Iwasaki flow
DBW2 flow
<Q^2>
m_u^PCAC = m_HL^PCAC - m_HH^PCAC/2
90
80
70
60
50
40
30
0
0.005
0.01
0.015
0.02
0.025

- We suggest that the $m_u = 0$ solution to the strong CP problem should be assessed in terms of χ_t and not m_u
- we have presented a strategy to estimate or bound the mistake the PCAC method could make
- We have presented preliminary results in $N_\mathrm{f} = 1 + 2$
- Unfortunately we have not been able to explore much of the expensive $\mathrm{Index}(D_\mathrm{ov})$ approach
- We have large statistical errors for the moment
- We need lighter quarks, finer ensembles, and probably larger volumes
- Investigating $m_u \sim 0$ ($\chi_t \sim 0$) may require specific methods (see hep-lat/1606.07175)

Thanks for your attention!