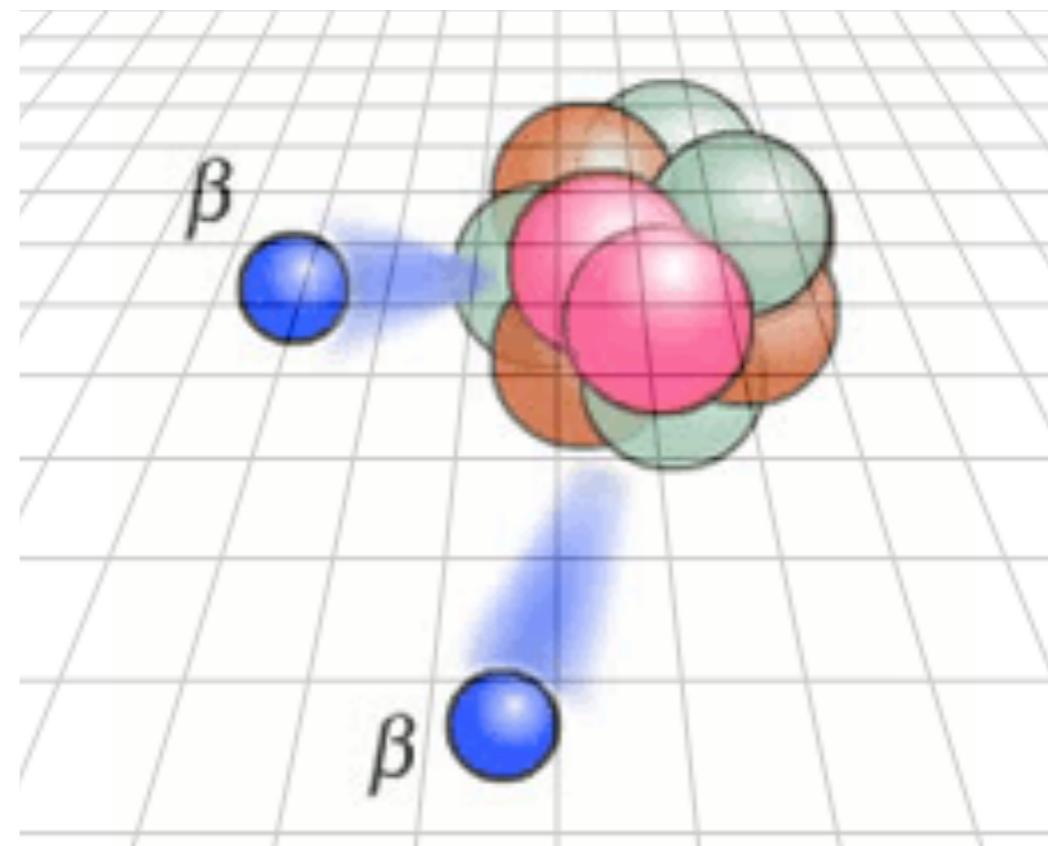


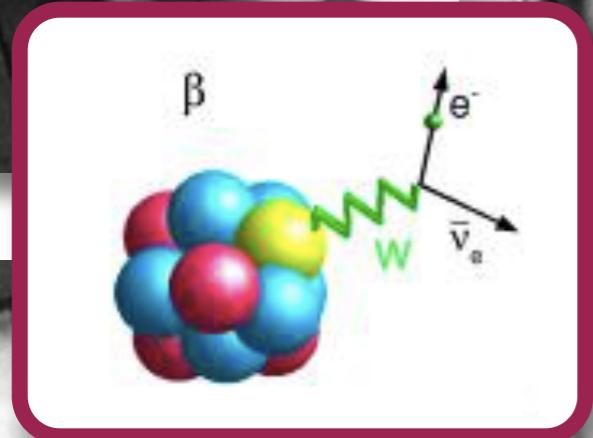
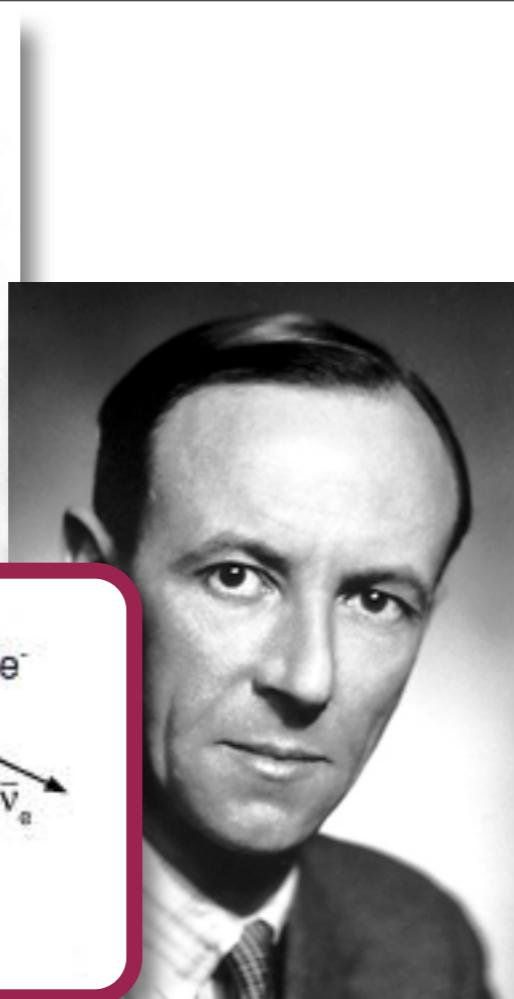
Neutrinoless Double Beta Decay from Lattice QCD



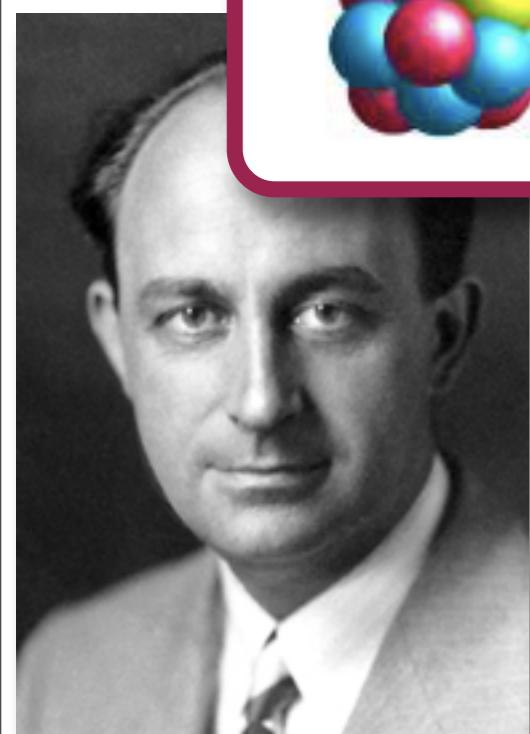
Amy Nicholson
UC Berkeley
Lattice 2016
Southampton, UK



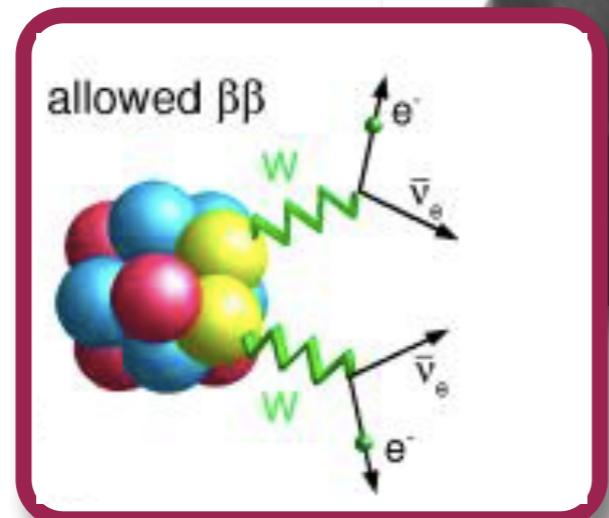
Pauli
1930



Chadwick
1932



Fermi
1934



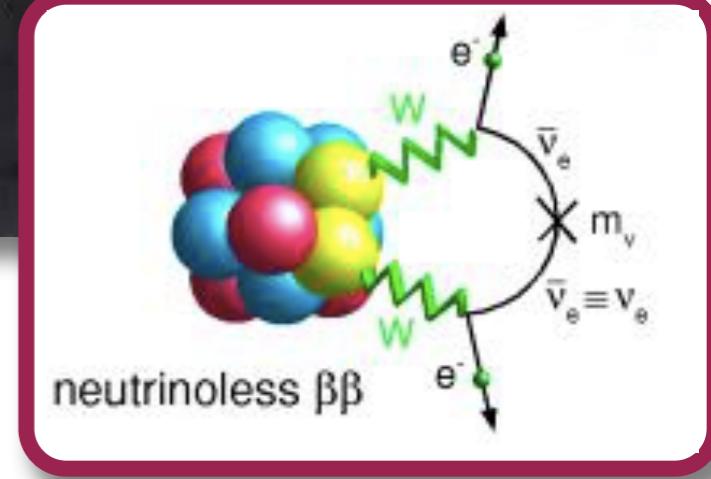
Goppert-Mayer
1935

History

Majorana
1937

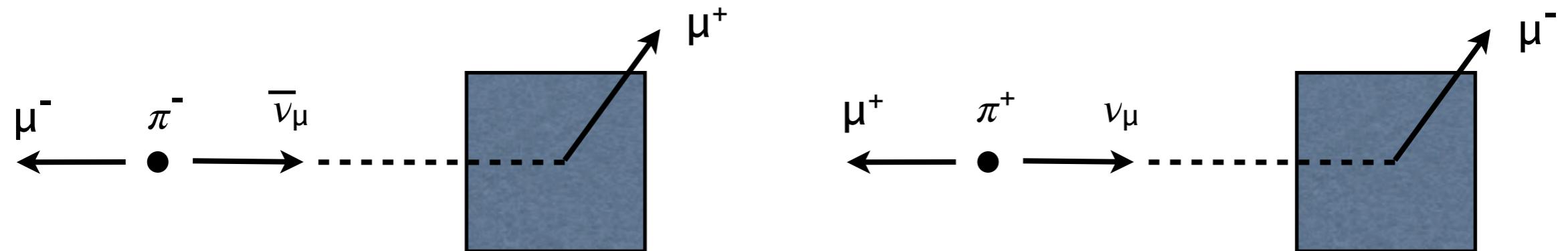
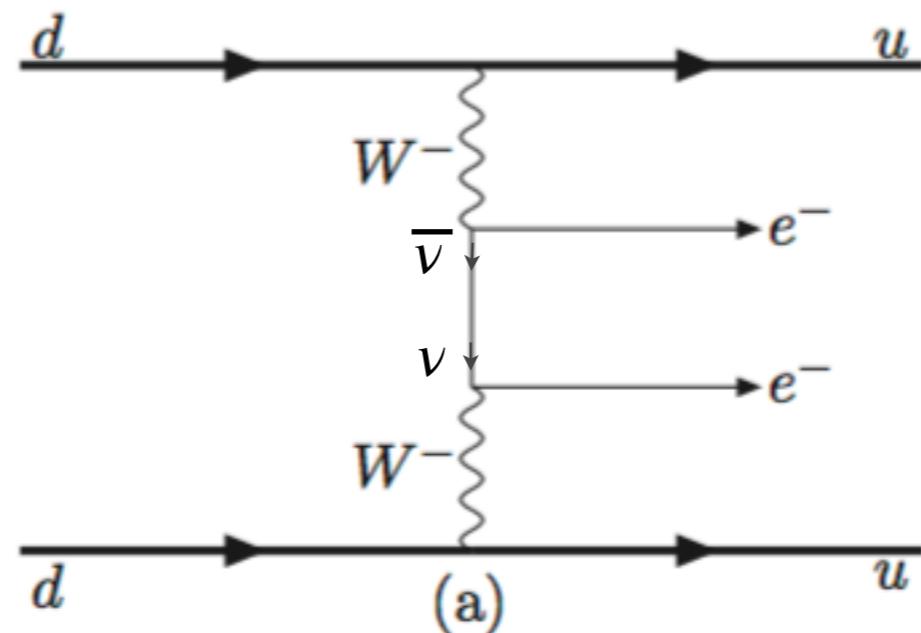


Racah
1937



Lepton Number

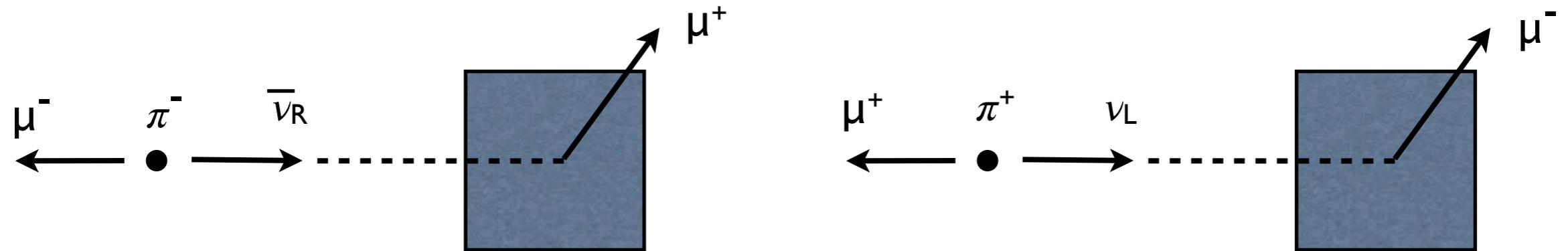
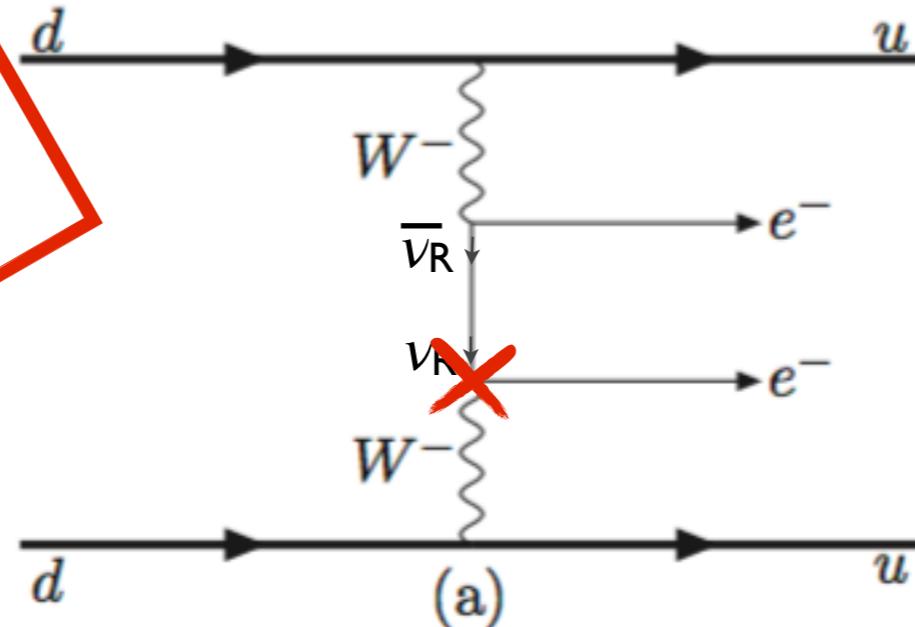
Neutrinos have no known charge or other additively conserved quantum number



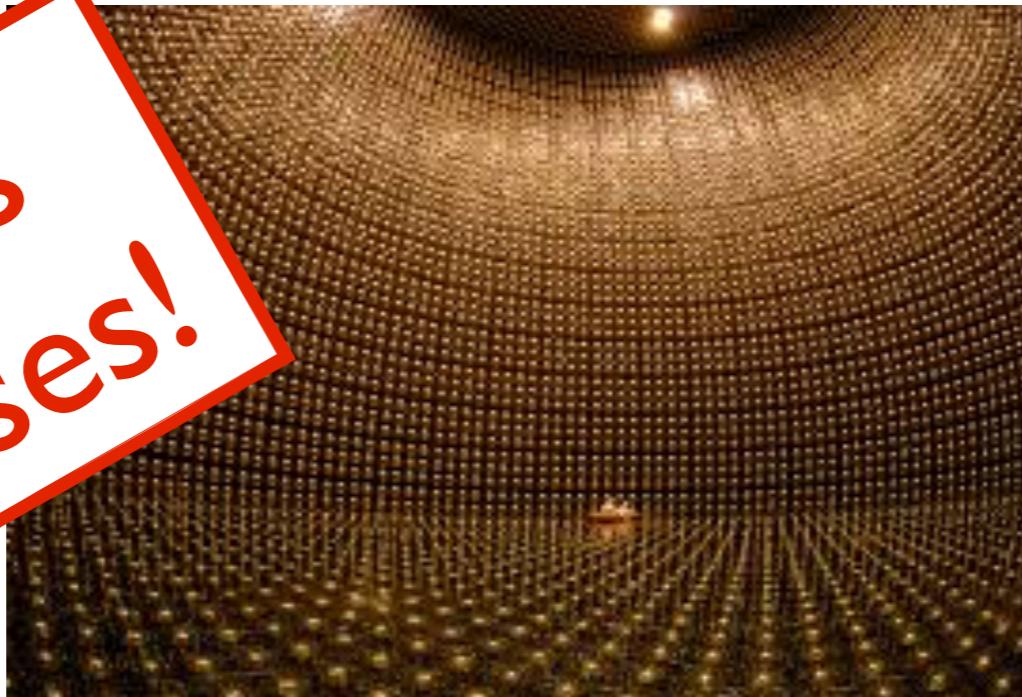
Lepton Number

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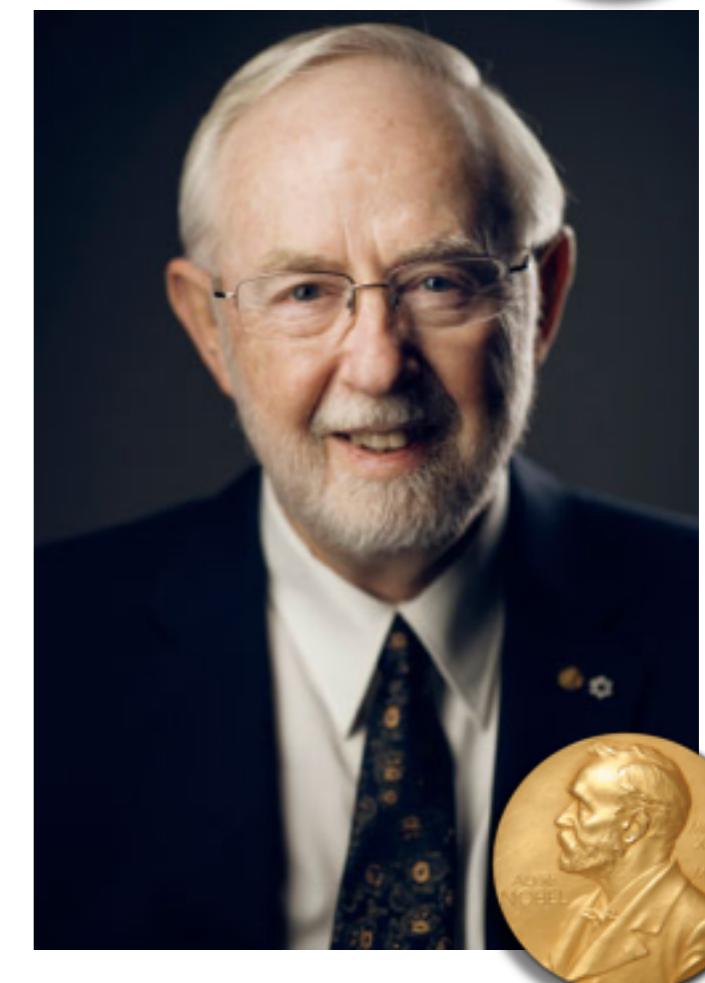
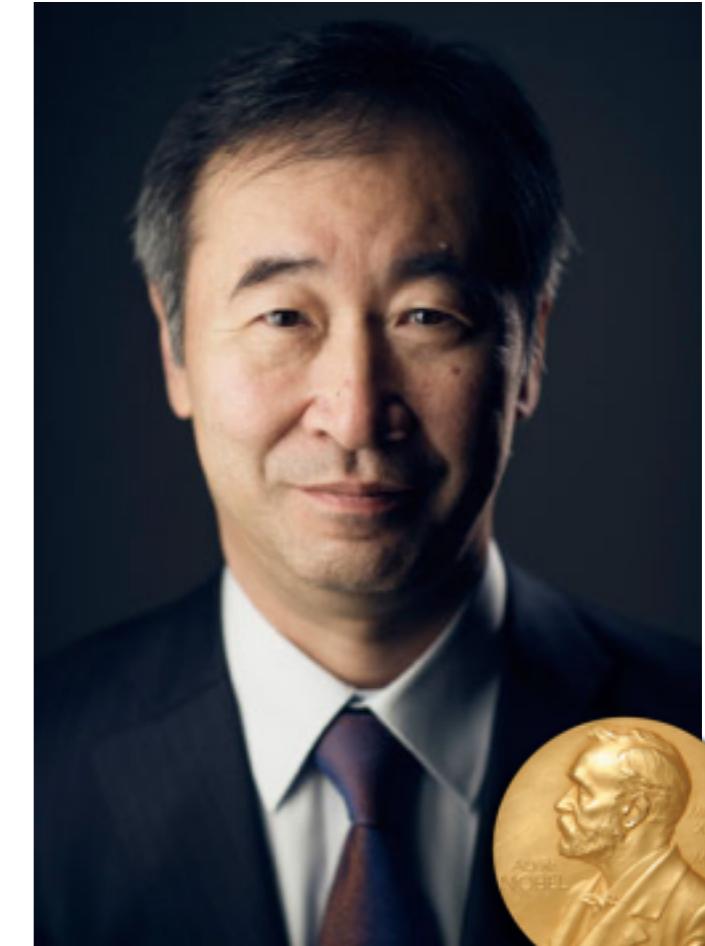
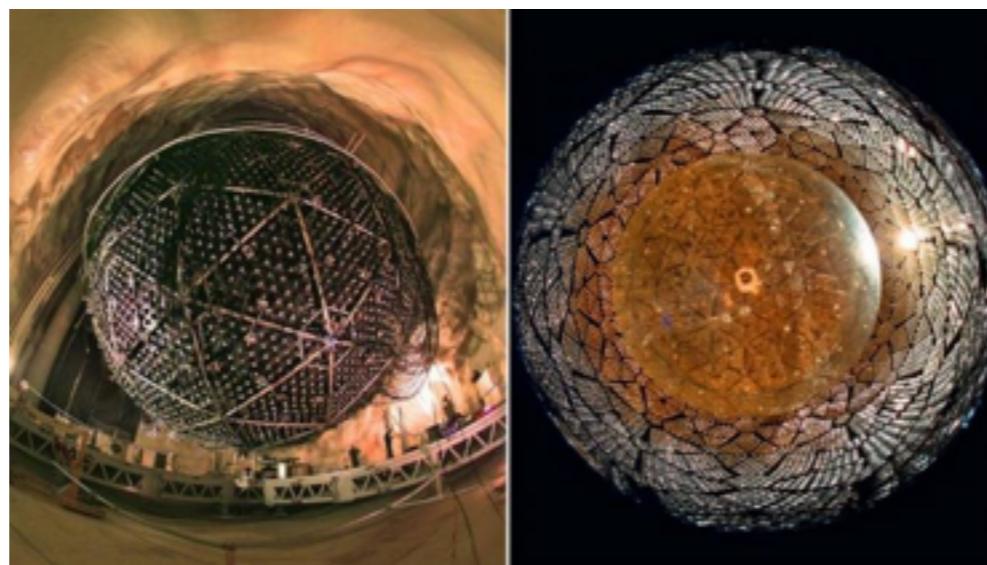
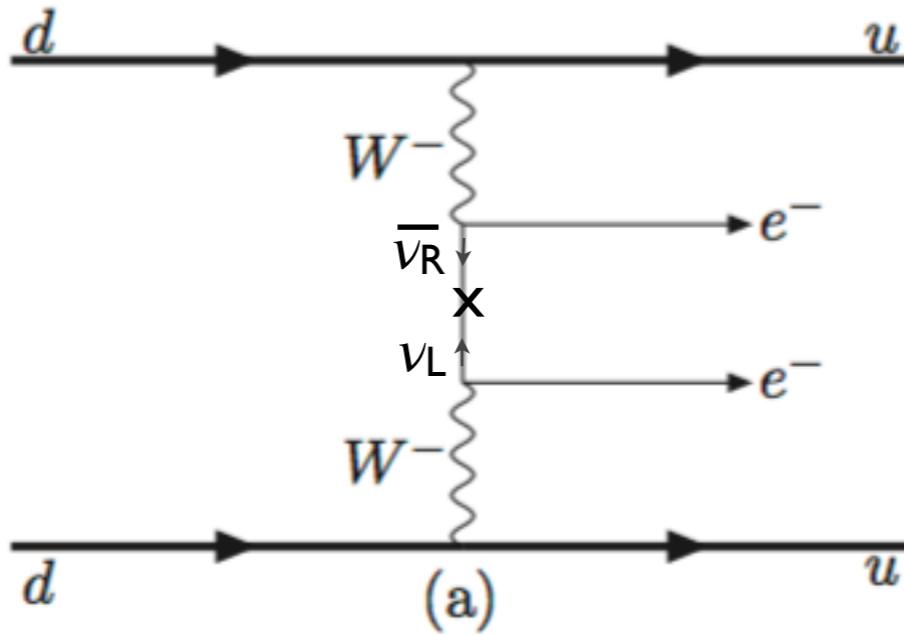
Forbidden by helicity?



Neutrinos
have masses!

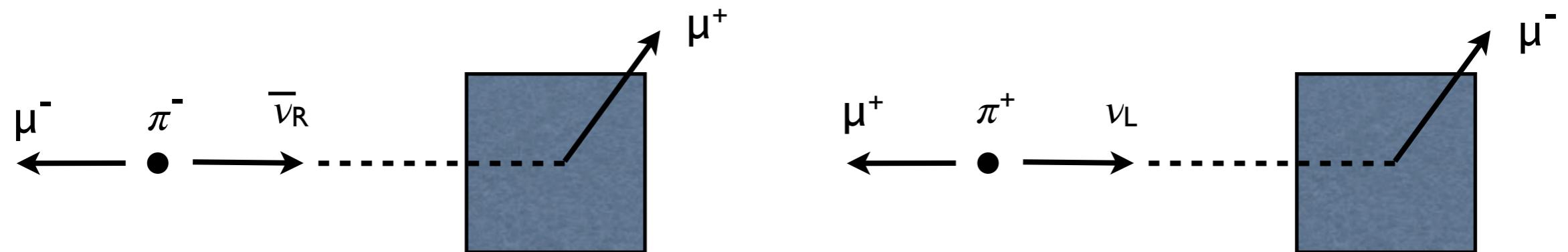
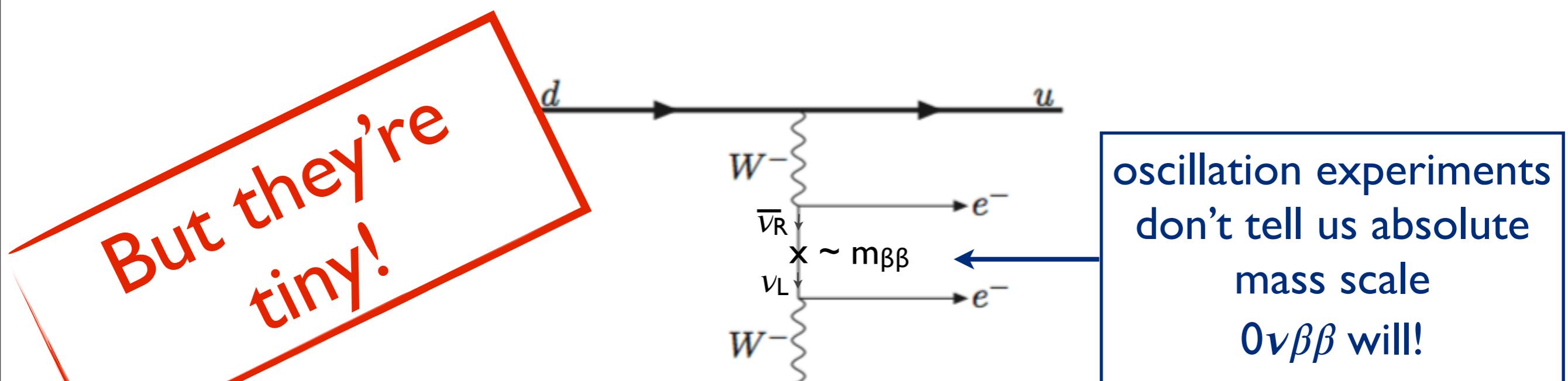


Takaaki Kajita
(Super-K)
Arthur B.
McDonald
(SNO)
Nobel Prize,
2015



Lepton Number

Neutrinos have no known charge or other additively conserved quantum number



Majorana or Dirac?

- Anything not forbidden by symmetry should occur in nature

$$\mathcal{L}_5 = -m \left(\bar{L} \tilde{H} \right) \left(\tilde{H} L \right)^\dagger$$

- Why are neutrinos so light?
 - Dirac mass on its own requires fine-tuning



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$$\begin{pmatrix} M_L & M_D \\ M_D & M_R \end{pmatrix}$$

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$$\left(\begin{array}{cc} \cancel{M_L} & M_D \\ M_D & M_R \end{array} \right)$$

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$$\begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix}$$

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$$m_l \sim M_D^2/M_R \quad m_h \sim M_R$$

Majorana or Dirac?

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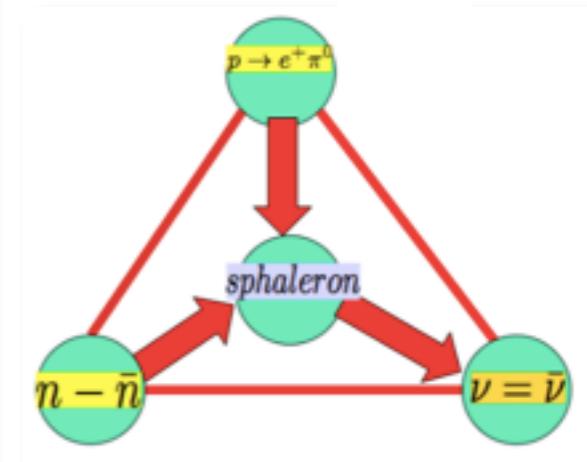
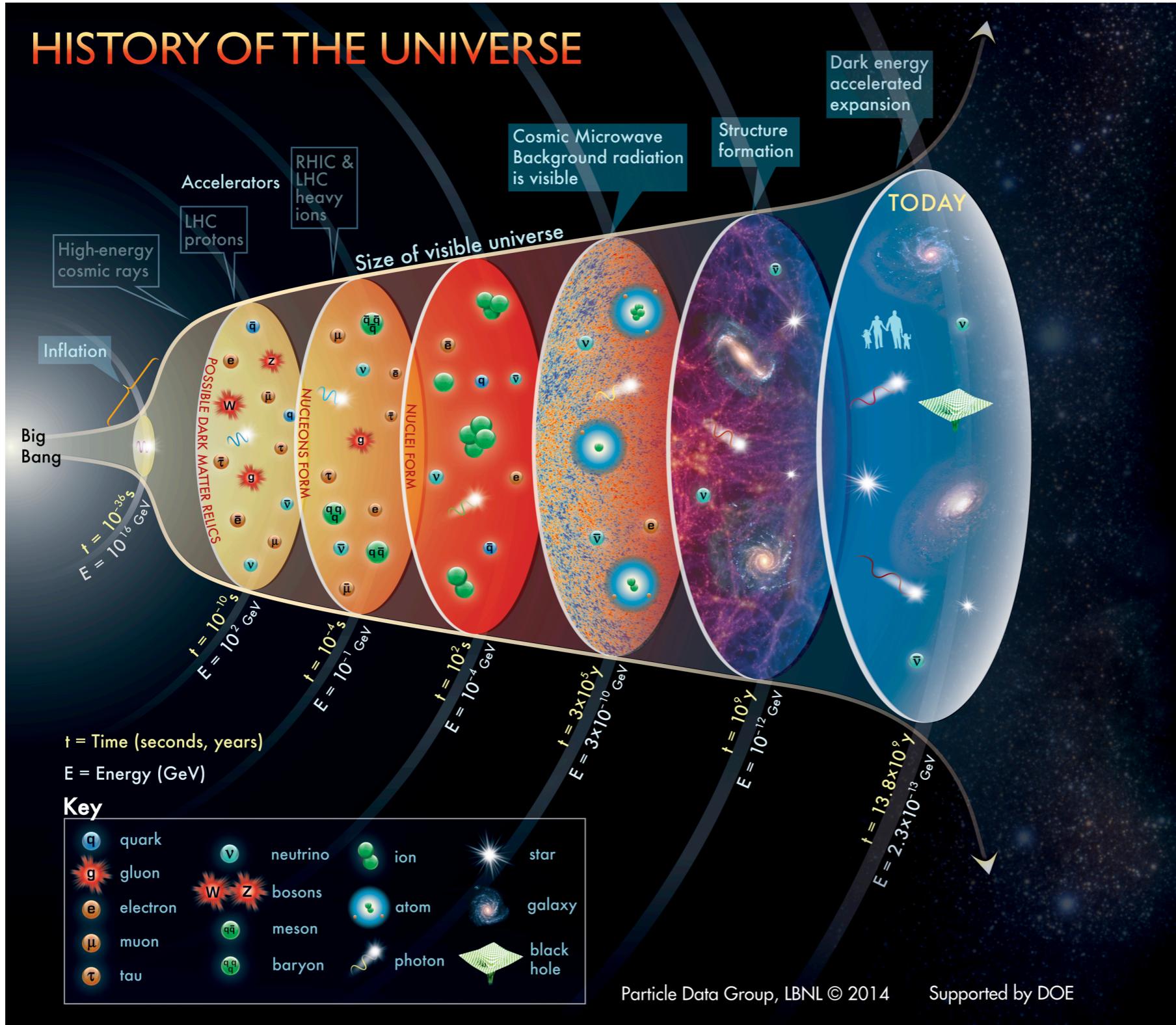
$$\begin{pmatrix} 0 & M_D \\ M_D & M_R \end{pmatrix}$$

$$m_l \sim M_D^2/M_R \quad m_h \sim M_R$$

$$M_D \sim 200\text{GeV} \quad m_l \sim 0.05\text{eV}$$

$$M_R \sim 10^{15}\text{GeV}$$

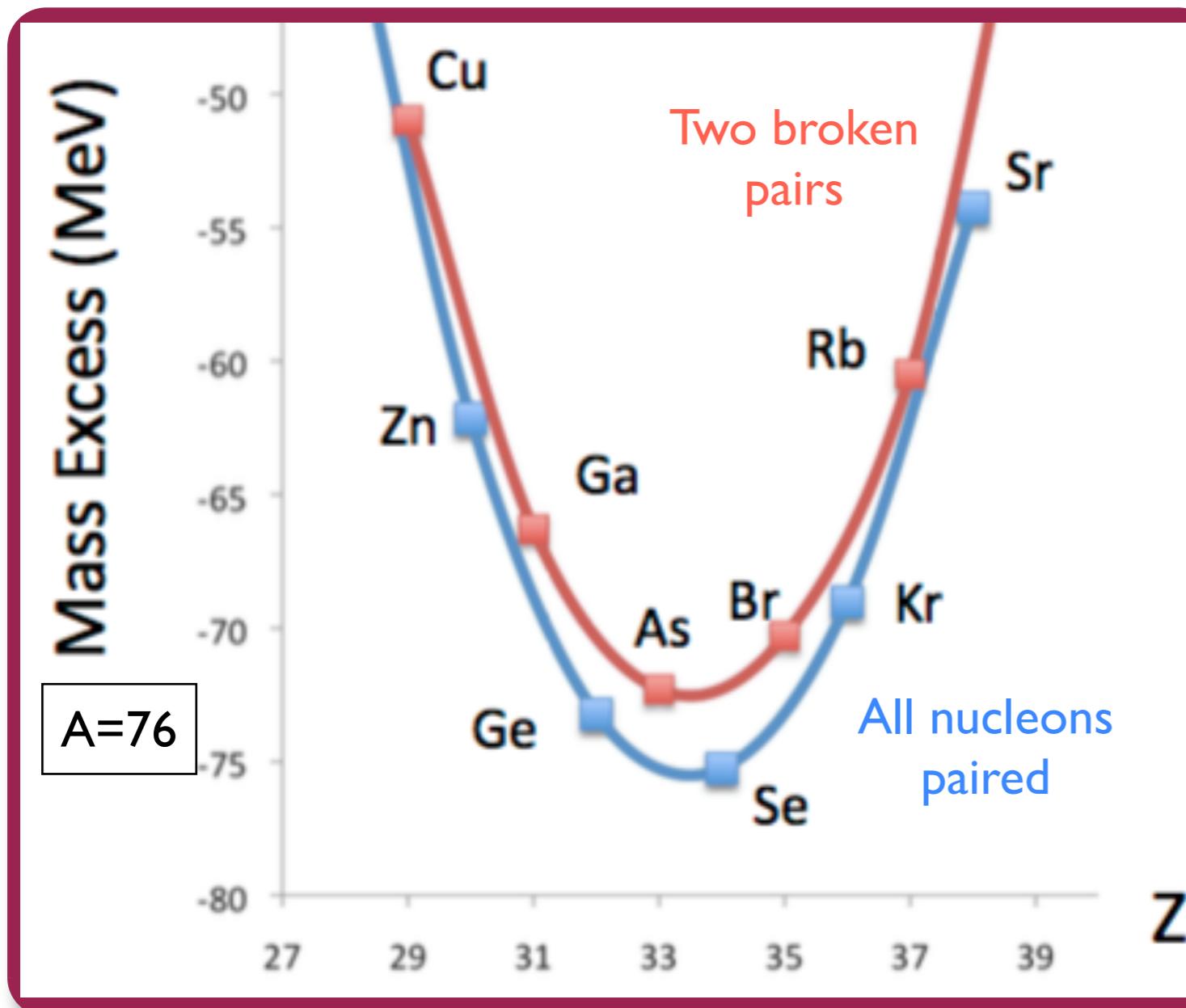
If observed, could help explain matter/anti-matter asymmetry in the universe!



Jansen (1996)
Bödeker,
Moore,
Rummukainen
(2000)
Fodor (2000)

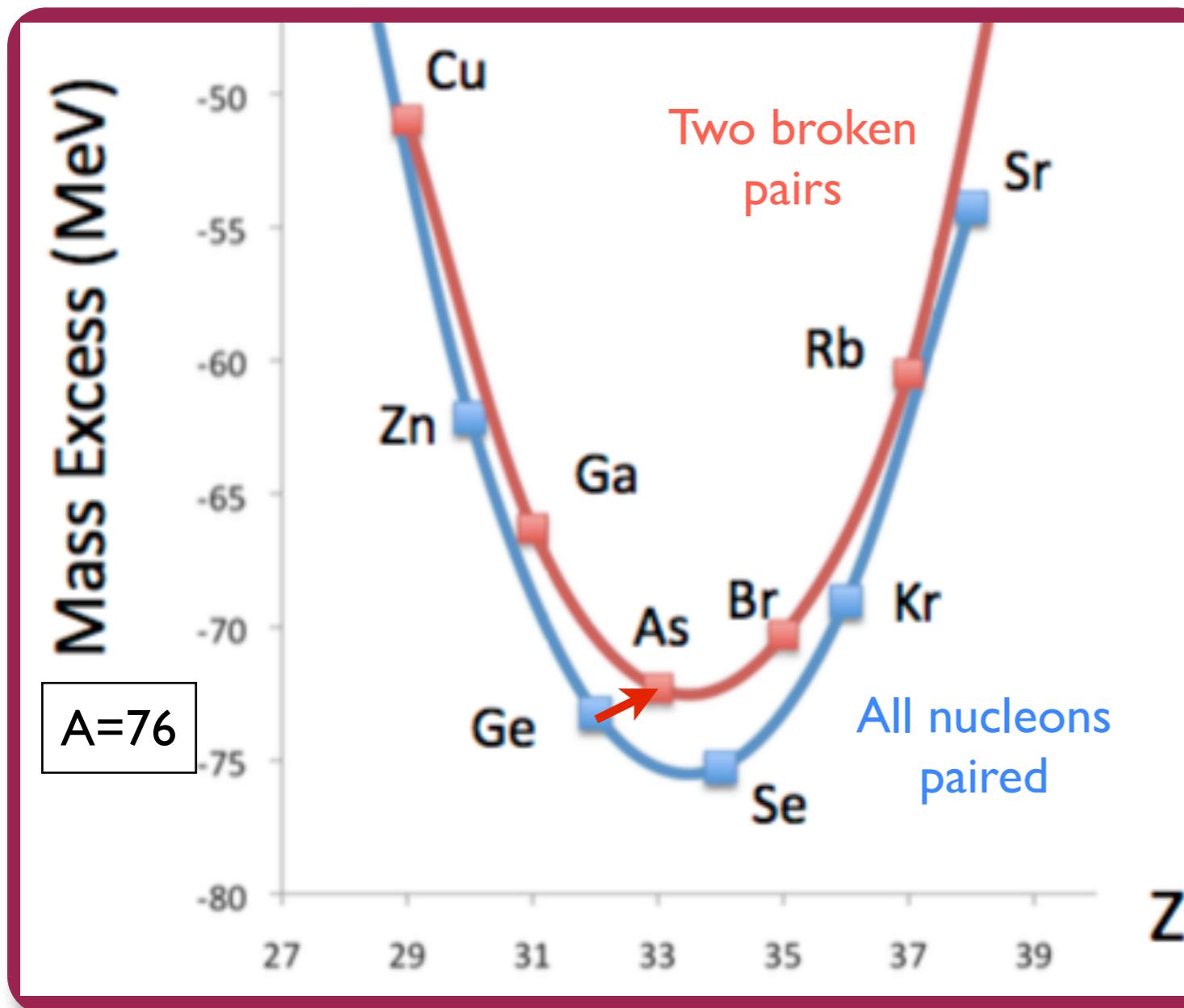
Experiment

Nuclear physics gives us a natural filter for the process

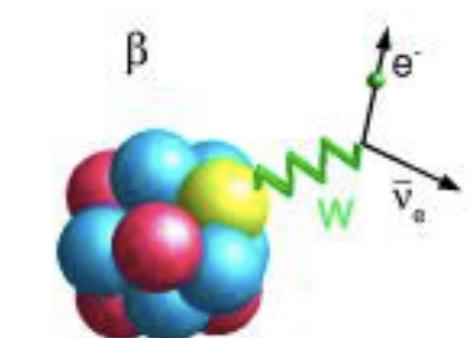


Experiment

Nuclear physics gives us a natural filter for the process

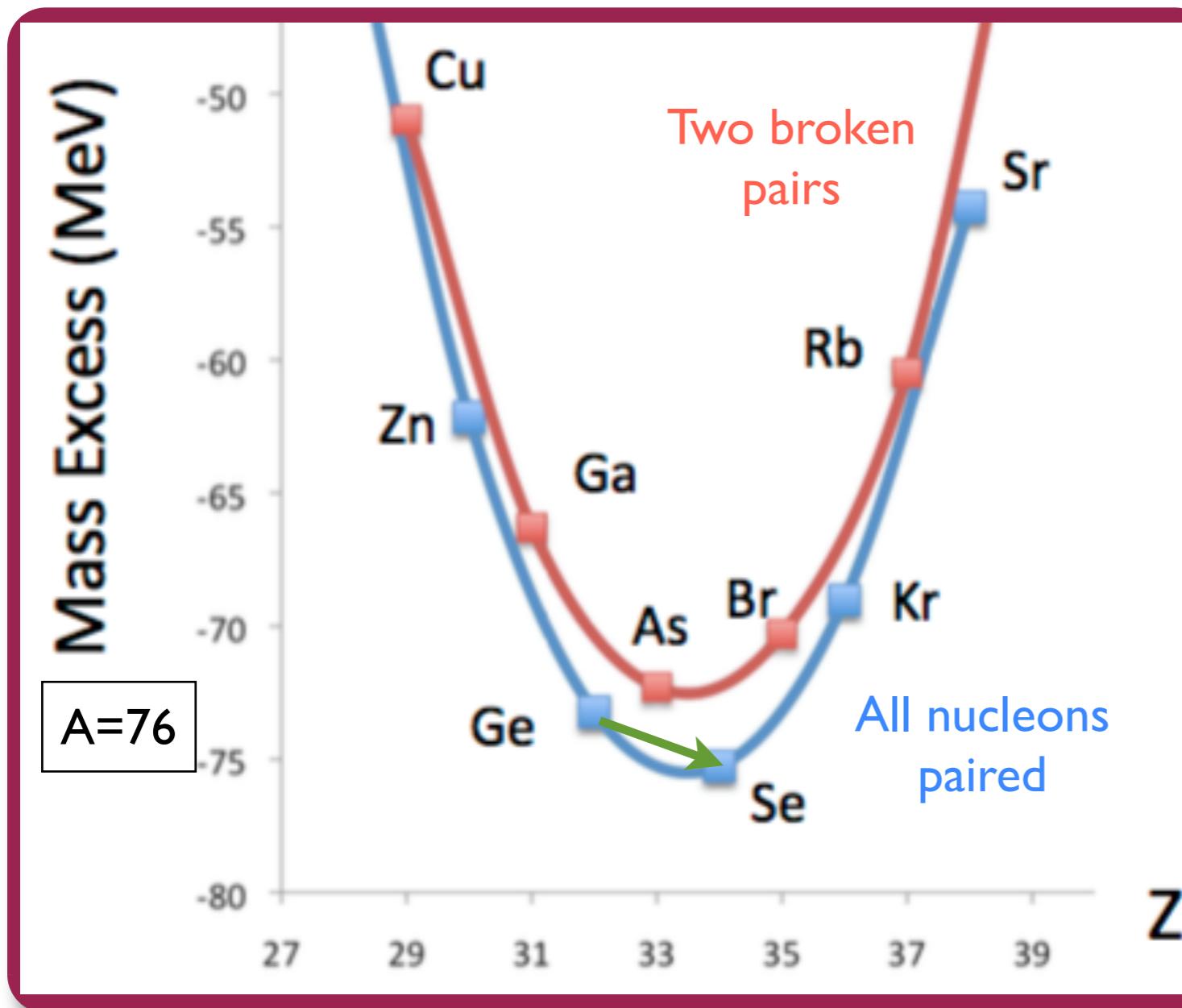


Energetically
forbidden

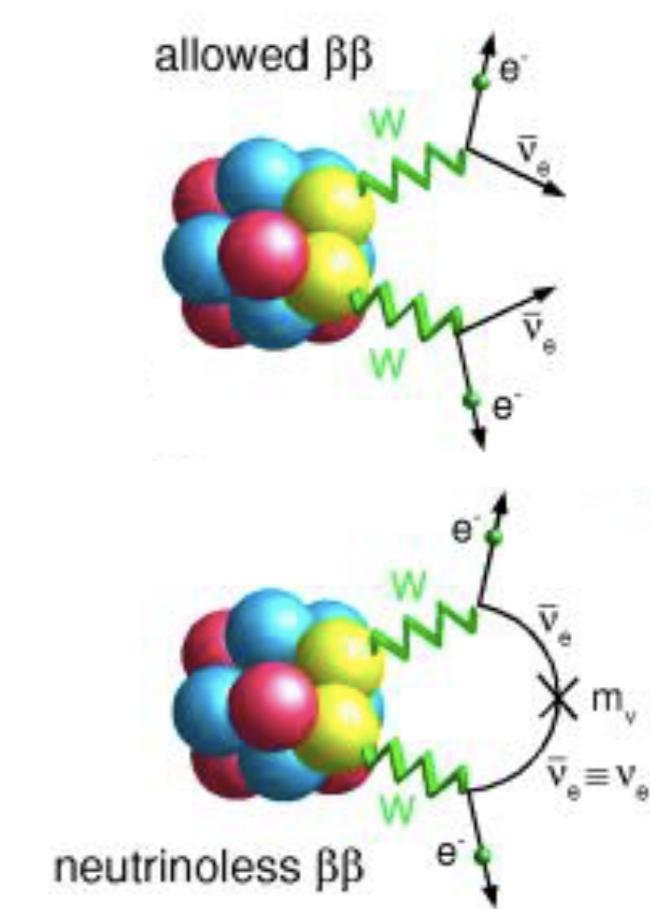


Experiment

Nuclear physics gives us a natural filter for the process

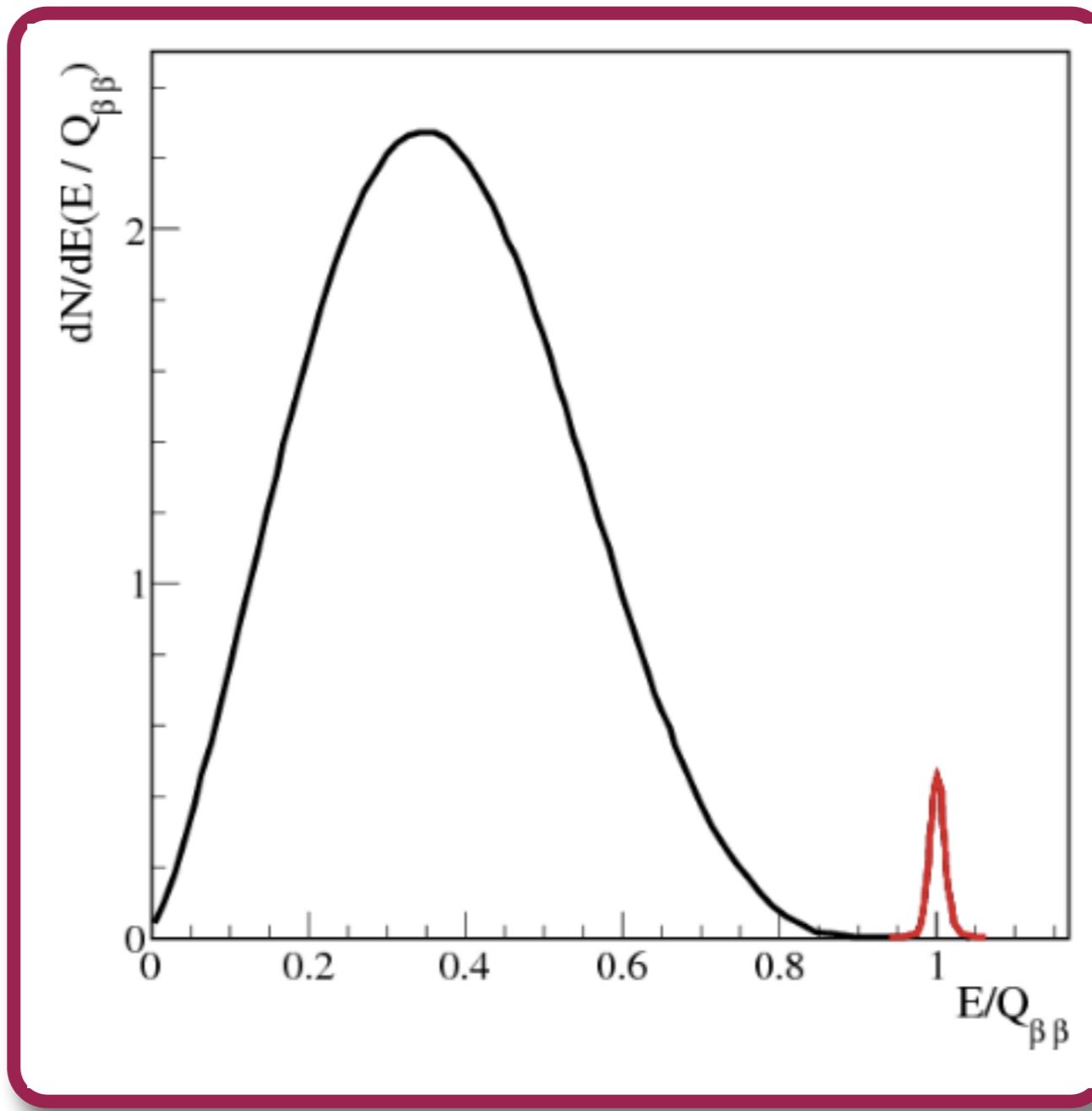


Second order,
allowed



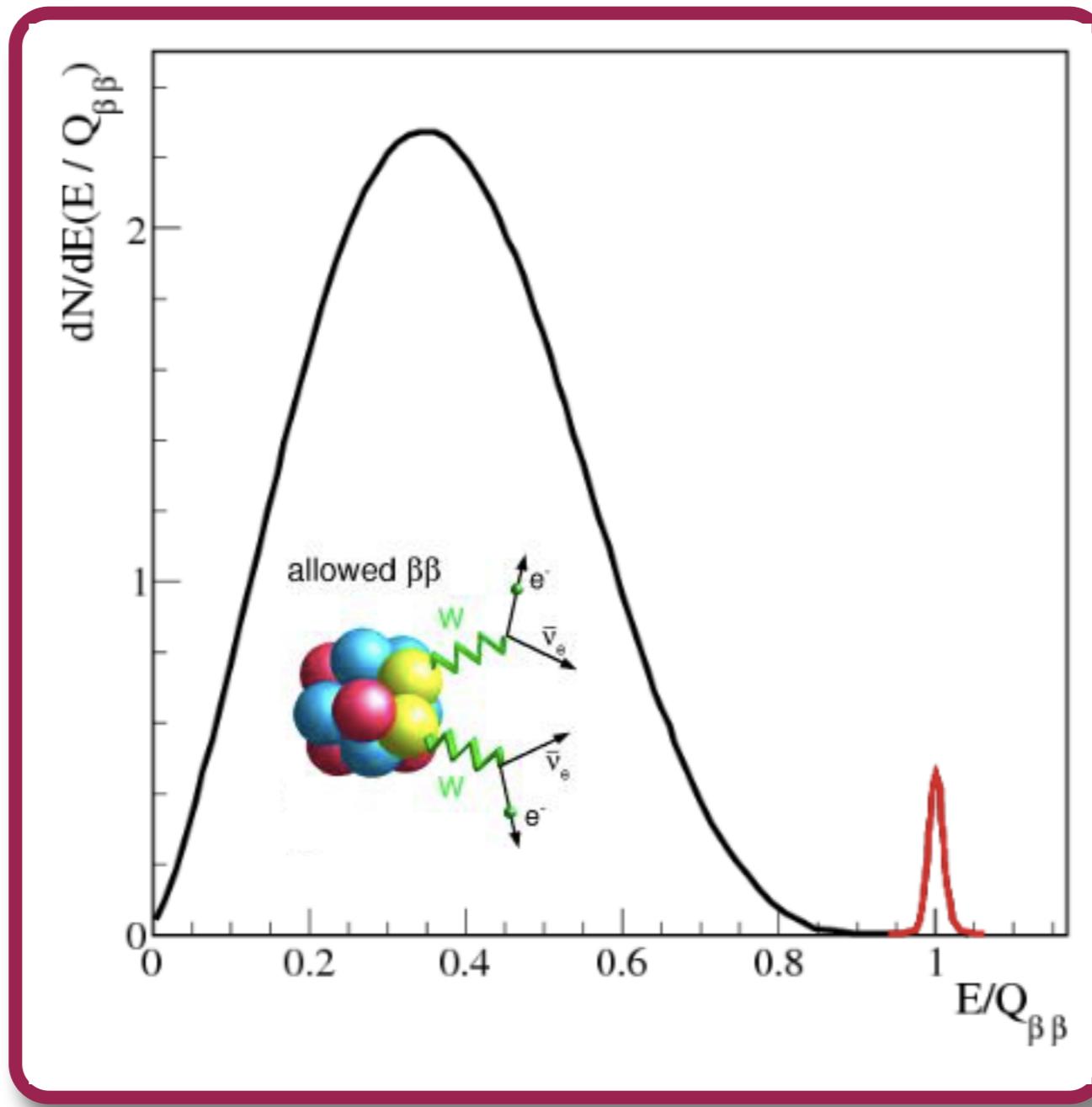
Experiment

Neutrinoless mode can be isolated using spectroscopic methods



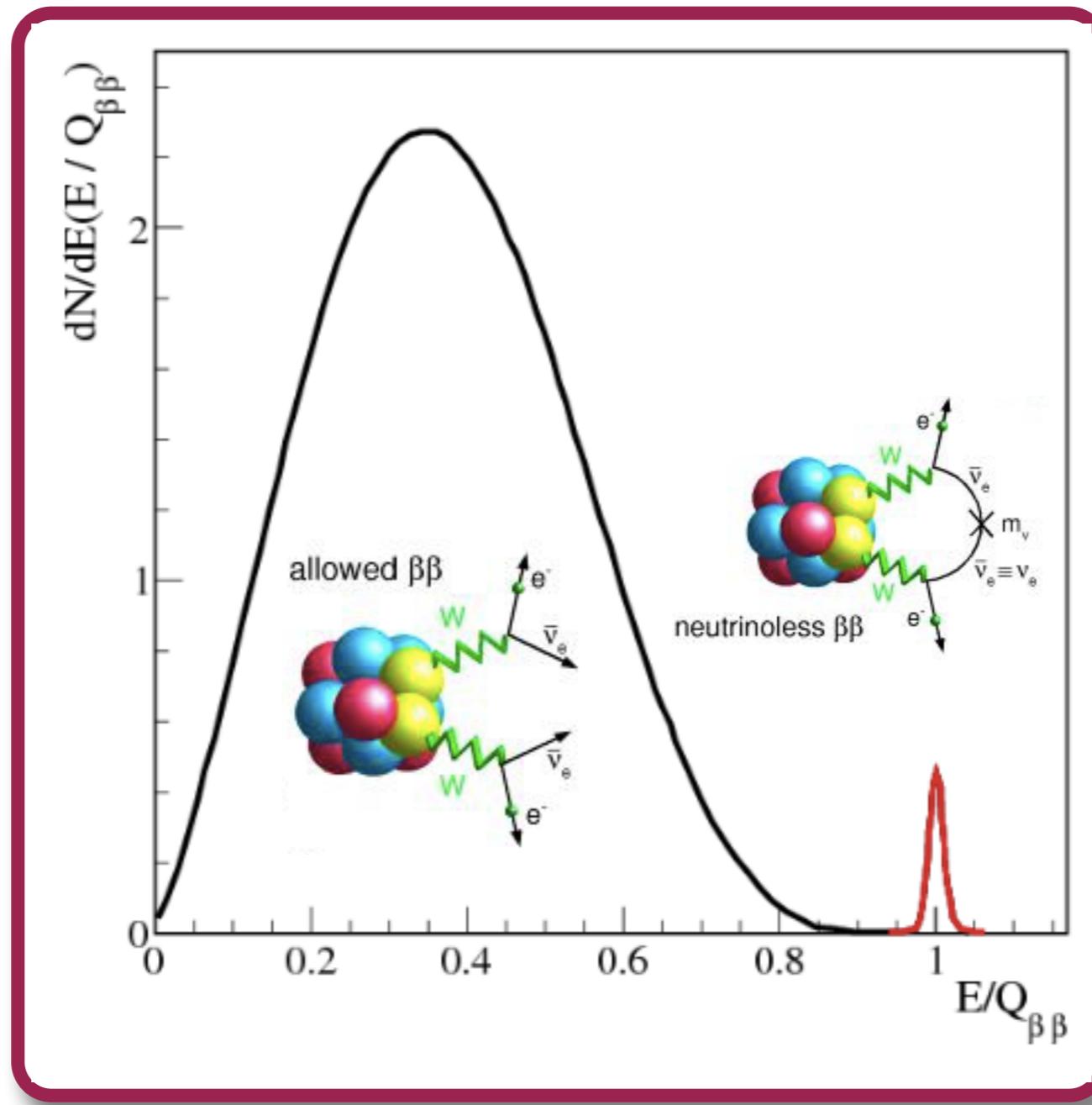
Experiment

Neutrinoless mode can be isolated using spectroscopic methods



Experiment

Neutrinoless mode can be isolated using spectroscopic methods



Cuore
 ^{130}Te



Experiment

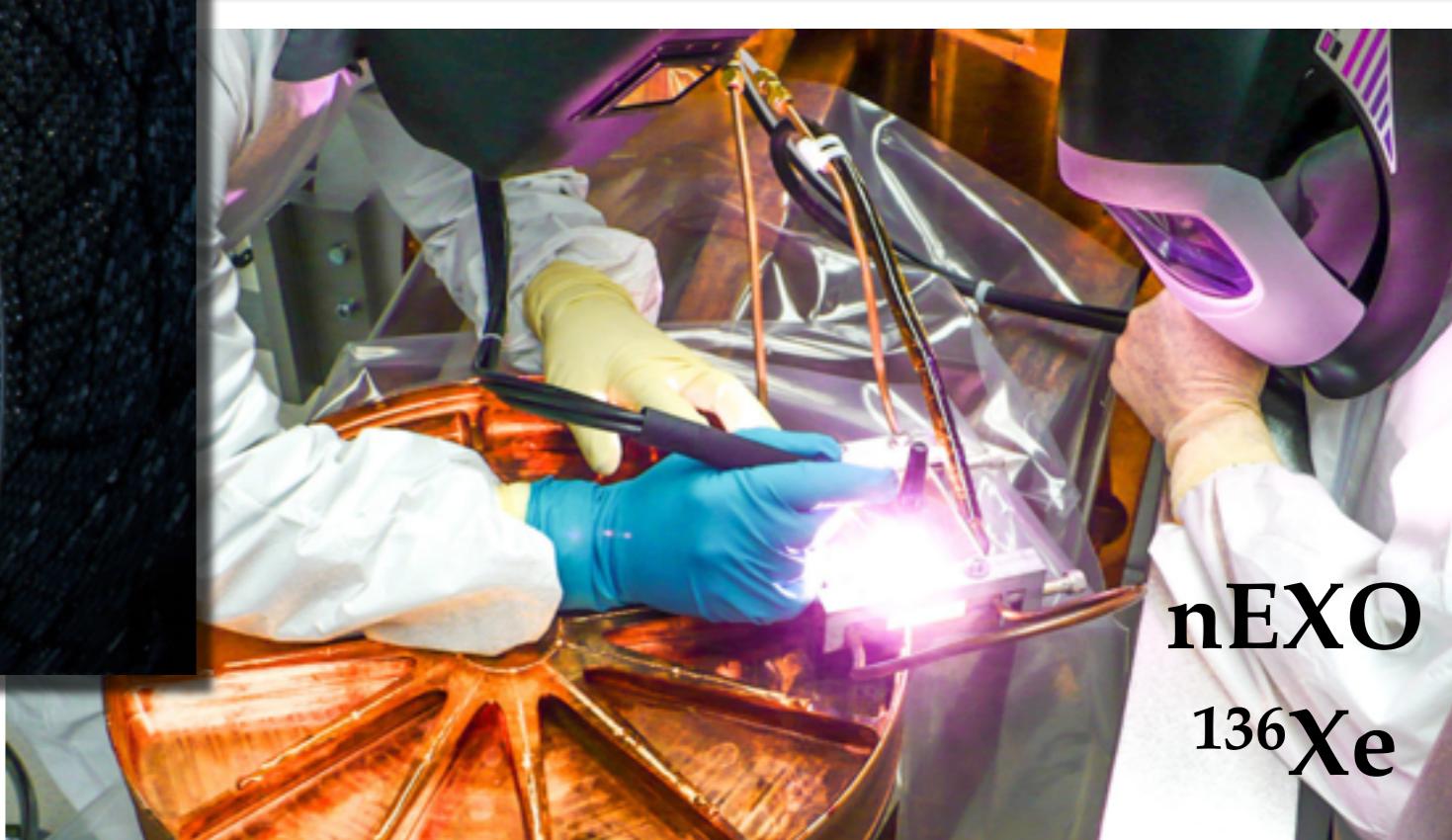
Gerda
 ^{76}Ge



Sno+
 ^{130}Te



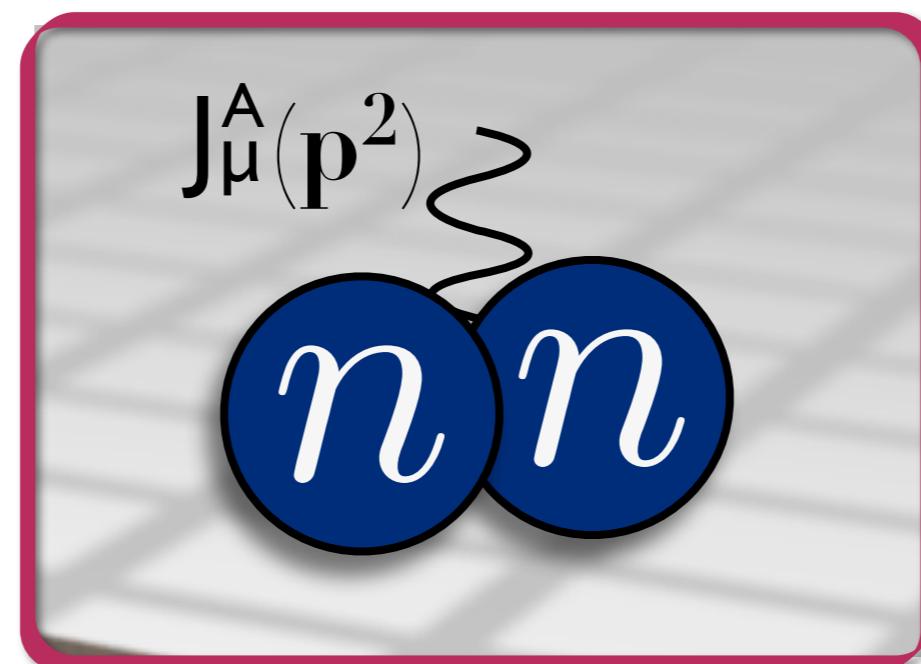
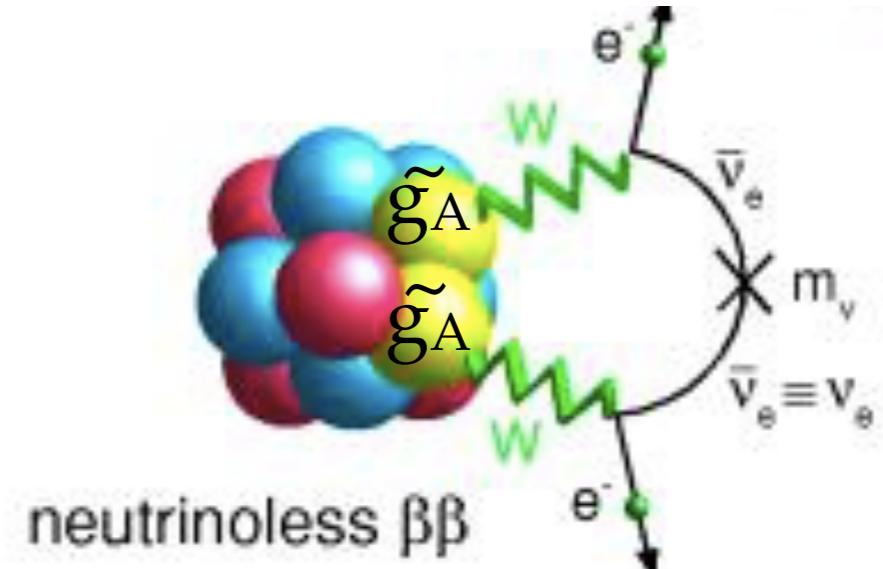
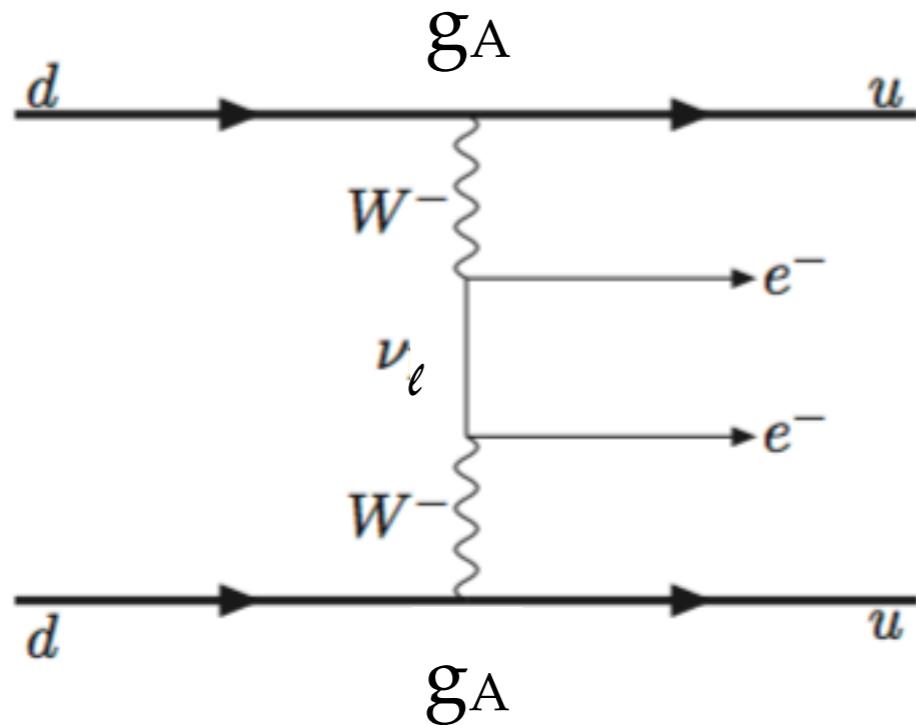
nEXO
 ^{136}Xe



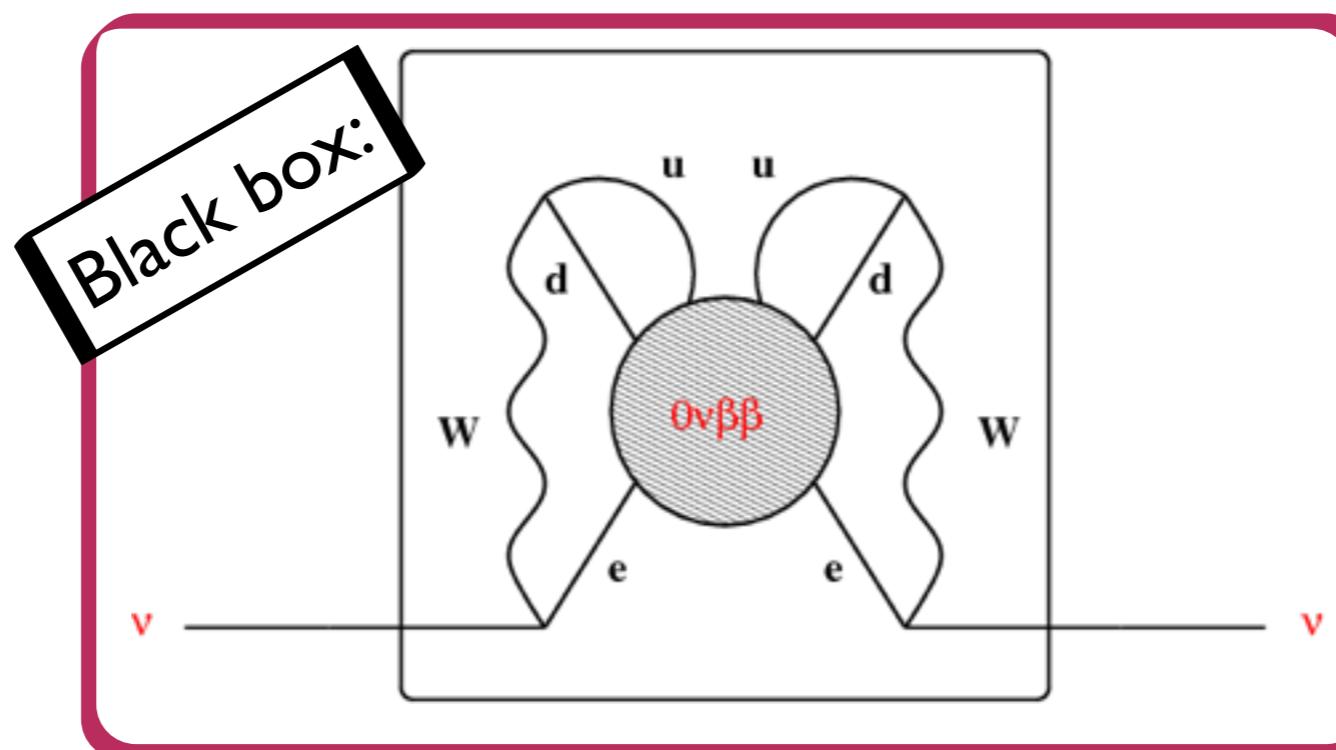
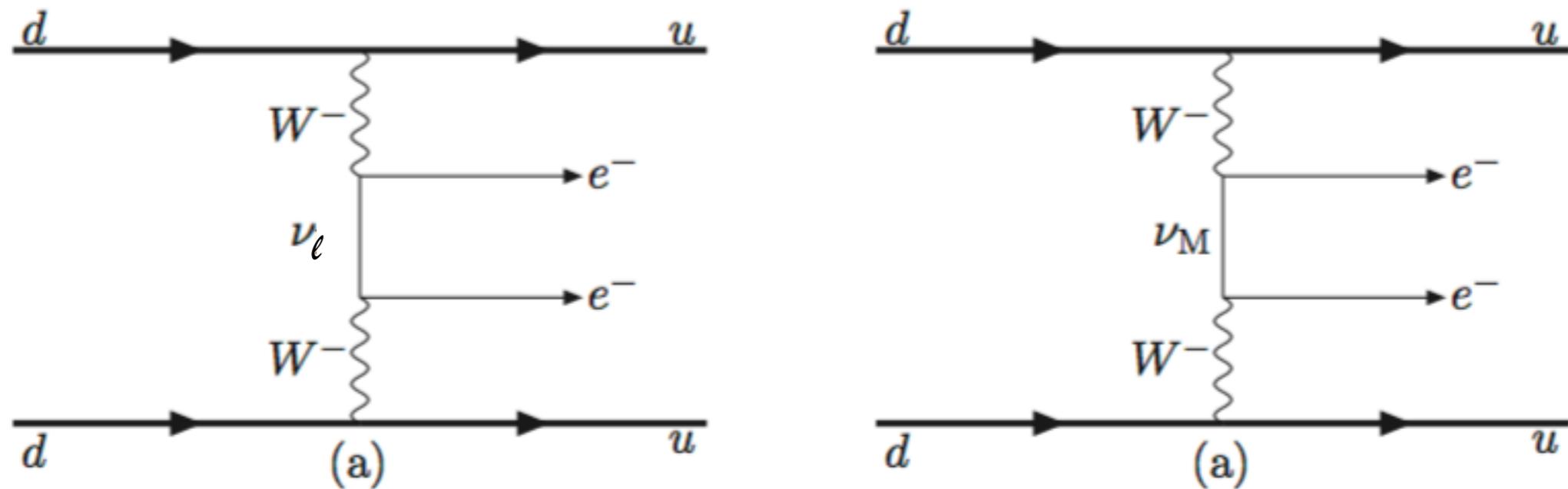


How can LQCD
contribute?

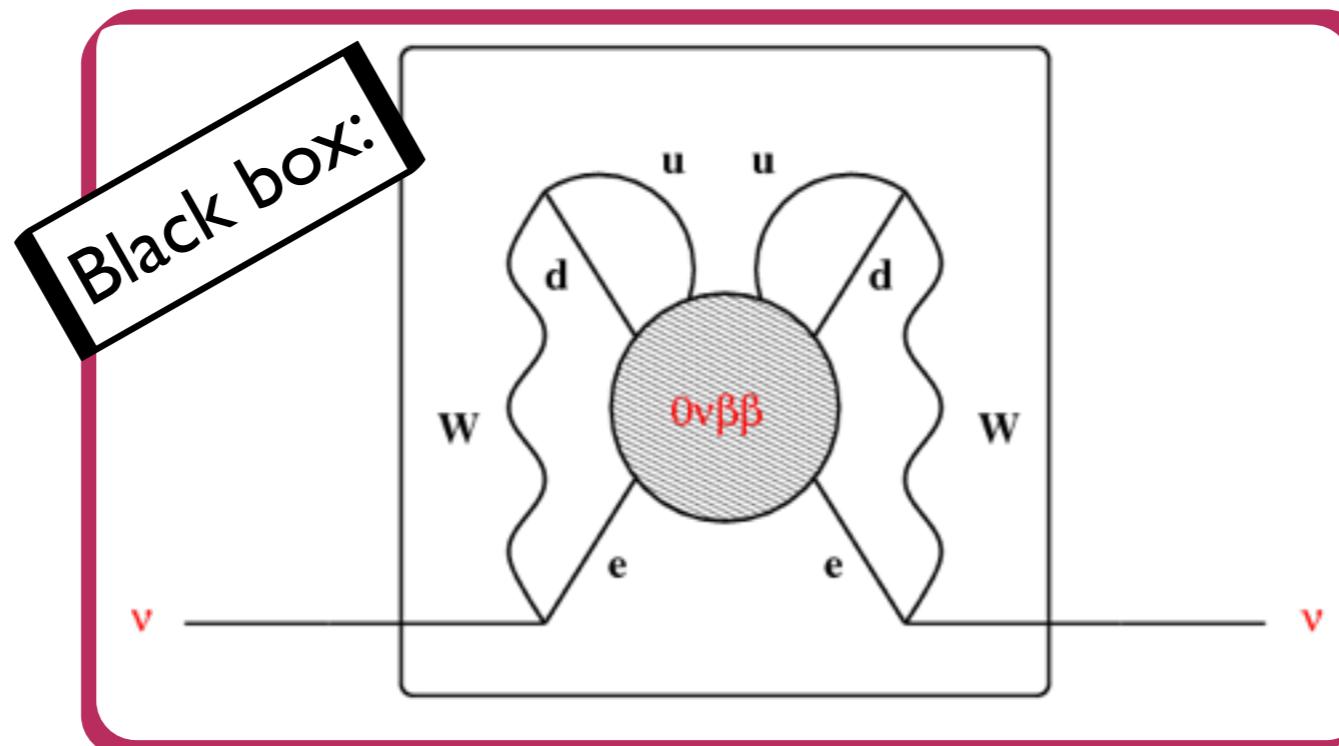
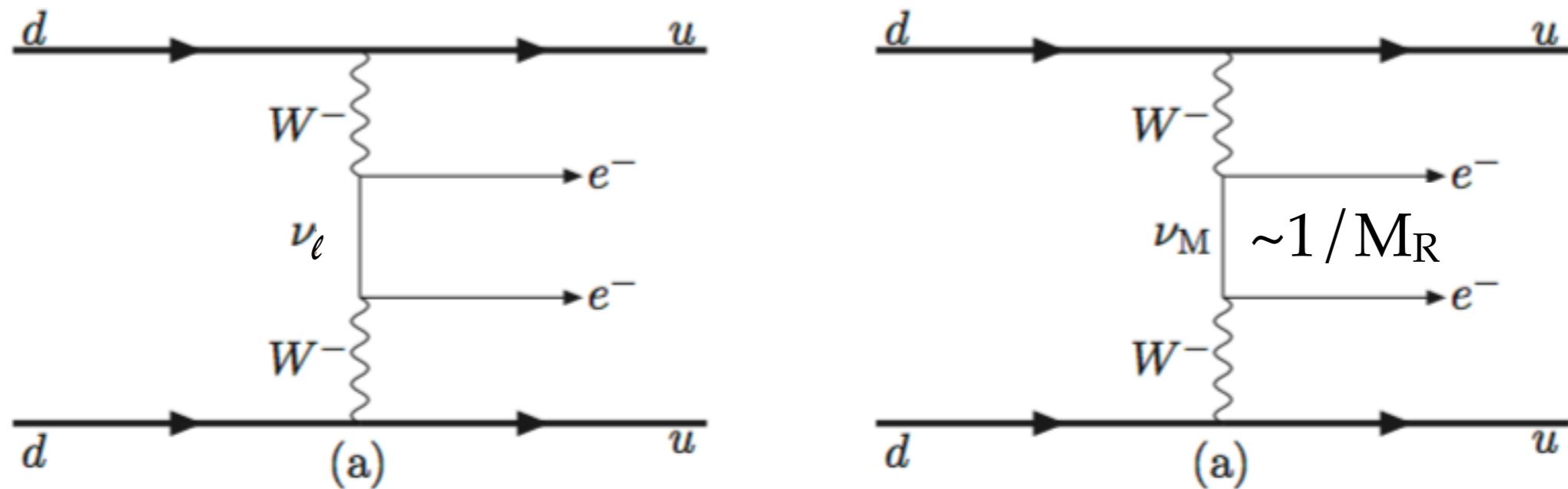
Standard picture: long-range contribution



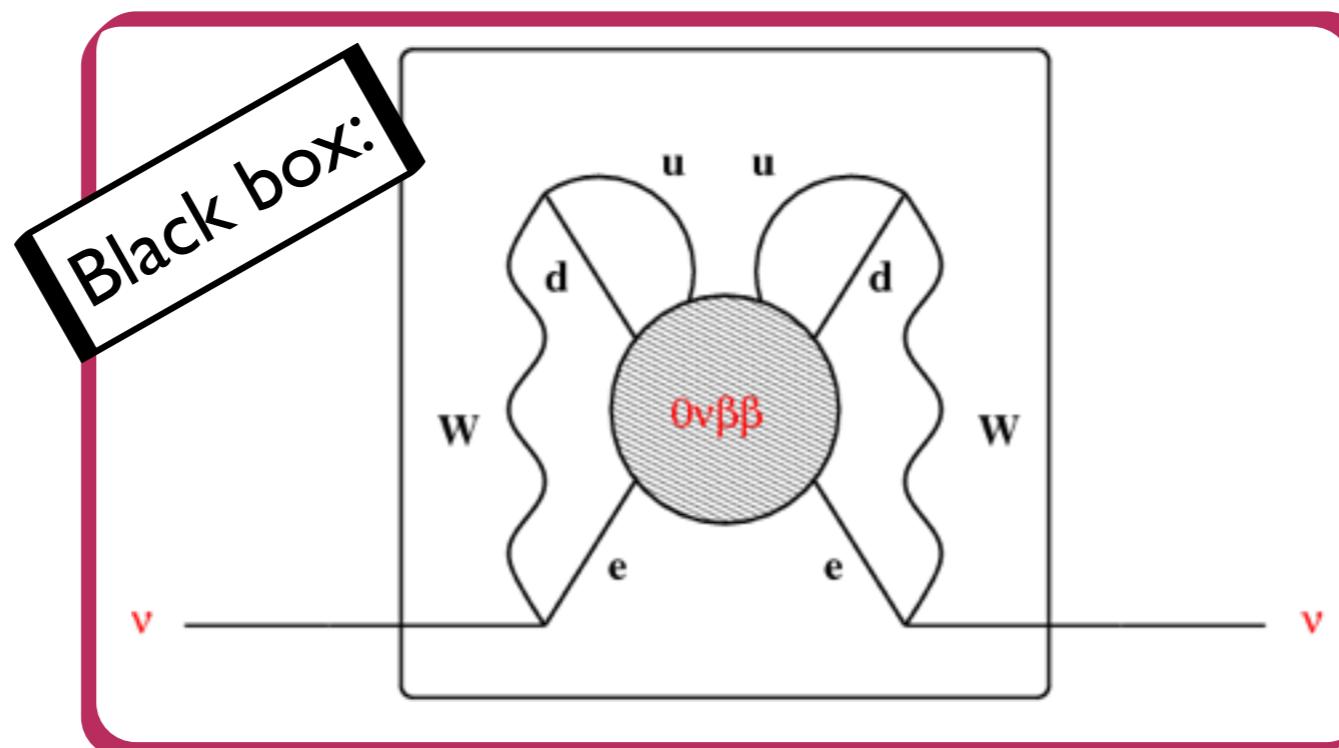
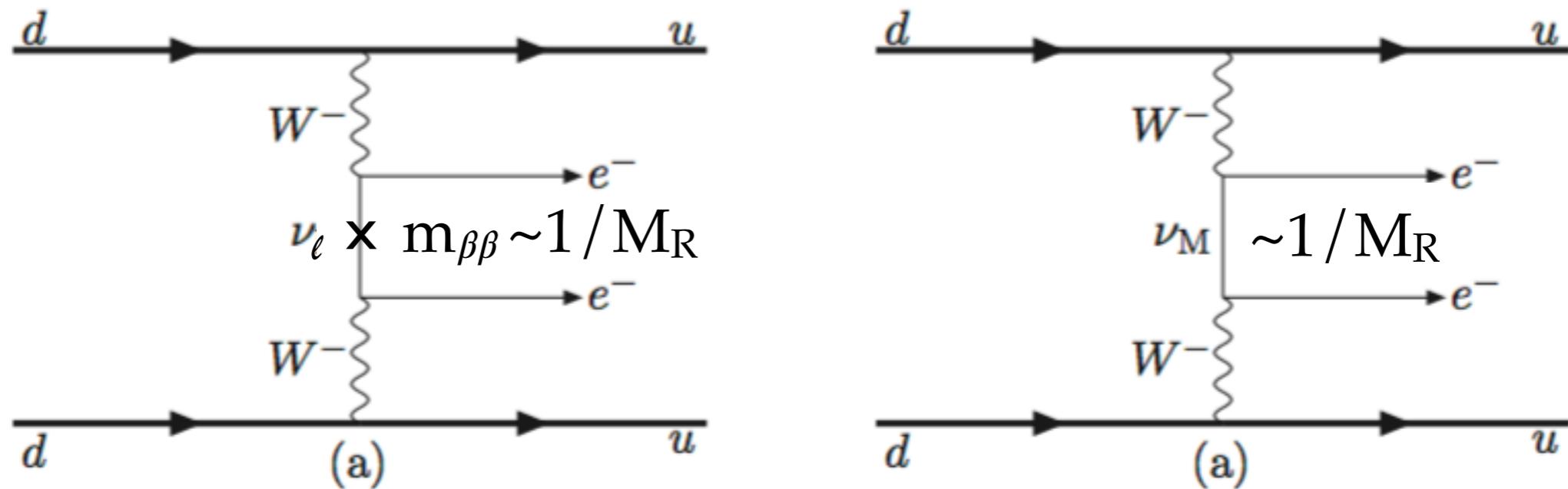
Short-range contribution: probe for heavy physics



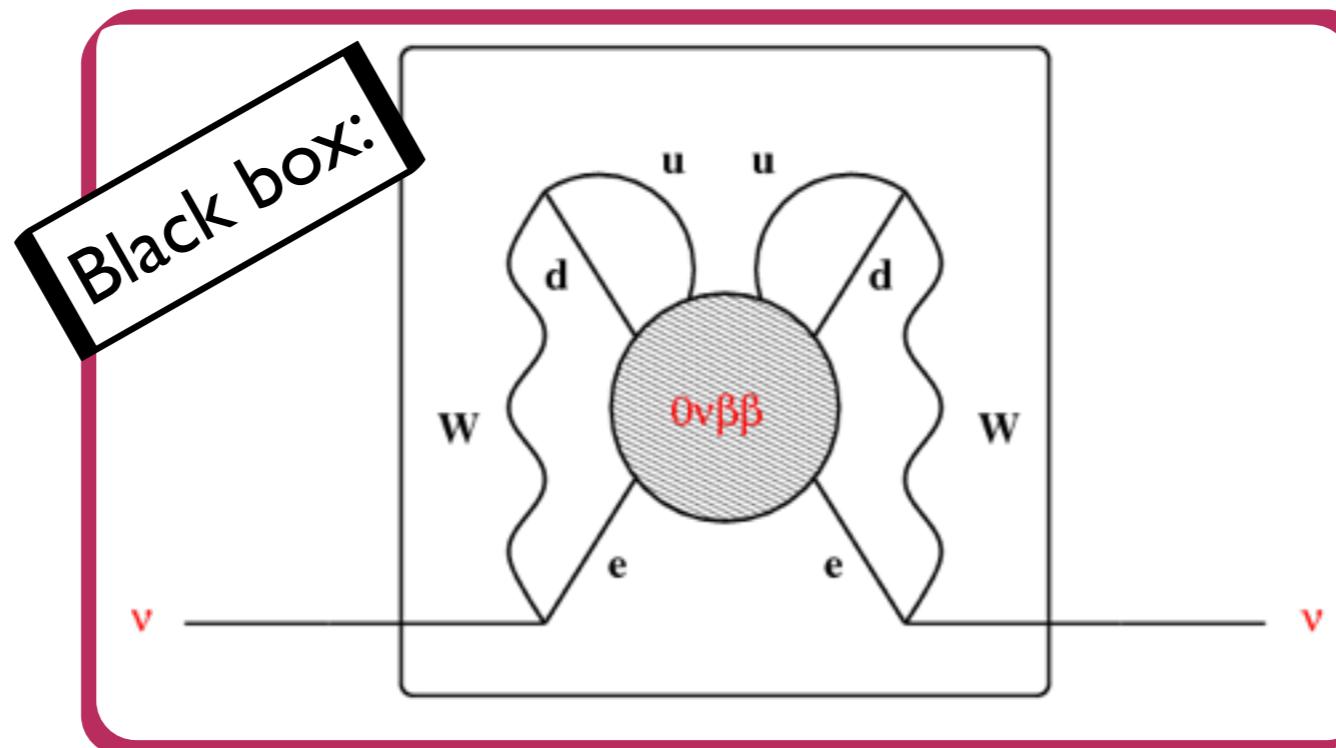
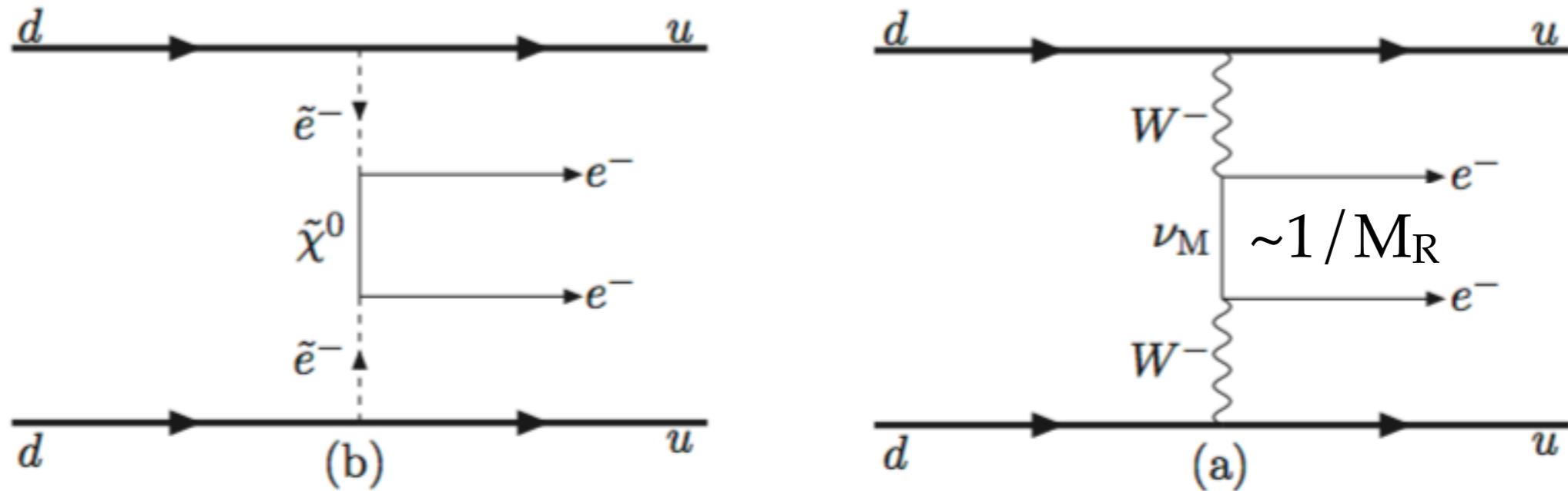
Short-range contribution: probe for heavy physics



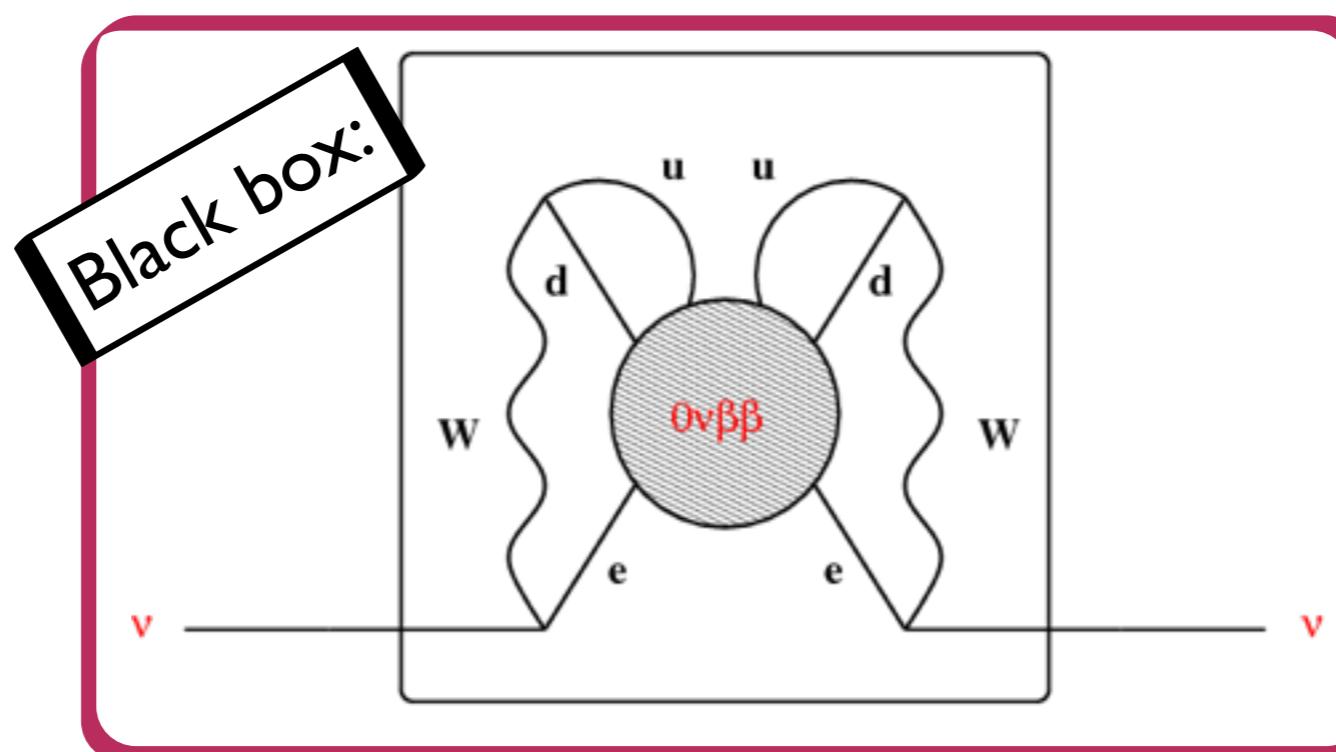
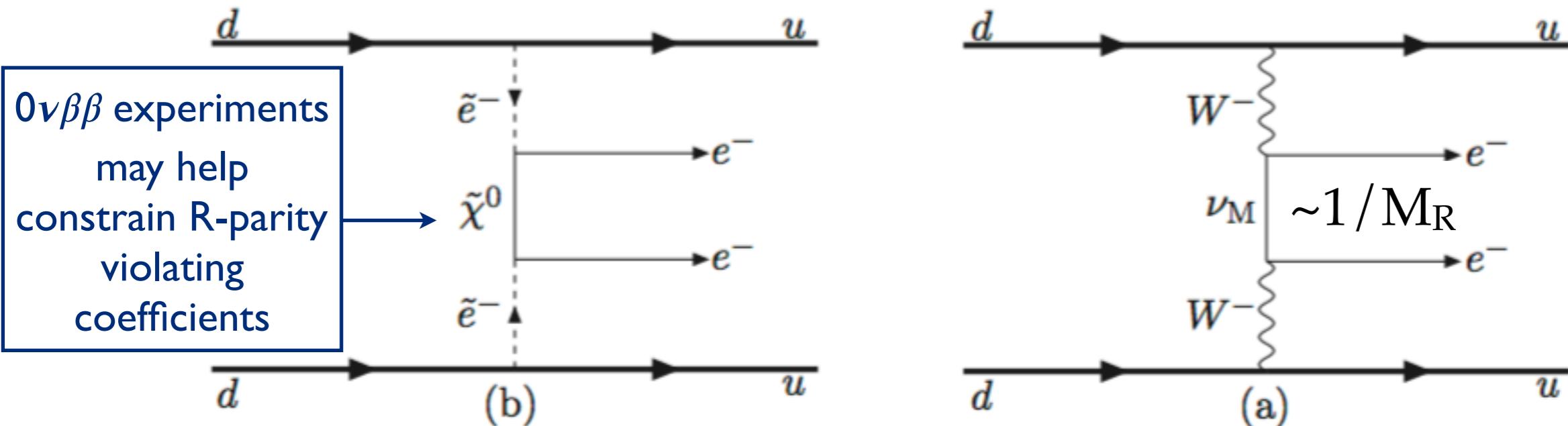
Short-range contribution: probe for heavy physics



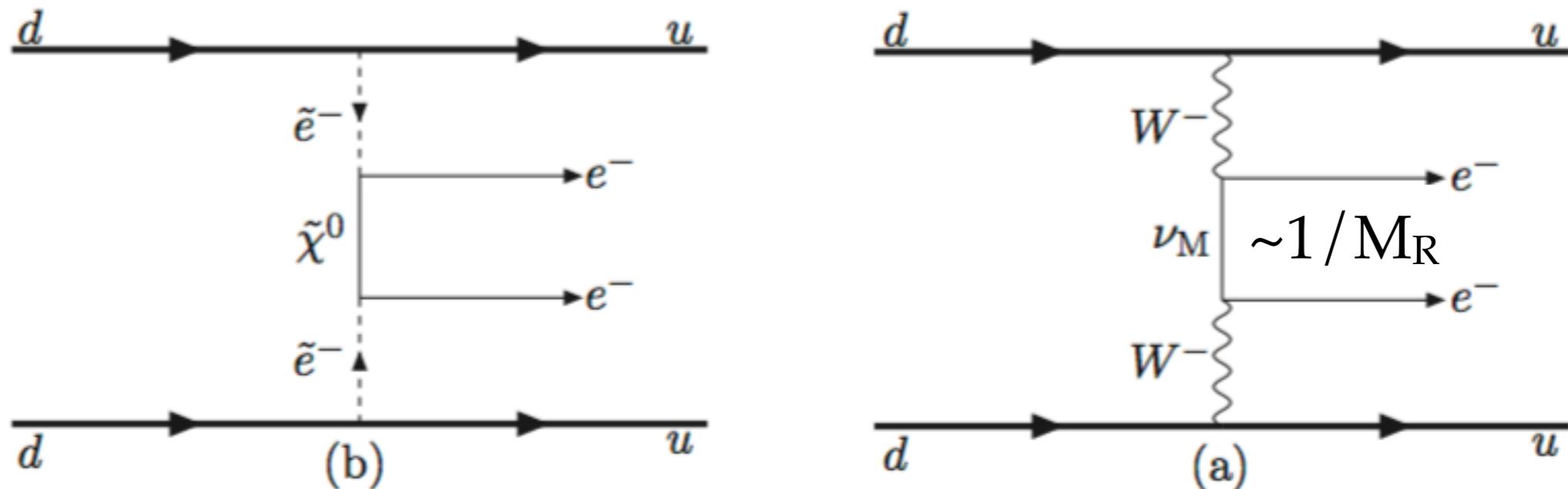
Short-range contribution: probe for heavy physics



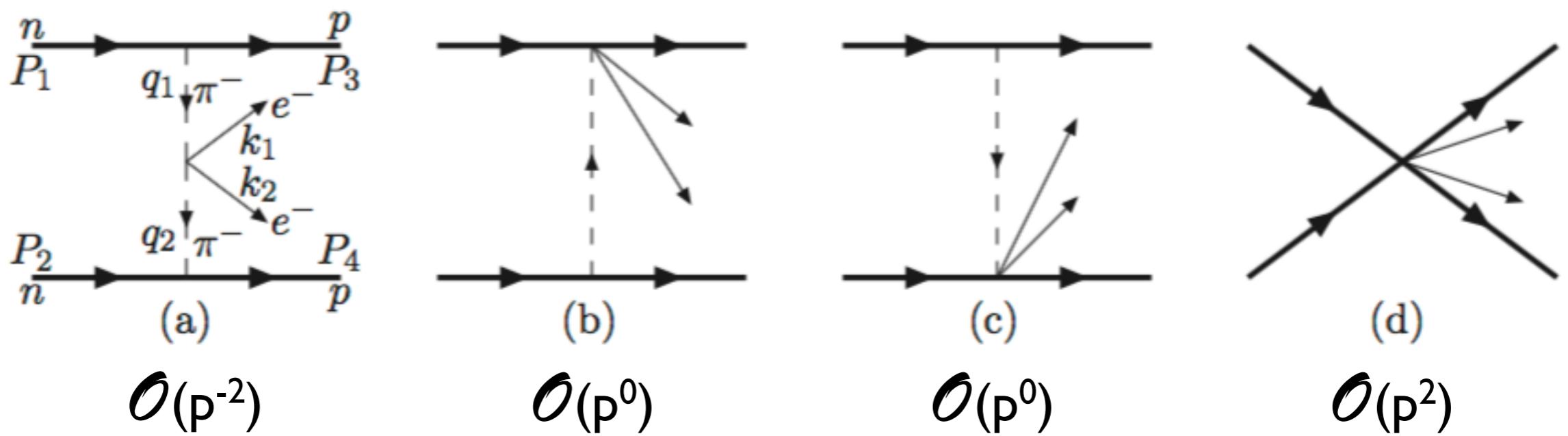
Short-range contribution: probe for heavy physics



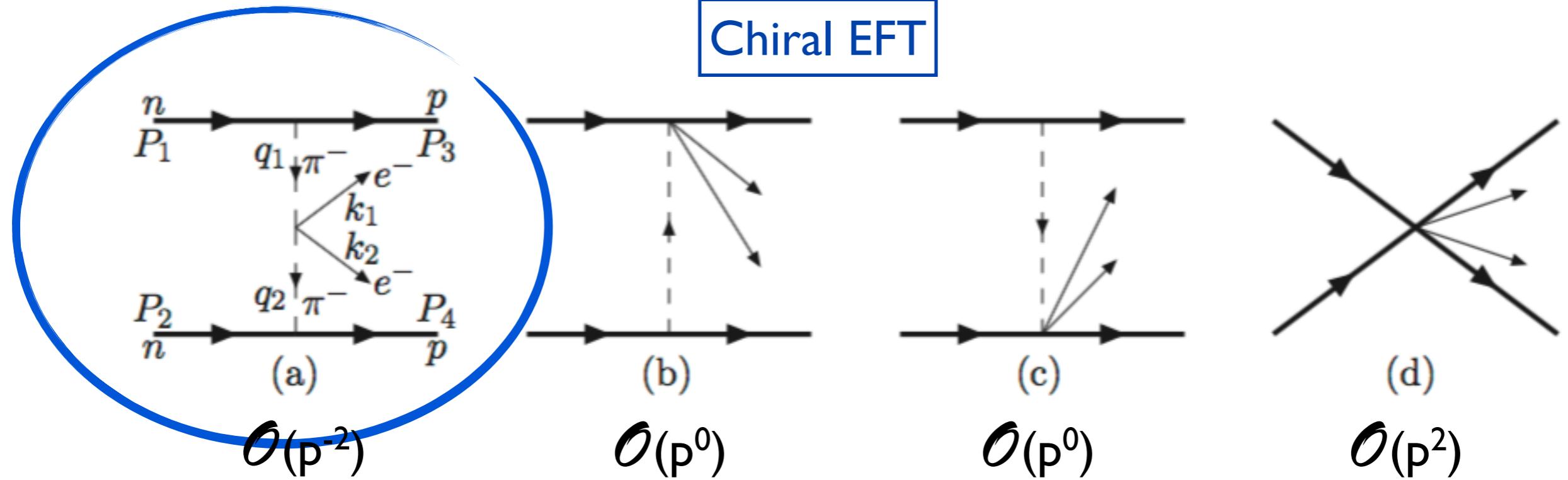
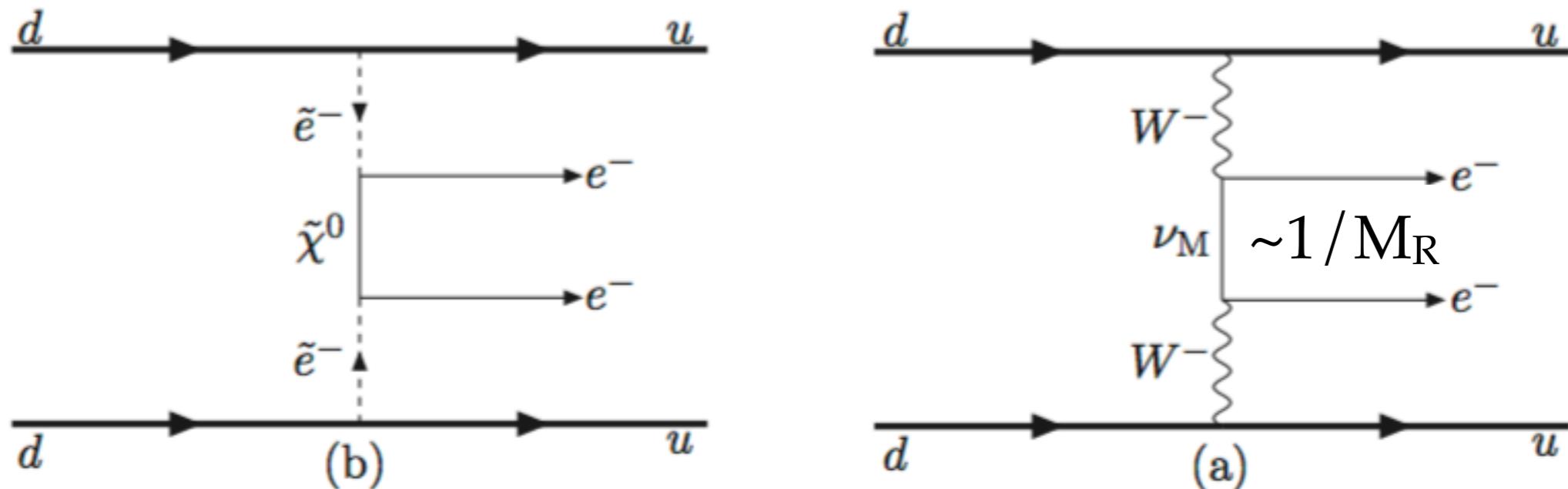
Short-range contribution: probe for heavy physics



Chiral EFT



Short-range contribution: probe for heavy physics



Effective Lagrangian

$$\begin{aligned} \mathcal{L}_{0\nu\beta\beta}^q = & \frac{G_F^2}{\Lambda_{\beta\beta}} \left\{ (o_1 \mathcal{O}_{1+}^{++} + o_2 \mathcal{O}_{2+}^{++} + o_3 \mathcal{O}_{2-}^{++} + o_4 \mathcal{O}_{3+}^{++} + o_5 \mathcal{O}_{3-}^{++}) \bar{e} e^c \right. \\ & + (o_6 \mathcal{O}_{1+}^{++} + o_7 \mathcal{O}_{2+}^{++} + o_8 \mathcal{O}_{2-}^{++} + o_9 \mathcal{O}_{3+}^{++} + o_{10} \mathcal{O}_{3-}^{++}) \bar{e} \gamma^5 e^c \\ & \left. + (o_{11} \mathcal{O}_{4+}^{++,\mu} + o_{12} \mathcal{O}_{4-}^{++,\mu} + o_{13} \mathcal{O}_{5+}^{++,\mu} + o_{14} \mathcal{O}_{5-}^{++,\mu}) \bar{e} \gamma_\mu \gamma^5 e^c + \text{h.c.} \right\} \end{aligned}$$

Prezeau, Ramsey-Musolf, Vogel (2003)

- Nine operators:
 - $\pi \rightarrow \pi$: only need parity even
 - Vector operators suppressed by m_e

$$\begin{aligned} \mathcal{O}_{1+}^{ab} &= (\bar{q}_L \tau^a \gamma^\mu q_L) (\bar{q}_R \tau^b \gamma_\mu q_R), \\ \mathcal{O}_{2\pm}^{ab} &= (\bar{q}_R \tau^a q_L) (\bar{q}_R \tau^b q_L) \pm (\bar{q}_L \tau^a q_R) (\bar{q}_L \tau^b q_R), \\ \mathcal{O}_{3\pm}^{ab} &= (\bar{q}_L \tau^a \gamma^\mu q_L) (\bar{q}_L \tau^b \gamma_\mu q_L) \pm (\bar{q}_R \tau^a \gamma^\mu q_R) (\bar{q}_R \tau^b \gamma_\mu q_R), \\ \mathcal{O}_{4\pm}^{ab,\mu} &= (\bar{q}_L \tau^a \gamma^\mu q_L \mp \bar{q}_R \tau^a \gamma^\mu q_R) (\bar{q}_L \tau^b q_R - \bar{q}_R \tau^b q_L), \\ \mathcal{O}_{5\pm}^{ab,\mu} &= (\bar{q}_L \tau^a \gamma^\mu q_L \pm \bar{q}_R \tau^a \gamma^\mu q_R) (\bar{q}_L \tau^b q_R + \bar{q}_R \tau^b q_L). \end{aligned}$$

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Effective Lagrangian

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Prezeau, Ramsey-Musolf, Vogel (2003)

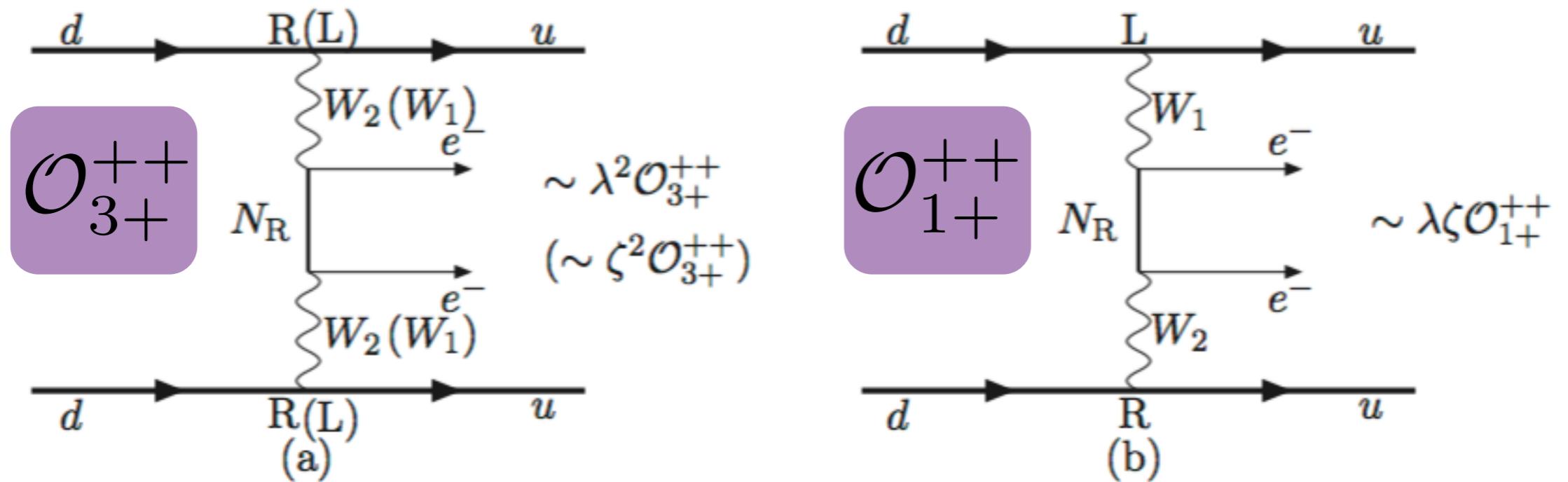
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Calculate LECs; EFT then determines $nn \rightarrow pp$ transition via pion exchange diagram

$0\nu\beta\beta$ -decay ops.	$\mathcal{O}_{1+}^{\pm\pm}$	$\mathcal{O}_{2+}^{\pm\pm}$	$\mathcal{O}_{2-}^{\pm\pm}$	$\mathcal{O}_{3+}^{\pm\pm}$	$\mathcal{O}_{3-}^{\pm\pm}$	$\mathcal{O}_{4+}^{\pm\pm,\mu}$	$\mathcal{O}_{4-}^{\pm\pm,\mu}$	$\mathcal{O}_{5+}^{\pm\pm,\mu}$	$\mathcal{O}_{5-}^{\pm\pm,\mu}$
$\pi\pi ee$ LO	✓	✓	X	X	X	X	X	X	X
$\pi\pi ee$ NNLO	✓	✓	X	✓	X	X	X	X	X
$NN\pi ee$ LO	X	X	✓	X	X	✓	✓	✓	✓
$NN\pi ee$ NLO	X	✓	X	✓	X	✓	✓	✓	✓
$NNNNee$ LO	✓	✓	X	✓	X	✓	✓	✓	✓

Left-right symmetric models



Contractions

- Exact momentum projection at source and sink
- Must add color mixed versions of Prezeau, Ramsey-Musolf, Vogel ops 1&2

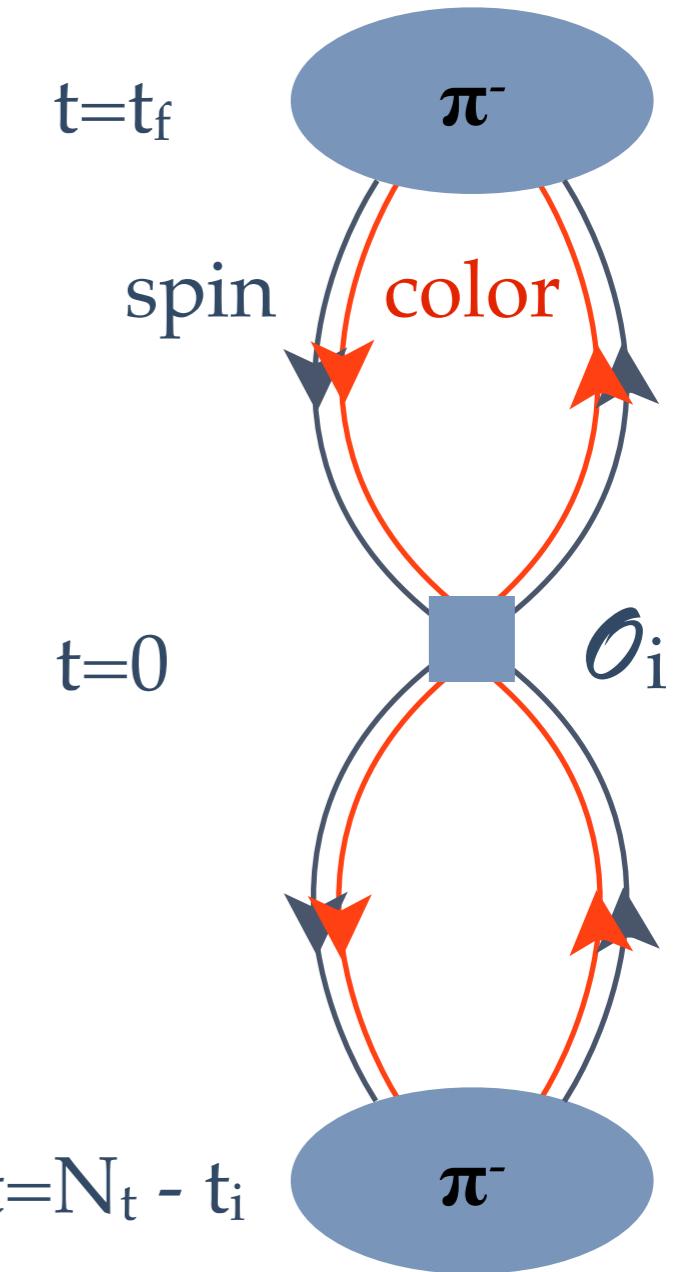
$$\mathcal{O}_{1+}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_R \tau^- \gamma_\mu q_R]$$

$$\mathcal{O}'_{1+}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_R \tau^- \gamma_\mu q_R]$$

$$\mathcal{O}_{2+}^{++} = (\bar{q}_R \tau^- q_L) [\bar{q}_R \tau^- q_L] + (\bar{q}_L \tau^- q_R) [\bar{q}_L \tau^- q_R]$$

$$\mathcal{O}'_{2+}^{++} = (\bar{q}_R \tau^- q_L) [\bar{q}_R \tau^- q_L] + (\bar{q}_L \tau^- q_R) [\bar{q}_L \tau^- q_R]$$

$$\mathcal{O}_{3+}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_L \tau^- \gamma_\mu q_L] + (\bar{q}_R \tau^- \gamma^\mu q_R) [\bar{q}_R \tau^- \gamma_\mu q_R]$$



Contractions

- Exact momentum projection at source and sink
- Must add color mixed versions of Prezeau, Ramsey-Musolf, Vogel ops 1&2

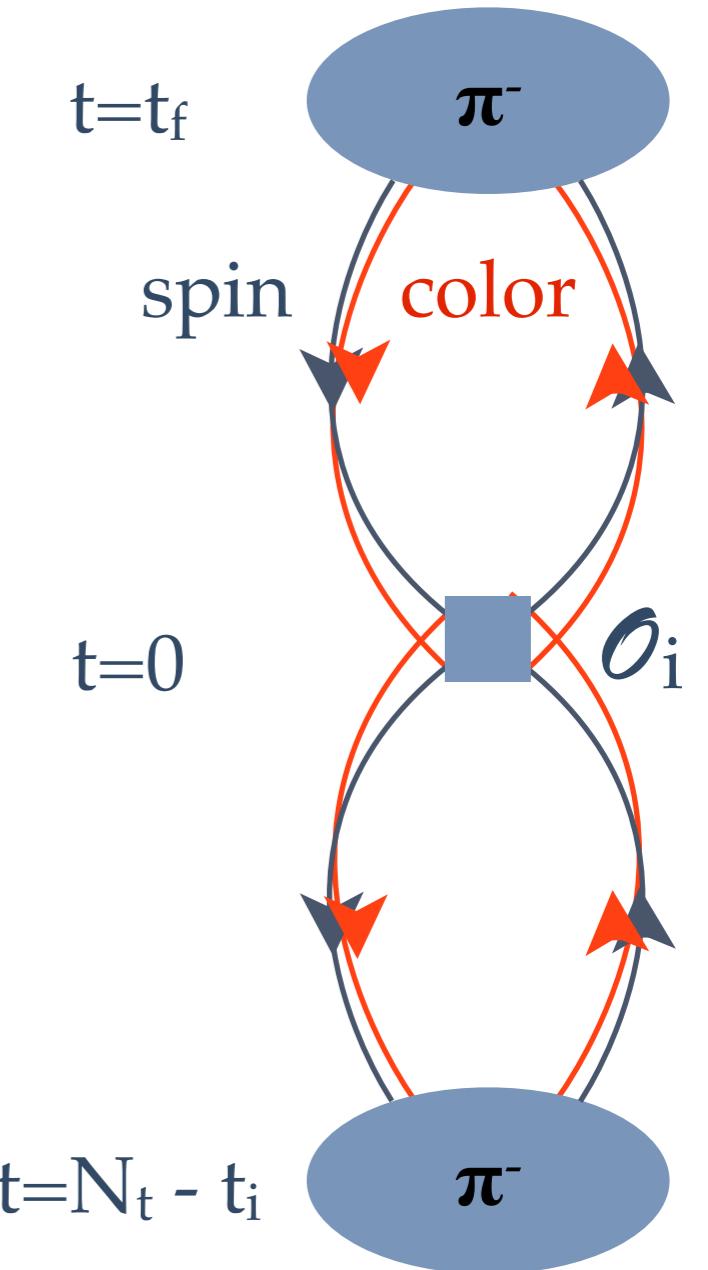
$$\mathcal{O}_{1+}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_R \tau^- \gamma_\mu q_R]$$

$$\mathcal{O}'_{1+}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_R \tau^- \gamma_\mu q_R]$$

$$\mathcal{O}_{2+}^{++} = (\bar{q}_R \tau^- q_L) [\bar{q}_R \tau^- q_L] + (\bar{q}_L \tau^- q_R) [\bar{q}_L \tau^- q_R]$$

$$\mathcal{O}'_{2+}^{++} = (\bar{q}_R \tau^- q_L) [\bar{q}_R \tau^- q_L] + (\bar{q}_L \tau^- q_R) [\bar{q}_L \tau^- q_R]$$

$$\mathcal{O}_{3+}^{++} = (\bar{q}_L \tau^- \gamma^\mu q_L) [\bar{q}_L \tau^- \gamma_\mu q_L] + (\bar{q}_R \tau^- \gamma^\mu q_R) [\bar{q}_R \tau^- \gamma_\mu q_R]$$



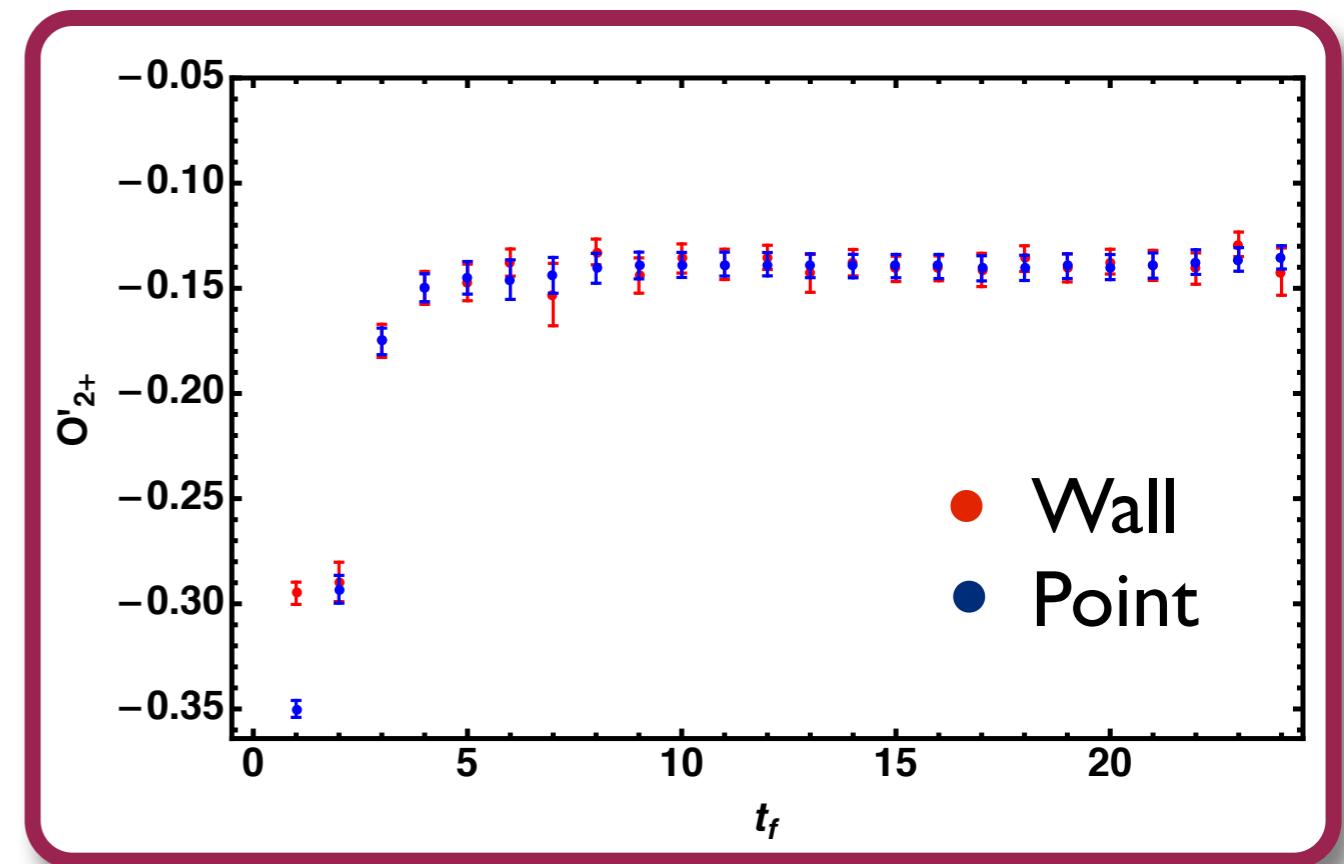
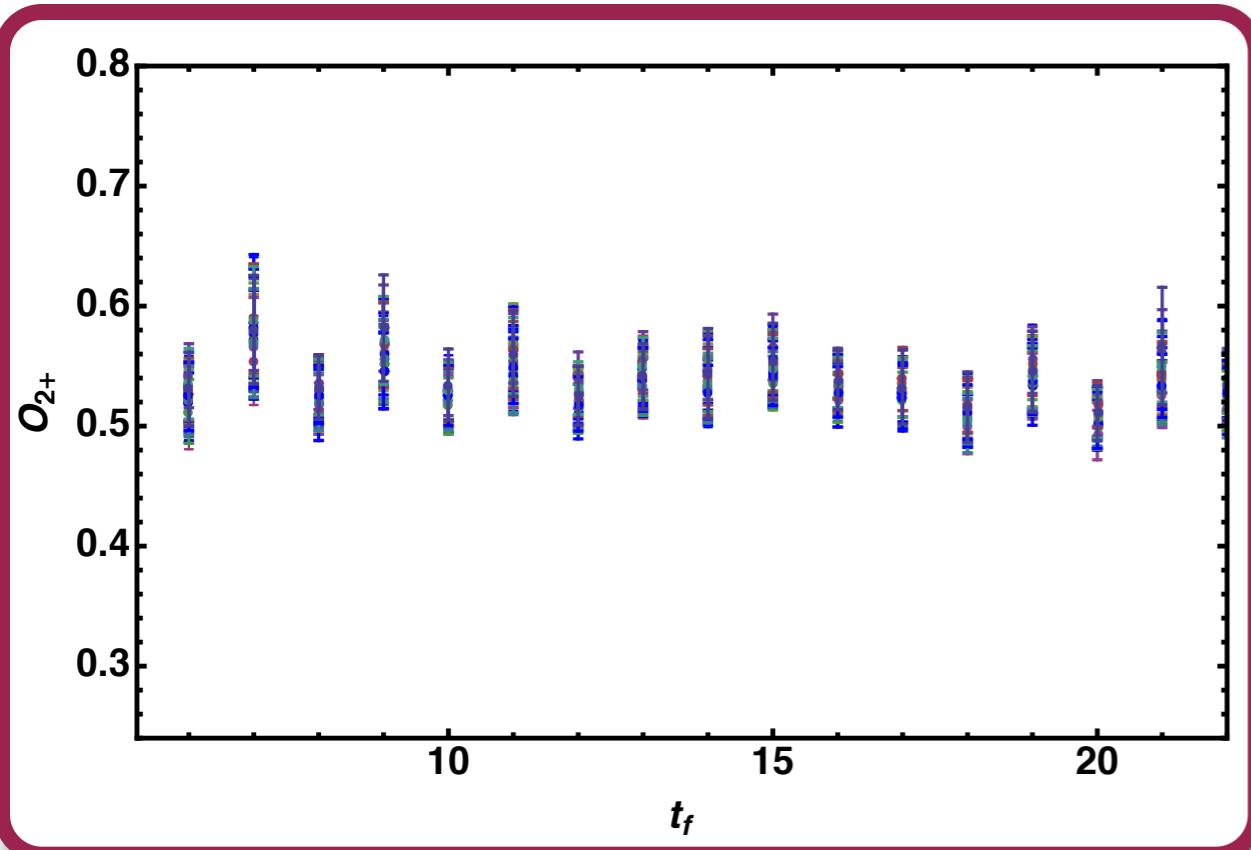
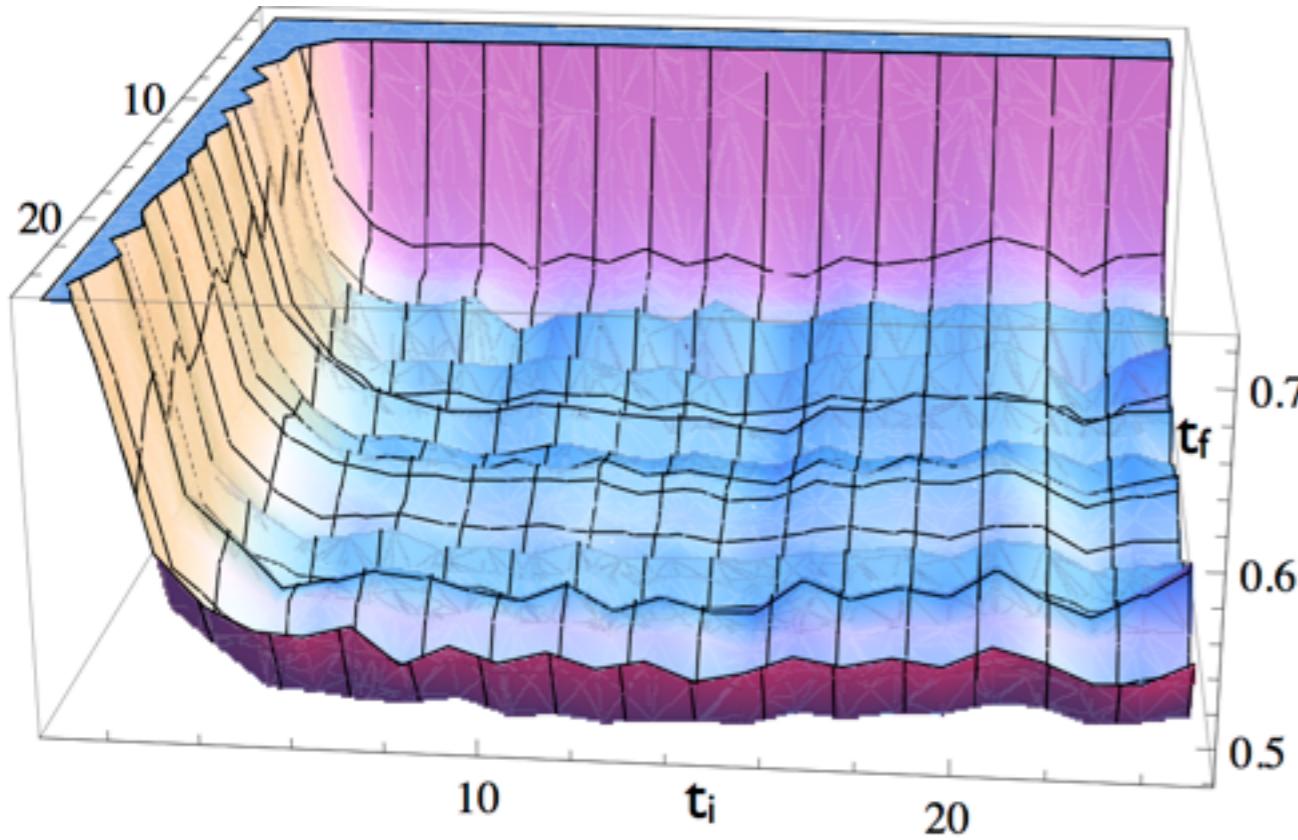
HISQ ensembles

$a[fm]$	$m_\pi [MeV]$	310	220	135
0.15	$16^3 \times 48, m_\pi L \sim 3.78$		$24^3 \times 48, m_\pi L \sim 3.99$	$32^3 \times 48, m_\pi L \sim 3.25$
0.12			$24^3 \times 64, m_\pi L \sim 3.22$	
0.12	$24^3 \times 64, m_\pi L \sim 4.54$		$32^3 \times 64, m_\pi L \sim 4.29$	$48^3 \times 64, m_\pi L \sim 3.91$
0.12			$40^3 \times 64, m_\pi L \sim 5.36$	
0.09	$32^3 \times 96, m_\pi L \sim 4.50$		$48^3 \times 96, m_\pi L \sim 4.73$	

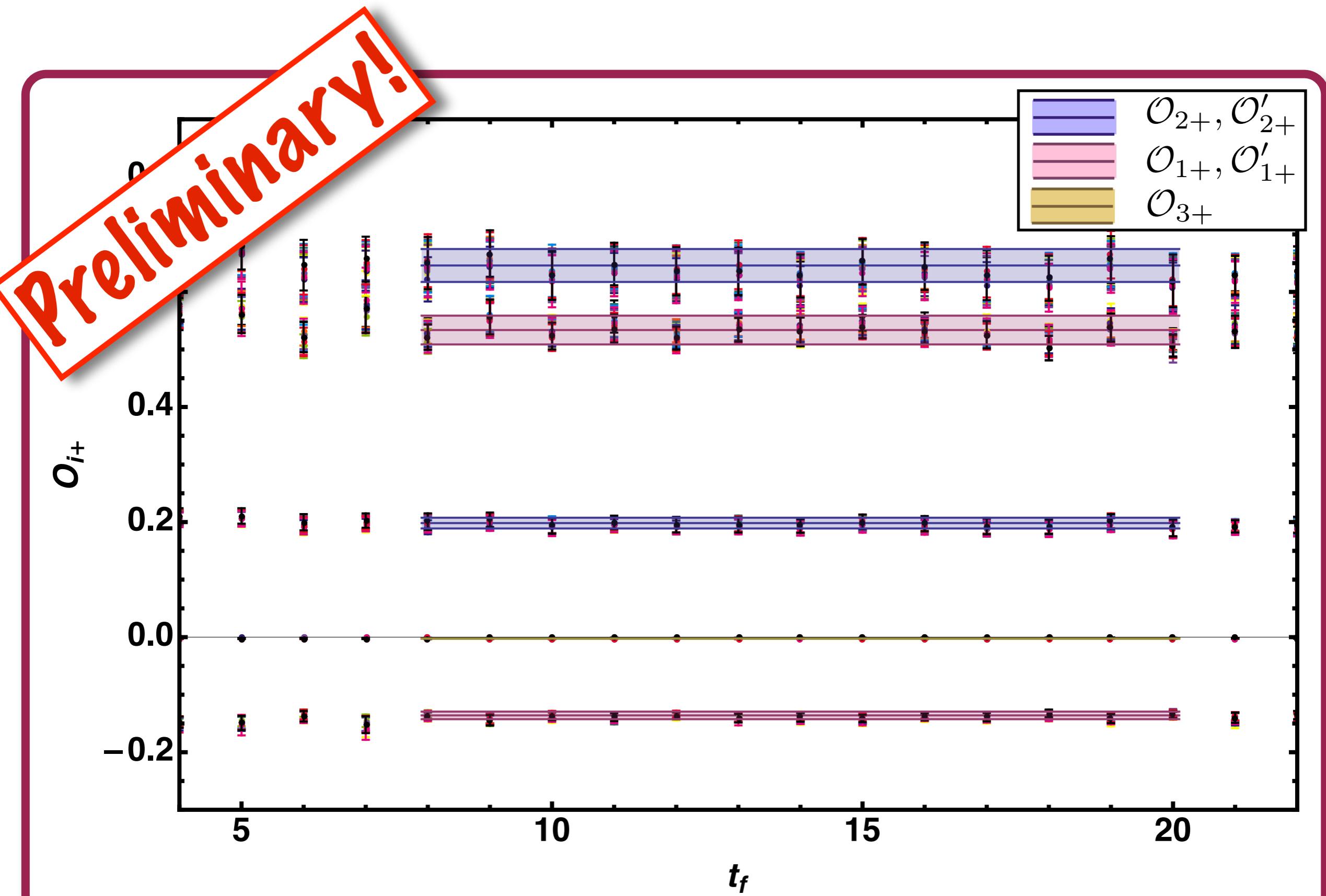
- Möbius DWF on HISQ
- Gradient flow method for smearing configs
 - $m_{\text{res}} < 0.1 m_\ell$ for moderate L_5
- Wall + point sources for pions
- ~ 1000 cfgs, 1 source/cfg

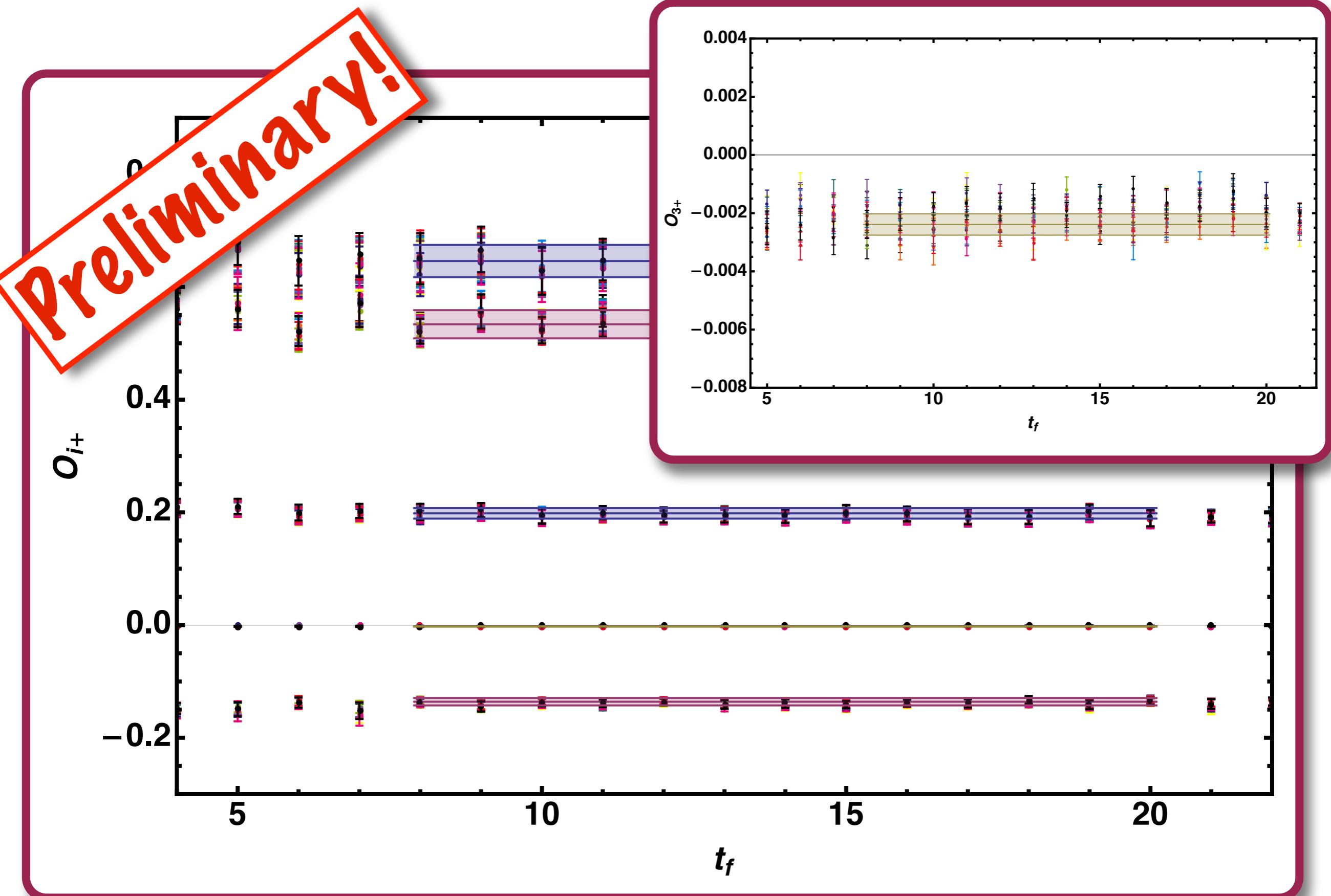
MILC Collaboration Phys. Rev. D87 (2013) 054505
 Narayanan, Neuberger (2006), Luscher (2010)
 K. Orginos, C. Monahan (private communication)

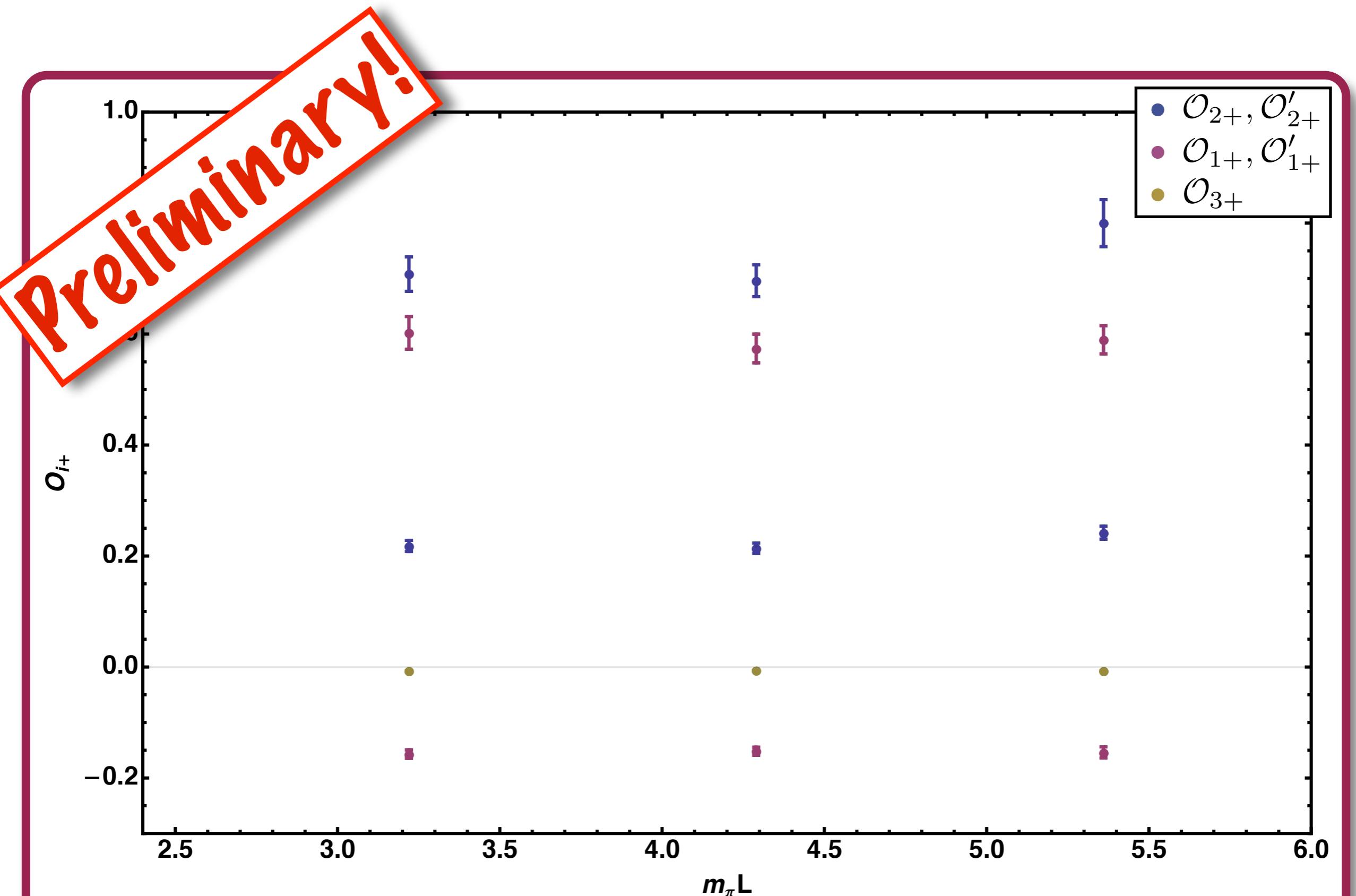
Signals

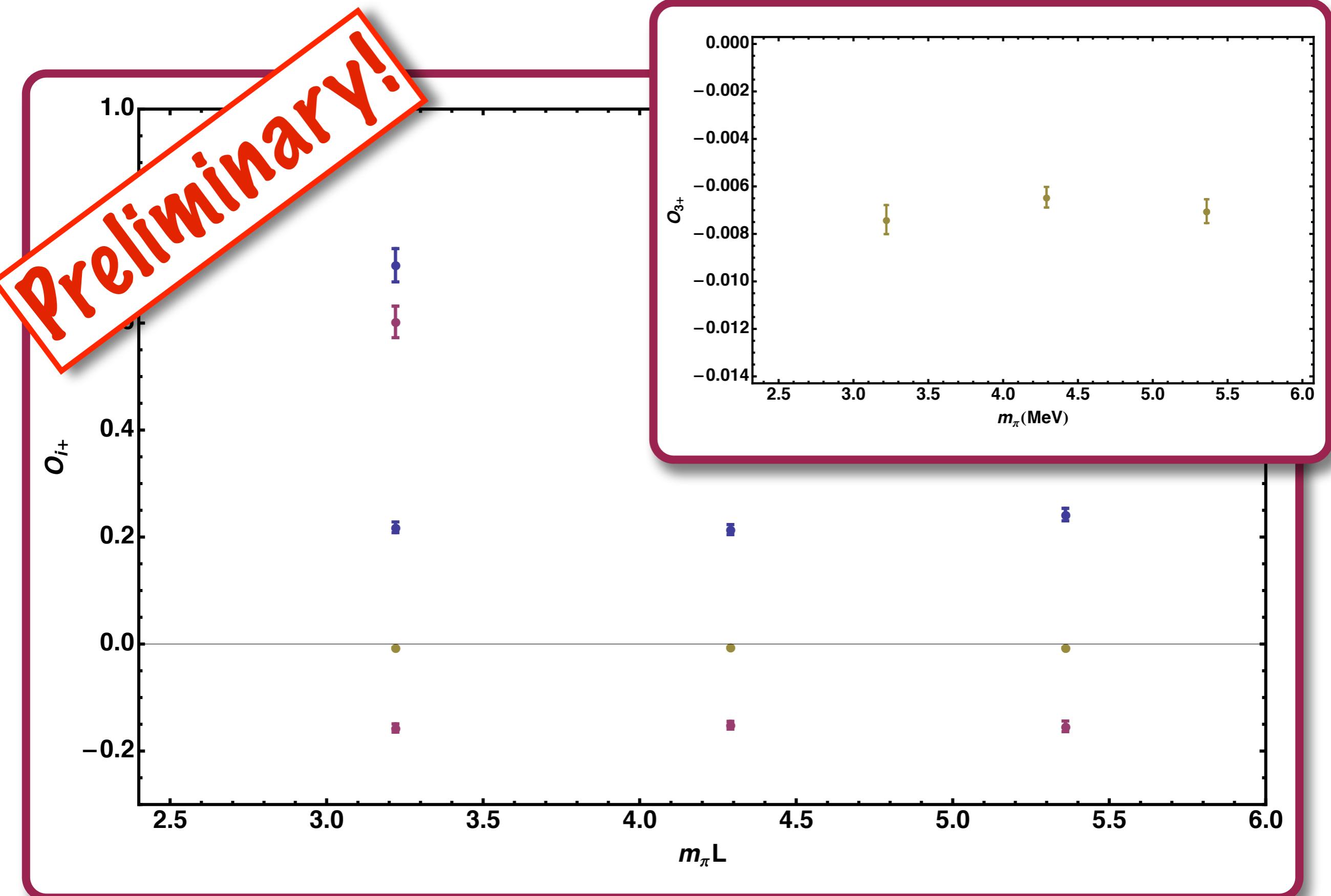


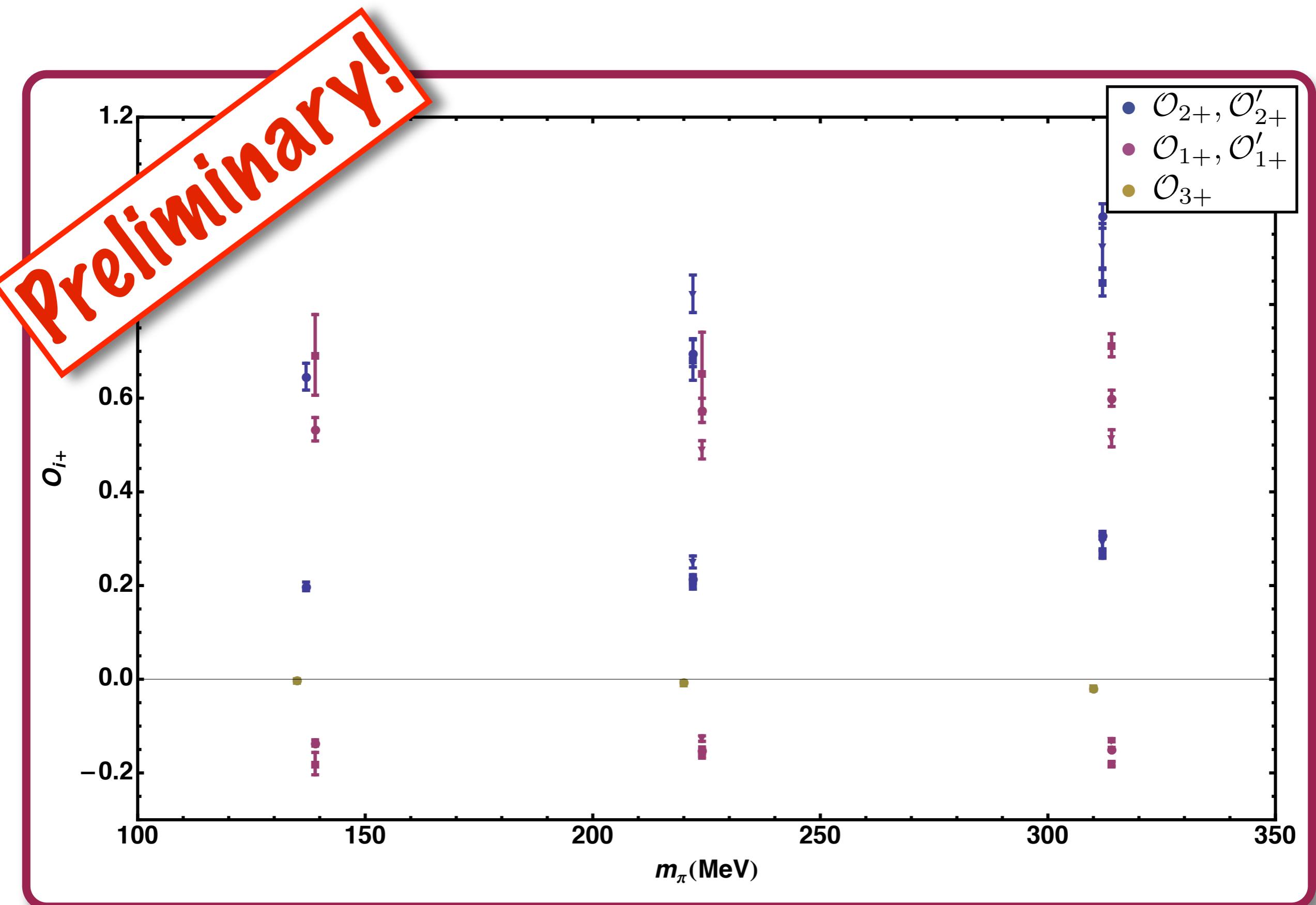
- $m_\pi \sim 135 \text{ MeV}$
- $L = 5.76 \text{ fm}$
- $a = 0.12 \text{ fm}$

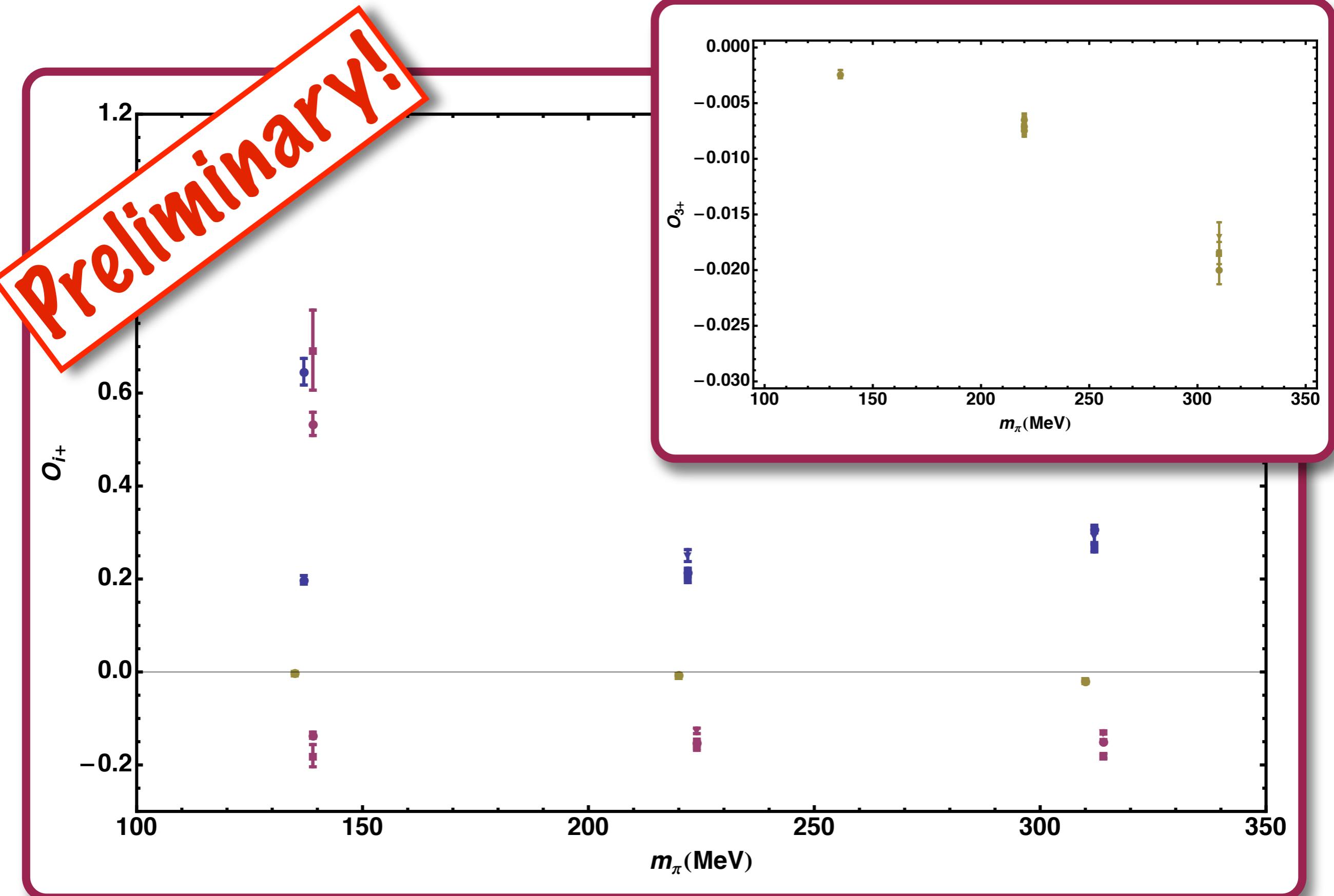


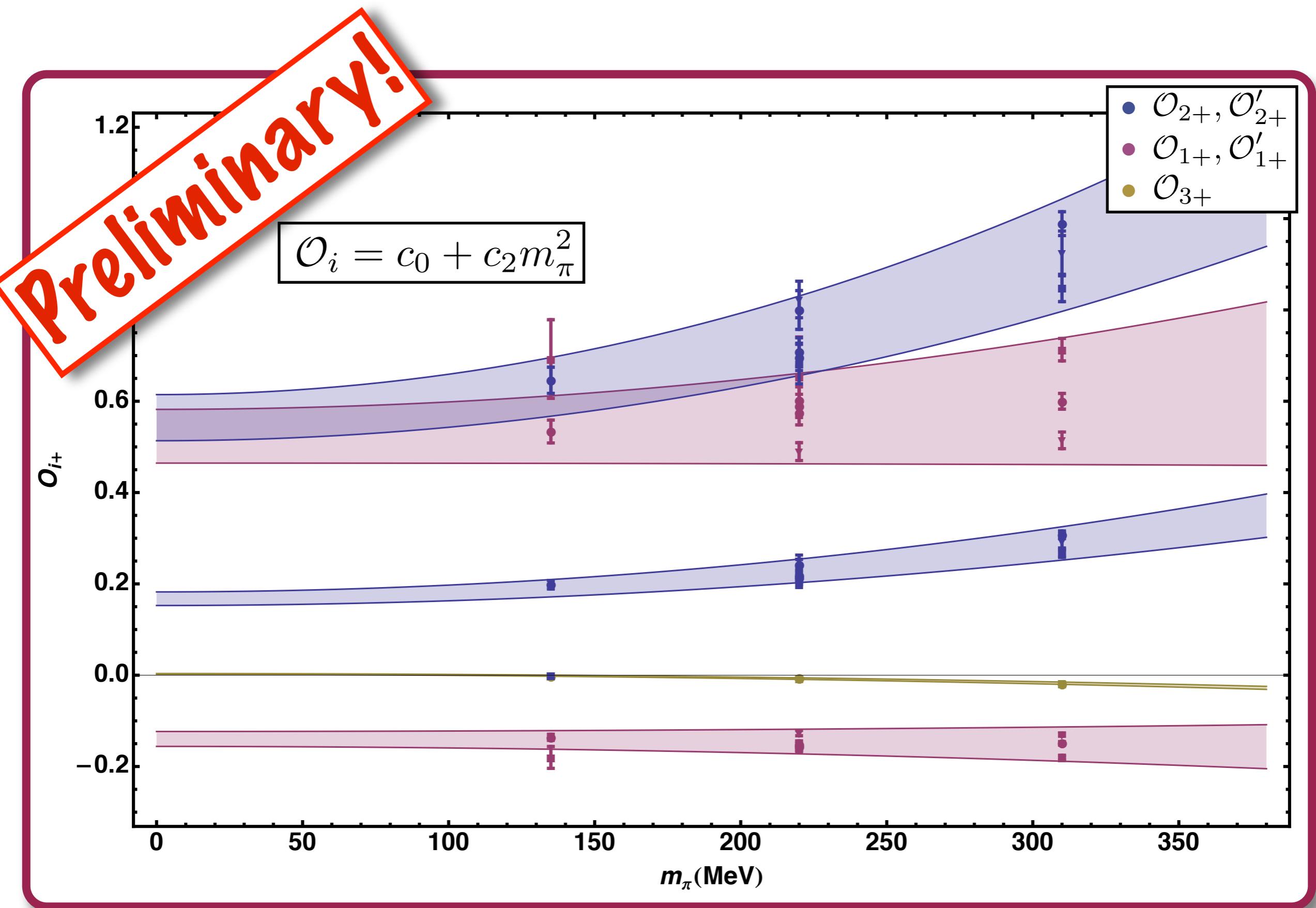


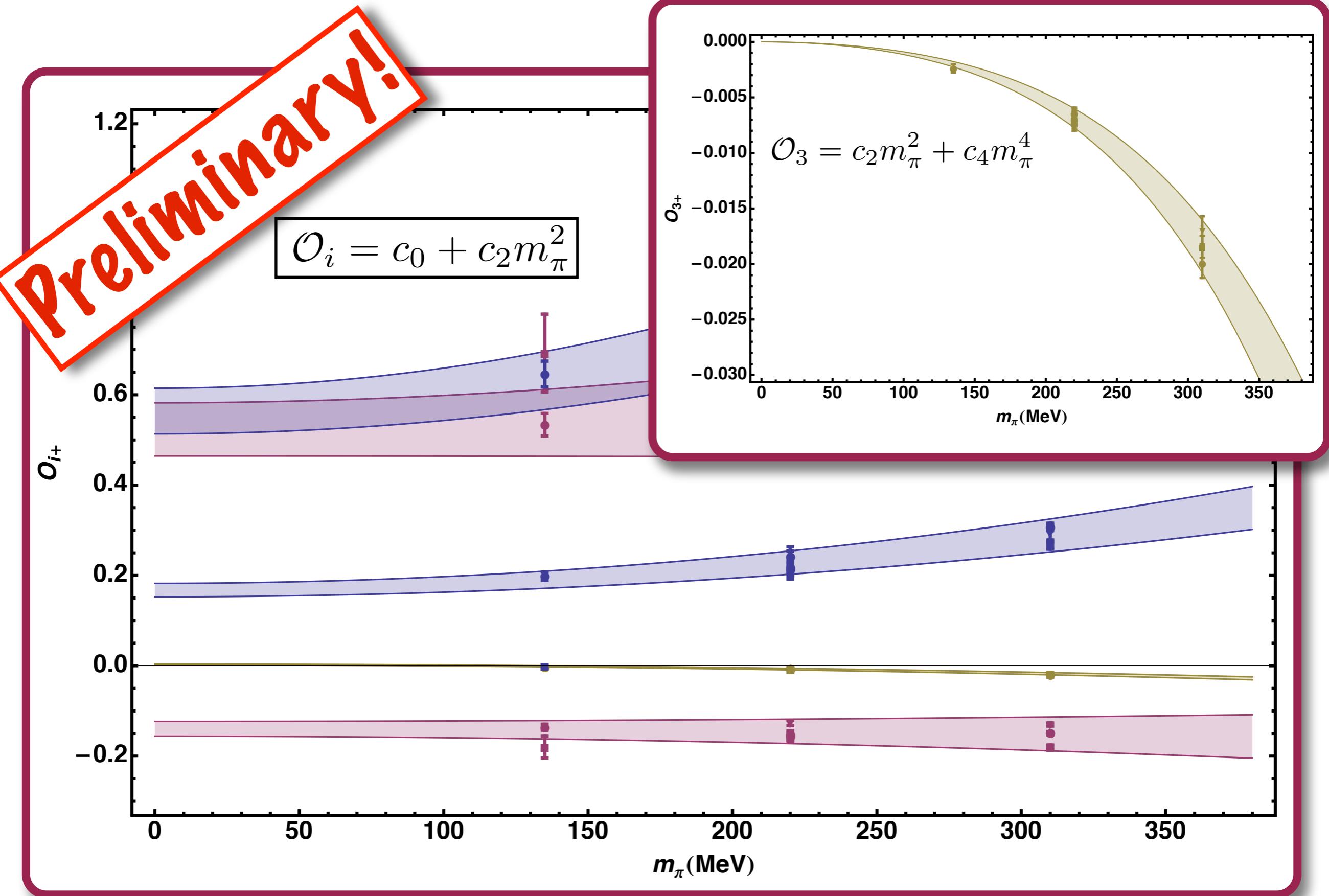








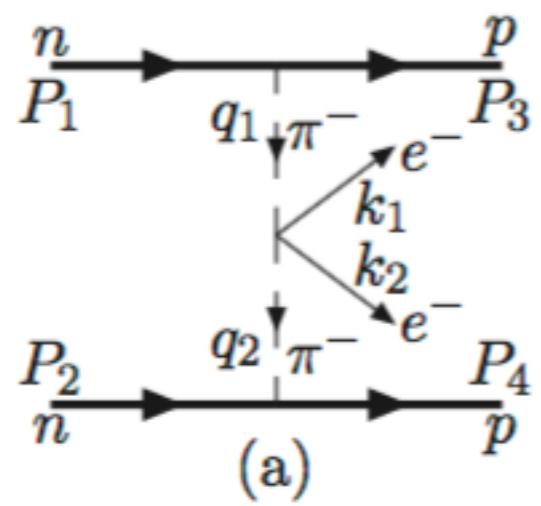




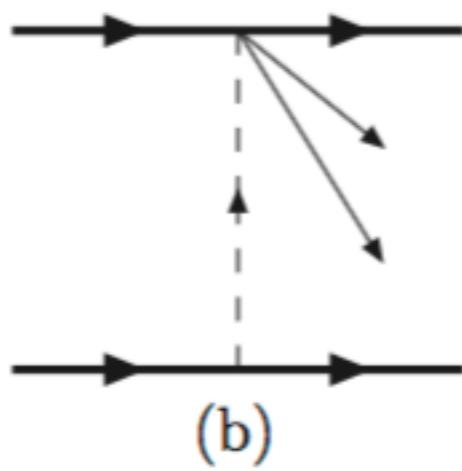
Summary

- $0\nu\beta\beta$: search for Majorana mass signature
 - Lepton number violation could be source of matter/anti-matter asymmetry
 - Huge experimental efforts planned/underway
 - LQCD can make major impact on understanding of short-range operators
- Preliminary results for $\pi^- \rightarrow \pi^+$ matrix element
 - Multiple pion masses, lattice spacings, volumes
 - Pion mass dependence as expected from chiral EFT counting
- To do:
 - Renormalization Buras, Misiak, Urban (2000), Tiburzi (2012)
 - Extrapolations in pion mass/lattice spacing
 - Other contact operators....

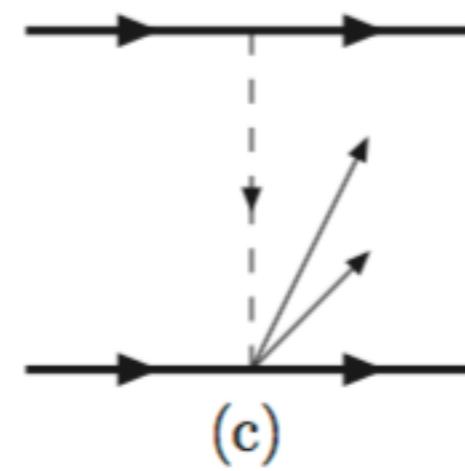
Contact operators



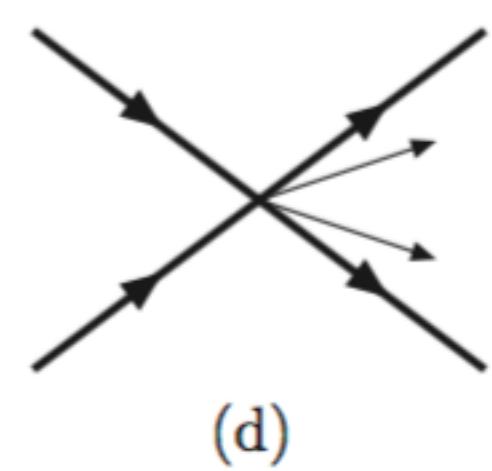
$\mathcal{O}(P^{-2})$



$\mathcal{O}(P^0)$



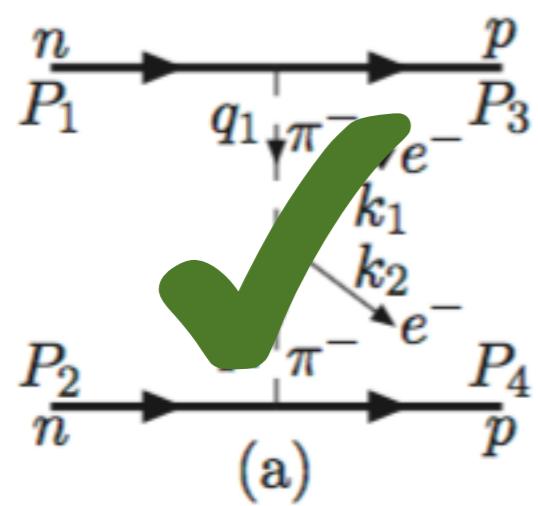
$\mathcal{O}(P^0)$



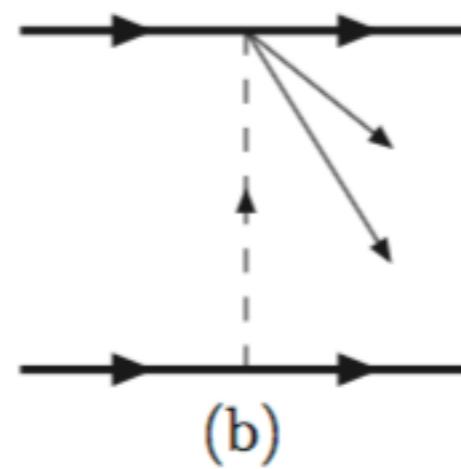
$\mathcal{O}(P^2)$

Contact operators

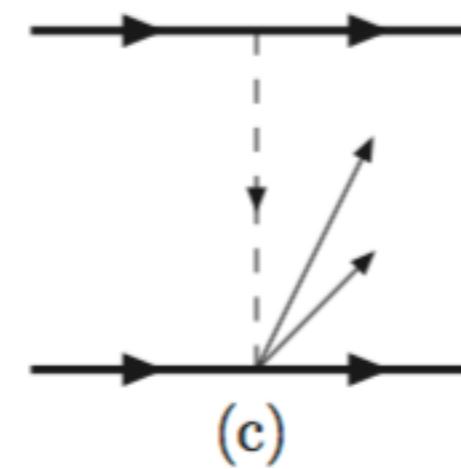
- LO almost complete!



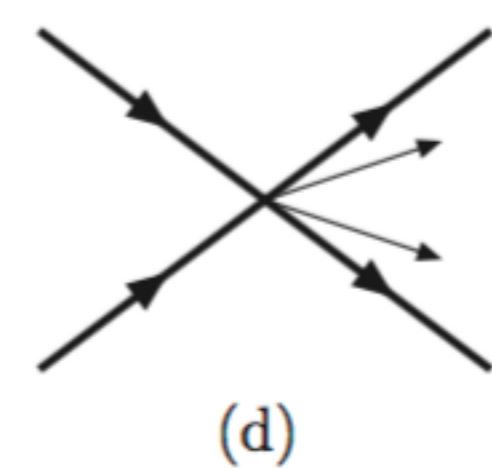
$\mathcal{O}(P^{-2})$



$\mathcal{O}(P^0)$



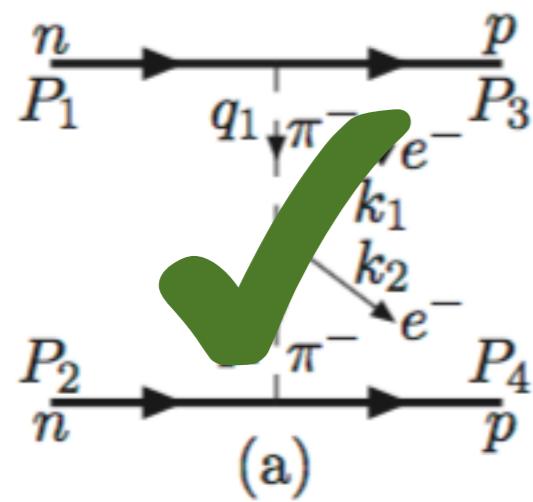
$\mathcal{O}(P^0)$



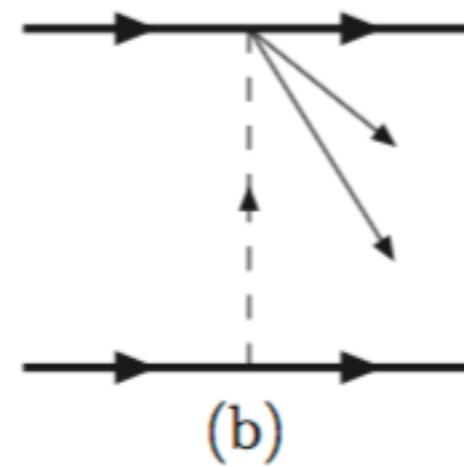
$\mathcal{O}(P^2)$

Contact operators

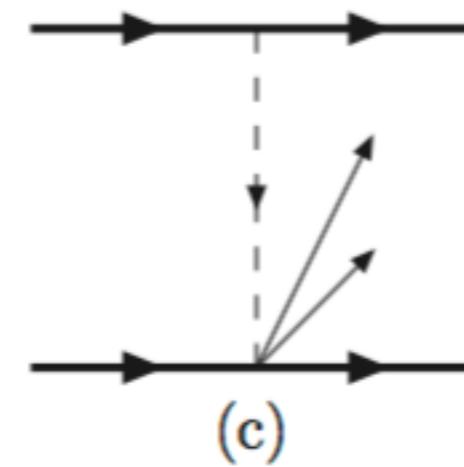
- LO almost complete!
- NLO: disconnected diagrams



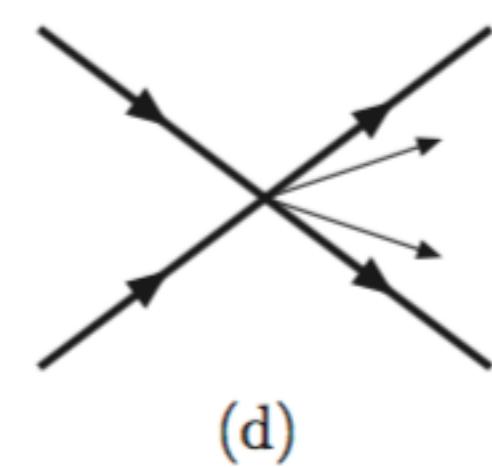
$\mathcal{O}(P^{-2})$



$\mathcal{O}(P^0)$



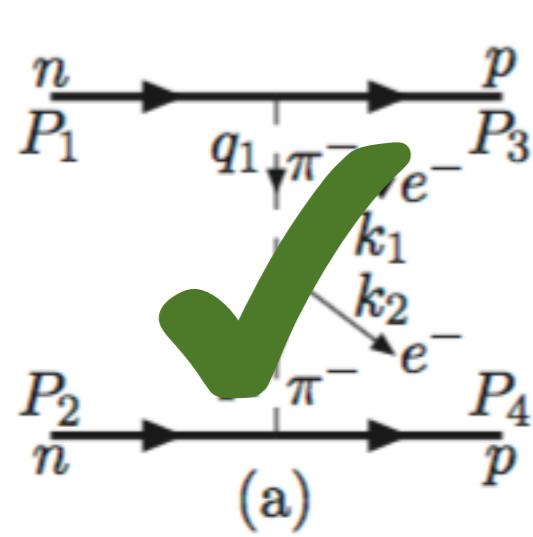
$\mathcal{O}(P^0)$



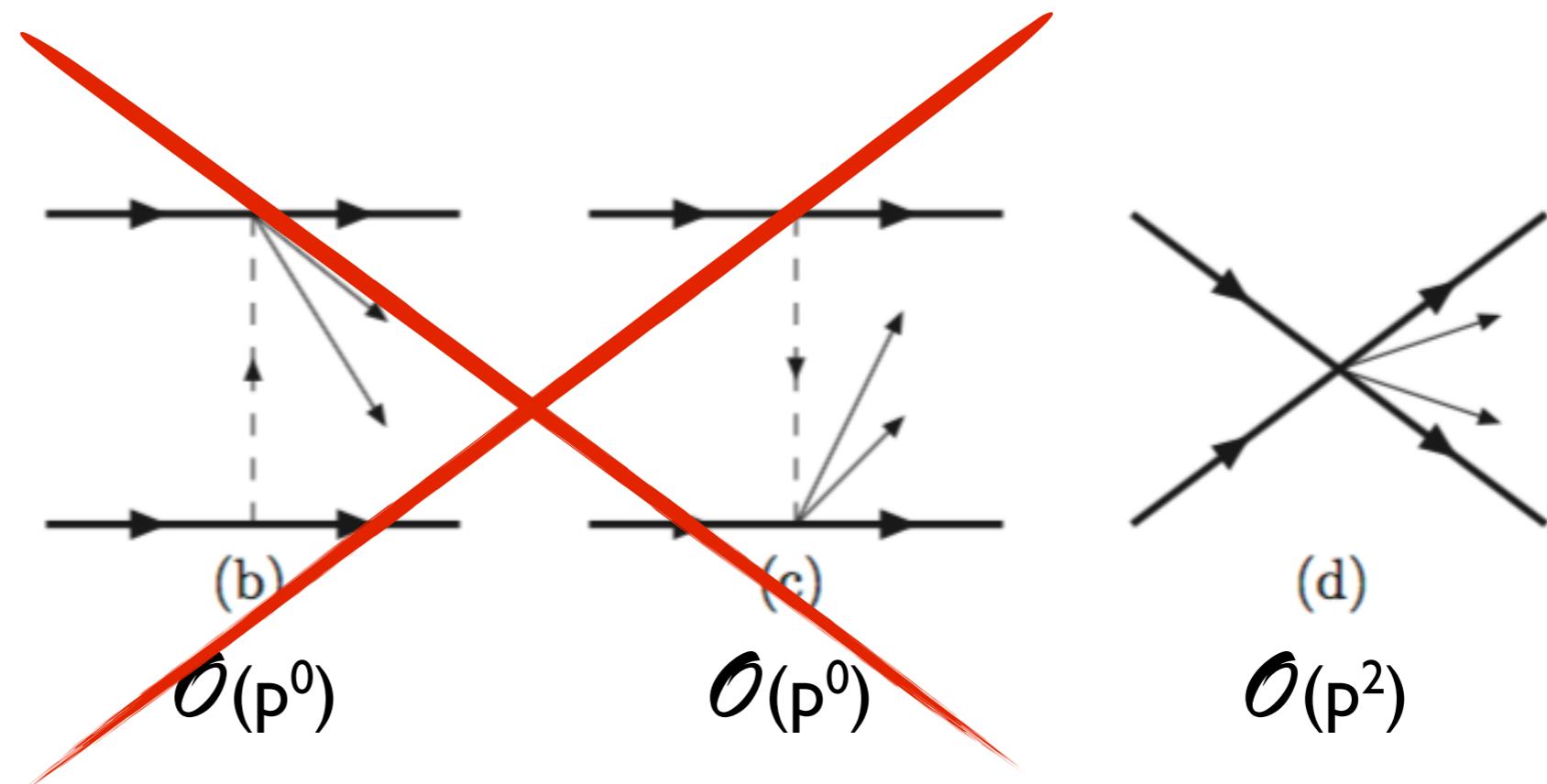
$\mathcal{O}(P^2)$

Contact operators

- LO almost complete!
- NLO: disconnected diagrams
 - Don't contribute to $0^+ \rightarrow 0^+$ nuclear transitions



$\mathcal{O}(p^{-2})$

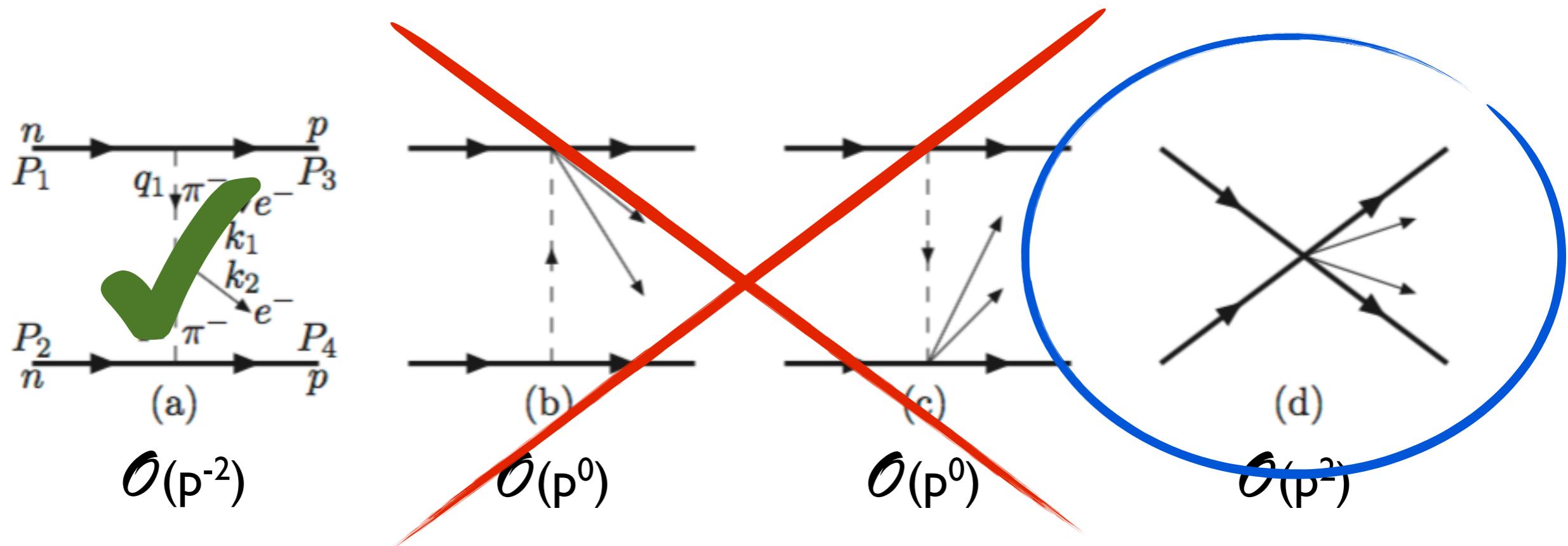


$\mathcal{O}(p^0)$

$\mathcal{O}(p^2)$

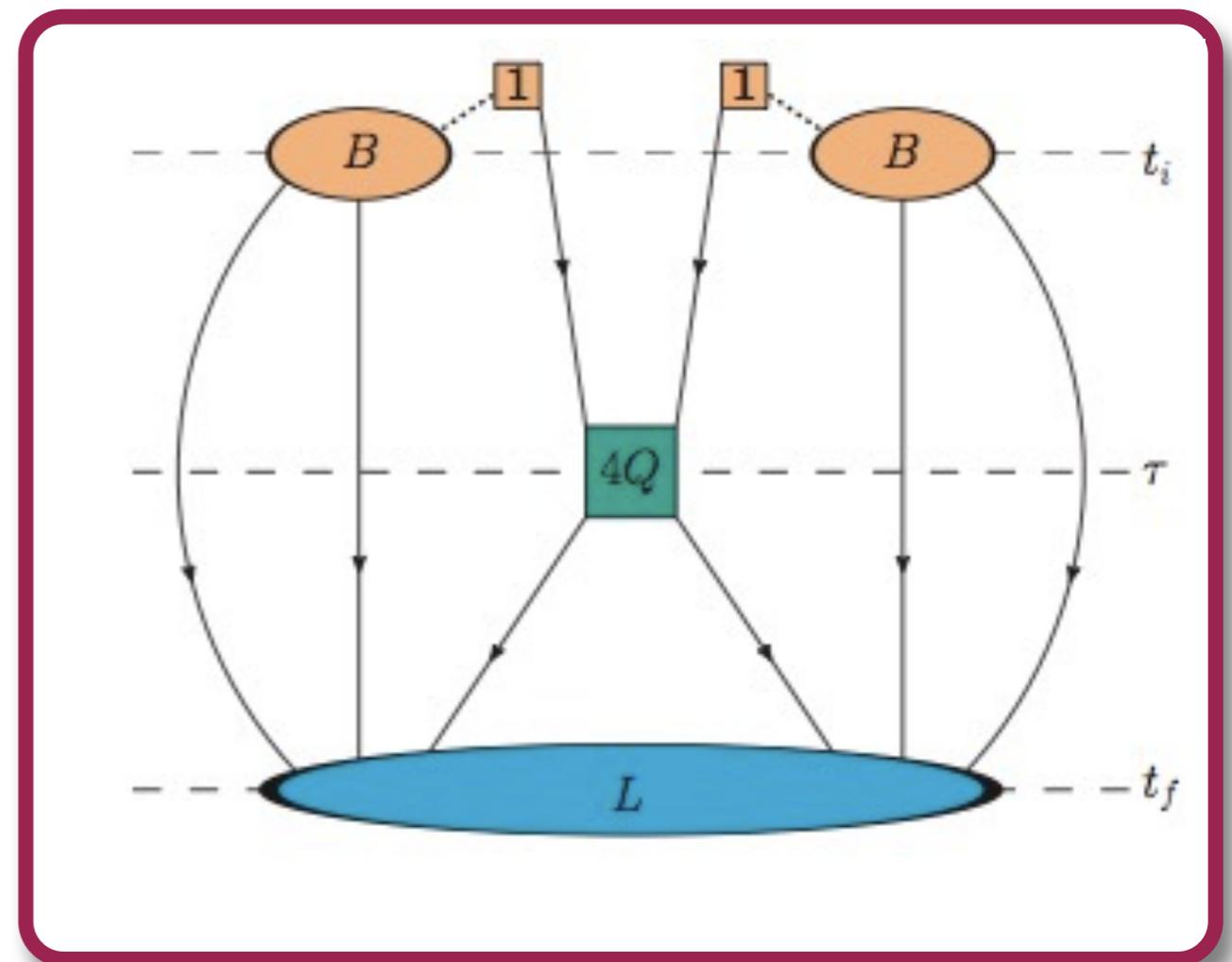
Contact operators

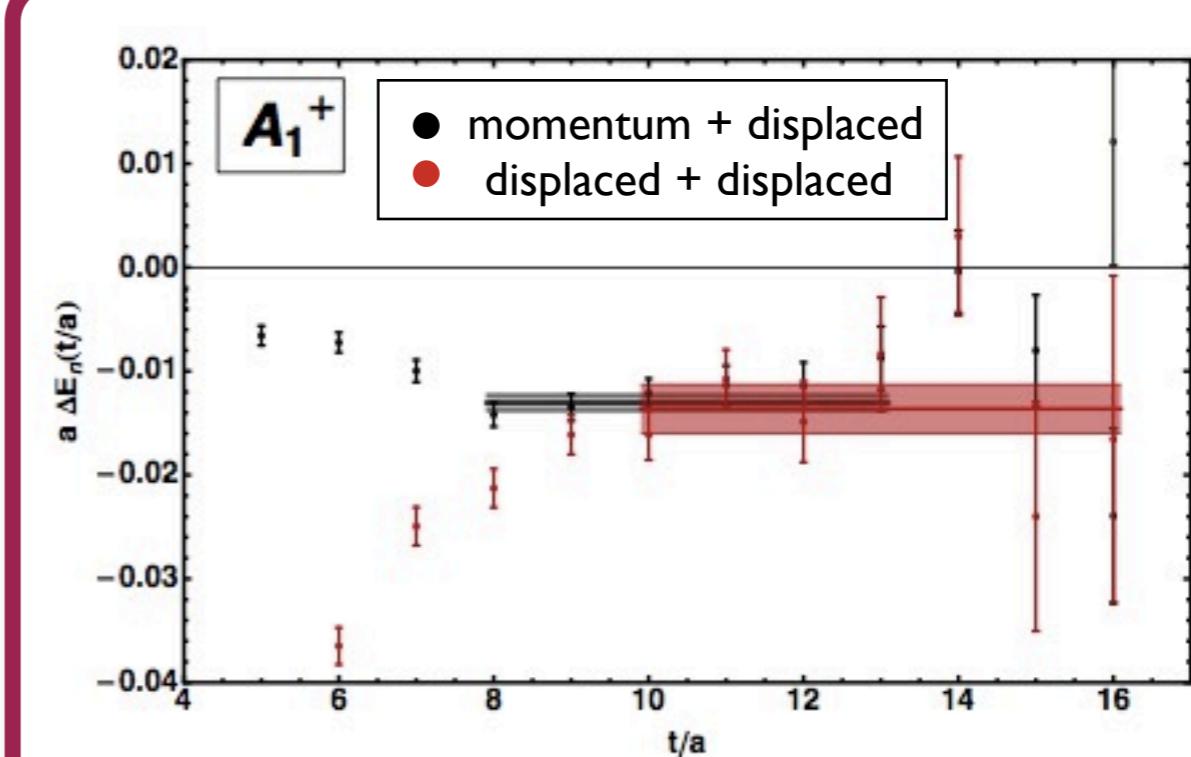
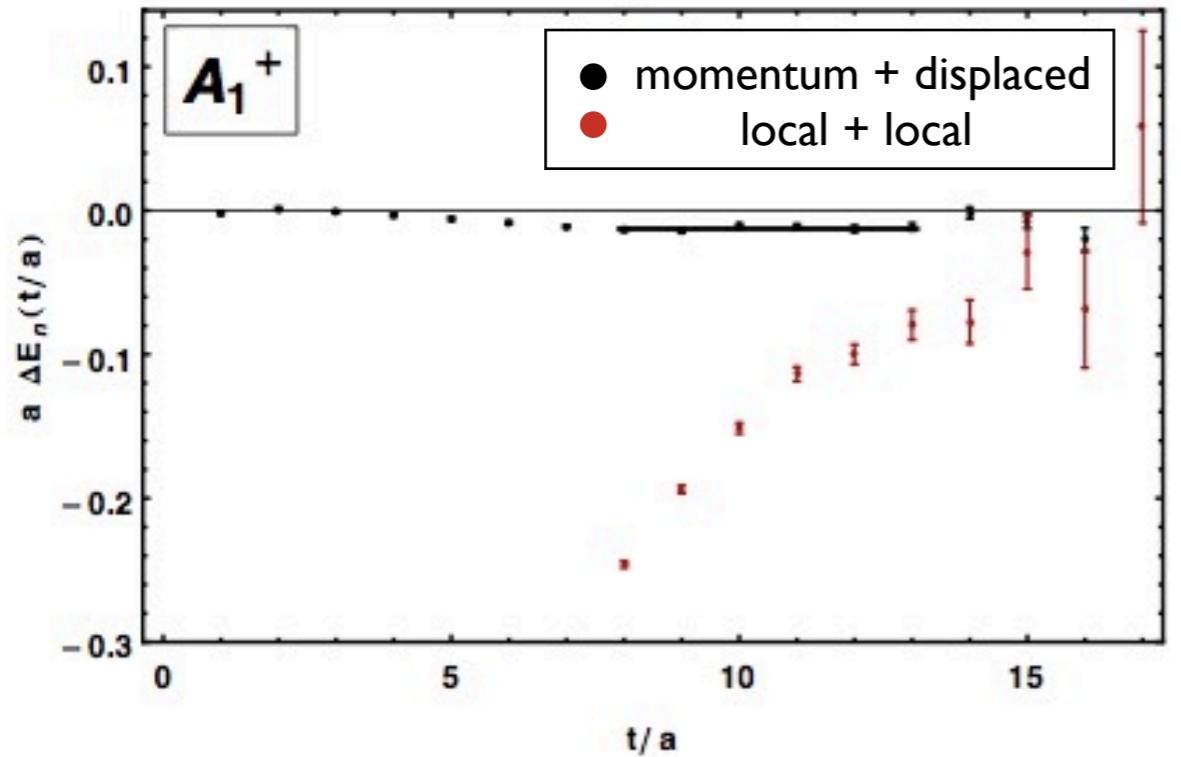
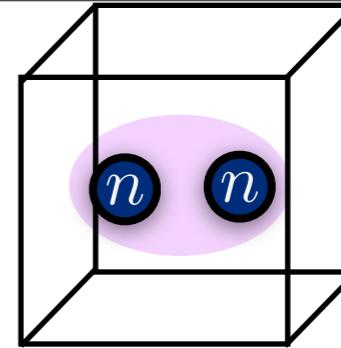
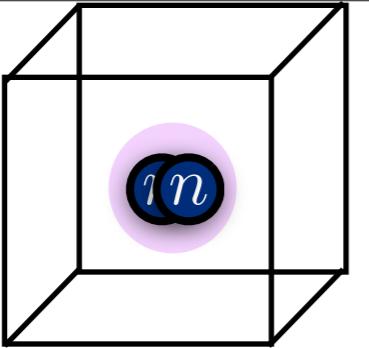
- LO almost complete!
- NLO: disconnected diagrams
 - Don't contribute to $0^+ \rightarrow 0^+$ nuclear transitions
- nn \rightarrow pp contact operators



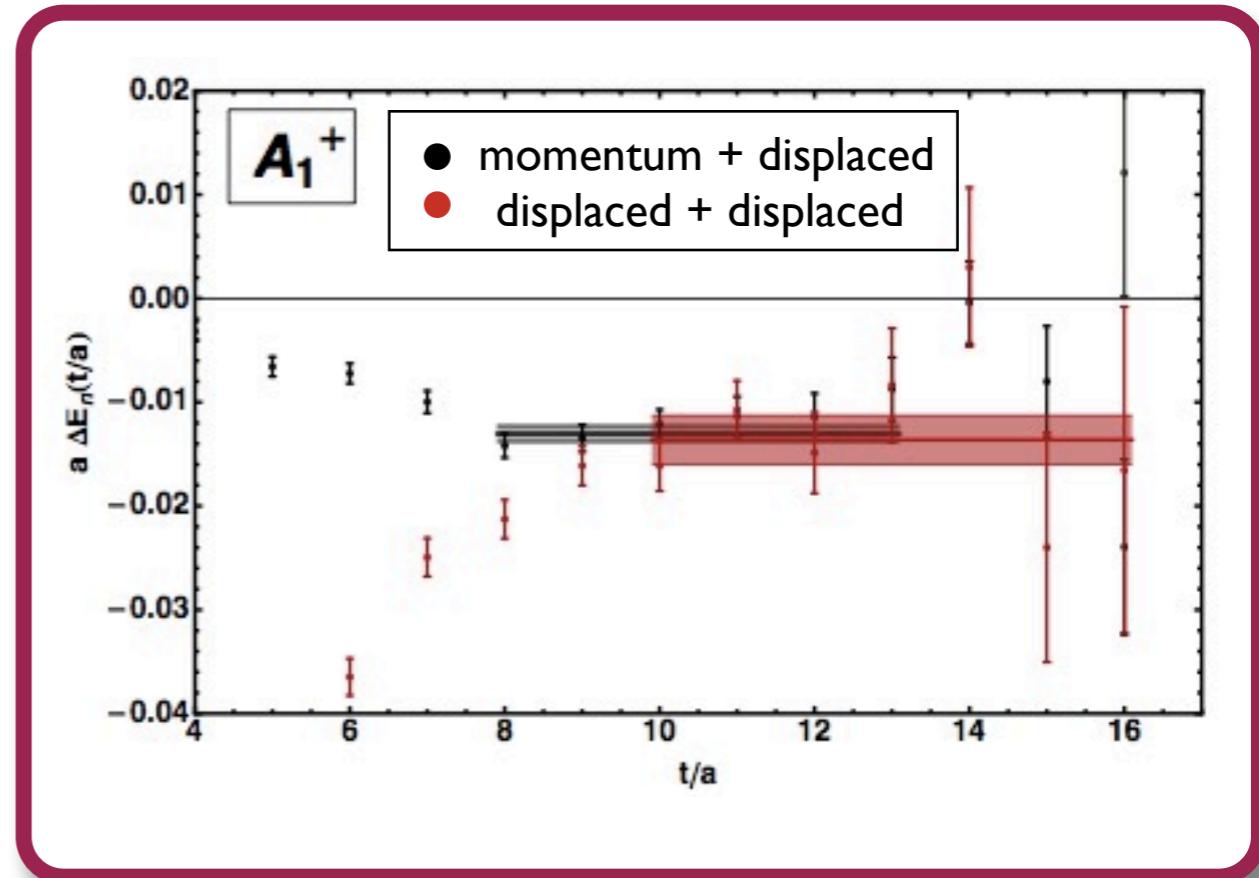
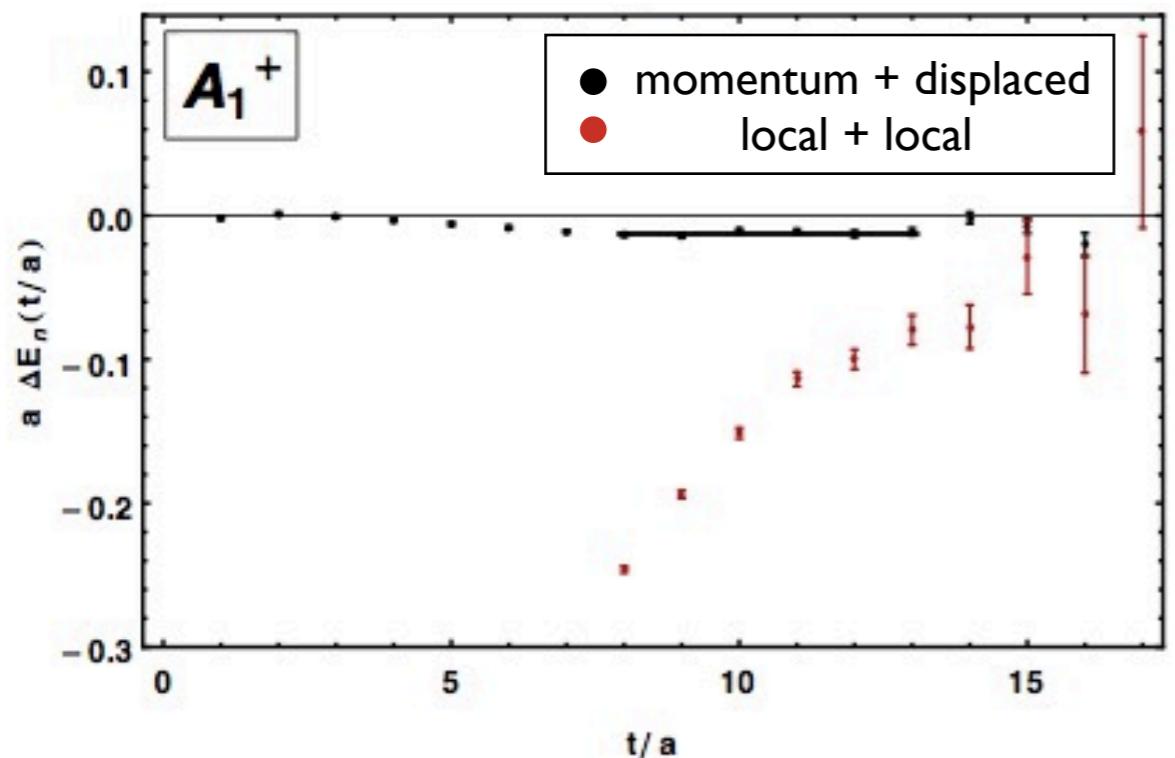
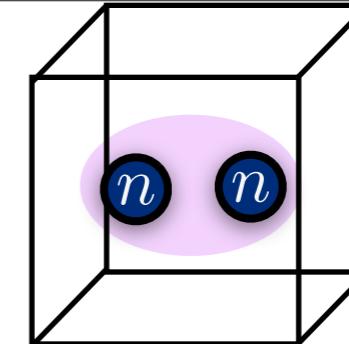
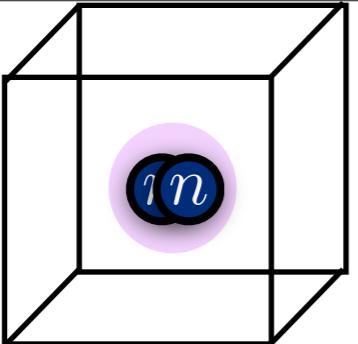
Contractions

- Isospin limit: 576 contractions
- Extension of unified contraction method*
- Need position space source & sink
 - otherwise all-to-all propagators connect to 4-quark operator
 - stochastically project onto zero total momentum



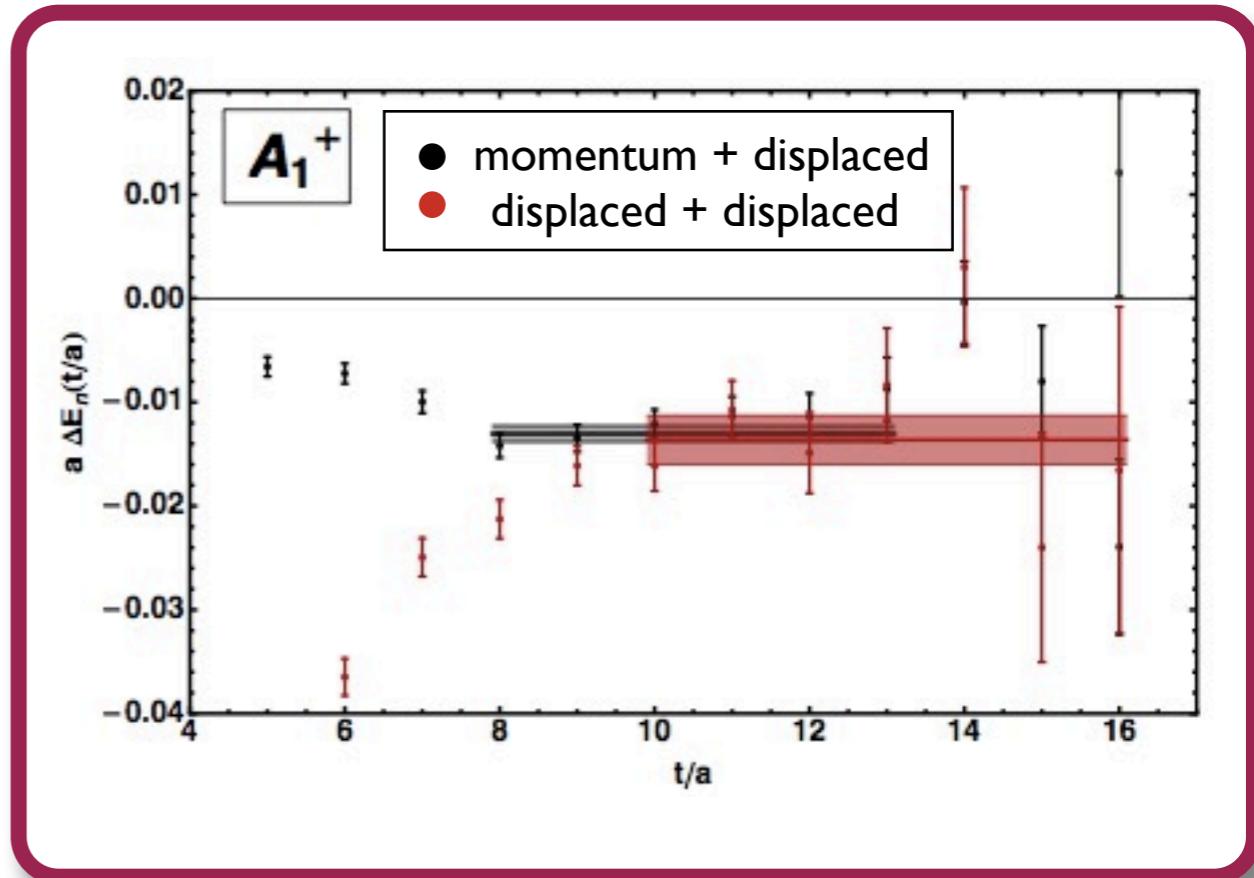
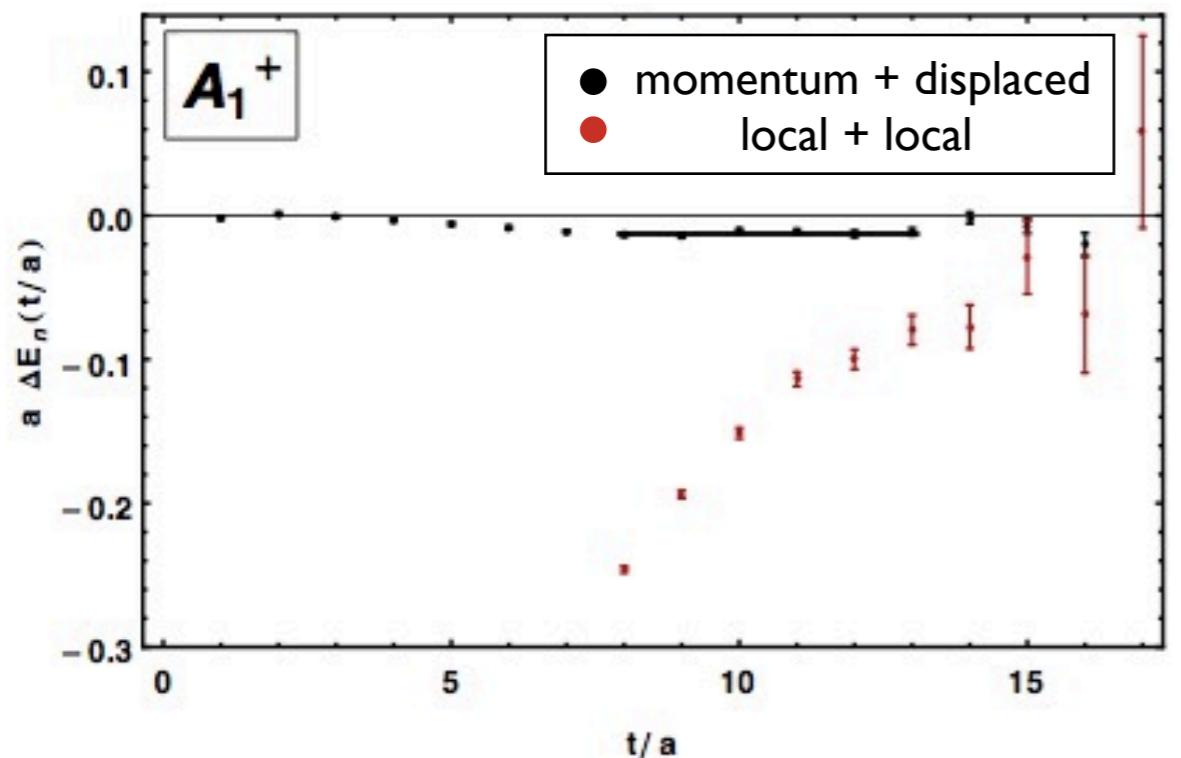
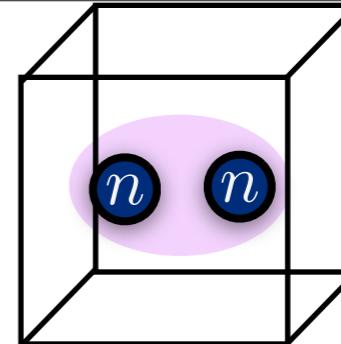
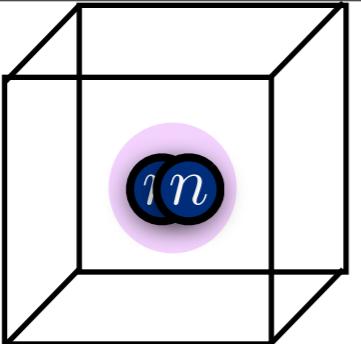


Iso-clover cfgs (W. Detmold,
R. Edwards, D. Richards, K. Orginos)



Need displaced operators!

Iso-clover cfgs (W. Detmold,
R. Edwards, D. Richards, K. Orginos)

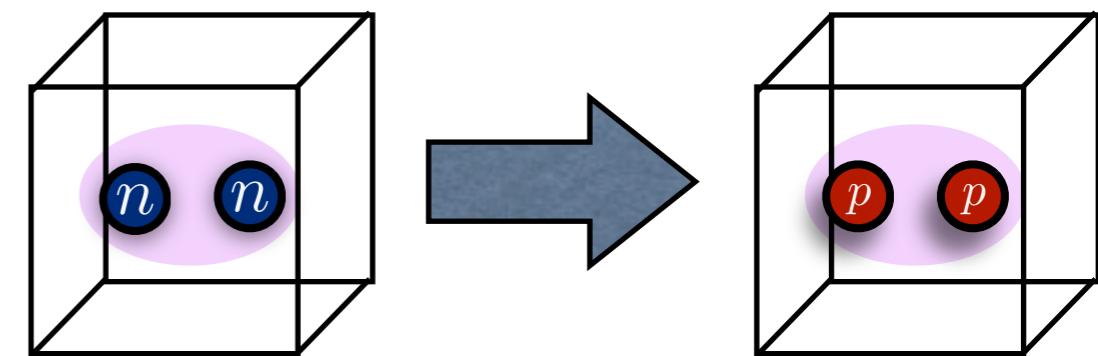


Finite volume formalism for $2 \rightarrow 2$ matrix elements completed:

R. Briceño, M. Hansen Phys.Rev. D94 (2016) no.1, 013008

Renormalization known in $\overline{\text{MS}}$:

B. Tiburzi Phys.Rev. D86 (2012) 097501

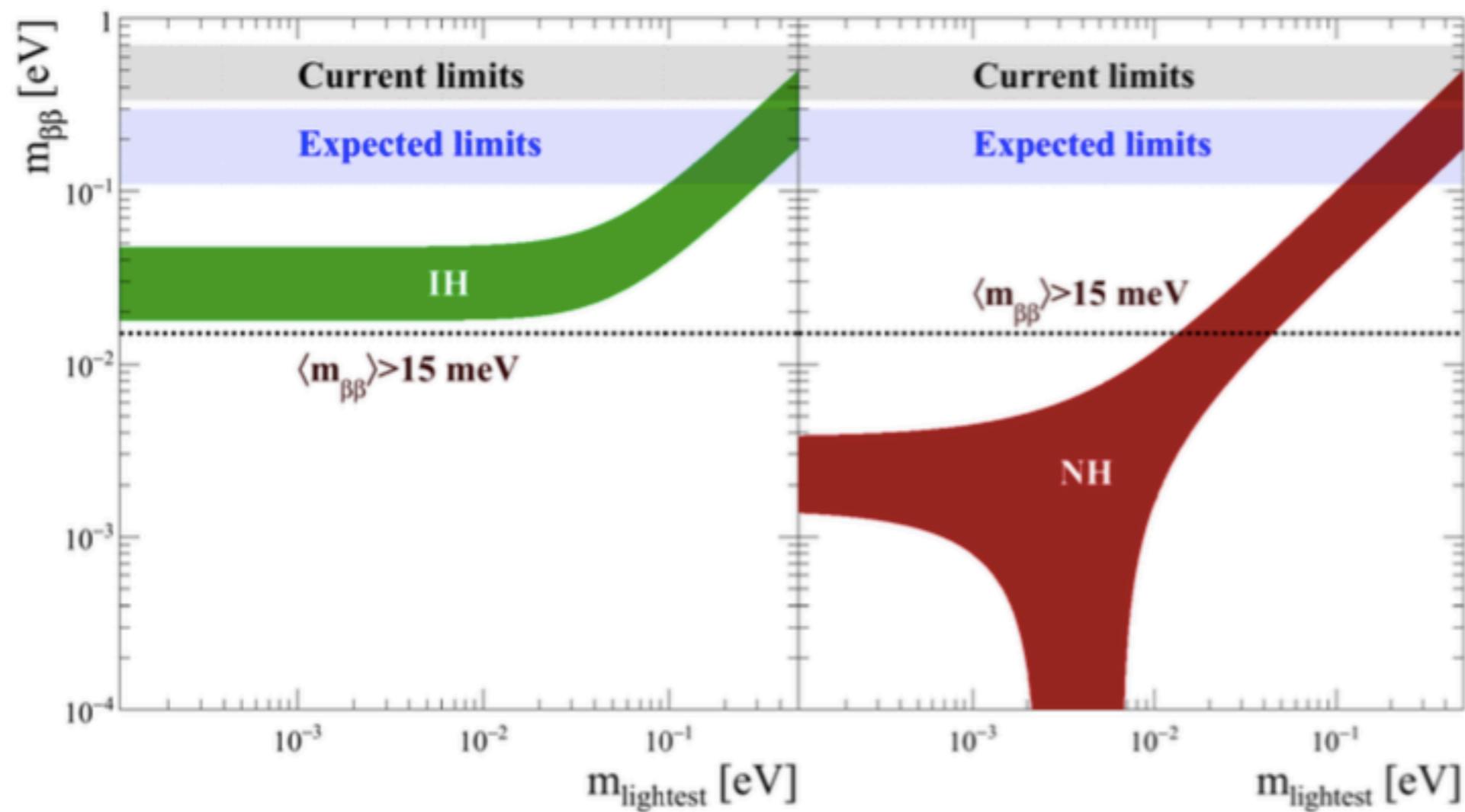


Stay tuned!



- LBL/UCB: Chia Cheng Chang, AN, André Walker-Loud,
- LLNL: Evan Berkowitz, Enrico Rinaldi, Pavlos Vranas
- NERSC: Thorsten Kurth
- JLab: Balint Joo
- CCNY: Brian Tiburzi
- nVidia: Kate Clark





Ton-scale Neutrinoless Double Beta Decay ($0\nu\beta\beta$) - A Notional Timeline

Search for Lepton Number Violation

