Using a new analysis method to extract excited states in the scalar meson sector

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  - Scalar channel $q\overline{q}$
* AMIAS (Athens Model Independent Analysis Scheme)
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  - Application
* AMIAS in practice
  - Search for a_0 excluding loops
  - Search for a_0 including loops

TARGET: extract mass states from correlation matrix

$\rightarrow$ for large time distances lowest state can be extracted, but ...

- how to extract if signal is only good for few time slices ?
- how to extract if signal is dominated by more than one states ?

We are using: ensemble generated with 2+1 dynamical clover fermions and Iwasaki gauge action by the PACS-CS Collaboration [Aoki et.al. 2008]
Lattice 64×32^3 with $a \sim 0.09$ fm and
~ 500 configurations at $M_\pi \sim 300$ MeV

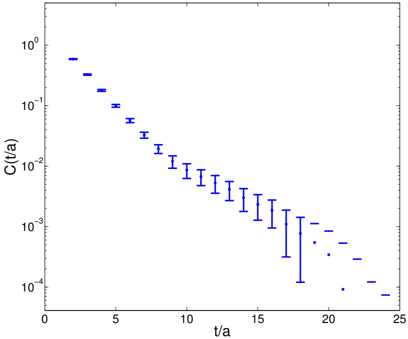
We are using a_0 interpolators with $J^P = 0^+$ [Joshua Berlin's Talk]
interpolators quark content: $\bar{d}u + \bar{s}s$

$\rightarrow$ 6x6 correlation matrix with $q\bar{q}, diQ\overline{diQ}, \pi\eta_s, 2K$

$\rightarrow$ we will start with the $q\bar{q}$ correlator

The Scalar channel: $q\overline{q}$

Operator with $J^P = 0^+$: $\mathcal{O}(x) = \overline{d}(x)u(x)$
Correlator $C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle$ on a periodic lattice:



Standard technique to extract effective masses using asymptotic behavior:

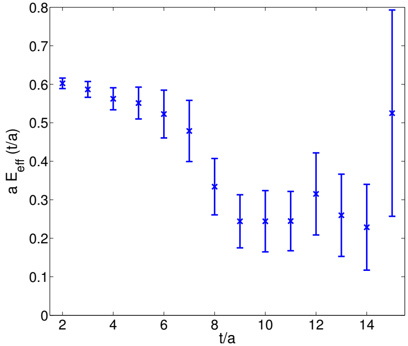
$$C(t/a) = \sum_i A_i \cosh(E_i(t - T/2)) \underset{t \gg 1, T \gg 1}{\longrightarrow} A_0 \cosh(E_0(t - T/2)) + \ldots \quad (1)$$

$\rightarrow$ here dominated by two states

The Scalar channel: $q\bar{q}$

Operator with $J^P = 0^+$: $\mathcal{O}(x) = \overline{d}(x)u(x)$

Effective mass on a periodic lattice:



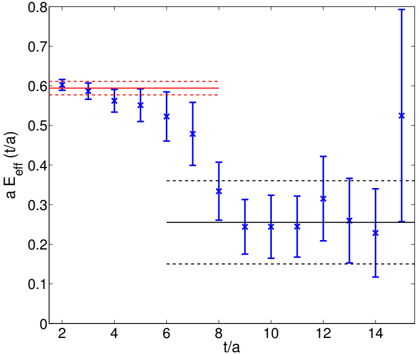
Standard technique to extract effective masses using asymptotic behavior:

$$\frac{C(t/a)}{C(t/a-1)} = \frac{\cosh(E_{eff}(t-T/2))}{\cosh(E_{eff}(t+1-T/2))} \tag{2}$$

$\rightarrow$ dominated by two states

The Scalar channel: $q\bar{q}$

here: lowest energy easy to identify (high statistics)
however: we are not interesting in lowest state (unphysical)
$\longrightarrow$ lattice artefact, backwards traveling pion

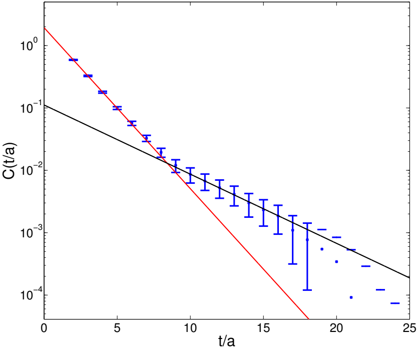


- higher state do not have a plateau
- can'nt be extracted by a simple plateau average

$\rightarrow$ what is with the correlator ?

The Scalar channel: $q\bar{q}$

here: we simply fit the logarithm behavior for $t \in [2;4]$ and $t \in [8;12]$



Another possiblity: fitting the correlator

$$f(t) = \sum_{n=0}^{N} A_n \cosh(E_n(t - T/2))$$

This is a non–linear minimization

AMIAS

- Standard least-squares algorithms:
  are numerically unstable,
  depend on the initial condition and
  fail for very low signal-to-noise ratio

$\Rightarrow$ to circumvent this we will use AMIAS

AMIAS (Athens Model Independent Analysis Scheme)

- relies on statistical concepts
- sample probability distributions of parameters
- also insensitive parameters are fully accounted and do not bias the results
- can access a large number of parameters by using Monte Carlo techniques

[Alexandrou et.al. arXiv:1411.6765],
[C. Papanicolas and E. Stiliaris, arXiv:1205.6505],
[E. Stiliaris and C. Papanicolas, AIP Conf. Proc. 904, 257 (2007)]

AMIAS: Basic Idea

- Using the χ^2 of the fit function $f(t) = \sum_{n=0}^{N} A_i \cosh(E_n(t - T/2))$

$$\chi^2 = \sum_{k=1}^{N_t} \frac{(C(t_k) - \sum_{n=0}^{\infty} A_n \cosh\{E_n(t_k - T/2)\})^2}{(\sigma_{t_k}^2/N)}.$$

- Using the central limit theorem
$\Rightarrow$ each value assigned to the model parameters has a statistical weight proportional to

$$P(C(t_n); n) = e^{-\frac{\chi^2}{2}}$$

Model parameters

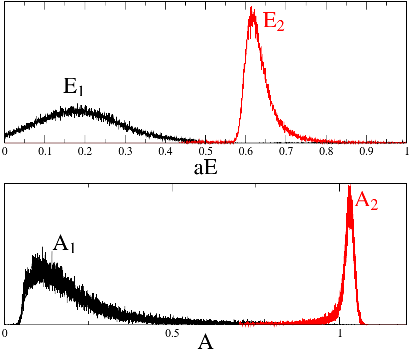
The probability for parameter $A_i, (E_i)$ to have a specific value a_i is given by

$$\Pi(A_i = a_i) = \frac{\int_{b_i}^{c_i} dA_i \int_{-\infty}^{\infty} \prod_{j \neq i} dA_j, A_i e^{-\tilde{\chi}^2/2}}{\int_{-\infty}^{\infty} \left(\prod_j dA_j\right) A_i e^{-\tilde{\chi}^2/2}}.$$

$\Rightarrow$ Multi-dimensional integrals $\rightarrow$ Monte Carlo sampling with $P(C(t_n); n)$

Scalar channel: $q\bar{q}$

Results from AMIAS:

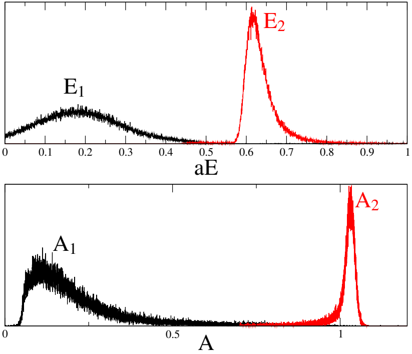


$\Rightarrow$ distribution of two energies and their corresponding apmlitudes

- clean distinction
- need to set intervalls for identifying them
  we use multiple tempering to explore the whole parameter region

Scalar channel: $q\bar{q}$

Results from AMIAS:



lattice artefact:

the π is propagating backwarts in time
generate a lattice artefact state with
$\rightarrow aE_{art} = a(m_{\eta_s} - m_\pi) \sim 0.2$

state around the a_0-region ~ 0.6:

in this region is the is $\pi + \eta_s$ and the 2 kaon channel
$\longrightarrow$ we need to couple the meson-meson interpolators to $q\bar{q}$

Using in search for a_0 particle

AMIAS: can be also adapted for correlation matrices ("Search for a_0 interpolators $J^P = 0^+$")

$$
\begin{aligned}
(j=0):\ & \mathcal{O}^{q\bar{q}} && = \sum_{\mathbf{x}} \left(\bar{d}_x u_x\right) \\
(j=1):\ & \mathcal{O}^{K\bar{K}}_{point} && = \sum_{\mathbf{x}} \left(\bar{s}_x \gamma_5 u_x\right)\left(\bar{d}_x \gamma_5 s_x\right) \\
(j=2):\ & \mathcal{O}^{\eta_{ss}\pi}_{point} && = \sum_{\mathbf{x}} \left(\bar{s}_x \gamma_5 s_x\right)\left(\bar{d}_x \gamma_5 u_x\right) \\
(j=3):\ & \mathcal{O}^{diQdi\bar{Q}} && = \sum_{\mathbf{x}} \epsilon_{abc}\left(\bar{s}_{x,b} C\gamma_5 \bar{d}^T_{x,c}\right)\epsilon_{ade}\left(u^T_{x,d} C\gamma_5 s_{x,e}\right) \\
(j=4):\ & \mathcal{O}^{K\bar{K}}_{2-part} && = \sum_{\mathbf{x},\mathbf{y}} \left(\bar{s}_x \gamma_5 u_x\right)\left(\bar{d}_y \gamma_5 s_y\right) \\
(j=5):\ & \mathcal{O}^{\eta_{ss}\pi}_{2-part} && = \sum_{\mathbf{x},\mathbf{y}} \left(\bar{s}_x \gamma_5 s_x\right)\left(\bar{d}_y \gamma_5 u_y\right)
\end{aligned}
$$

$\Rightarrow 6 \times 6$ correlation matrix:
$C_{jk}(t) = \langle \mathcal{O}_j(t)\mathcal{O}^\dagger_k(0)\rangle \underset{t \ll T}{\sim} \sum_{n=0}^{\infty} \langle 0|\mathcal{O}_j(t)|n\rangle\langle n|\mathcal{O}^\dagger_k(0)|0\rangle e^{-E_n t}.$

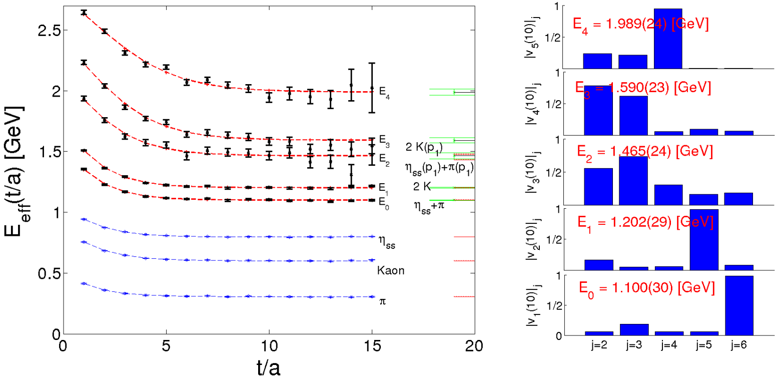
To analyze Correlation matrix:
Generalized Eigenvalue Problem (GEVP) and AMIAS
cross check with GEVP by solving:

$$[C(t)]\ v_n(t,t_0) = \lambda_n(t,t_0)\ [C(t_0)]\ v_n(t,t_0),$$

on the otherside: using Amias by fitting every matrix element with:

$$C_{jk}(t) = \sum_{n=0}^{\infty} A_j^{(n)} A_k^{*(n)} \cosh\{-E_n(t - T/2)\}.$$

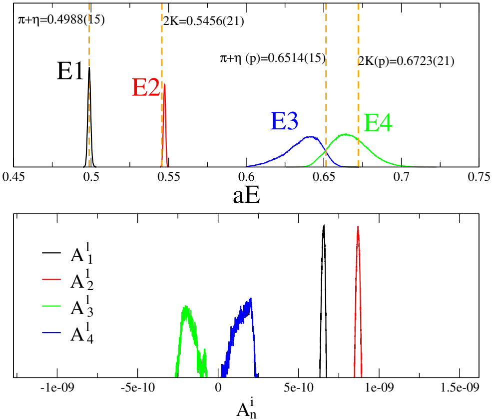
Measurements: GEVP 5x5 without loops



- Lowest states coincidence with $\eta_s\pi$ and 2–kaon
- Diquark-antidiquark interpolating field ($j = 4$) mixed with excited states
- No a_0 candidate $\longrightarrow$ expected to be around the $\eta_s\pi$ and the 2–kaon states

Measurements: AMIAS 4x4 without loops

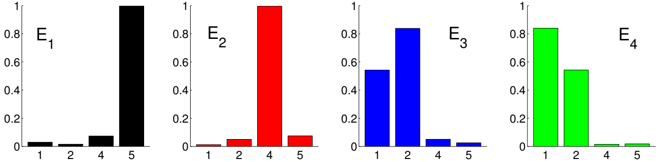
Using AMIAS for correlation matrix
with 2–meson interpolators ($2 \times (\pi + \eta_s)$ and $2 \times 2\mathrm{K}$)



- Energies correspondes to expectations from single channels
- No a_0 candidate $\longrightarrow$ expected to be around the $\eta_s\pi$ and the 2–kaon states
- Clear signal for energies and corresponding amplitudes

Measurements: AMIAS 4x4 without loops

Overlap of states with different interpolating fields ?
$\rightarrow$ using energies and corresponding amplitude to reconstruct correlation matrix
$\rightarrow$ extract overlap from eigenvectors of the orthogonalized GEVP

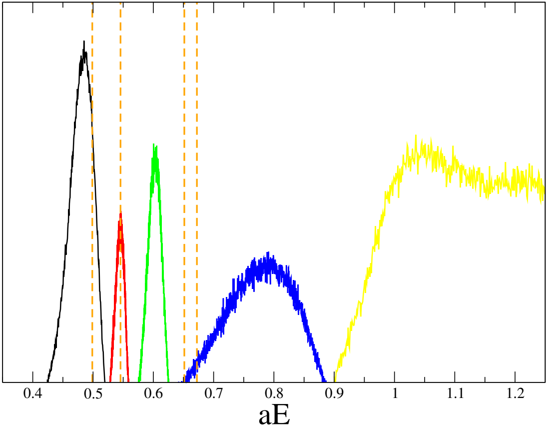


$\rightarrow$ agree with results of *pure* GEVP

$\rightarrow$ needs to identify the amplitudes
$\rightarrow$ can be difficult due to overlapping amplitudes/energies

Measurements: AMIAS 6x6 with loops *PRELIMINARY*

4 lowest states can be resolved (higher states can not be identify)



- strange quark loops introduces large noise (higher states very difficult to resolve)
- indication for an addtional state in the a_0 region
  $\rightarrow$ measurements are ongoing *(this are PRELIMINARY results)*

Conclusions

AMIAS:

- sampling of fit parameters by using χ^2
- can be used for single channels and correlation matrices

avoid plateau fits
can extrace states from channels which are dominated by more than one state
$\rightarrow$ like it is done in case of $q\overline{q}$

AMIAS in the search for a0:

without loops: resolve four lowest state
with loops **:** a_0 cannidate

Ongoing:

- increasing statistics
- investigate AMIAS: for example:
  - excluding noisy matrix elements
  - identification of amplitudes and energies

Thanks