Physical Spectra and the Limits of Perturbative Estimates in a Theory with a Higgs

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The Problem

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- The Higgs sector is a gauge theory

\[ L = -\frac{1}{4} W^a_{\mu \nu} W^a_{\mu \nu} \]

\[ W^a_{\mu \nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f^{a}_{\ b\ c} W^b_{\mu} W^c_{\nu} \]

- Ws

- Coupling \( g \) and some numbers \( f^{abc} \)
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\[
L = -\frac{1}{4} W_a^{\mu \nu} W^{a \mu \nu} + (D_{\mu}^{ij} h^j)^+ D_{ik}^{\mu} h_k
\]

\[
W_a^{\mu \nu} = \partial_{\mu} W_a^{\nu} - \partial_{\nu} W_a^{\mu} + gf_{bc}^{a} W_b^{\mu} W_c^{\nu}
\]

\[
D^{ij}_{\mu} = \delta^{ij}_{\mu} \partial_{\mu} - ig W_a^{\mu} t_a^{ij}
\]

- Ws \quad W^{a}_{\mu}
- Higgs \quad h_i
- Coupling \ g \ and \ some \ numbers \ f^{abc} \ and \ t_a^{ij}
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\[
L = -\frac{1}{4} W_{\mu \nu}^a W^{\mu \nu}_a + (D^{ij}_\mu h^j)^+ D_{ik}^\mu h_k + \gamma (h^a h_a^+ - \nu^2)^2
\]

\[
W_{\mu \nu}^a = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + gf_{bc}^a W^b_\mu W^c_\nu
\]

\[
D^{ij}_\mu = \delta^{ij} \partial_\mu - ig W^a_\mu t^{ij}_a
\]

- **Ws** \( W^a_\mu \)
- **Higgs** \( h_i \)
- No QED: Ws and Zs are degenerate
- Couplings \( g, \nu, \gamma \) and some numbers \( f^{abc} \) and \( t^{ij}_a \)
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\[
L = -\frac{1}{4} W^a_{\mu \nu} W^a_{\mu \nu} + (D^j_i h^j) + D^\mu_{ik} h^k + \gamma (h^a h^a + v^2)^2
\]

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W^a_{\mu \nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + gf^a_{bc} W^b_\mu W^c_\nu
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- Local SU(2) gauge symmetry

\[ W^a_\mu \rightarrow W^a_\mu + (\delta^a_b \partial_\mu - gf^a_{bc} W^c_\mu) \phi^b \]

\[ h_i \rightarrow h_i + g t^i_j \phi^a h_j \]
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\[ L = -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} + (D^i_j h^j)^+ D^i_{\mu} h_k + \gamma (h^a h_a^+ - \nu^2)^2 \]

\[ W^a_{\mu\nu} = \partial_{\mu} W^a_{\nu} - \partial_{\nu} W^a_{\mu} + gf^a_{bc} W^b_{\mu} W^c_{\nu} \]

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- Global SU(2) Higgs custodial (flavor) symmetry
  - Acts as right-transformation on the Higgs field only

\[ W^a_{\mu} \rightarrow W^a_{\mu} \]

\[ h_i \rightarrow h_i + a^i_j h_j + b^i_j h^*_j \]
Physical states

- Physical spectrum: Observable particles
  - Experiments measure peaks in cross-sections

[Fröhlich et al. PLB 80, 't Hooft ASIB 80, Bank et al. NPB 79]
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  - Not asymptotic states in perturbation theory
  - Higgs-Higgs, W-W, Higgs-Higgs-W etc.
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- Mass spectrum?
- Why does perturbation theory work?

[Fröhlich et al. PLB 80, 't Hooft ASIB 80, Bank et al. NPB 79]
Mass relation - Higgs

- Lattice result: Lightest $0^+$ composite state has the same mass as Higgs at tree-level [Maas MPLA 12, Maas & Mufti JHEP 14]
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\[ h = v + \eta \approx \text{const.} + \langle h^+ (x) h (y) \rangle + O(\eta^3) \]

- Same poles to leading order
- Fröhlich-Morchio-Strocchi (FMS) mechanism
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\[
\langle (h^+ h)(x)(h^+ h)(y) \rangle \approx \text{const.} + \langle h^+ (x) h(y) \rangle + O(\eta^3)
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- Perturbative tool to calculate bound state masses
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- Fröhlich-Morchio-Strocchi (FMS) mechanism
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- Deeply-bound relativistic state
  - Mass defect $\sim$ constituent mass
  - Cannot be described with quantum mechanics
Mass relation - W

• $W$ is a $1^-$ (degenerate) gauge triplet

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Mass relation - $W$

- $W$ is a $1^-$ (degenerate) gauge triplet
- No physical gauge triplets – but custodial triplets!
- Same mechanism [Fröhlich et al. PLB 80]

$$\langle (h^+ D_\mu h)(x)(h^+ D_\mu h)(y) \rangle$$

$h = v + \eta$

$$\approx \text{const.} + \langle W_\mu(x) W_\mu(y) \rangle + O(\eta^3)$$

$$\partial v = 0$$

- Same poles at leading order
Mass relation - W

- W is a 1\(^{-}\) (degenerate) gauge triplet
- No physical gauge triplets – but custodial triplets!
- Same mechanism \[\text{[Fröhlich et al. PLB 80]}\]
  \[\langle (h^+ D_\mu h)(x)(h^+ D_\mu h)(y) \rangle \]
  \[h=v+\eta \approx \text{const.}\+\langle W_\mu (x)W_\mu (y)\rangle+O(\eta^3)\]
  \[\partial v=0\]
- Same poles at leading order
- Also confirmed in lattice calculations \[\text{[Maas MPLA 12, Maas & Mufti JHEP 14]}\]
• Structural
  • Degeneracy patterns: Local vs. global symmetries
  • Works for 1HDM and 2HDM models [Maas & Pedro PRD 16]
  • May not hold for other theories [Maas MPLA 15, Maas & Törek 15]
    • Talk by Pascal Törek directly afterwards
  • Implications for Technicolor-type theories [Maas MPLA 15]
Limits?

- **Structural**
  - Degeneracy patterns: Local vs. global symmetries
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- **Dynamical** [Maas & Mufti PRD 15]
  - When is this identification possible?
  - When is perturbation theory predictive?
FMS validity

Effective mass

$\frac{m_t}{m_\tau}$
FMS validity

FMS prediction

Effective mass

$\langle \text{m}_W [\text{GeV}] \rangle$

$\langle \text{m}_t/\text{m}_0 \rangle$

[Maas & Mufti JHEP 14]
FMS validity

Too low: Finite volume effect

FMS prediction

Effective mass

\[ m_{\text{eff}} \text{ (GeV)} \]

\[ m_t/m_0 \]

[Maas & Mufti JHEP 14]
FMS validity

Too low: Finite volume effect

Elastic decay threshold
Higgs as resonance
Expensive, signal very bad
FMS validity

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Elastic decay threshold
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FMS prediction

Higgs and W mass agrees
FMS stops working
So does Brout-Englert-Higgs!
Higgs mass

Standard mass-cutoff plot

Elastic decay threshold

No strong dependence of mass range on cutoff - expected
Phase diagram
- Quantum effects remove BEH effect
  - Opposite does not happen
Phase diagram

Critical end-line? [Bonati et al. NPB 10]

QCD like

Higgs like

LCP direction?

Quantum effects remove BEH effect

Opposite does not happen


LCP: 0^+, 1^- masses, $\alpha(200 \text{ GeV})$(miniMOM scheme)
Phase diagram

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- Quantum effects remove BEH effect
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  - LCP: $0^+, 1^-$ masses, $\alpha(200 GeV)$ (miniMOM scheme)
Perturbative predictivity: Coupling

Higgs-like

QCD-like
Perturbative predictivity: Coupling

1/a [GeV]

Higgs-like

Large cutoff, QCD: Larger couplings

QCD-like

$\alpha(200 \text{ GeV})$
Perturbative predictivity: Coupling

Large cutoff, BEH: Small couplings

Higgs-like

Large cutoff, QCD: Larger couplings

QCD-like

Large cutoff, BEH: Small couplings
Perturbative predictivity: Coupling

1/a [GeV]

Higgs-like

Large cutoff, QCD: Larger couplings

BEH/QCD at Similar couplings

QCD-like

Large cutoff, BEH: Small couplings
Perturbative predictivity: Mass ratios

Tree-level perturbation theory is right

QCD-like

Elastic threshold

$0^+/1^-$ mass ratio

$0^+/1^-$ mass ratio
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Physical ratio

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FMS mechanism and perturbation theory predicts spectrum of observable states well in the SM

Predictions: Structure and dynamics important
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- Cannot predict reliably
  - Presence of BEH effect
  - Size of quantum corrections to the potential
  - Possible mass ranges for states
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- FMS mechanism and perturbation theory predicts spectrum of observable states well in the SM
- Predictions: Structure and dynamics important
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- Cannot predict reliably
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  - Size of quantum corrections to the potential
  - Possible mass ranges for states
- What happens beyond the SM case?
55th International Winter School on Theoretical Physics

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