The QCD equation of state at finite density from analytical continuation

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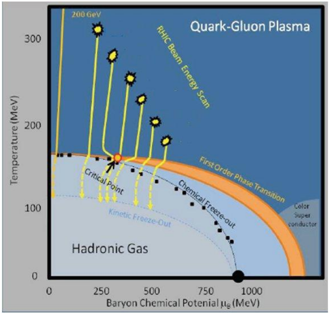
SFB TR55

BERGISCHE UNIVERSITÄT WUPPERTAL

WB collaboration

The (T, μ_B)-phase diagram of QCD



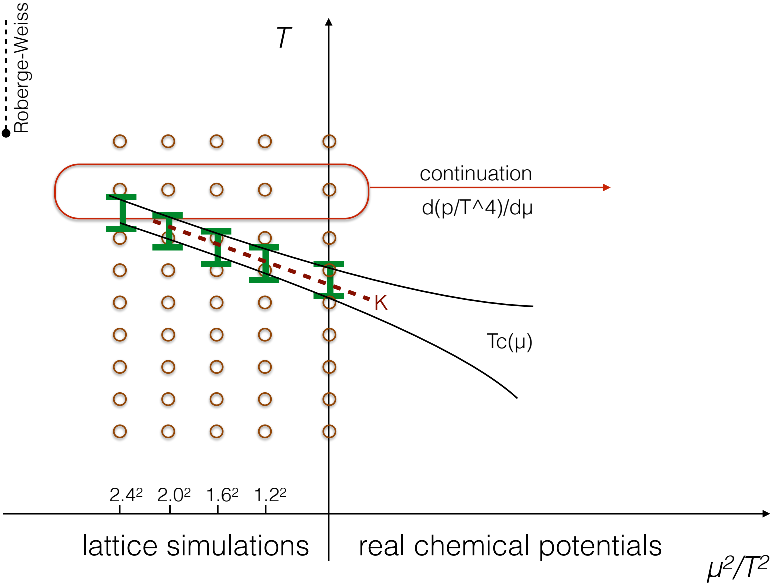
Our observables:
Last Year: T_c

R. Bellwied et al., Phys. Lett. B751, 559 (2015), arXiv:1507.07510

This year: The Equation of State along trajectories of constant $\frac{S}{N_B}$ and its Taylor coefficients determined by the method of analytical continuation

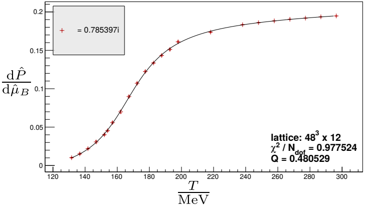
J. Günther et al., arXiv:1607.02493

Analytic continuation

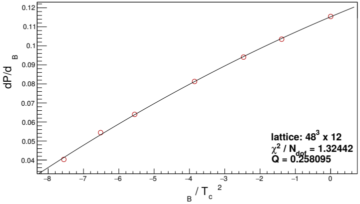


Overview over the Analysis

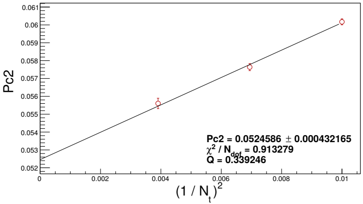
1. Do the simulations at $\langle n_s \rangle \approx 0$
2. Extrapolate to $\langle n_s \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$
3. Make a fit in the T direction



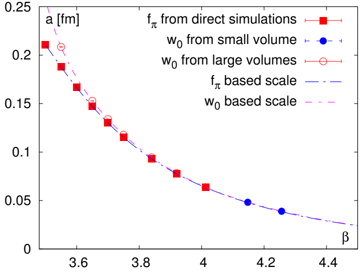
4. Determine everything you need for the observables
5. Make a fit in the μ_B direction



6. Make a fit in the $\frac{1}{N_t^2}$ direction
7. Determine the observables



Simulation details



- Action: tree-level Symanzik improved gauge action, with four times stout smeared staggered fermions
- 2+1+1 flavour, on LCP with pion and kaon mass
- Simulation at $\langle n_S \rangle = 0$ (as for heavy ion collisions, in contrast to simulations with $\mu_s = 0$ or $\mu_S = 0$ where $\mu_S = \frac{1}{3}\mu_B - \mu_s$)
- Lattice sizes: $40^3 \times 10$, $48^3 \times 12$ and $64^3 \times 16$
- $\frac{\mu_B}{T} = \mathrm{i}\frac{j\pi}{8}$ with $j = 0, 3, 4, 5, 6, 6.5$ and 7
- Two methods of scale setting: f_π and w_0, $Lm_\pi > 4$

Tuning to $\langle n_S \rangle = 0$

Aim: For a given μ_B determine μ_S so that $\langle n_S \rangle = 0$. This means solving the differential equation

$$\langle n_S \rangle = 0 \Leftrightarrow \frac{\partial \log Z}{\partial \mu_S} = 0$$

Notation:

$$\chi_{udsc} = -\frac{1}{T^4} \frac{\partial^4}{\partial(\mu_u/T)\partial(\mu_d/T)\partial(\mu_s/T)\partial(\mu_c/T)} \frac{T}{V} \log Z$$

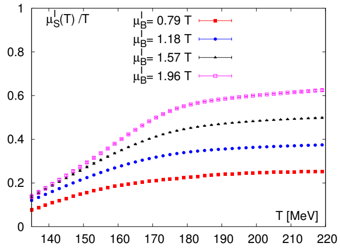
Assuming we know the value for $\mu_S(\mu_B)$ so that $\langle n_s \rangle = 0$ for $\mu_S(\mu_B^0)$ and $\mu_S(\mu_B^0 - \Delta\mu_B)$ with all the derivatives. Then (Runge-Kutta):

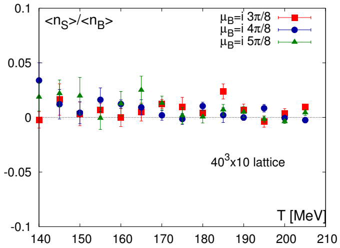
$$\mu_S(\mu_B^0 + \Delta\mu_B) = \mu_S(\mu_B^0 - \Delta\mu_B) + 2\Delta\mu_B \frac{\mathrm{d}\mu_S}{\mathrm{d}\mu_B}(\mu_B^0).$$

In the simulations with μ_B^0 and $\mu_B^0 - \Delta\mu_B$, μ_S might not precisely tuned. There we want to extrapolate to a better value. We assume that correct value of μ_S is $\tilde{\mu}_S = \mu'_S + \Delta\mu'_S$. Then:

$$\langle n_S \rangle = \frac{\partial \log Z}{\partial \tilde{\mu}_S} = \frac{\partial \log Z}{\partial \mu'_S} + \frac{\partial^2 \log Z}{\partial \mu'^2_S} \Delta\mu'_S = 0$$

Tuning to $\langle n_S \rangle = 0$





This yields

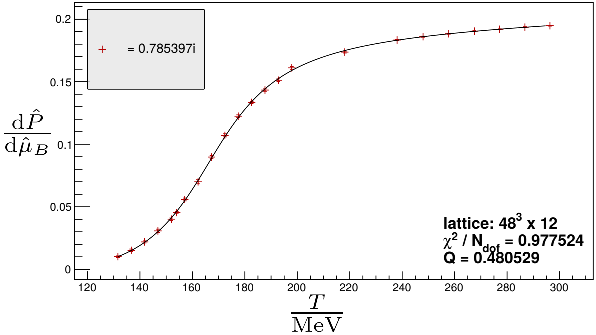
$$\Delta\mu'_S = -\frac{\chi_S}{\chi_{SS}}$$

Similar for the derivative we get:

$$\frac{\mathrm{d}\tilde{\mu}_S}{\mathrm{d}\mu_B} = -\frac{\tilde{\chi}_{SB}}{\tilde{Z}_{SS}}\Big|_{\langle n_S\rangle=0} = -\frac{\chi_{SB}}{\chi_{SS}} - \frac{\chi_{SSB}\chi_{SS} - \chi_{SSS}\chi_{SB}}{(\chi_{SS})^2}\Delta\mu'_S + \mathcal{O}(\Delta\mu'^2_S)$$

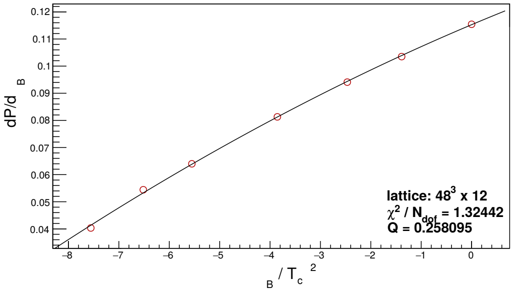
With similar arguments we get to extrapolate to $\langle n_Q \rangle = 0.4\langle n_B \rangle$

Fit in the T direction



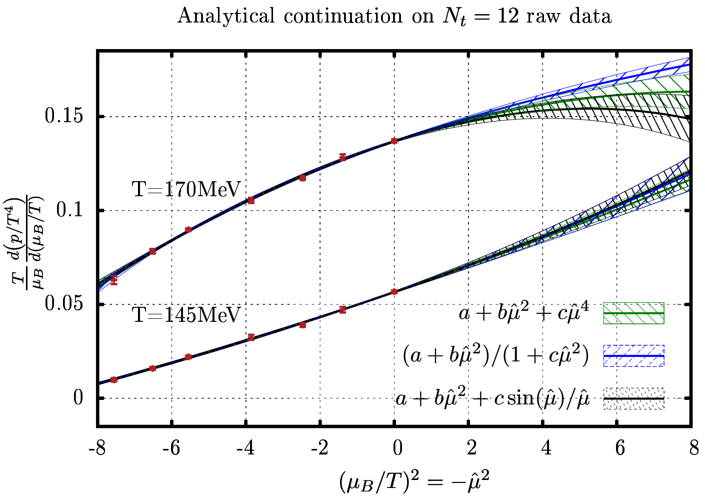
$$
\begin{aligned}
A_1(T) &= a + bT + c/T + d\arctan(e(T-f)) \\
A_2(T) &= a + bT + c/T + d/(1 + e(T-f)^g)^{1/g}, \\
A_3(T) &= a + bT + cT^2 + d\arctan(e(T-f)) \\
A_4(T) &= a + bT + cT^2 + d/(1 + e(T-f)^g)^{1/g}.
\end{aligned}
$$

Fit in the μ_B direction



$$
\begin{aligned}
B_1(\hat{\mu}) &= a + b\hat{\mu}^2 + c\hat{\mu}^4 \\
B_2(\hat{\mu}) &= (a + b\hat{\mu}^2)/(1 + c\hat{\mu}^2) \\
B_3(\hat{\mu}) &= a + b\hat{\mu}^2 + c\sin(\hat{\mu})/\hat{\mu}
\end{aligned}
$$

Extrapolation from different fit functions



Entropy

- $S = \left.\frac{\partial P}{\partial T}\right|_{\mu_i}$
- We have $\frac{\mathrm{d}\hat{P}}{\mathrm{d}\hat{\mu}_B}$
- And we can only do a total derivative in T

$$\frac{\mathrm{d}\hat{P}}{\mathrm{d}T} = \left.\frac{\partial \hat{P}}{\partial T}\right|_{\mu_i} + \frac{\mathrm{d}\mu_B}{\mathrm{d}T}\frac{\hat{n}_B}{T} + \frac{\mathrm{d}\mu_Q}{\mathrm{d}T}\frac{\hat{n}_Q}{T}$$

How we correct this:

$$\frac{\mathrm{d}\mu_B}{\mathrm{d}T} = \frac{\mathrm{d}(\hat{\mu}_B T)}{\mathrm{d}T} = \hat{\mu}_B$$

$$\frac{\mathrm{d}\mu_Q}{\mathrm{d}T} = \frac{\mathrm{d}(\hat{\mu}_Q T)}{\mathrm{d}T} = \hat{\mu}_Q + T\frac{\mathrm{d}\hat{\mu}_Q}{\mathrm{d}T}$$

$$\hat{S} = 4\hat{P} + T\frac{\mathrm{d}\hat{P}}{\mathrm{d}T} - \hat{\mu}_B\hat{n}_B - \hat{\mu}_Q\hat{n}_Q - T\frac{\mathrm{d}\hat{\mu}_Q}{\mathrm{d}T}$$

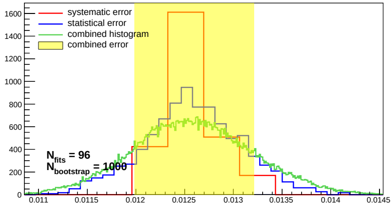
$$= 4\hat{P} + T\frac{\mathrm{d}\hat{P}}{\mathrm{d}T} - \hat{n}_B\left(\hat{\mu}_B + 0.4\hat{\mu}_Q\right) - T\frac{\mathrm{d}\hat{\mu}_Q}{\mathrm{d}T}$$

New terms we have to determine

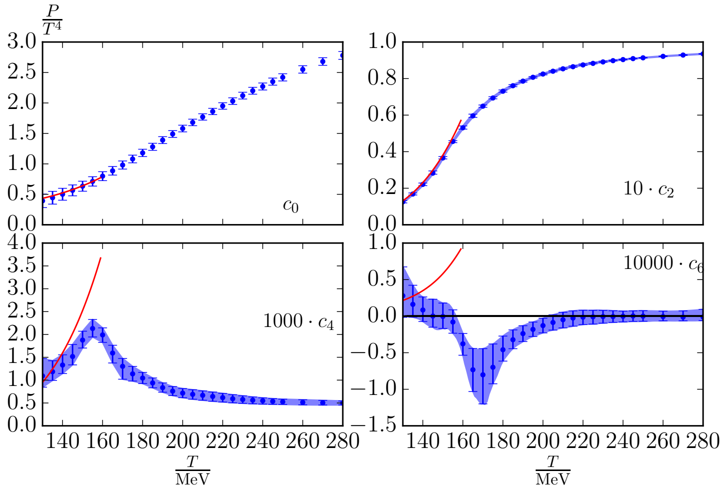
Error estimation

- Statistical error:
  Bootstrap method
- Systematic error:
  Using different way of analysis, combining them in a histogram:
  - 4 fit functions for the T direction
  - 3 fit functions in the μ_B direction
  - Doing continuum extrapolation and μ_B-fit in one or two steps
  - 2 methods of scale setting: f_π and w_0
  - 2 temperatures from where we use the extrapolated data

  This adds up to 96 ways of analysis



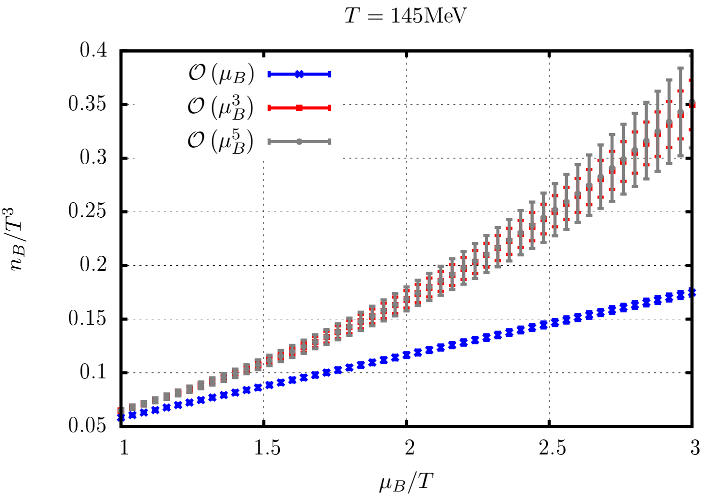
Taylor coefficients



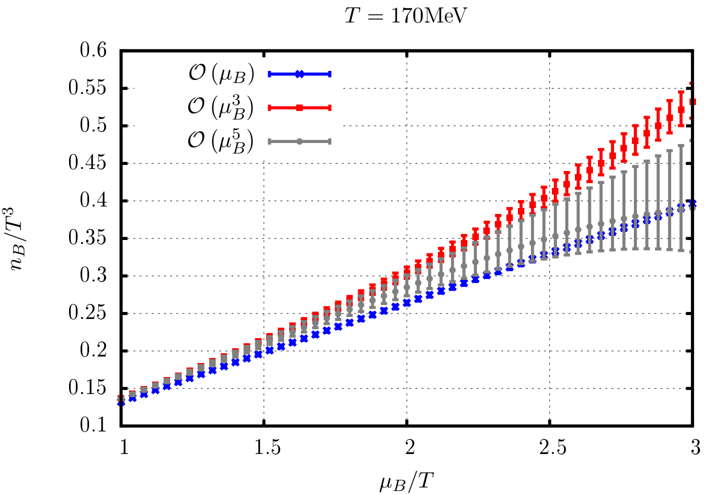
The Taylor coefficients of $\frac{P}{T^4} = c_0 + c_2\left(\frac{\mu_B}{T}\right)^2 + c_4\left(\frac{\mu_B}{T}\right)^4 + c_6\left(\frac{\mu_B}{T}\right)^6$

c_0 is from *S. Borsanyi et al, Phys. Lett. B730, 99 (2014), arXiv:1309.5258*

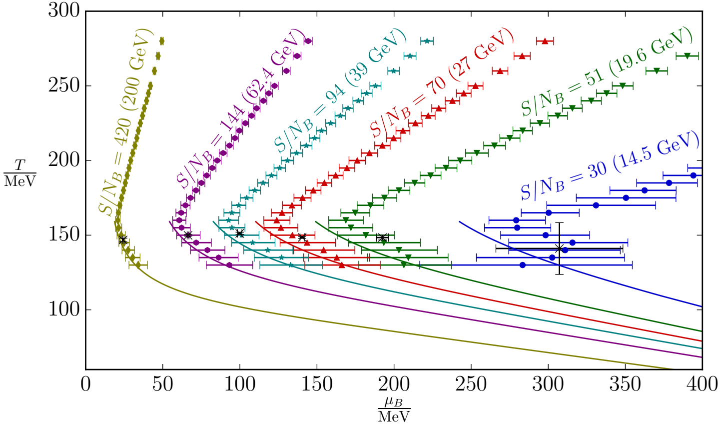
Influence of different orders



Influence of different orders



Trajectories



Equation of state