Parity doubling in two-color and two-flavor theory at high temperature

Jong-Wan Lee

Swansea University
Prifysgol Abertawe

In collaboration with B. Lucini & M. Piai

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Motivation - SU(2) gauge theory

Shares some nonperturbative properties with QCD, such as confinement and chiral symmetry breaking.

SU(2) gauge theory with even number of fundamental fermions
Finite density calculations are free from sign problem

Alford, Kapustin, Wilczek (1999)
Hands, Kogut, Lombardo, Morrison (1999)

SU(2) gauge theory with two fundamental fermions
minimal model for Composite Higgs dynamics

Lewis, Pica, Sannino (2012)
Hietanen, Lewis, Pica, Sannino (2014)

Finite T calculations on an anisotropic lattice - finer temporal spacing

$L = \frac{1}{N_T a}$
Lattice spacing $a$ is fixed
Change $T$ by changing $N_T$
SU(2) gauge theory with 2 Dirac fermions in fundamental representation

\[ \mathcal{L} = -\frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu} + \bar{u} (i \gamma^\mu D_\mu - m) u + \bar{d} (i \gamma^\mu D_\mu - m) d \]

Global symmetry: \( \text{SU(4)} \) \text{ broken} \rightarrow \text{Sp(4)}

\[ \langle \bar{u} u + \bar{d} d \rangle \neq 0 \text{ at chiral limit} \]
\[ \langle \bar{u} u + \bar{d} d \rangle \neq 0, m\bar{u}u, m\bar{d}d \text{ at non-zero mass} \]

5 Goldstone bosons: 3 pseudoscalar mesons + 2 diquark baryons

Degenerate (two-point correlation functions are identical)

\textit{Aloisio, Azcoiti, Di Carlo, Galante, Grillo (2000)}

\textit{Hands, Montvay, Morrison, Oevers, Scorzato, Skullerud (2000)}

Observables: Isovector mesons \( O^{(\Gamma)}_{\bar{u}d} \equiv \bar{u}(x) \Gamma d(x) \),

where \( \Gamma = 1, \gamma^5, \gamma^\mu, \gamma^\mu \gamma^5 \)
\[ S_g[U] = \frac{\beta}{\xi_0} \left[ \sum_i (\xi_0^i)^2 \left( 1 - \frac{1}{N} \text{Re} \, \text{tr} P_{0i} \right) \right] \]

\[ S_f[U, \tilde{\psi}, \psi] = a_s^3 a_t \sum_x \tilde{\psi}(x) D_m \psi(x) \]

Bare gauge anisotropy

\[ D_m \psi(x) \equiv (D + m_0) \psi(x) \]

Bare Fermion anisotropy

\[ = \frac{1}{a_t} \left[ \left( a_t m_0 + 1 + \frac{3}{\xi_0} \right) \psi(x) - \frac{1}{2} \left( (1 - \gamma_0) U_0(x) \psi(x + \hat{0}) + (1 + \gamma_0) U_0^\dagger(x - \hat{0}) \psi(x - \hat{0}) \right) \right. \]

\[ - \frac{1}{2\xi_0} \sum_j \left( (1 - \gamma_j) U_j(x) \psi(x + \hat{j}) + (1 + \gamma_j) U_j^\dagger(x - \hat{j}) \psi(x - \hat{j}) \right) \]

Bare parameters need to be tuned in order that the renormalized gauge and fermion anisotropies are same for a given quark mass.
Anisotropy - Gauge sector

Gauge anisotropy $\xi_g$ is determined by using Klassen’s method \textit{Klassen (2000)}

\[ R_t(x, t = \xi_g y) = R_s(x, y) \quad \text{where} \quad R_s(x, y) = \frac{W_{ss}(x, y)}{W_{ss}(x + 1, y)}, \quad R_t(x, t) = \frac{W_{st}(x, t)}{W_{st}(x + 1, t)} \]

In practical, we minimize

\[ L(\xi_g) = \sum_{x, y} \frac{(R_{ss}(x, y) - R_{st}(x, \xi_g y))^2}{(\Delta R_s)^2 + (\Delta R_t)^2} \]

\textit{Umeda et. al. (CP-PACS) (2003)}

Small Wilson loops suffer from short range lattice artifacts, while large ones suffer from very large noise.

Fix max$(x * y)$

Then, scan min$(x * y)$

$\xi_g$ should approach the asymptotic value.
Anisotropy - Gauge sector

Gauge anisotropy $\xi_g$ is determined by using Klassen’s method \textit{Klassen (2000)}

\[ R_t(r, t = \xi_g y) = R_s(r, y) \quad \text{where} \quad R_s(r, y) = \frac{W_{ss}(r, y)}{W_{ss}(r + 1, y)}, \quad R_t(r, t) = \frac{W_{st}(r, t)}{W_{st}(r + 1, t)} \]

In practice, we minimize

\[ L(\xi_g) = \sum_{r,y} \frac{(R_{ss}(r, y) - R_{st}(r, \xi_g y))^2}{(\Delta R_s)^2 + (\Delta R_t)^2} \]

\[ \xi_g^0 = 4.9, \quad \xi_f^0 = 4.7, \quad m_0 = -0.209 \]

2D path in x-z plane (Bresenham algorithm)

For 3D, \textit{Boldor et. al. (2001)}
Fermion anisotropy $\xi_f$ is determined by using meson dispersion relations (Pseudo Goldstone Boson)

$$E^2(\vec{p}^2) = m^2 + \frac{\vec{p}^2}{\xi_f^2}, \quad \vec{p} = 2\pi \vec{n}/L_s$$

Point sources for both source and sink

We use the first four momentum vectors for fitting.

$$\vec{n} = (0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1)$$

Higher momentum states are consistent with the fit result.
Fermion anisotropy $\xi_f$ is determined by using meson dispersion relations (Pseudo Goldstone Boson)

$$E^2(p^2) = m^2 + \frac{p^2}{\xi_f}, \quad p = 2\pi \vec{n}/L_s$$

Point sources for both source and sink

We use the first four momentum vectors for fitting.

$$\vec{n} = (0, 0, 0), \ (1, 0, 0), \ (0, 1, 0), \ (0, 0, 1)$$

Also, $\xi_f$ from the dispersion relation for vector meson agrees.
### Anisotropy Tuning - Simulation details

#### Table 1: Simulation parameters and results.

<table>
<thead>
<tr>
<th>$m_0$</th>
<th>$\xi_g^0$</th>
<th>$\xi_f^0$</th>
<th>$N_{\text{traj}}/\ell_{\text{auto}}$</th>
<th>$N_{\text{conf}}$</th>
<th>$m_\pi$</th>
<th>$m_\nu$</th>
<th>$\xi_g$</th>
<th>$\xi_f$</th>
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<td>4.7</td>
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<td>0.1659(8)</td>
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<td>0.77(3)</td>
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</table>

Configurations are generated using HMC algorithms (modified HiRep code).

Del Debbio, Patella, Pica (2010)

Thermalization time is determined by monitoring Plaquette values.

Two adjacent configurations are separated by roughly one autocorrelation time (8~16 trajectories).

Implemented periodic boundary conditions for all directions.
Anisotropy tuning - results

Linear Ansatz for renormalized parameters

\[
\xi_g (\xi_g^0, \xi_f^0, m_0) = a_0 + a_1 \xi_g^0 + a_2 \xi_f^0 + a_3 m_0, \\
\xi_f (\xi_g^0, \xi_f^0, m_0) = b_0 + b_1 \xi_g^0 + b_2 \xi_f^0 + b_3 m_0, \\
M_{ps}^2 (\xi_g^0, \xi_f^0, m_0) = c_0 + c_1 \xi_g^0 + c_2 \xi_f^0 + c_3 m_0.
\]

with

\[
a_0 = 0.6(16), \quad a_1 = 0.97(13), \quad a_2 = 0.31(23), \quad a_3 = 2(4), \\
b_0 = 1.8(24), \quad b_1 = 0.06(18), \quad b_2 = 1.1(3), \quad b_3 = 4(7), \\
c_0 = 0.475(5), \quad c_1 = -0.0168(4), \quad c_2 = -0.0375(6), \quad c_3 = 0.986(11).
\]

Edwards, Joo, Lin (2008) very mild dependence
Renormalized conditions

\[ \xi_g(\xi_g^0, \xi_f^0, m_0^*) = \xi_f(\xi_g^0, \xi_f^0, m_0^*) = \xi, \quad M_{ps}^2(\xi_g^0, \xi_f^0, m_0^*) = m_{ps}^2. \]

Our target anisotropy is \( \xi = 6.3 \) with \( (m_{ps}a)^2 = 0.005. \)

\[ \xi_g^0 = 4.84(8), \quad \xi_f^0 = 4.72(12), \quad m_0^* = -0.2148(37) \]

Results of 128x16^3 Lattice

\[ \xi_g = 6.29(4), \quad \xi_f = 6.1(2) \]

\( (m_{ps}a)^2 = 0.00517(14) \)

Consistent with our target parameters!
Finite T calculations

Ensembles (~200 configurations)

- $N_T \times 16^3$ lattice with $N_T = 16, 20, 24, 28, 30, 36, 40, 48, 128$
- $N_T \times 16^2 \times 24$ lattice with $N_T = 16, 20, 24, 28, 36, 42, 48, 56$

Boundary conditions: antiperiodic temporal fermion boundary conditions
periodic b.c. for all others

Critical temperature: deconfining transition

Rapid change of temporal Polyakov loop

\[ T^c = \frac{1}{N_T^c a} \]

Temporal and spacial correlators of isovector mesons (Stochastic wall sources)
pseudoscalar(PS), scalar(S), vector(V), and axial vector(AV) mesons
Degeneracy between parity partners in hadron multiplets

Chiral symmetry restoration
Renormalized Polyakov Loop

Multiplicative renormalization of Polyakov Loop

\[ L(T) = e^{-F(T)/T} \]

Free energy \[ F_R = F + \Delta F \]

Additive renormalization

Renormalized Polyakov Loop

\[ L_R(T) \equiv Z_L^{N_\tau} L(T) \]

Renormalization condition

\[ L_R(T_R) \equiv \text{constant} \]

*Borsanyi et. al. (2012)*

*Aarts et. al. (2014)*

Deconfining critical temperature

\[ T^c = 1/N_{\tau}^c = 0.0254(14) \text{ or } N_{\tau}^c = 39.5(2) \]
Temporal correlators

Below the critical temperature, the temporal correlation functions decay in the order of

$$\log \left[ \frac{C_{PS}(t)}{C_{PS}(N_\tau/2)} \right] < \log \left[ \frac{C_V(t)}{C_V(N_\tau/2)} \right] < \log \left[ \frac{C_{AV}(t)}{C_{AV}(N_\tau/2)} \right]$$

Onset of the critical temperature, the correlation functions for vector and axial-vector mesons are degenerate.
Spacial correlators - Vector channel

Screening mass for vector and axial vector mesons

\[
R(T) = \frac{M_{AV}(T) - M_V(T)}{M_{AV}(T) + M_V(T)}
\]

Black: \(N_\tau \times 16^3\) lattice
Red: \(N_\tau \times 16^2 \times 24\) lattice

\(M_\tau\) and AV mesons are degenerate at \(T \gtrsim T_c\)
In this section, we define the gauge and fermion actions used in this calculation. For the gauge sector, we use the Wilson action modified as

\[ S_{\text{gauge}} = \sum_{x} \{ \sum_{\mu} (U_{\mu}(x) + U_{\mu}(x+\hat{\mu})) - 2 \} \]

and

\[ S_{\text{fermion}} = \sum_{x} \{ \sum_{i,j} \bar{c}_{i}(x) D_{ij} c_{j}(x+\hat{\mu}) + \text{terms involving quark masses} \} \]

where

\[ x = (x_{0}, x_{1}, x_{2}, x_{3}) \]

and

\[ \mu \]

are the link variables.

Spacial correlators - Vector channel

Screening mass for vector and axial vector mesons

Black: \( N_{\tau} \times 16^{3} \) lattice

Red: \( N_{\tau} \times 16^{2} \times 24 \) lattice

Red dotted line = \( 2\pi \) (free quark)

Above 2\( T_{c} \), screening mass of vector and axial-vector mesons begins to deviate from the plateau.

Lattice artifacts due to too small \( N_{\tau} \)?
In this section, we define the gauge and fermion actions used in this calculation. For the fermion action, we use the Wilson action modified as

$$S = \sum U_i \sum_{\mu=0}^{3} \bar{c}_i U_\mu l_i \nu_i c_j U_\mu l_j$$

where $U_i$ are the link variables. The plaquette is defined by

$$P = \frac{1}{2} \sum_{\mu<\nu} (U_{\mu\nu} U_{\nu\mu})^T$$

and the screening mass for scalar and pseudoscalar mesons are calculated as

$$M_{PS} = \frac{1}{2} \sum_{\mu<\nu} (U_{\mu\nu} U_{\nu\mu})^T$$

and

$$M_{S} = \frac{1}{2} \sum_{\mu<\nu} (U_{\mu\nu} U_{\nu\mu})^T$$

The lattice bare coupling and the bare gauge parameter are $g$ and $\mu$, respectively. The screening mass is defined by

$$M_{PS}(T) = \frac{1}{N} \sum_{\mu<\nu} (U_{\mu\nu} U_{\nu\mu})^T$$

where $N$ is the number of lattice sites. The forward and backward covariant derivatives are denoted as

$$\nabla^+ = \nabla - \gamma^5$$

and

$$\nabla^- = \nabla + \gamma^5$$

The screening mass for scalar and pseudoscalar mesons are degenerate at $T \gtrsim 1.5 T_c$. The screening mass for scalar and pseudoscalar mesons are shown in the graph below.

![Screening mass for scalar and pseudoscalar mesons](image)
**Spacial correlators - Scalar channel**

**Screening mass for scalar and pseudoscalar mesons**

![Graph showing screening mass for scalar and pseudoscalar mesons](image)

- **Black**: $N_T \times 16^3$ lattice
- **Red**: $N_T \times 16^2 \times 24$ lattice

Red dotted line = $2\pi$ (free quark)

Above $2T_C$, screening mass of pseudoscalar and scalar mesons begins to deviate from the plateau.

Lattice artifacts due to too small $N_T$?
Conclusion and future work

SU(2) gauge theory with 2 fund. Wilson fermions on an anisotropic lattice

Anisotropy tuning works!

Non-plain Wilson loops are helpful for determining gauge anisotropy.

Finite T results

Parity doubling in the temporal and spacial correlators for vector channel just above $T_c$

Parity doubling in the spacial correlators for scalar channel above $1.5T_c$

Systematic errors

For $T \leq T_c$ fitting errors in scalar and axial-vector screening mass due to very limited numbers of data points in the asymptotic region

Mistuned bare anisotropy generate up to $\sim3\%$ errors.

Anisotropy tuning works at any temperature?

Massless, infinite volume, and continuum limit needs to be investigated.

How does the meson spectrum change at finite chemical potential?
Thank you for your attention!