# Parity doubling in two-color and two-flavor theory at high temperature


**Jong-Wan Lee**

Swansea University
Prifysgol Abertawe

*In collaboration with B. Lucini & M. Piai*


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# ✤ Motivation - SU(2) gauge theory

Shares some nonperturbative properties with QCD, such as confinement and chiral symmetry breaking.

SU(2) gauge theory with even number of fundamental fermions

Finite density calculations are free from sign problem

***Alford, Kapustin, Wilczek (1999)***
***Hands, Kogut, Lombardo, Morrison (1999)***
***Aloisio, Azcoiti, Di Carlo, Galante, Grillo (2000)***

SU(2) gauge theory with two fundamental fermions

minimal model for Composite Higgs dynamics

***Lewis, Pica, Sannino (2012)***
***Hietanen, Lewis, Pica, Sannino (2014)***

Finite T calculations on an anisotropic lattice - finer temporal spacing

This work!

$$T = \frac{1}{N_\tau a}$$

Lattice spacing $a$ is fixed

Change $T$ by changing $N_\tau$

# ✤ Model

SU(2) gauge theory with 2 Dirac fermions in fundamental representation

$$\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F_{a\mu\nu} + \bar{u}(i\gamma^\mu D_\mu - m)u + \bar{d}(i\gamma^\mu D_\mu - m)d$$

Global symmetry: SU(4) $\xrightarrow{\textbf{broken}}$ *Sp*(4)

$\langle \bar{u}u + \bar{d}d \rangle \neq 0$ at chiral limit

$\langle \bar{u}u + \bar{d}d \rangle \neq 0, \ m\bar{u}u, \ m\bar{d}d$ at non-zero mass

5 Goldstone bosons: 3 pseudoscalar mesons + 2 diquark baryons

Degenerate (two-point correlation functions are identical)

***Aloisio, Azcoiti, Di Carlo, Galante, Grillo (2000)***

***Hands, Montvay, Morrison, Oevers, Scorzato, Skullerud (2000)***

Observables: Isovector mesons $\mathcal{O}^{(\Gamma)}_{\bar{u}d} \equiv \bar{u}(x)\Gamma d(x)$,

where $\Gamma = 1, \ \gamma^5, \ \gamma^\mu, \ \gamma^\mu\gamma^5$

# ✤ Anisotropic Lattice Action

Standard Wilson action on an anisotropic lattice

$$S_g[U] = \frac{\beta}{\xi_g^0}\left[\sum_i (\xi_g^0)^2\left(1-\frac{1}{N}\mathrm{Re}\ \mathrm{tr}\mathcal{P}_{0i}\right)+\sum_{i<j}\left(1-\frac{1}{N}\mathrm{Re}\ \mathrm{tr}\mathcal{P}_{ij}\right)\right]$$

$\boxed{\beta = 2.0}$

$$S_f[U,\bar{\psi},\psi] \;=\; a_s^3 a_t \sum \bar{\psi}(x) D_m \psi(x)$$

Bare gauge anisotropy

Bare Fermion anisotropy

$$\begin{aligned}
D_m\psi(x) &\equiv (D+m_0)\psi(x)\\
&= \frac{1}{a_t}\Bigg[\left(a_t m_0 + 1 + \frac{3}{\xi_f^0}\right)\psi(x) - \frac{1}{2}\Big((1-\gamma_0)U_0(x)\psi(x+\hat{0}) + (1+\gamma_0)U_0^\dagger(x-\hat{0})\psi(x-\hat{0})\Big)\\
&\quad -\frac{1}{2\xi_f^0}\sum_j\Big((1-\gamma_j)U_j(x)\psi(x+\hat{j}) + (1+\gamma_j)U_j^\dagger(x-\hat{j})\psi(x-\hat{j})\Big)\Bigg]
\end{aligned}$$

Bare parameters need to be *tuned* in order that the renormalized gauge and fermion anisotropies are same for a given quark mass.

# ✤ Anisotropy - Gauge sector

Gauge anisotropy $\xi_g$ is determined by using Klassen's method ***Klassen (2000)***

$$R_t(x, t = \xi_g y) = R_s(x, y) \text{ where } R_s(x,y) = \frac{W_{ss}(x,y)}{W_{ss}(x+1,y)}, R_t(x,t) = \frac{W_{st}(x,t)}{W_{st}(x+1,t)}$$

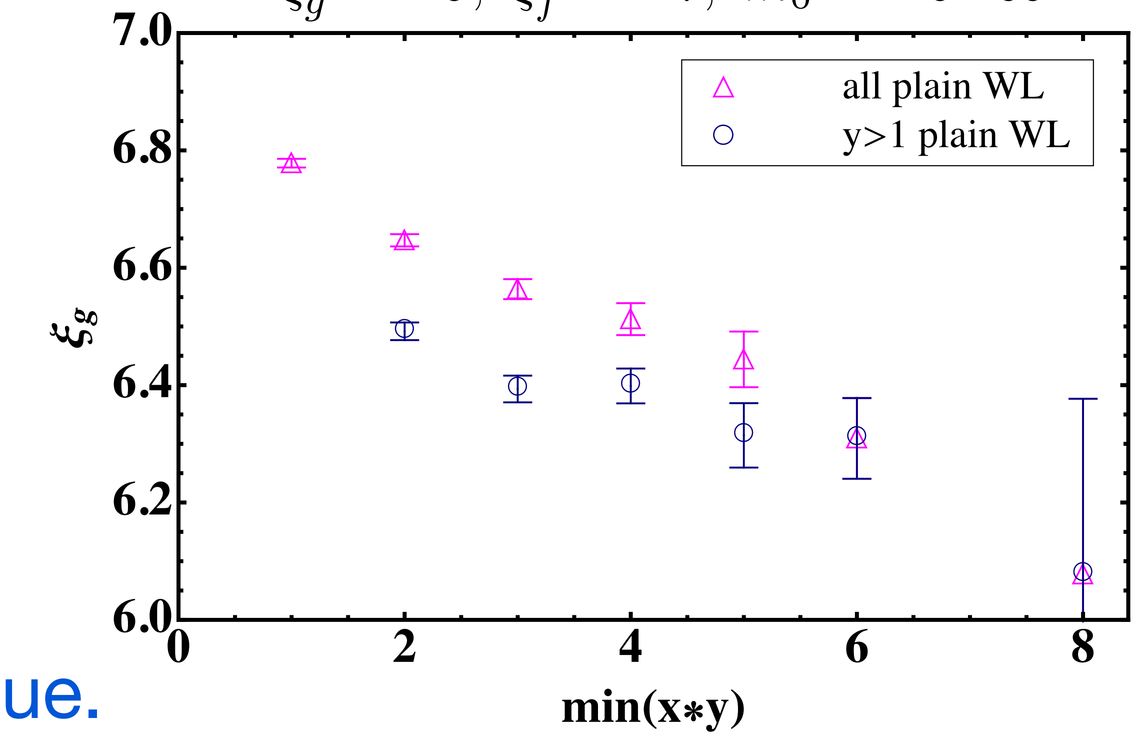
In practical, we minimize

$$L(\xi_g) = \sum_{x,y} \frac{(R_{ss}(x,y) - R_{st}(x,\xi_g y))^2}{(\Delta R_s)^2 + (\Delta R_t)^2}$$

***Umeda et. al. (CP-PACS) (2003)***

Small Wilson loops suffer from short range lattice artifacts, while large ones suffer from very large noise.

$\xi_g^0 = 4.9,\ \xi_f^0 = 4.7,\ m_0 = -0.209$




Fix $\max(x * y)$

Then, scan $\min(x * y)$

$\xi_g$ should approach the asymptotic value.

# ✤ Anisotropy - Gauge sector

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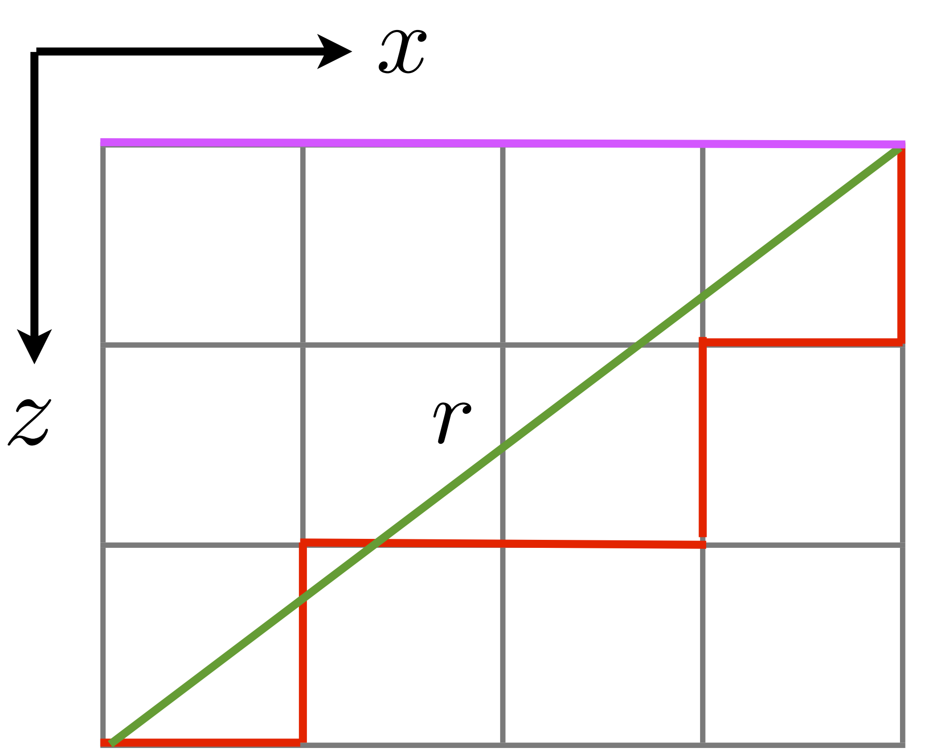
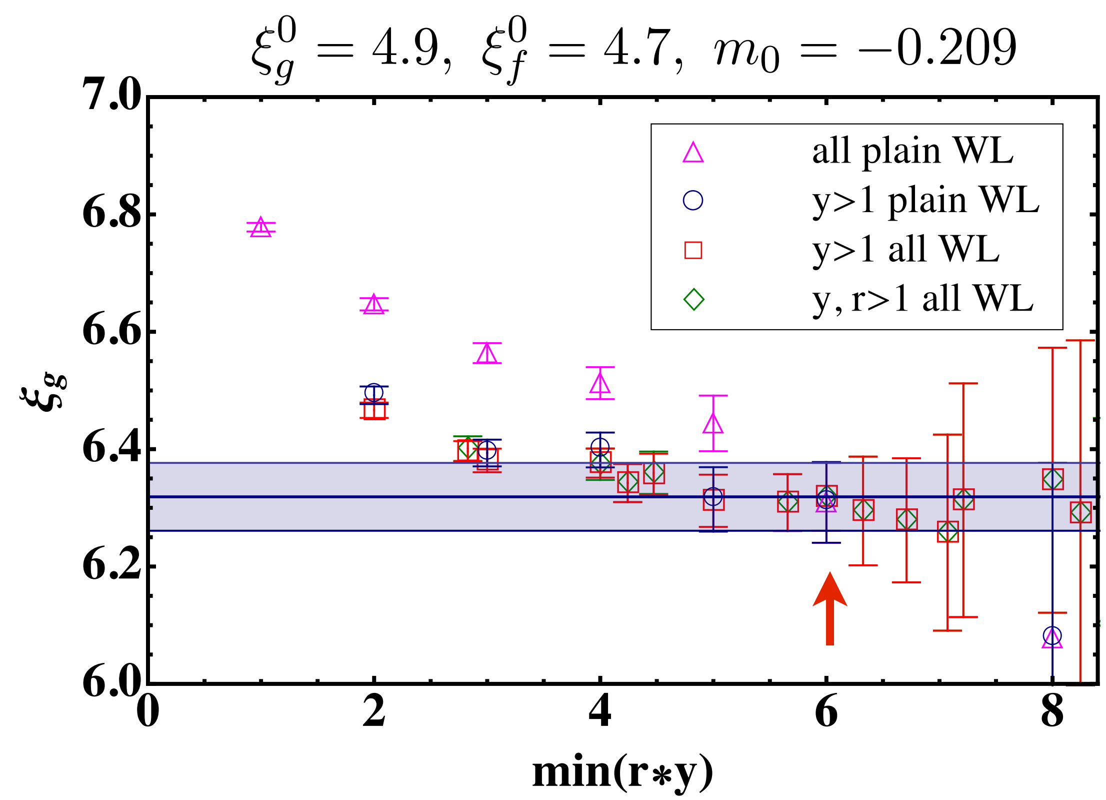



**2D path in x-z plane**(Bresenham algorithm)

For 3D, ***Bolder et. al. (2001)***



# ✤ Anisotropy - Fermion sector

Fermion anisotropy $\xi_f$ is determined by using meson dispersion relations

(Pseudo Goldstone Boson)

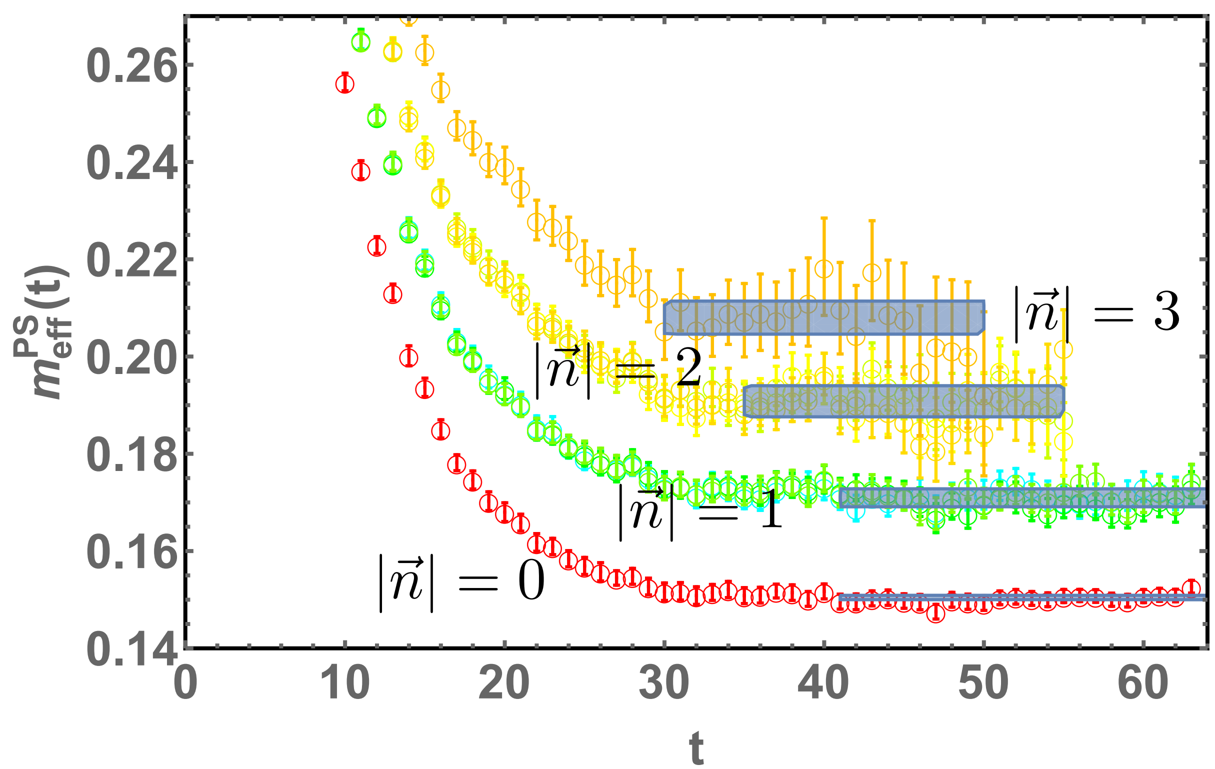
$$E^2(\vec{p}^2) = m^2 + \frac{\vec{p}^2}{\xi_f^2},\ \vec{p} = 2\pi\vec{n}/L_s$$

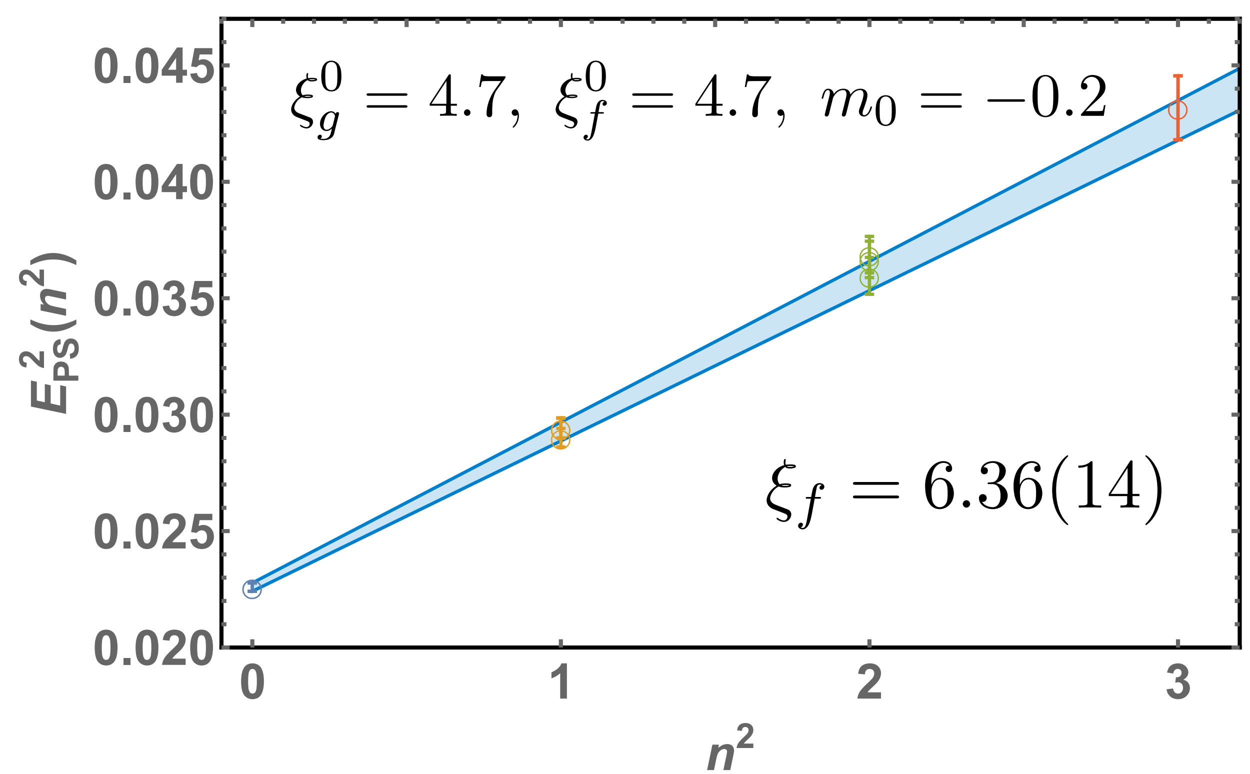
Point sources for both source and sink

We use the first four momentum vectors for fitting.

$$\vec{n} = (0,0,0),\ (1,0,0),\ (0,1,0),\ (0,0,1)$$

Higher momentum states are consistent with the fit result.



# ✤ Anisotropy - Fermion sector

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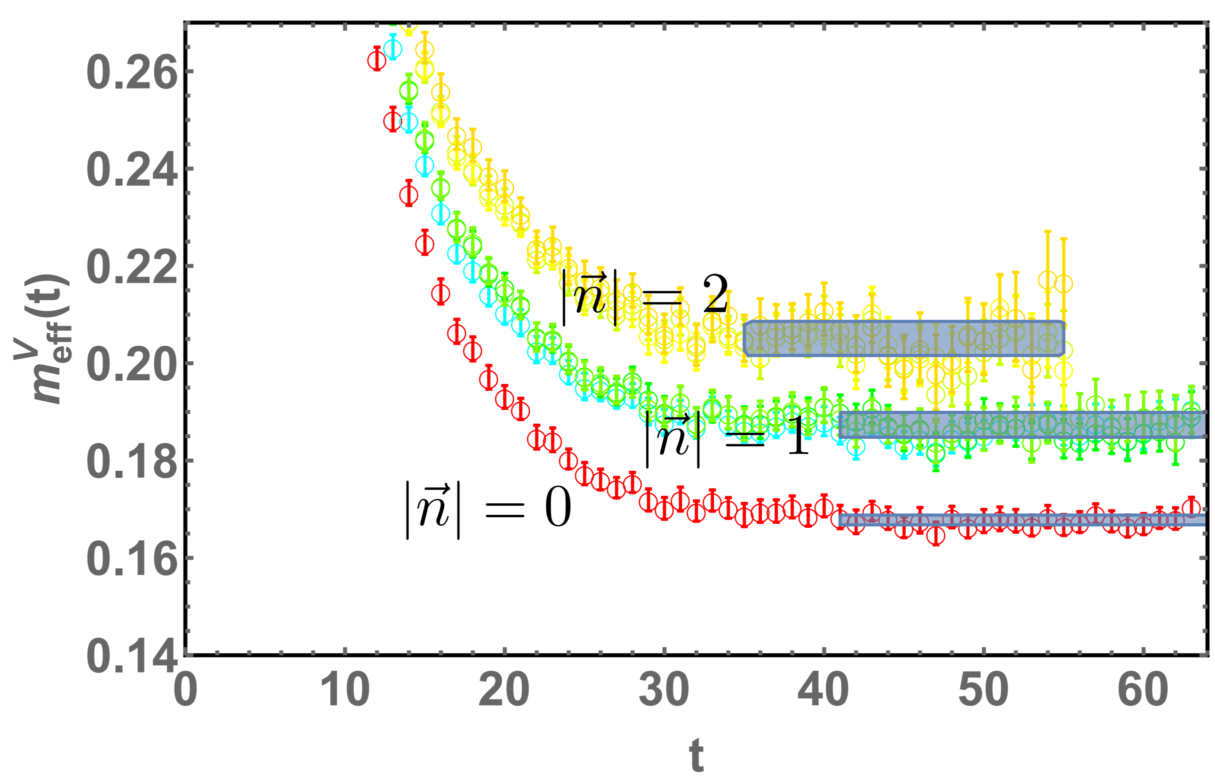
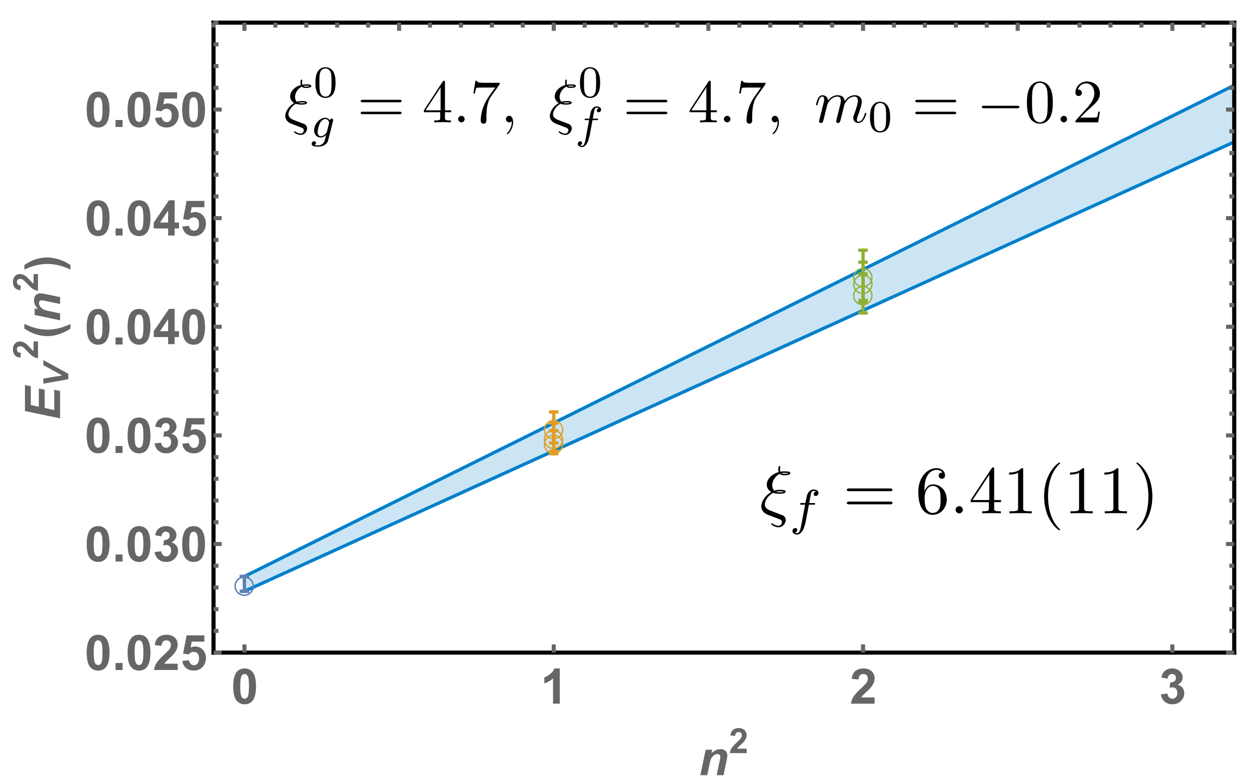
Point sources for both source and sink

We use the first four momentum vectors for fitting.

$$\vec{n} = (0,0,0),\ (1,0,0),\ (0,1,0),\ (0,0,1)$$

Also, $\xi_f$ from the dispersion relation for vector meson agrees.



# ✤ Anisotropy Tuning - Simulation details

Ensembles($128 \times 12^3$ lattice, $\beta = 2.0$)

| $m_0$ | $\xi_g^0$ | $\xi_f^0$ | $N_{traj}/\ell_{auto}$ | $N_{conf}$ | $m_\pi$ | $m_v$ | $\xi_g$ | $\xi_f$ | $m_\pi/m_\rho$ |
|---|---|---|---|---|---|---|---|---|---|
| -0.195 | 4.7 | 4.7 | 1600/8 | 200 | 0.1659(8) | 0.1823(10) | 6.19(7) | 6.34(10) | 0.910(7) |
| -0.195 | 4.9 | 4.7 | 2400/12 | 200 | 0.1544(6) | 0.1709(13) | 6.33(8) | 6.33(9) | 0.903(8) |
| -0.2 | 4.5 | 4.7 | 2400/8 | 300 | 0.1616(5) | 0.1784(8) | 6.03(6) | 6.28(7) | 0.906(5) |
| -0.2 | 4.7 | 4.5 | 2400/8 | 300 | 0.1743(5) | 0.1910(7) | 6.07(7) | 6.12(6) | 0.913(4) |
| -0.2 | 4.7 | 4.7 | 2400/12 | 200 | 0.1504(6) | 0.1678(10) | 6.13(6) | 6.41(11) | 0.896(6) |
| -0.2 | 4.9 | 4.7 | 3000/10 | 300 | 0.1399(5) | 0.1589(7) | 6.42(6) | 6.35(7) | 0.880(5) |
| -0.2 | 5.1 | 4.7 | 2250/14 | 160 | 0.1279(13) | 0.1479(19) | 6.58(9) | 6.34(17) | 0.865(14) |
| -0.209 | 4.7 | 4.5 | 2400/16 | 150 | 0.1455(7) | 0.1643(11) | 6.10(6) | 6.04(10) | 0.885(7) |
| -0.209 | 4.7 | 4.7 | 3000/10 | 300 | 0.1169(7) | 0.1392(13) | 6.22(6) | 6.35(12) | 0.840(10) |
| -0.209 | 4.9 | 4.5 | 3000/10 | 300 | 0.1336(6) | 0.1533(9) | 6.34(7) | 6.11(9) | 0.872(6) |
| -0.209 | 4.9 | 4.7 | 2100/14 | 150 | 0.1023(9) | 0.1243(15) | 6.35(6) | 6.25(12) | 0.823(12) |
| -0.215 | 4.7 | 4.7 | 1650/12 | 138 | 0.0904(21) | 0.118(5) | 6.04(9) | · | 0.77(3) |

Configurations are generated using HMC algorithms(modified HiRep code).

***Del Debbio, Patella, Pica (2010)***

Thermalization time is determined by monitoring Plaquette values.

Two adjacent configurations are separated by roughly one autocorrelation time(8~16 trajectories).

Implemented periodic boundary conditions for all directions.

# ✤ Anisotropy tuning - results

Linear Ansatz for renormalized parameters

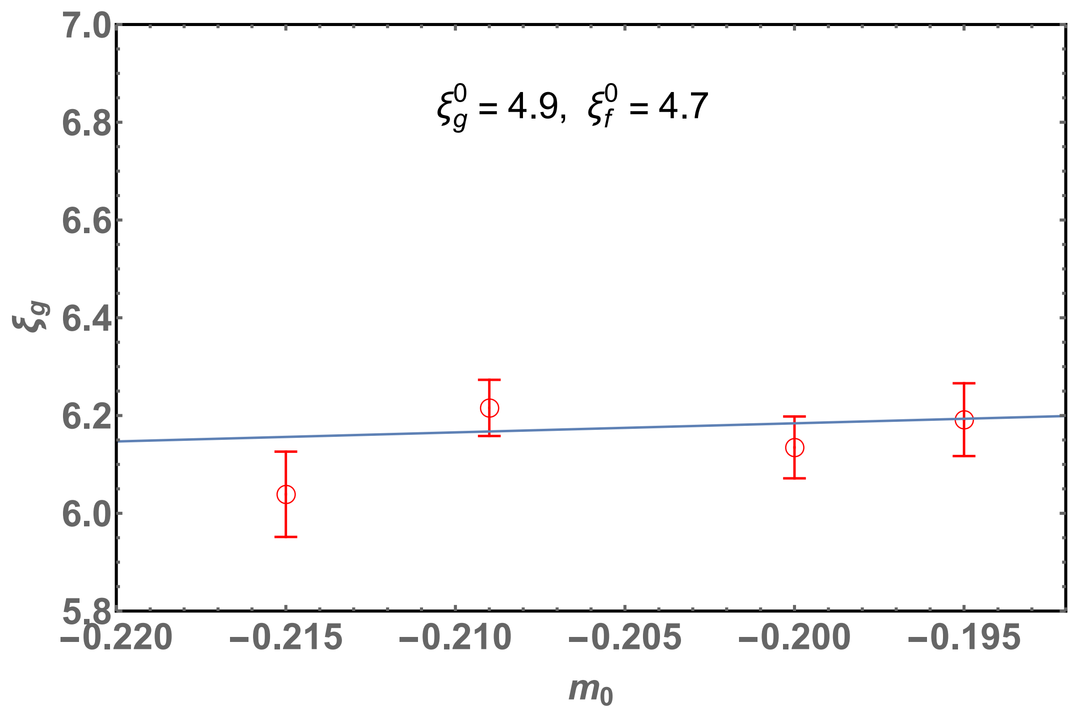
$$\xi_g(\xi_g^0, \xi_f^0, m_0) = a_0 + a_1\xi_g^0 + a_2\xi_f^0 + a_3 m_0,$$
$$\xi_f(\xi_g^0, \xi_f^0, m_0) = b_0 + b_1\xi_g^0 + b_2\xi_f^0 + b_3 m_0,$$
$$M_{ps}^2(\xi_g^0, \xi_f^0, m_0) = c_0 + c_1\xi_g^0 + c_2\xi_f^0 + c_3 m_0.$$

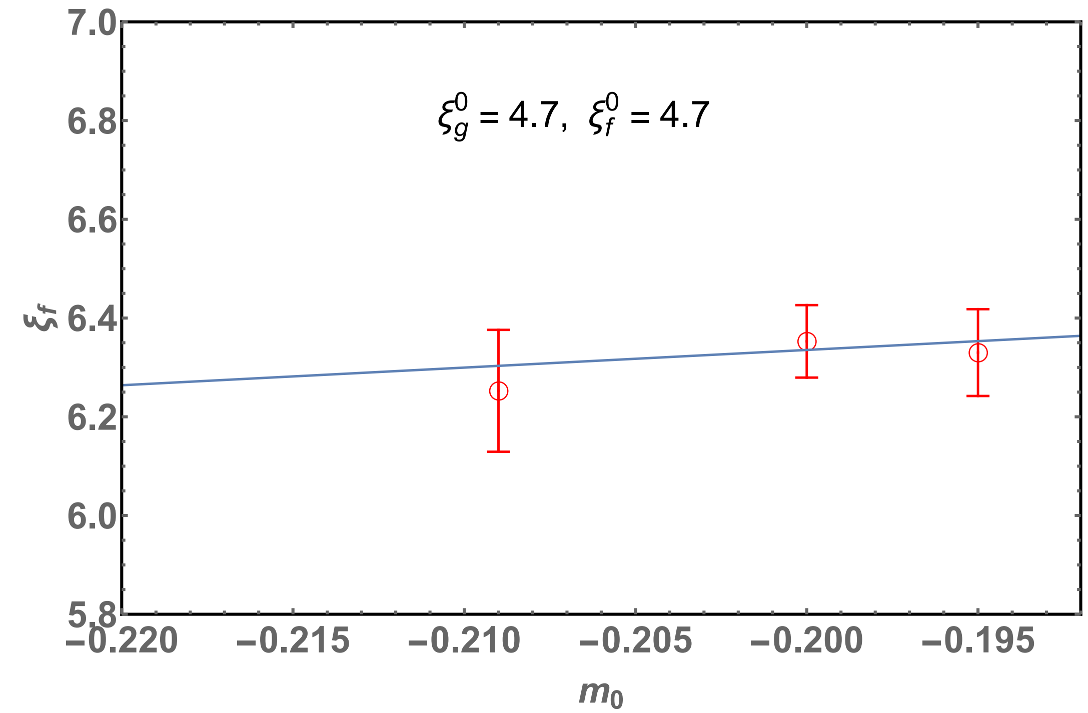
with

***Edwards, Joo, Lin (2008)***

$$a_0 = 0.6(16),\ a_1 = 0.97(13),\ a_2 = 0.31(23),\ a_3 = 2(4),$$
$$b_0 = 1.8(24),\ b_1 = 0.06(18),\ b_2 = 1.1(3),\ b_3 = 4(7),$$
$$c_0 = 0.475(5),\ c_1 = -0.0168(4),\ c_2 = -0.0375(6),\ c_3 = 0.986(11).$$

very mild dependence



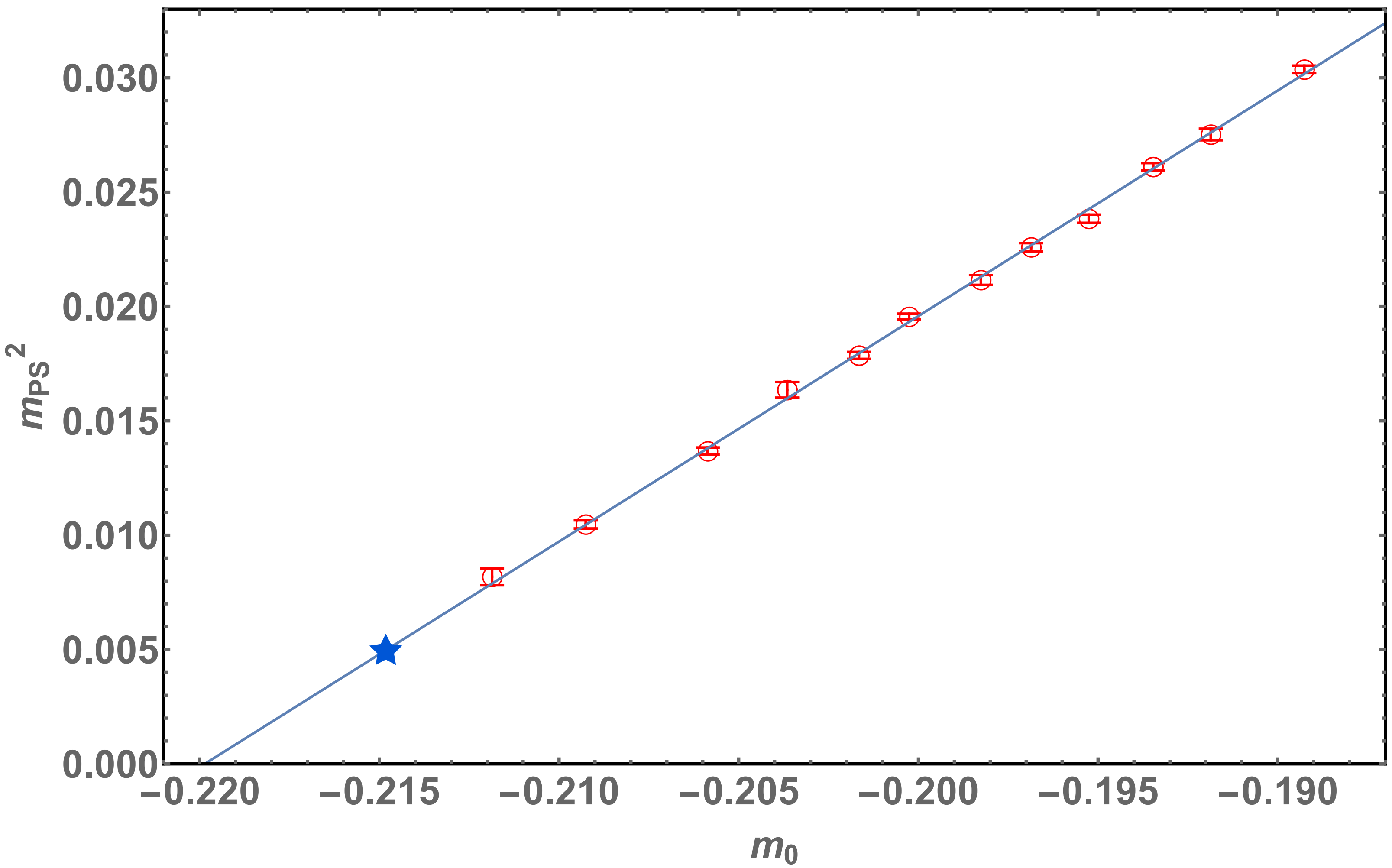
# ✤ Anisotropy tuning - results

Renormalized conditions

$$\xi_g(\xi_g^{0*},\xi_f^{0*},m_0^*)=\xi_f(\xi_g^{0*},\xi_f^{0*},m_0^*)=\xi,\quad M_{ps}^2(\xi_g^{0*},\xi_f^{0*},m_0^*)=m_{ps}^2.$$

Our target anisotropy is $\xi=6.3$ with $(m_{ps}a)^2=0.005$.

$$\boxed{\xi_g^{0*}=4.84(8),\ \xi_f^{0*}=4.72(12),\ m_0^*=-0.2148(37)}$$




Results of 128x16³ Lattice

$$\xi_g=6.29(4),\ \xi_f=6.1(2)$$

$$(m_{ps}a)^2=0.00517(14)$$

**Consistent with our target parameters!**

# ✤ Finite T calculations

Ensembles (~200 configurations)

$$N_\tau \times 16^3 \text{ lattice with } N_\tau = 16,\ 20,\ 24,\ 28,\ 30,\ 36,\ 40,\ 48,\ 128$$

$$N_\tau \times 16^2 \times 24 \text{ lattice with } N_\tau = 16,\ 20,\ 24,\ 28,\ 36,\ 42,\ 48,\ 56$$

Boundary conditions: antiperiodic temporal fermion boundary conditions

periodic b.c. for all others

Critical temperature: deconfining transition

Rapid change of temporal Polyakov loop $\Rightarrow$ $T^c = \dfrac{1}{N_\tau^c a}$

Temporal and spacial correlators of isovector mesons (Stochastic wall sources)

pseudoscalar(PS), scalar(S), vector(V), and axial vector(AV) mesons

Degeneracy between parity partners in hadron multiplets

$$V \xleftrightarrow{SU(2)_A} AV \quad \Rightarrow \quad \text{Chiral symmetry restoration}$$

$$PS \xleftrightarrow{U(1)_A} S$$

# Renormalized Polyakov Loop

Multiplicative renormalization of Polyakov Loop

$$L(T) = e^{-F(T)/T}$$

Free energy $F_R = F + \Delta F$

Additive renormalization

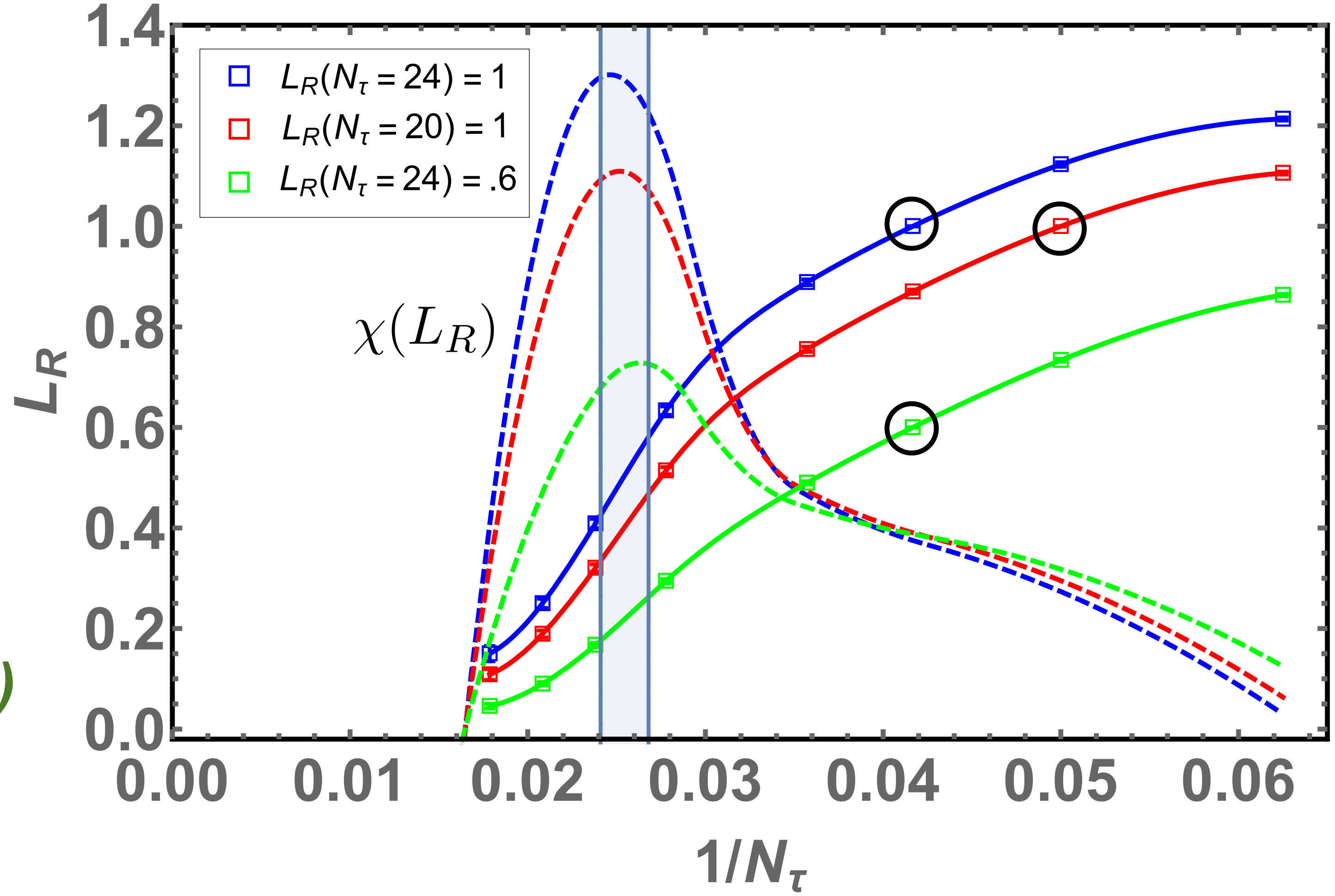
Renormalized Polyakov Loop

$$L_R(T) \equiv Z_L^{N_\tau} L(T)$$

Renormalization condition

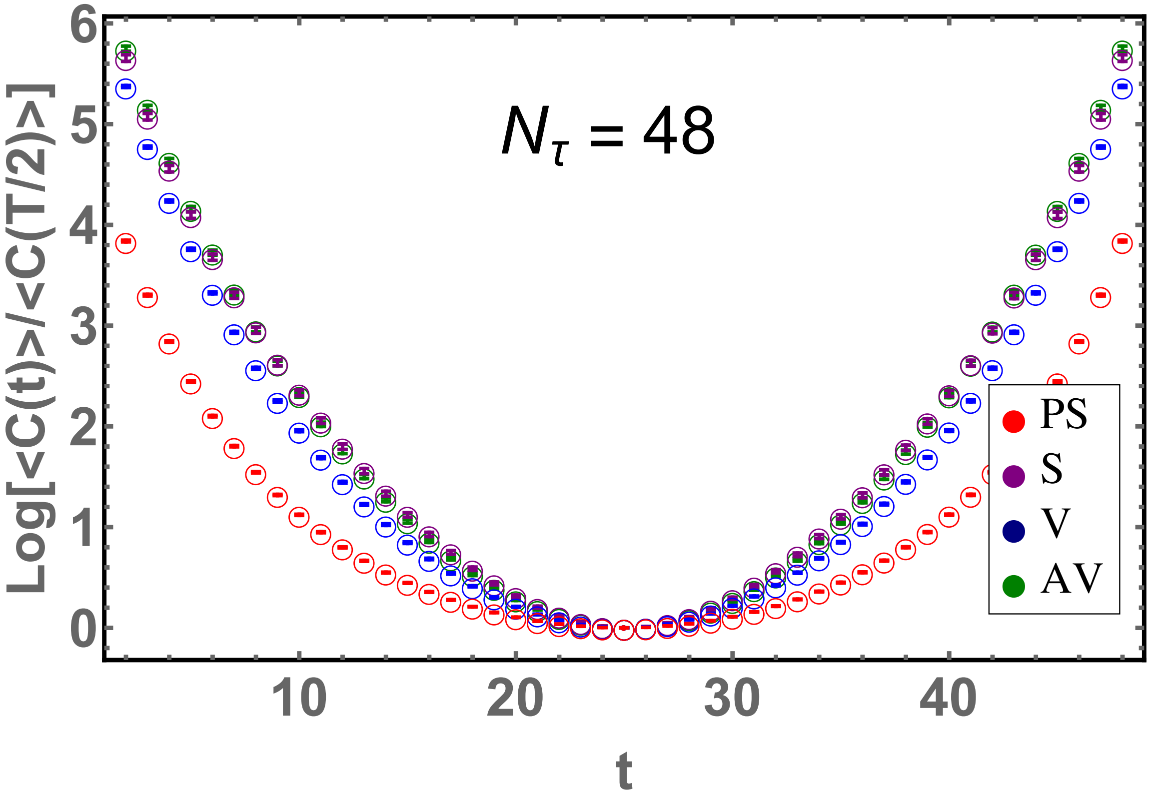
$$L_R(T_R) \equiv \text{constant}$$

***Borsanyi et. al. (2012)***
***Aarts et. al. (2014)***




**Deconfining critical temperature** $T^c = 1/N_\tau^c = 0.0254(14)$ or $N_\tau^c = 39.5(2)$

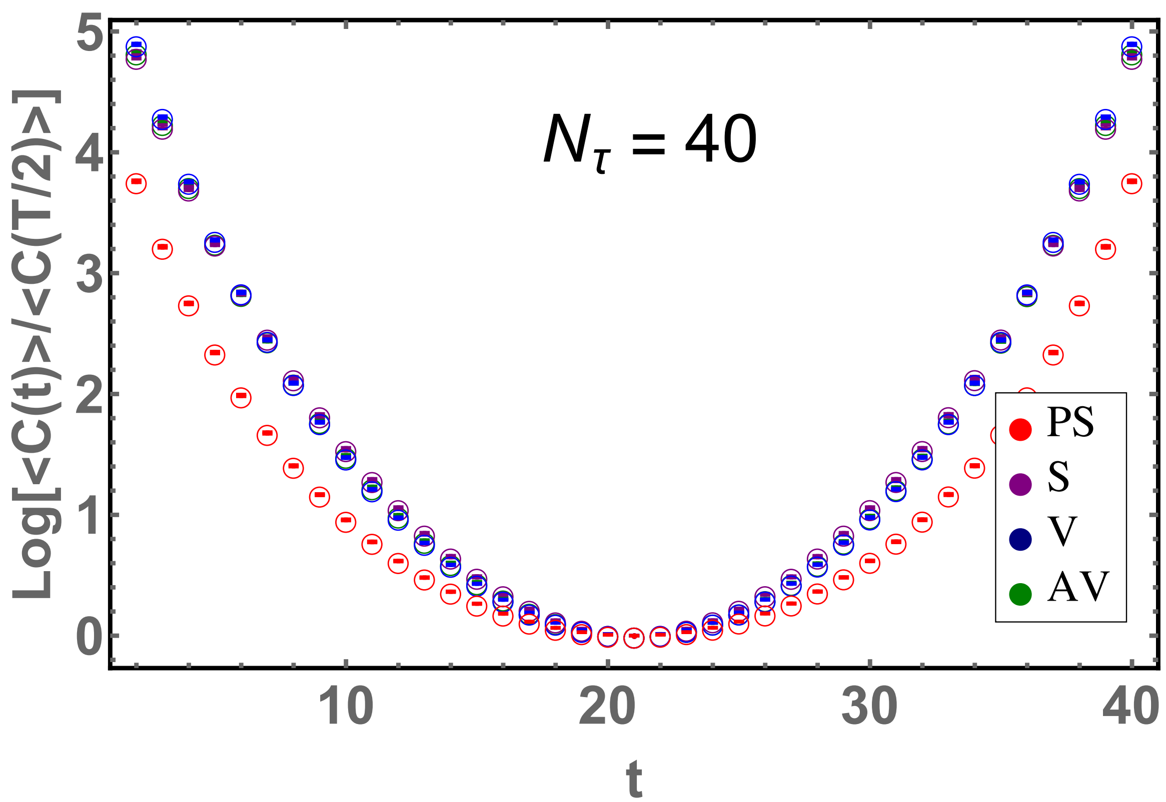
# ✤ Temporal correlators




Below the critical temperature, the temporal correlation functions decay in the order of

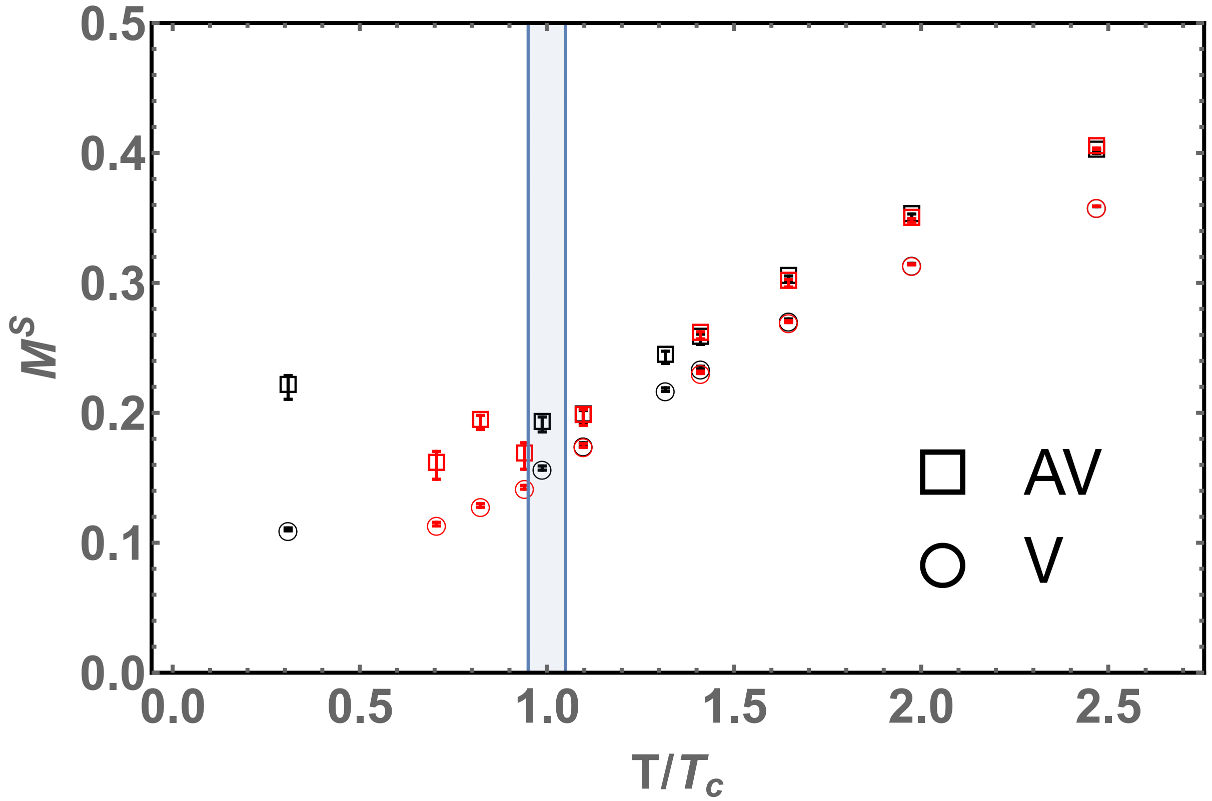
$$\mathrm{Log}\left[\frac{C_{PS}(t)}{C_{PS}(N_\tau/2)}\right] < \mathrm{Log}\left[\frac{C_V(t)}{C_V(N_\tau/2)}\right] < \mathrm{Log}\left[\frac{C_{AV}(t)}{C_{AV}(N_\tau/2)}\right]$$

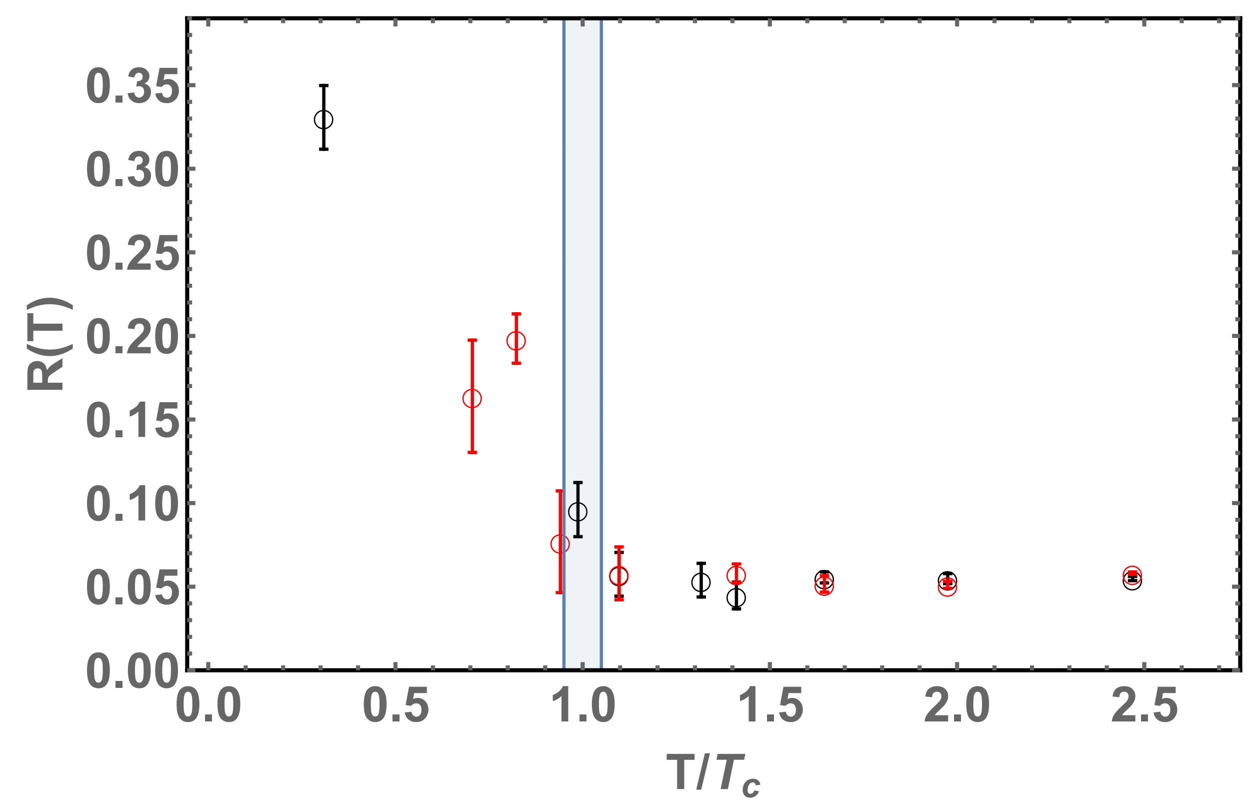
Onset of the critical temperature, the correlation functions for vector and axial-vector mesons are degenerate.



# ✤ Spacial correlators - Vector channel

Screening mass for vector and axial vector mesons




Black: $N_\tau \times 16^3$ lattice

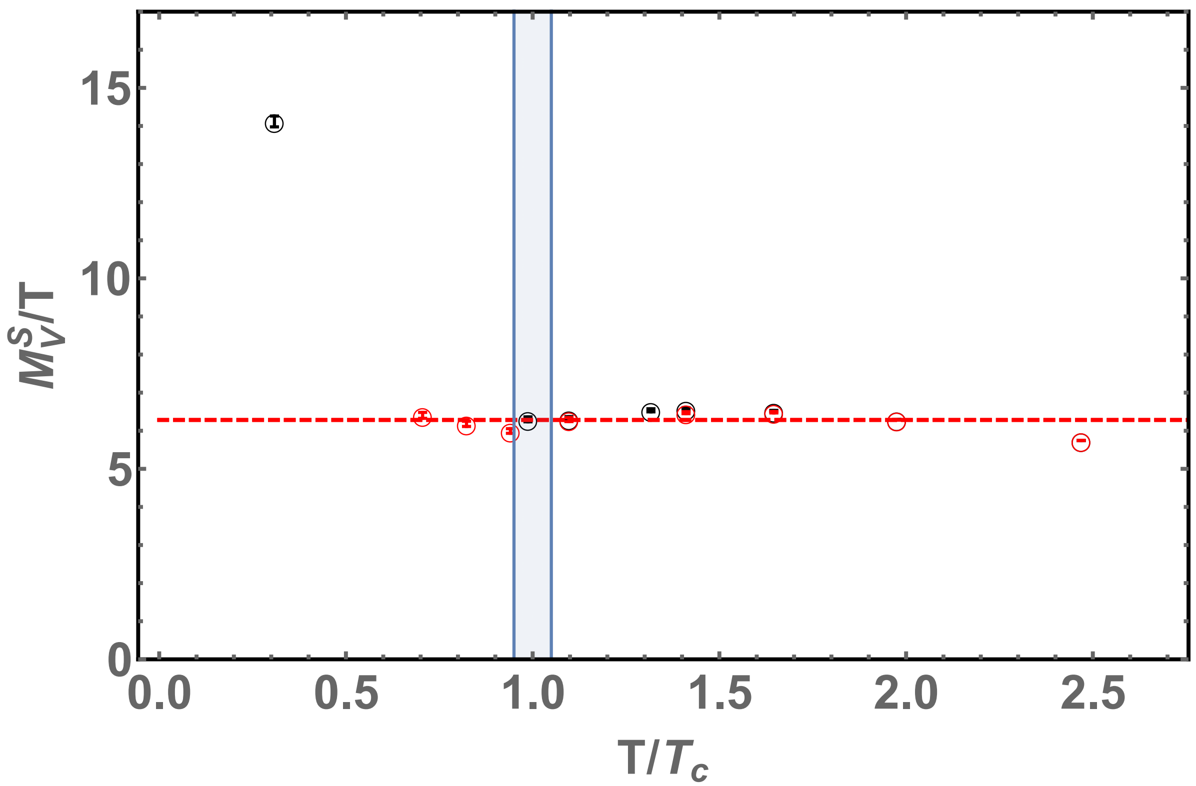
Red: $N_\tau \times 16^2 \times 24$ lattice

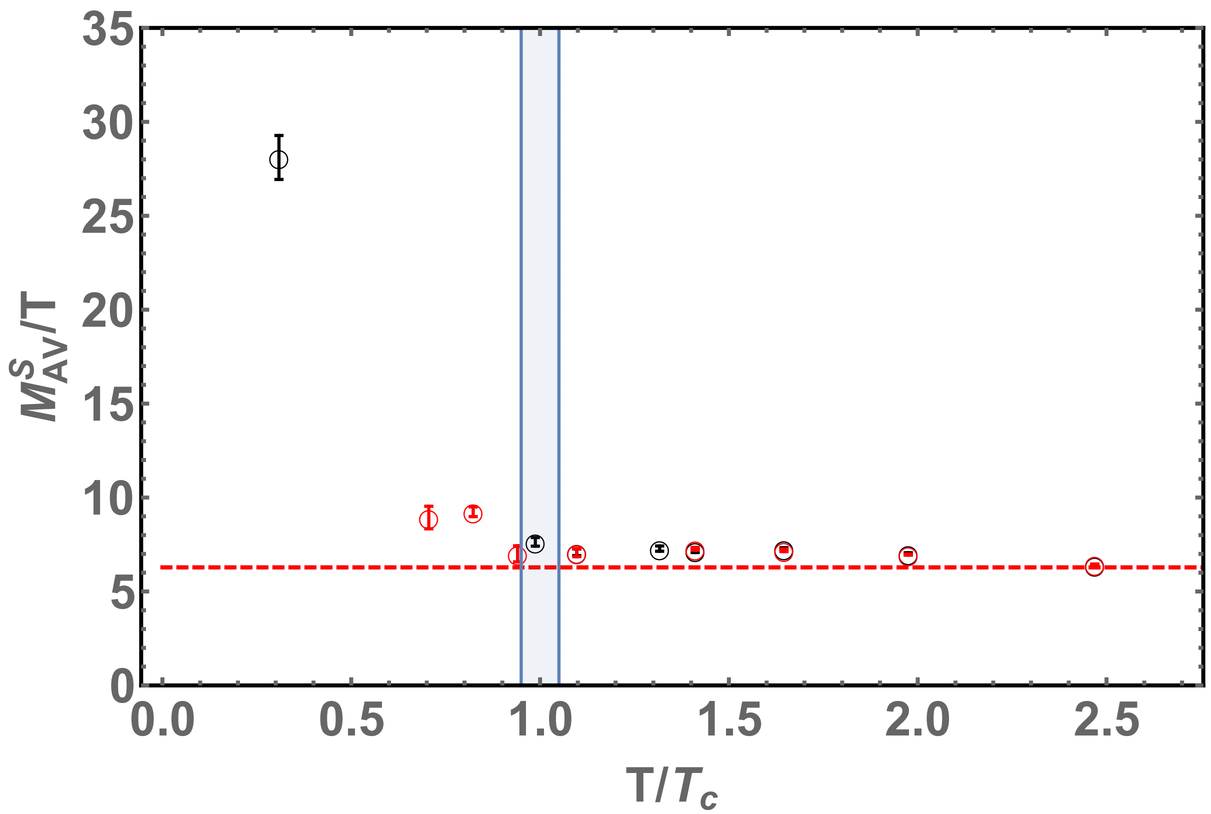
$$R(T) = \frac{M_{AV}(T) - M_V(T)}{M_{AV}(T) + M_V(T)}$$

V and AV mesons are degenerate at $T \gtrsim T_c$

# ✤ Spacial correlators - Vector channel

Screening mass for vector and axial vector mesons




Black: $N_\tau \times 16^3$ lattice

Red: $N_\tau \times 16^2 \times 24$ lattice

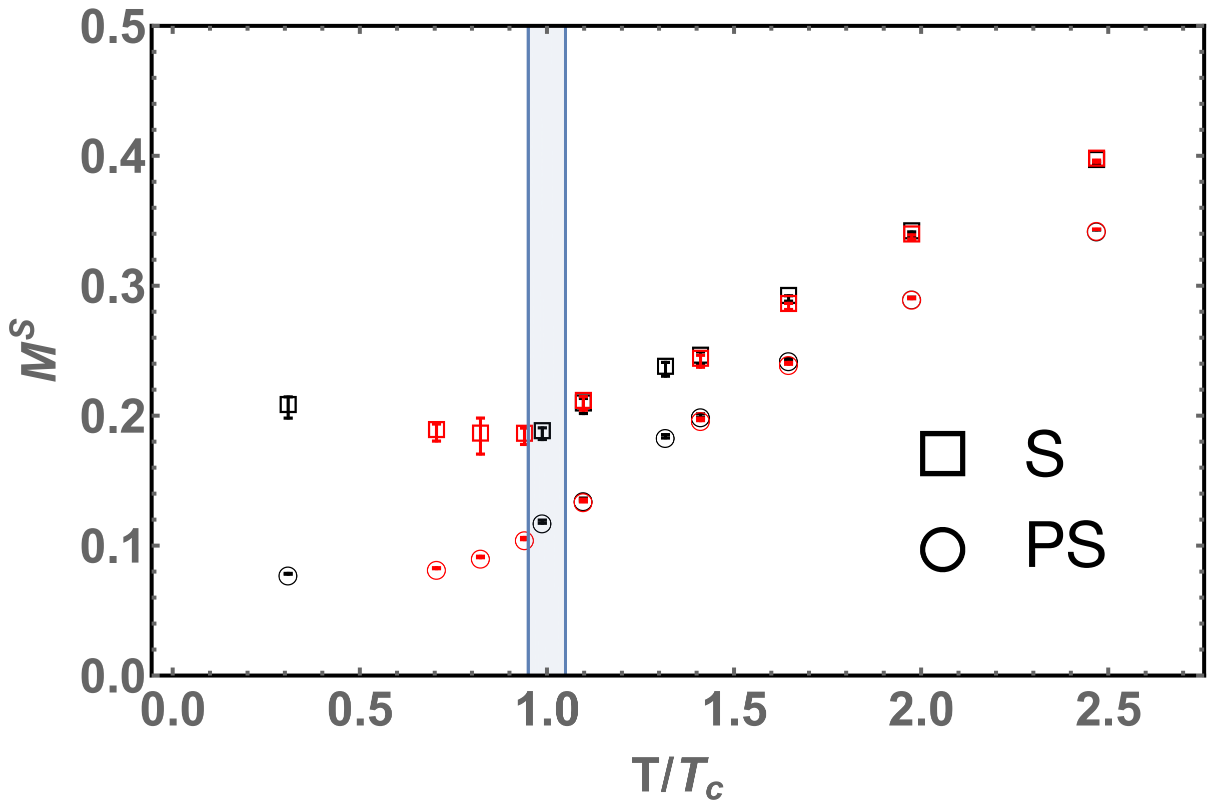
Red dotted line = $2\pi$ (free quark)

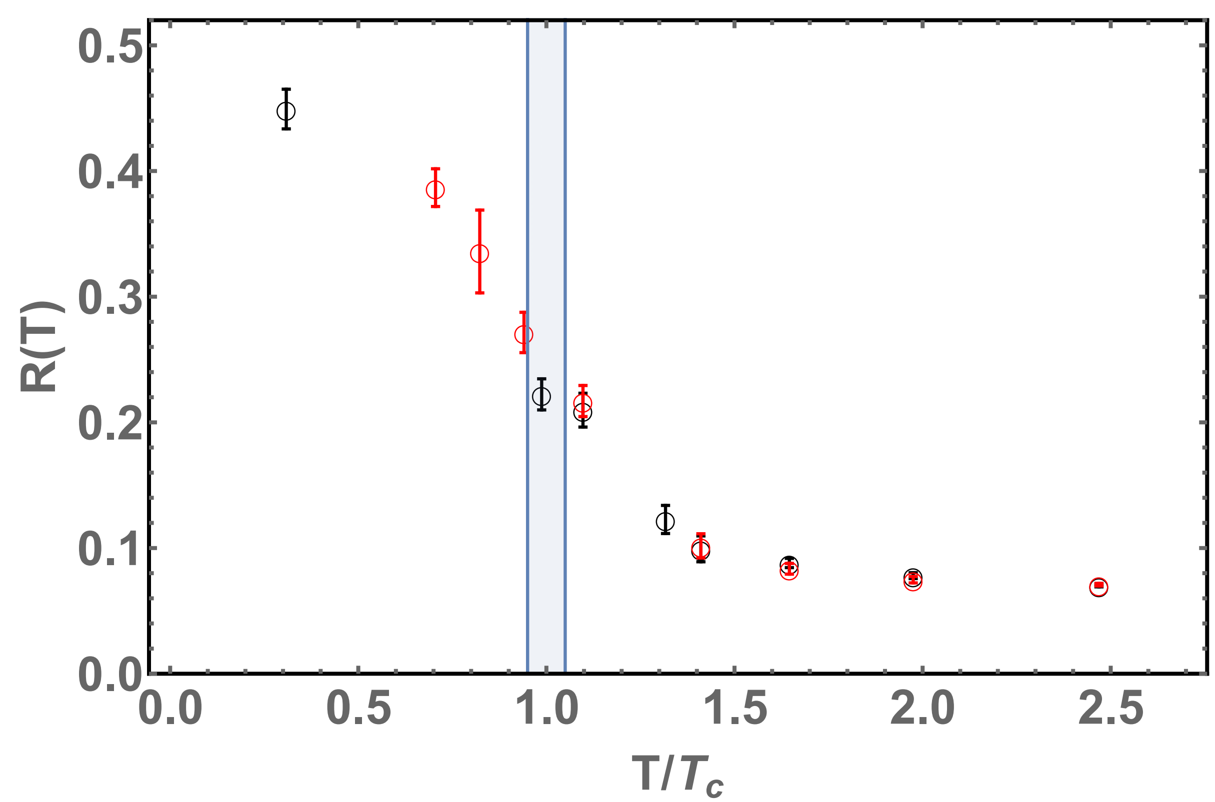
Above 2Tc, screening mass of vector and axial-vector mesons begins to deviate from the plateau.

Lattice artifacts due to too small $N_\tau$?

# Spacial correlators - Scalar channel

Screening mass for scalar and pseudoscalar mesons




Black: $N_\tau \times 16^3$ lattice

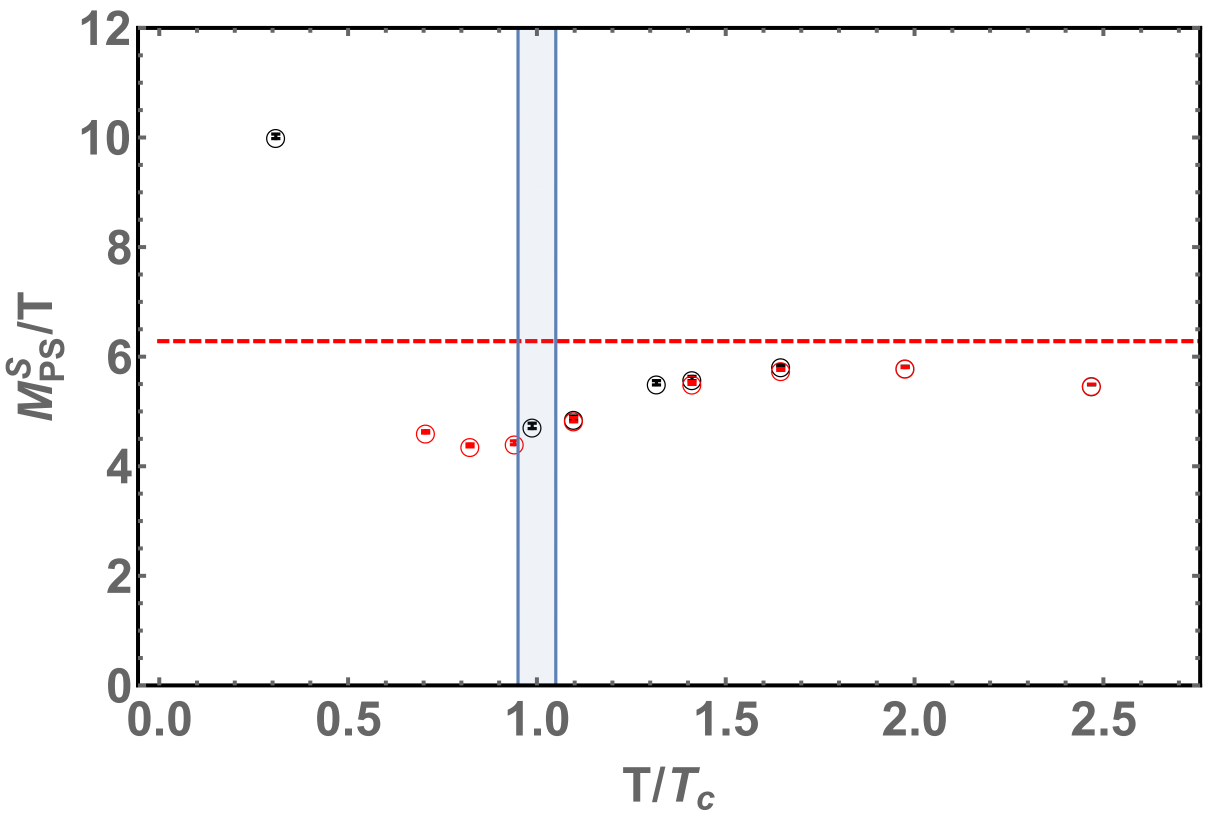
Red: $N_\tau \times 16^2 \times 24$ lattice

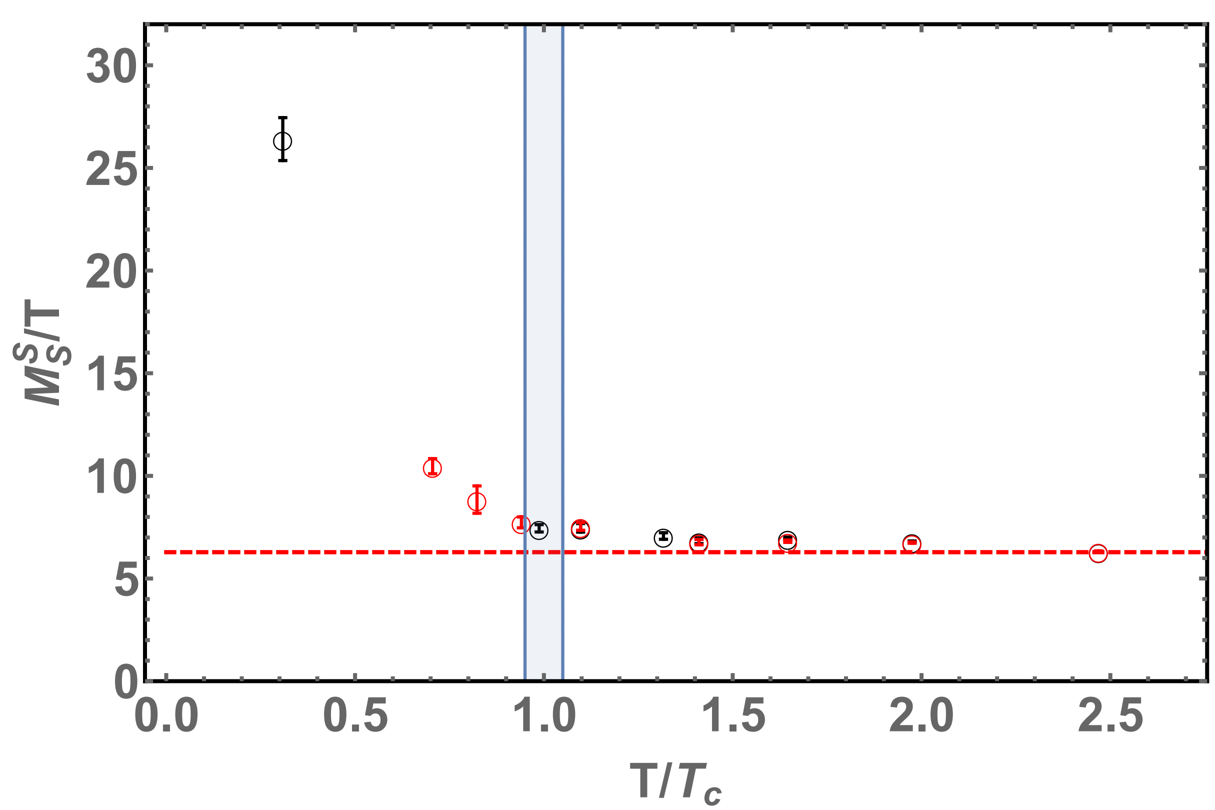
$$R(T) = \frac{M_S(T) - M_{PS}(T)}{M_S(T) + M_{PS}(T)}$$

S and PS mesons are degenerate at $T \gtrsim 1.5 T_c$

# ✤ Spacial correlators - Scalar channel

Screening mass for scalar and pseudoscalar mesons




Black: $N_\tau \times 16^3$ lattice

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Red dotted line = $2\pi$ (free quark)

Above 2TC, screening mass of pseudoscalar and scalar mesons begins to deviate from the plateau.

Lattice artifacts due to too small $N_\tau$?

# ✤ Conclusion and future work

SU(2) gauge theory with 2 fund. Wilson fermions on an anisotropic lattice

Anisotropy tuning works!

Non-plain Wilson loops are helpful for determining gauge anisotropy.

Finite T results

Parity doubling in the temporal and spacial correlators for vector channel just above Tc

Parity doubling in the spacial correlators for scalar channel  above 1.5Tc

Systematic errors

For $T \leq T_c$ fitting errors in scalar and axial-vector screening mass due to very limited numbers of data points in the asymptotic region

Mistuned bare anisotropy generate up to ~3% errors.

Anisotropy tuning works at any temperature?

Massless, infinite volume, and continuum limit needs to be investigated.

How does the meson spectrum change at finite chemical potential?

**Thank you for your attention!**