

B_(s) to D_(s) semileptonic decays with NRQCD-HISQ valence quarks

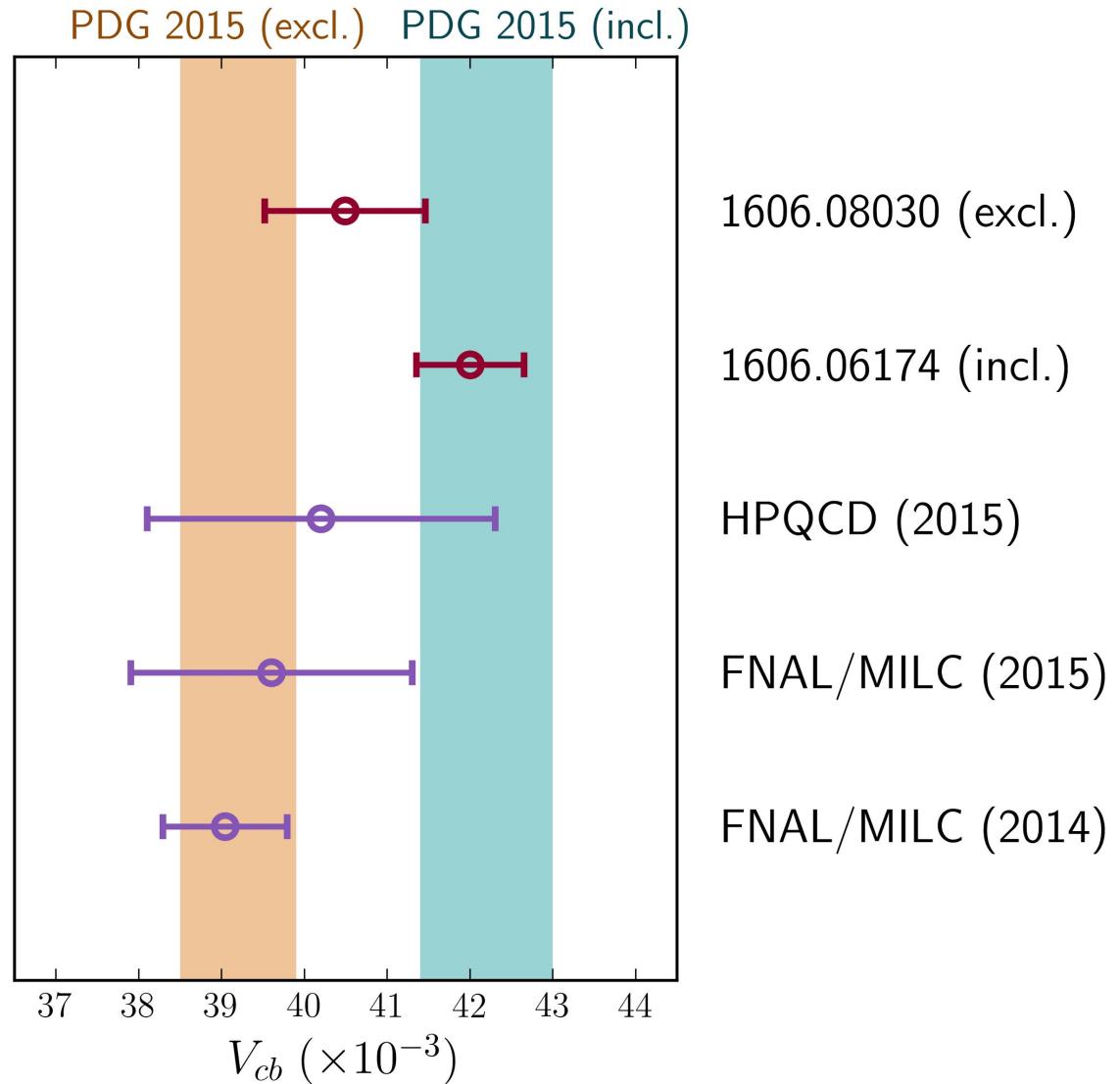
Christopher Monahan

New High Energy Theory Center
Rutgers, The State University of New Jersey

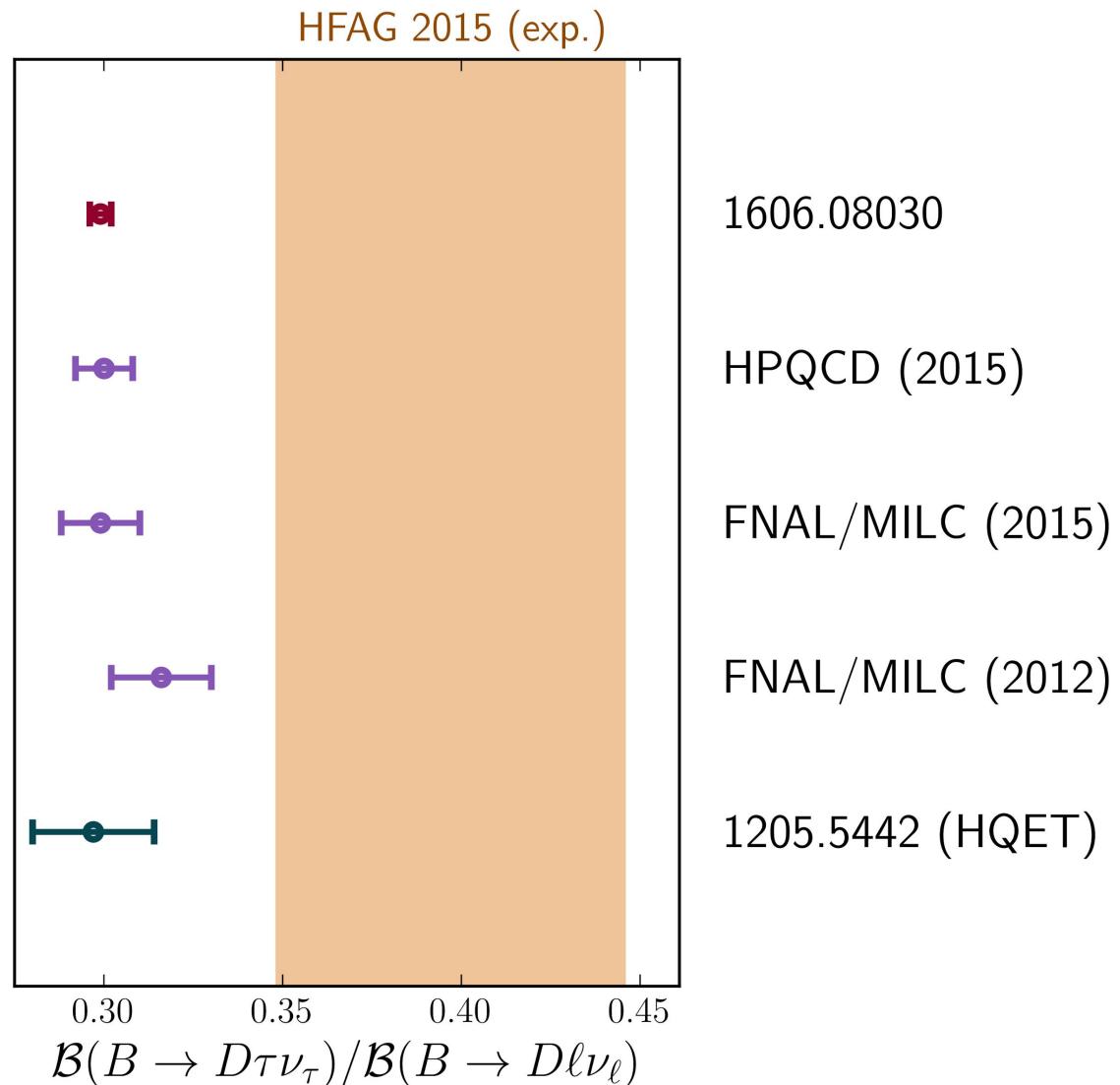
HPQCD Collaboration

Heechang Na, Chris Bouchard, G. Peter Lepage, and Junko Shigemitsu

Semileptonic B to D decays



Semileptonic B to D decays



Motivation

$B_{(s)} \rightarrow D_{(s)} \ell \nu$ semileptonic decays provide determination of V_{cb}

Two lattice calculations of $B_s \rightarrow D_s \ell \nu$ at or near zero recoil

FNAL/MILC

Bailey et al [FNAL/MILC], PRD 85 (2012) 114502

Twisted mass fermions

Atoui et al, EPJ C 74 (2014) 2861

First unquenched analysis at nonzero recoil

Combined analysis of $B_s \rightarrow D_s \ell \nu$ and $B \rightarrow D \ell \nu$ to reduce theoretical uncertainties in determination of $\mathcal{B}(B_s \rightarrow \mu^+ \mu^-)$

Forthcoming LHCb analysis of $B_s \rightarrow D_s \ell \nu$ and $B_s \rightarrow K \ell \nu$ for $|V_{ub}/V_{cb}|$

Form factors

Form factors

$$q^\mu = p_{B_s}^\mu - p_{D_s}^\mu$$

$$\langle D_s(p_{D_s}) | V^\mu | B_s(p_{B_s}) \rangle = f_+(q^2) \left[p_{D_s}^\mu + p_{B_s}^\mu - \frac{M_{B_s}^2 - M_{D_s}^2}{q^2} q^\mu \right] + f_0(q^2) \frac{M_{B_s}^2 - M_{D_s}^2}{q^2} q^\mu$$

Convenient to determine

$$\langle D_s(p_{D_s}) | V^\mu | B_s(p_{B_s}) \rangle = \sqrt{2M_{B_s}} \left[f_{\parallel}(q^2) \frac{p_{B_s}^\mu}{M_{B_s}} + f_{\perp}(q^2) \left(p_{D_s}^\mu - \frac{p_{B_s} \cdot p_{D_s}}{M_{B_s}^2} p_{B_s}^\mu \right) \right]$$

In B_s rest frame

$$\langle D_s(p_{D_s}) | V^0 | B_s(p_{B_s}) \rangle = \sqrt{2M_{B_s}} f_{\parallel}(q^2), \quad \langle D_s(p_{D_s}) | V^k | B_s(p_{B_s}) \rangle = \sqrt{2M_{B_s}} p_{D_s}^k f_{\perp}(q^2)$$

Reconstruct form factors

$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_s}}} [f_{\parallel}(q^2)(M_{B_s} - E_{D_s})f_{\perp}(q^2)]$$

$$f_0(q^2) = \frac{\sqrt{2M_{B_s}}}{M_{B_s}^2 - M_{D_s}^2} [(M_{B_s} - E_{D_s})f_{\parallel}(q^2)(E_{B_s}^2 - M_{D_s}^2)f_{\perp}(q^2)]$$

Heavy quark currents

NRQCD bottom and HISQ charm valence quarks

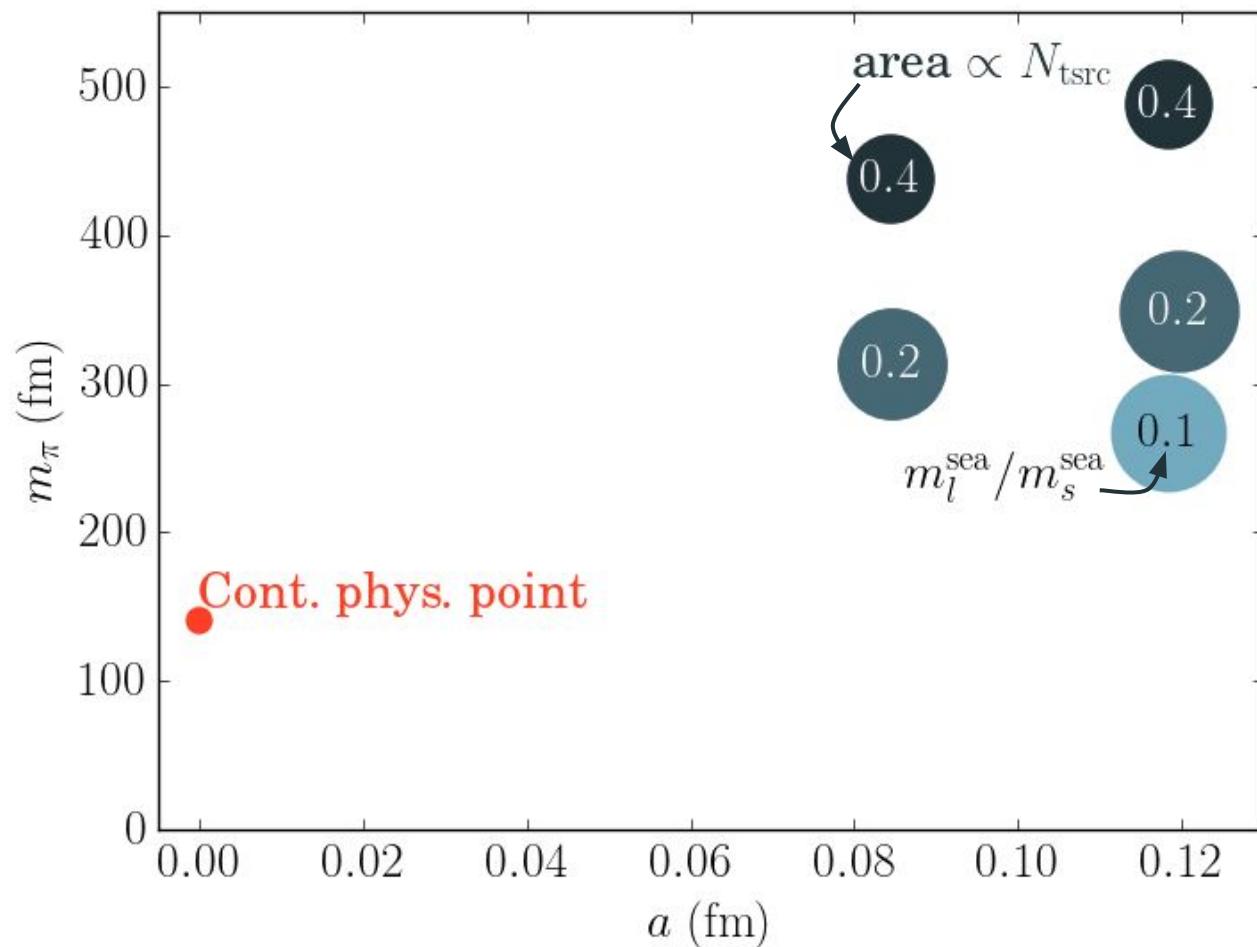
$$J_\mu^{(0)} = \bar{\psi}_c \gamma_\mu \Psi_b, \quad J_\mu^{(1)} = -\frac{1}{M_b} \bar{\psi}_c \gamma_\mu \boldsymbol{\gamma} \cdot \boldsymbol{\nabla} \Psi_b \quad \mathcal{O}(\alpha_s, \Lambda_{\text{QCD}}/M_b, \alpha_s/(aM_b))$$

Match currents via lattice perturbation theory  CJM et al, PRD 87 (2013) 034017

$$\langle V_\mu \rangle_{\text{QCD}} = (1 + \alpha_s \rho_\mu) \langle J_\mu^{(0)} \rangle + \langle J_\mu^{(\text{sub})} \rangle$$
$$J_\mu^{(\text{sub})} = J_\mu^{(1)} - \alpha_s \zeta_\mu J_\mu^{(0)}$$

Ensembles

MILC 2+1 asqtad ensembles



Correlators

B_s meson

$$\Phi_{B_s}^{\alpha\dagger}(\mathbf{x}, t_0) = a^3 \sum_{\mathbf{x}'} \bar{\Psi}_b(\mathbf{x}', t_0) \phi^\alpha(\mathbf{x}' - \mathbf{x}) \gamma_5 \psi_s(\mathbf{x}, t_0)$$

$\delta^{(3)}(\mathbf{x}' - \mathbf{x})$ $e^{-|\mathbf{x}' - \mathbf{x}|/(2r_0^2)}$ $r_0/a = 5, 7$

Construct 2x2 correlator matrix

$$C_{B_s}^{\beta,\alpha}(t, t_0) = \frac{1}{L^3} \sum_{\mathbf{x}, \mathbf{y}} \langle \Phi_{B_s}^\beta(\mathbf{y}, t) \Phi_{B_s}^{\alpha\dagger}(\mathbf{x}, t_0) \rangle$$

D_s meson

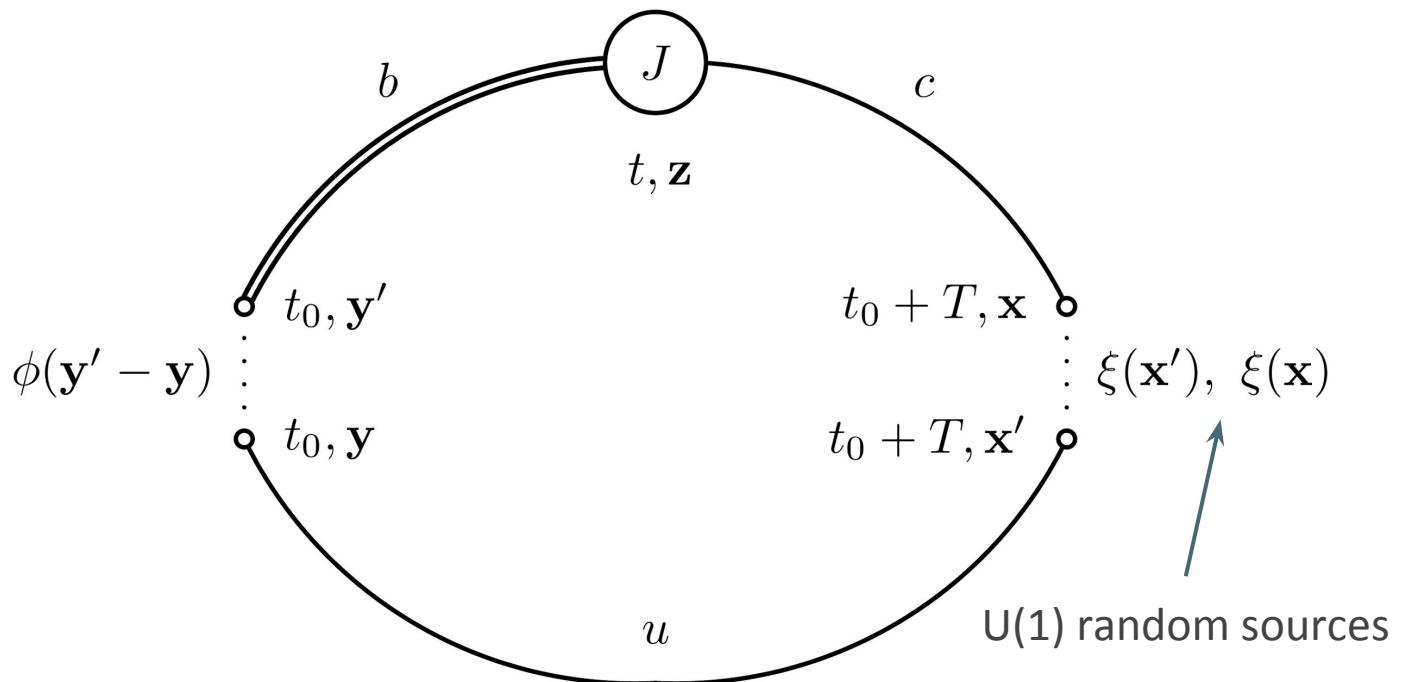
$$\Phi_{D_s}^\dagger(\mathbf{x}, t_0) = a^3 \bar{\psi}_c(\mathbf{x}, t_0) \gamma_5 \psi_s(\mathbf{x}, t_0)$$

Two-point correlator function

$$C_{D_s}(t, t_0; \mathbf{p}) = \frac{1}{16L^3} \sum_{\mathbf{x}, \mathbf{y}} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \langle \Phi_{D_s}(\mathbf{y}, t) \Phi_{D_s}^\dagger(\mathbf{x}, t_0) \rangle$$

Correlators

$$C_J^\alpha(t, t_0, T; \mathbf{p}) = \frac{1}{L^3} \sum_{x,y,z} e^{i\mathbf{p} \cdot (\mathbf{z} - \mathbf{x})} \langle \Phi_{D_s}(\mathbf{x}, t_0 + T) J_\mu(\mathbf{z}, t) \Phi_{B_s}^{\alpha\dagger}(\mathbf{y}, t_0) \rangle$$



Four momenta $[(0,0,0), (1,0,0), (1,1,0), (1,1,1)]$

Four values of T : $[12, 13, 14, 15]$ and $[21, 22, 23, 24]$

Correlator fits

Bayesian multi-exponential fitting strategy [`corrfitter`, `lsqfit`]

$$C_J^\alpha(t, T; \mathbf{p}) = \sum_{j=0}^{N_{B_s}-1} \sum_{k=0}^{N_{D_s}-1} A_{jk}^\alpha e^{-E_j^{D_s} t} e^{-E_k^{\text{sim}}(T-t)} + \dots$$

Extract three-point amplitudes

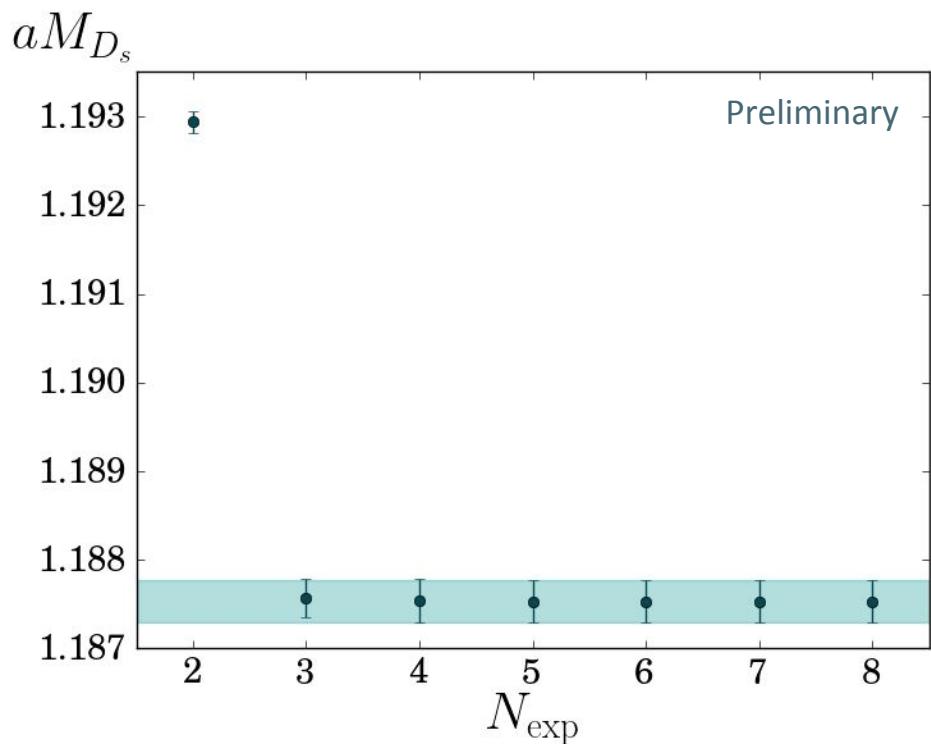
$$A_{jk}^\alpha = \frac{\langle 0 | \Phi_{D_s} | E_j^{D_s} \rangle \langle E_j^{D_s} | J_\mu | E_k^{B_s} \rangle \langle E_k^{B_s} | \Phi_{B_s}^{\alpha\dagger} | 0 \rangle}{(2a^3 E_j^{D_s})(2a^3 E_k^{B_s})}$$

Obtain required matrix element

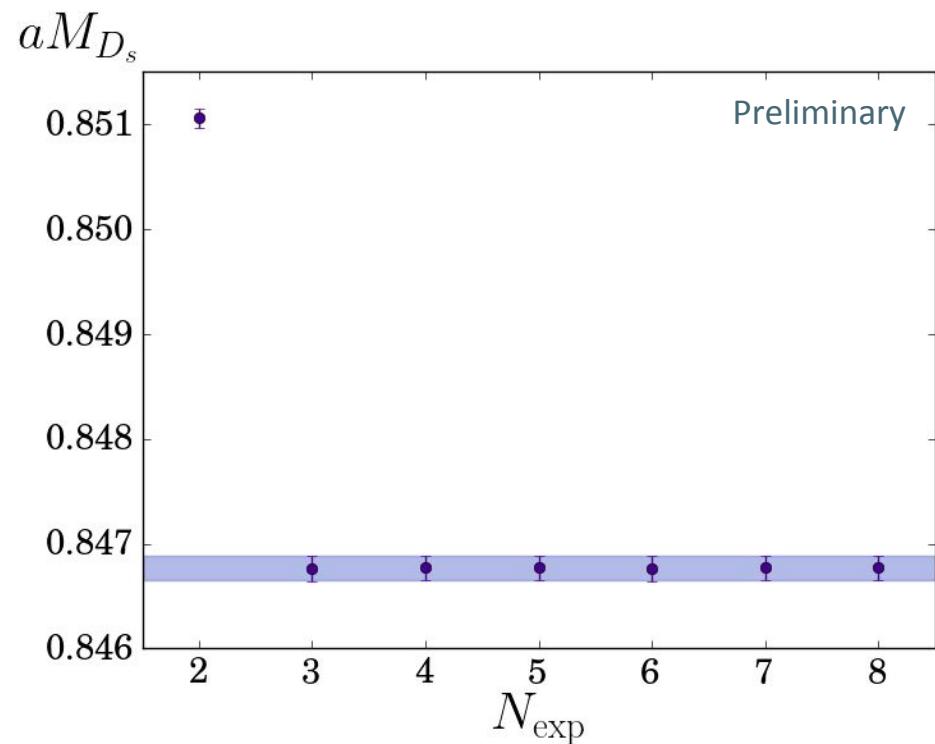
$$\langle D_s | J_\mu | B_s \rangle = A_{00}^\alpha \frac{(2a^3 E_0^{D_s})(2a^3 M_{B_s})}{\langle 0 | \Phi_{D_s} | E_0^{D_s} \rangle \langle M_{B_s} | \Phi_{B_s}^{\alpha\dagger} | 0 \rangle}$$

Two-point fits

Stability plots: multi-exponential fits



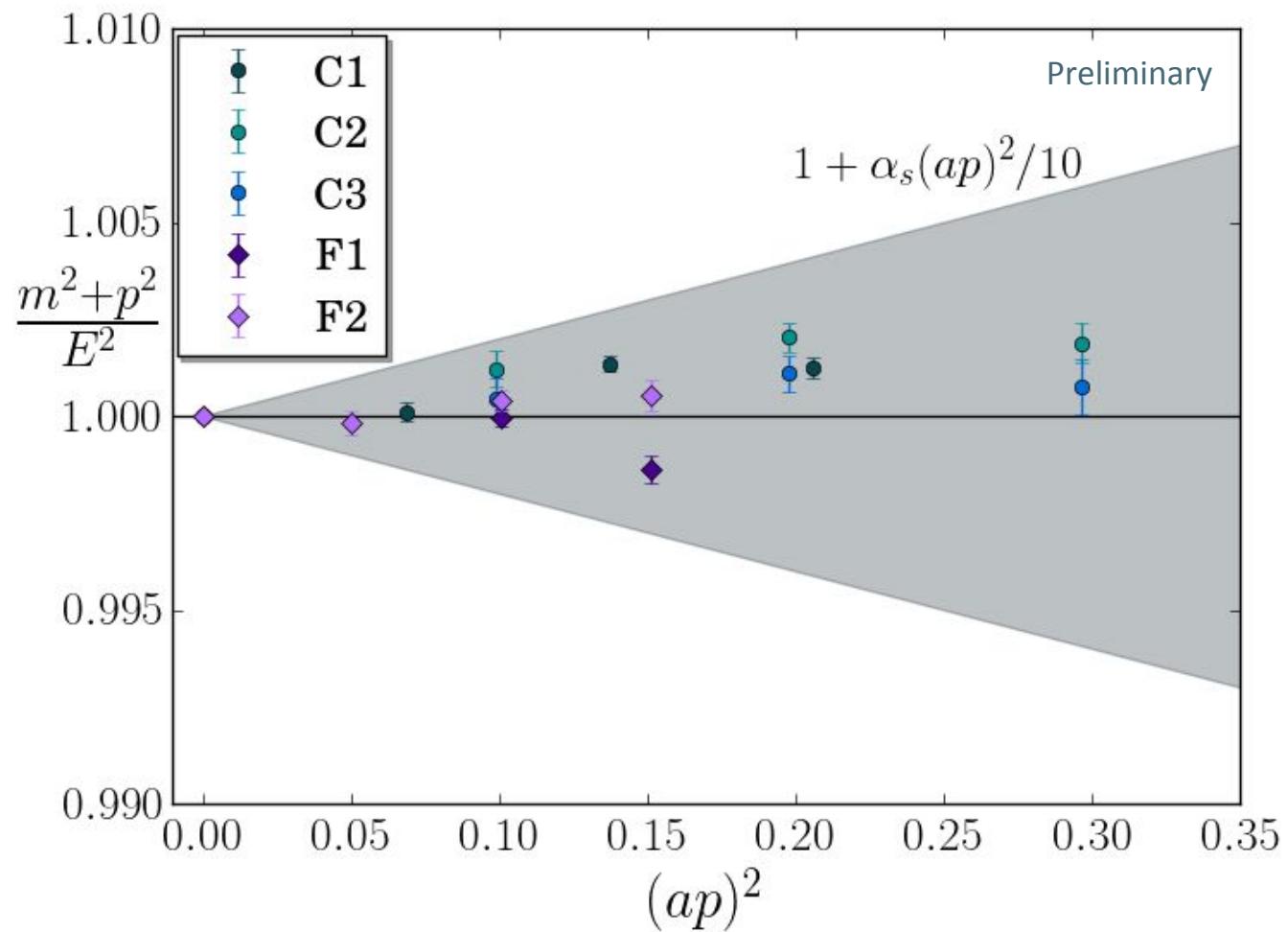
Ensemble C1



Ensemble F1

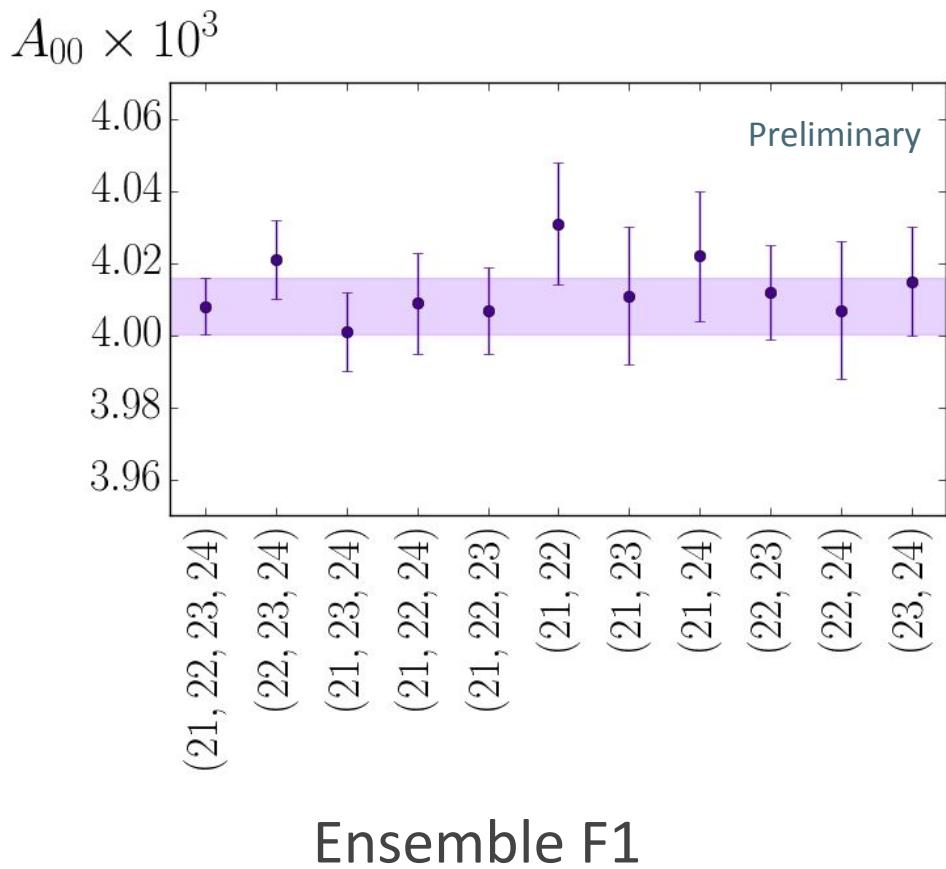
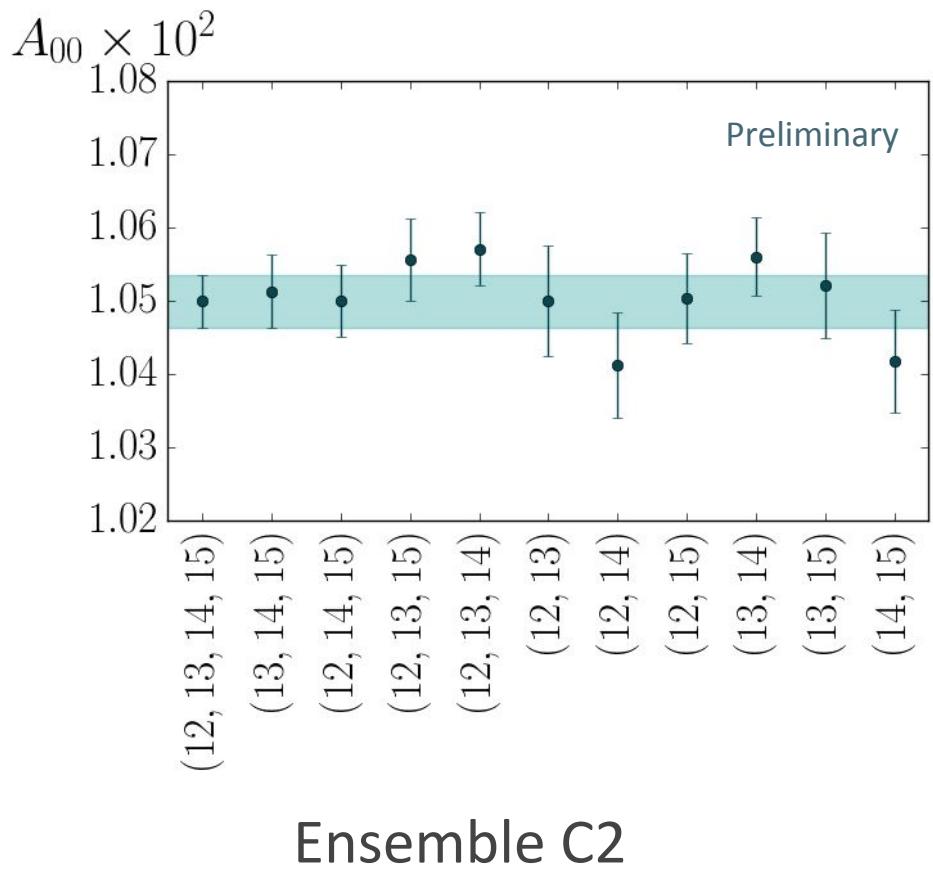
Two-point fits

D_s dispersion relation



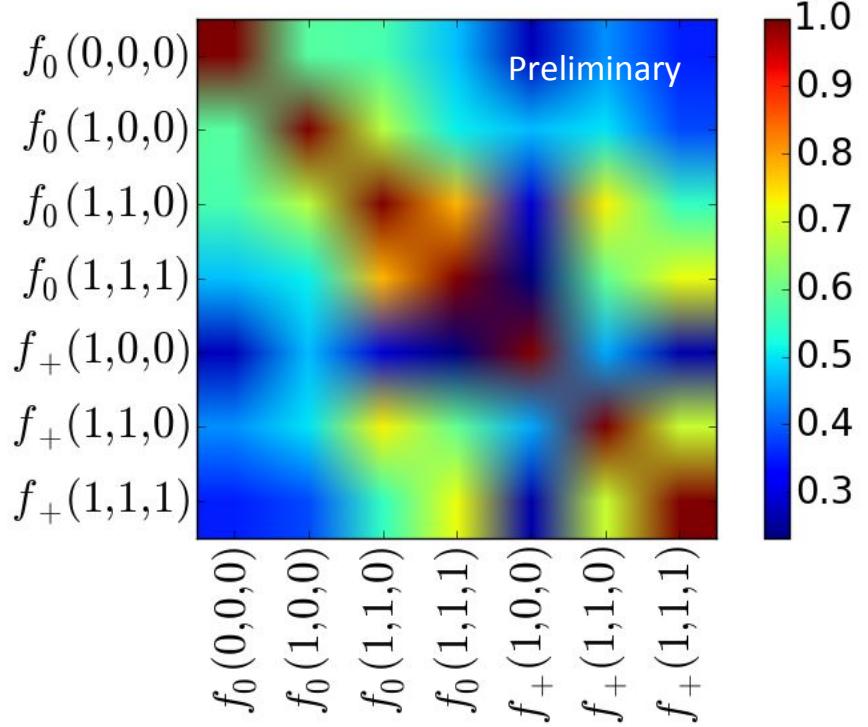
Three-point fits

Stability plots: T-combinations

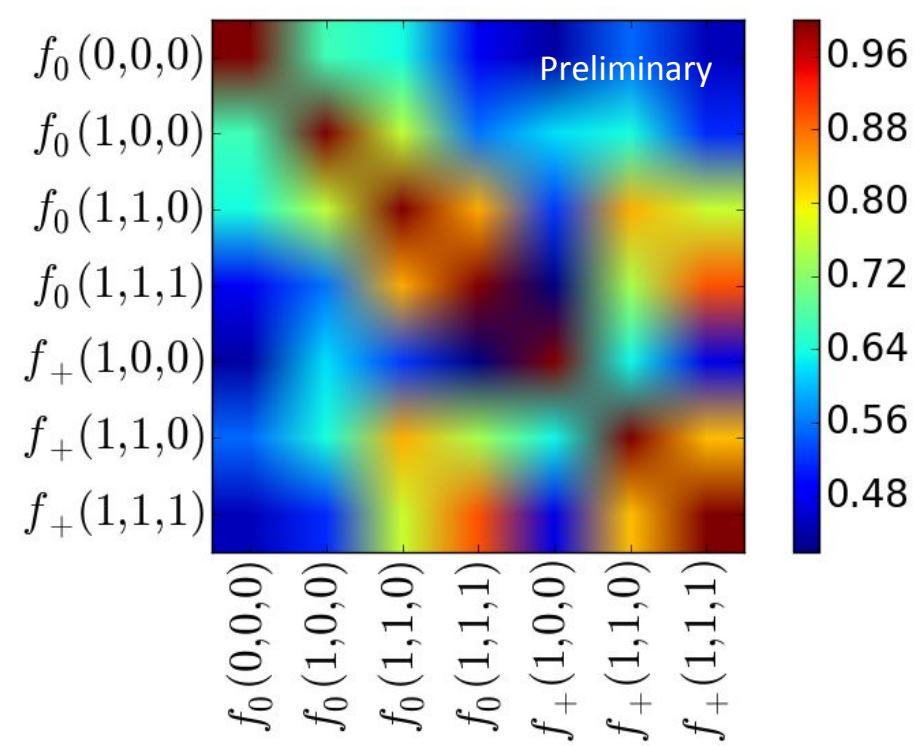


Three-point fits

Correlations between momenta



Ensemble C2



Ensemble F1

Chiral-continuum fits

z -expansion

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - q_{\max}^2}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - q_{\max}^2}}$$

$t_+ = (M_{B_s} + M_{D_s})^2$
 $q_{\max}^2 = (M_{B_s} - M_{D_s})^2$

Fit to BCL parameterisation

Bourrely et al, PRD 79 (2009) 013008

$$f_0 = \frac{1}{1 - q^2/M_0} \sum_{k=0}^{K-1} a_0^{(k)} z^k$$

$$f_+ = \frac{1}{1 - q^2/M_{B_c^*}} \sum_{k=0}^{K-1} a_+^{(k)} \left[z^k - (-1)^{(k-K)} \frac{k}{K} z^K \right]$$

Modified expansion coefficients

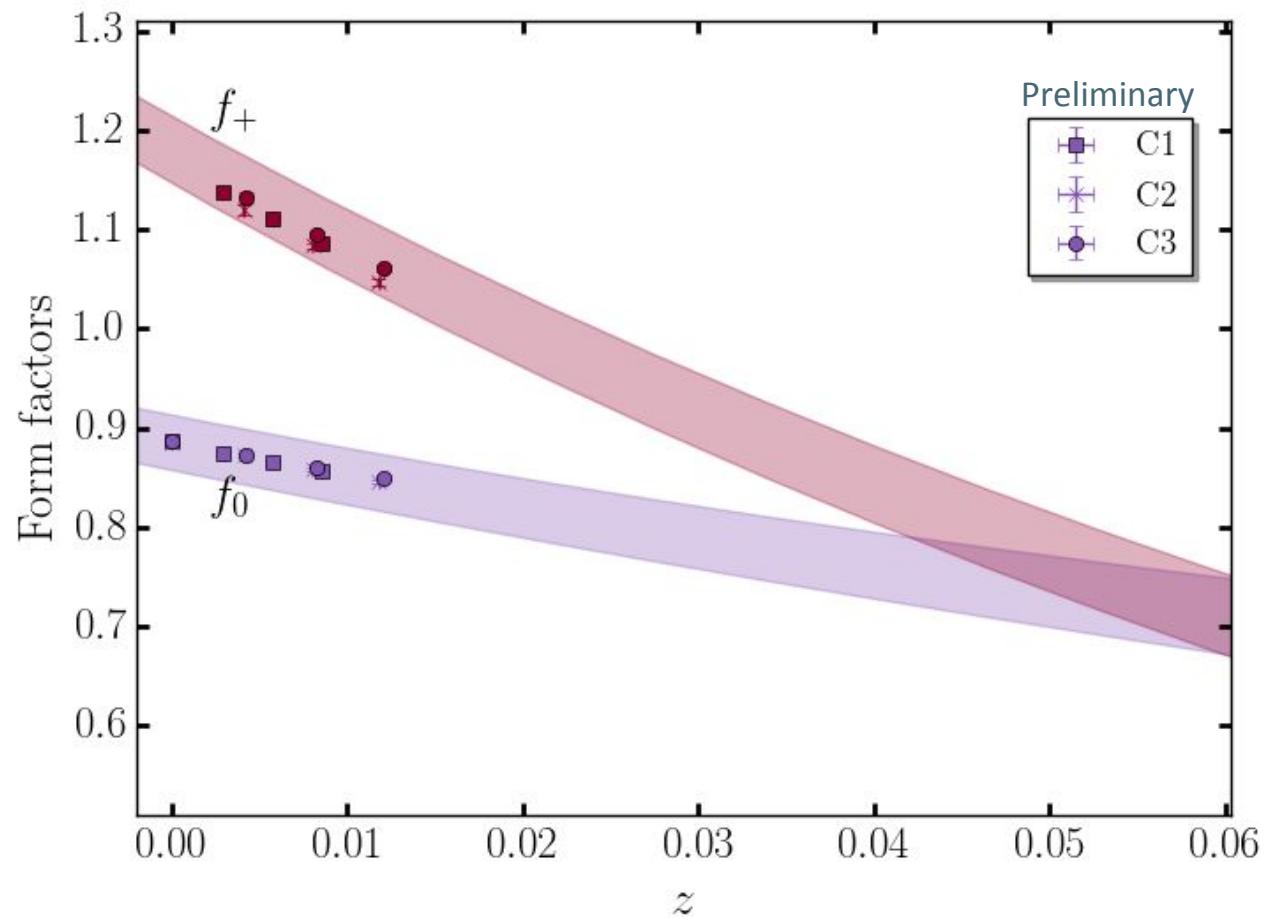
Na et al [HPQCD], PRD 82 (2010) 114506
Bouchard et al [HPQCD], PRL 111 (2013) 162002

$$a_{0,+}^{(k)} = \tilde{a}_{0,+}^{(k)} D_{0,+}^{(k)}(m^{\text{val}}, m^{\text{sea}}, a)$$

chiral logs and discretisation effects

Modified z-expansion

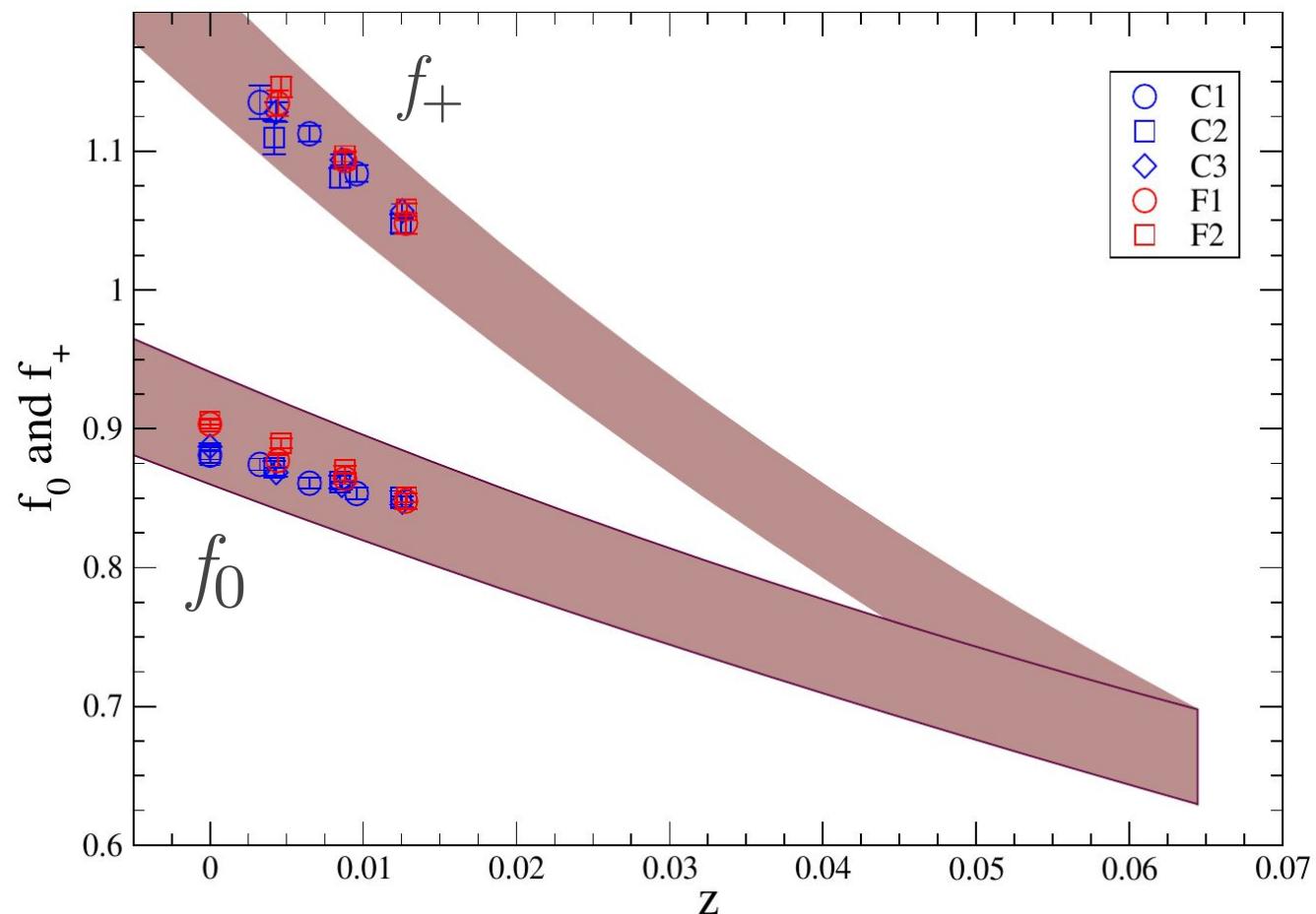
Results from coarse ensembles



Modified z-expansion

$B \rightarrow D\ell\nu$ form factors

Na et al [HPQCD], PRD 92 (2015) 054510

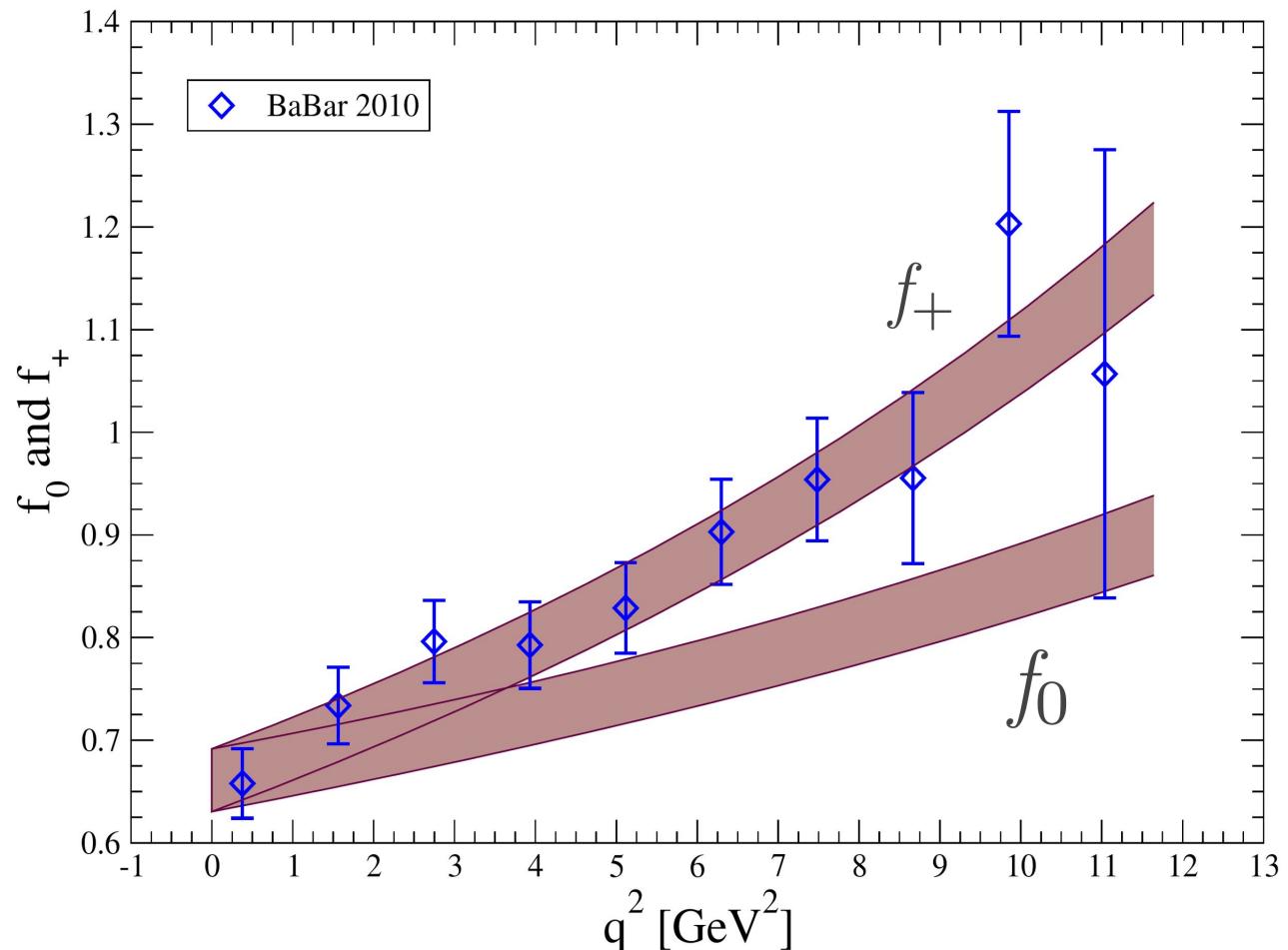


B to D form factors

With BaBar data

$$|V_{cb}| = 0.0402(17)(13) \quad R(D) = \frac{\mathcal{B}(B \rightarrow D\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D\ell\nu_\ell)} = 0.300(8)$$

Aubert et al [BaBar], PRL 104 (2010) 011802



Existing results

Experimental data not yet available

Form factor ratio at (almost) zero recoil

$$\frac{f_0^{(s)}(M_\pi^2)}{f_0^{(d)}(M_\pi^2)} = 1.054(50)$$

Bailey et al [FNAL/MILC], PRD 85 (2012) 114502

Form factor normalisation at zero recoil with twisted mass fermions

Atoui et al, EPJ C 74 (2014) 2861

Conclusion...

$B_{(s)} \rightarrow D_{(s)} \ell \nu$ semileptonic decays provide determination of V_{cb}

Preliminary results for $B_s \rightarrow D_s \ell \nu$ form factors at non-zero recoil

... and outlook

Finalise chiral/continuum extrapolation

Simultaneous fit to determine form factor ratio at zero recoil

Combine data to extract $|V_{ub}/V_{cb}|$

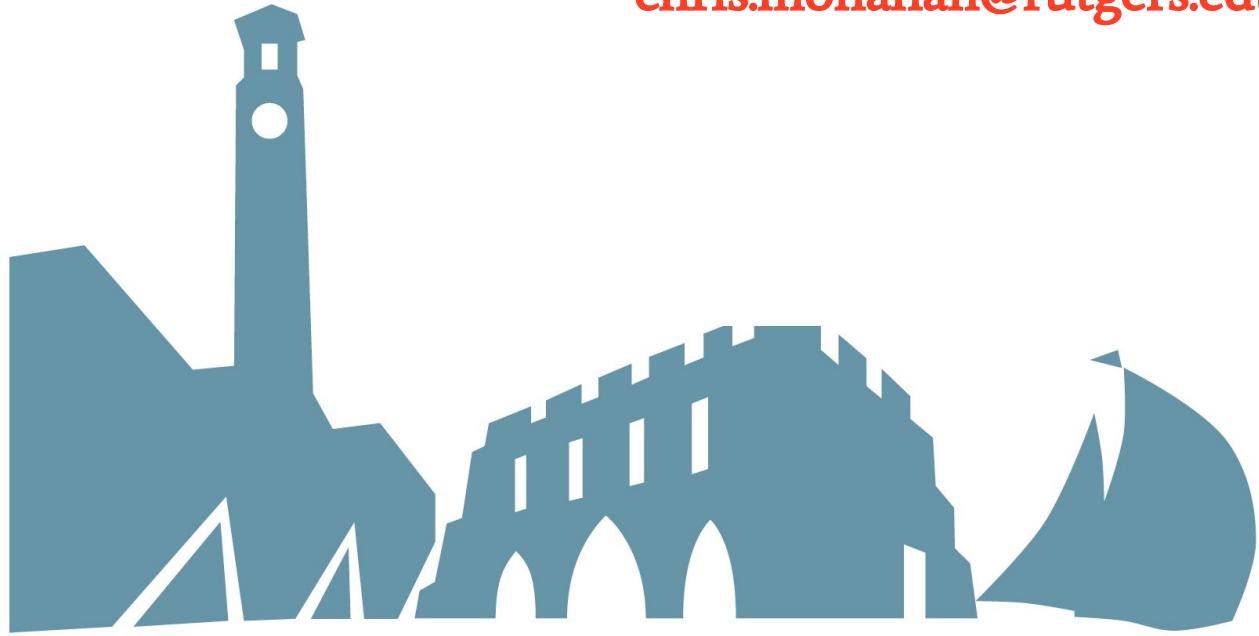
previous HPQCD $B_s \rightarrow K \ell \nu$ analysis

Bouchard et al [HPQCD], PRD 90 (2014) 054506

forthcoming LHCb analysis

Thank you

chris.monahan@rutgers.edu



EXTRA SLIDES

ENSEMBLES

MILC 2+1 asqtad ensembles

Set	r_1/a	$m_l^{\text{sea}}/m_s^{\text{sea}}$	N_{conf}	N_{tsrc}	$L^3 \times N_t$	aM_b	am_l^{val}	am_s^{val}	am_c^{val}
C1	2.647	0.005/0.050	2096	4	$24^3 \times 64$	2.650	0.0070	0.0489	0.6207
C2	2.618	0.010/0.050	2256	2	$20^3 \times 64$	2.688	0.0123	0.0492	0.6300
C3	2.644	0.020/0.050	1200	2	$20^3 \times 64$	2.650	0.0246	0.0491	0.6235
F1	3.699	0.0062/0.031	1896	4	$28^3 \times 96$	1.832	0.00674	0.0337	0.4130
F2	3.712	0.0124/0.031	1200	4	$28^3 \times 96$	1.826	0.01350	0.0336	0.4120

CORRELATOR FITS

Meson correlators

$$C_{B_s}^{\beta,\alpha}(t) = \sum_{j=0}^{N_{B_s}-1} b_k^\beta b_k^{\alpha*} e^{-E_j^{B_s^{\text{sim}}} t} + \sum_{j=0}^{N'_{B_s}-1} b'_k{}^\beta b'_k{}^{\alpha\dagger} (-1)^t e^{-E'_j{}^{B_s^{\text{sim}}} t}$$

$$\begin{aligned} C_{D_s}(t, \mathbf{p}) &= \sum_{j=0}^{N_{D_s}-1} |d_k|^2 \left(e^{-E_j^{D_s} t} + e^{-E_j^{D_s} (N_t - t)} \right) \\ &+ \sum_{j=0}^{N'_{D_s}-1} |d'_k|^2 (-1)^t \left(e^{-E'_j{}^{D_s} t} + e^{-E'_j{}^{D_s} (N_t - t)} \right) \end{aligned}$$

CORRELATOR FITS

Three-point correlator

$$\begin{aligned} C_J^\alpha(t, T; \mathbf{p}) = & \sum_{j=0}^{N_{B_s}-1} \sum_{k=0}^{N_{D_s}-1} A_{jk}^\alpha e^{-E_j^{D_s} t} e^{-E_k^{\text{sim}}(T-t)} \\ & + \sum_{j=0}^{N'_{B_s}-1} \sum_{k=0}^{N'_{D_s}-1} B_{jk}^\alpha e^{-E_j^{D_s} t} e^{-E_k'^{\text{sim}}(T-t)} (-1)^{(T-t)} \\ & + \sum_{j=0}^{N_{B_s}-1} \sum_{k=0}^{N'_{D_s}-1} C_{jk}^\alpha e^{-E_j'^{D_s} t} e^{-E_k^{\text{sim}}(T-t)} (-1)^t \\ & + \sum_{j=0}^{N'_{B_s}-1} \sum_{k=0}^{N'_{D_s}-1} D_{jk}^\alpha e^{-E_j'^{D_s} t} e^{-E_k'^{\text{sim}}(T-t)} (-1)^T \end{aligned}$$

Chiral-continuum fits

“Modified z-expansion”

$$f_0 = \frac{1}{1 - q^2/M_0} \sum_{k=0}^{K-1} a_0^{(k)} z^k \quad f_+ = \frac{1}{1 - q^2/M_{B_c^*}} \sum_{k=0}^{K-1} a_+^{(k)} \left[z^k - (-1)^{(k-K)} \frac{k}{K} z^K \right]$$

Where

$$a_{0,+}^{(k)} = \tilde{a}_{0,+}^{(k)} D_{0,+}^{(k)}(m^{\text{val}}, m^{\text{sea}}, a)$$

$$\begin{aligned} D^{(k)} &= 1 + \frac{M_\pi^2}{(4\pi f_\pi)^2} \left\{ c_1^{(k)} + c_2^{(k)} \log \left[\frac{M_\pi^2}{(4\pi f_\pi)^2} \right] \right\} \\ &\quad + c_3^{(k)} \left\{ \frac{(M_\pi^{\text{asqtad}})^2 - (M_\pi^{\text{HISQ}})^2}{2(4\pi f_\pi)^2} + \frac{(M_K^{\text{asqtad}})^2 - (M_K^{\text{HISQ}})^2}{(4\pi f_K)^2} \right\} \\ &\quad + d_1^{(k)} (am_c^2) + d_2^{(k)} (am_c)^4 + e_1^{(k)} \left(\frac{aE_{D_s}}{\pi} \right)^2 + e_2^{(k)} \left(\frac{aE_{D_s}}{\pi} \right)^4 \end{aligned}$$