Flux tubes at Finite Temperature
Pedro Bicudo, Nuno Cardoso and Marco Cardoso
Portuguese Lattice QCD Collaboration
CFTP, Departamento de Física, Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais 1, 1049-001 Lisbon, Portugal

Abstract
We show the flux tubes produced by static quark-antiquark, quark-quark and quark-antiquark charges at finite temperature. The sources are placed in the lattice with fundamental and adjacent Polyakov loops. We compute the square densities of the chromomagnetic and chromoelectric fields above and below the phase transition. Our results are gauge invariant and produced in pure gauge SU(3). The codes are written in CUDA and the computations are performed with GPUs.

1. Introduction
We study the chromo-fields distributions inside the flux tubes formed by polyakov loops in the static QCD, QQ and QG systems. We address how the flux tube evolves with the distance between quarks and when the temperature increase beyond the deconfinement temperature. In section 2, we describe the lattice formulation. We briefly review the Polyakov loop for these systems and show how to compute the color fields as well as the Lagrangian distributions. In section 3, the numerical results are shown. Finally, we conclude in section 4.

2. Computation of the chromo-fields in the flux tube
The central observables that govern the event in the flux tube can be extracted from the correlation of a plaquette, \( \langle O_{\sigma} \rangle \), with the Polyakov loops, \( \lambda_i \),

\[
\langle O_{\sigma} \rangle = \frac{1}{N_L} \frac{\sum_{\lambda_i} \langle O_{\sigma} \lambda_i \rangle}{\sum_{\lambda_i}}
\]

where \( \lambda_i \) denotes the distance of the plaquette from the line connecting quark sources, \( \sigma \) is the quark separation, \( \langle O_{\sigma} \rangle = \frac{1}{N_L} \sum_{\lambda_i} \langle O_{\sigma} \lambda_i \rangle \) and \( N_L \) is the number of time slices of the lattice. We also analyze the periodicity in the time direction for the Polyakov loops, \( \langle O_{\sigma} \rangle = \frac{1}{N_L} \sum_{\lambda_i} \langle O_{\sigma} \lambda_i \rangle \), to average the plaquette over the time direction.

To reduce the fluctuations of the \( \langle O_{\sigma} \rangle \), we measure the following quantity,

\[
\langle O_{\sigma} \rangle = \frac{1}{N_L} \frac{\sum_{\lambda_i} \langle O_{\sigma} \lambda_i \rangle}{\sum_{\lambda_i}}
\]

where \( \mu_{\lambda_i} \) is the reference point placed far from the quark sources. Therefore, using the plaquette orientation \( \langle O_{\sigma} \rangle = \frac{1}{N_L} \frac{\sum_{\lambda_i} \langle O_{\sigma} \lambda_i \rangle}{\sum_{\lambda_i}} \), we can relate the six components in Eq. (2) to the components of the chromoelectric and chromomagnetic fields.

\[
\langle O_{\sigma} \rangle = \frac{1}{N_L} \frac{\sum_{\lambda_i} \langle O_{\sigma} \lambda_i \rangle}{\sum_{\lambda_i}}
\]

and also calculate the total actions (Lagrangian) densities, \( \langle O_{\sigma} \rangle = \frac{1}{N_L} \frac{\sum_{\lambda_i} \langle O_{\sigma} \lambda_i \rangle}{\sum_{\lambda_i}} \).

In order to improve the signal over noise ratio in the \( \langle Q_{\sigma} \rangle \) and \( \langle Q^*_{\sigma} \rangle \) systems, we use the multithit technique, [2, 3], replacing each temporal link by it’s thermal average, and the extended multihit technique, [4], which consists in replacing each temporal link by it’s thermal average with the first \( N \) neighbors fixed. Instead of taking the thermal average of a temporal link with the first neighbor, we fix the higher order neighbors, and apply the heat-bath algorithm to all the links inside, averaging the central link,

\[
\langle O_{\sigma} \rangle = \frac{1}{N_L} \frac{\sum_{\lambda_i} \langle O_{\sigma} \lambda_i \rangle}{\sum_{\lambda_i}}
\]

By using \( N = 2 \) we are able to greatly improve the signal, when compared with the error reduction achieved with the simple multihit. Of course, this technique is more computer intensive than simple multihit, while being simpler to implement than multilevel. The only restriction is \( R > 2 \) for this technique to be valid.

3. Results
In this section, we present the results for different \( \beta \) values using a fixed lattice volume of \( 48^3 \times 8 \). Table 1. All our computations are performed in NVIDIA GPUs using our CUDA codes.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( T/T_c )</th>
<th>( \chi^2 )</th>
<th>( \chi_0^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.037</td>
<td>0.945</td>
<td>0.2304</td>
<td>5990</td>
</tr>
<tr>
<td>0.034</td>
<td>0.963</td>
<td>0.21044</td>
<td>5990</td>
</tr>
<tr>
<td>0.028</td>
<td>1.127</td>
<td>0.17036</td>
<td>5990</td>
</tr>
<tr>
<td>0.025</td>
<td>1.408</td>
<td>0.14513</td>
<td>5990</td>
</tr>
<tr>
<td>0.042</td>
<td>1.690</td>
<td>0.11713</td>
<td>5990</td>
</tr>
</tbody>
</table>

Table 1: Lattice data for a \( N = 4 \) volume. The lattice spacing was computed using the parametrization from [5] in units of the string tension at zero temperature. We denote with \( \chi^2 \) the number of remaining configurations after we remove the configurations in the other plane. The QQ and QG are located at \( (0, R/2, 0) \) and \( (R/2, 0, 0) \) for \( R = 6, 8, 10, 12 \) lattice spacing units. In Figs. 3 and 4, we show the results for the QQ system. As expected the strength of the fields decrease with the temperature. Also, in the confined phase the width is in the middle of the flux tube increases with the distance between the sources, while above the phase transition the width decreases with the distance.

Just below the phase transition, we need to make sure that we don’t have contaminated configurations as already mentioned in [6]. By plotting the histogram of Polyakov loop history for \( \beta = 0.055 \), Fig. 1, we were able to identify a second peak which then we were able to remove all the thermodynamic configurations that lie on the second peak. Therefore, in Table 2 the value with asterisk corresponds to the configurations after removing these contaminated configurations.

![Image](image.jpg)

Figure 1: Histogram of the Polyakov loop history for \( \beta = 0.055 \).

![Image](image2.jpg)

Figure 2: The results for the QQ system. The results in the left column correspond to the fields along the sources (plane XY) and the right column to the results in the middle of the flux tube (plane XZ). \( R \) is the distance between the sources in lattice units.

![Image](image3.jpg)

Figure 3: The results for the QQ system. The results in the left column correspond to the fields along the sources (plane XY) and the right column to the results in the middle of the flux tube (plane XZ). \( R \) is the distance between the sources in lattice units.
3. Results (cont.)

As the distance increase between the sources, the fields square densities decrease. Below the deconfinement critical temperature, this decrease is moderate and is consistent with the widening of the flux tube as already seen in studies at zero temperature [6]; moreover the field strength clearly decreases as the temperature increases as expected from the critical curve for the string tension [6]. Above the deconfinement critical temperature, the fields rapidly decrease to zero as the quarks are pulled apart, qualitatively consistent with screened Coulomb-like fields. While the width of the flux tube below the phase transition temperature increases with the separation between the quark-antiquark, above the phase transition the width seems to decrease.

Acknowledgments
Nano Cardoso and Marco Cardoso are supported by FCT under the contracts SFRH/BPD/10944/2015 and SFRH/BPD/73340/2010 respectively. We also acknowledge the use of CPU and GPU servers at PQCD, supported by NVIDIA, CFTP and FCT grant UIDB/00177/2015.

References