Continuing the Saga of Fluffy Mirror Fermions

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work with David B. Kaplan
arXiv:1608.xxxx
Lattice Regularization of Chiral Gauge Theories

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Motivation: Well-Defined Chiral Gauge Theories

Big Question 1: Basic ingredients necessary for self-consistent chiral gauge theories ($\chi$GT)

- Perturbative regulator provides controlled theoretical description of perturbative phenomena
- Electroweak experiments probe weakly coupled $\chi$GT
- Currently no experimental access to nonperturbative behavior

Big Question 2: Properties of strongly coupled $\chi$GT

- Lattice methods allow for numerical simulation of nonperturbative systems

To address these questions, must find a lattice regulator
Requirements for Lattice Regulated $\chi$GT

- **Complex** fermion representation with decoupling of doublers
- **Road to failure** for anomalous fermion representations
  - Continuum: Only anomaly-free $\chi$GT are well-defined
  - Lattice: Symmetries cannot be anomalous
- **Gauge invariant** in the continuum limit
- **Unambiguous** definition of fermion determinant phase
  - Continuum: Kinetic operator for LH Weyl fermion maps between two different spaces
    - Ill-defined eigenvalue problem
Domain Wall Fermions

- Light modes localized on domain wall
  - Exactly massless in limit of infinite extra dimension

\[ m(s) \]

\[ \text{LH} \quad \text{RH} \]

\[ s \]

*Kaplan ’92*
Domain Wall Fermions

- Light modes localized on domain wall
  - Exactly massless in limit of infinite extra dimension
- Constant $s$-independent gauge field $A(x)$ throughout the bulk
- $U(1)_A$ anomaly due to Callan-Harvey Mechanism: incomplete decoupling of heavy bulk modes \((\text{Callan and Harvey, 84})\)
- Gives rise to solution of Ginsparg-Wilson equation

\[ m(s) \quad A(x) \]

\[ \text{LH} \quad \text{RH} \]
Ginsparg-Wilson Equation

- Derived by spin-blocking continuum theory

\[ \gamma_5 D^{-1} + D^{-1} \gamma_5 = a \gamma_5 \]

- Operator that satisfies Ginsparg-Wilson equation
  - Preserves all chiral symmetries except $U(1)_A$
  - Violates $U(1)_A$ by amount required to reproduce continuum anomaly

$DWF$ on compact extra dimension in limit of infinite wall separation give rise to overlap operator (Narayanan & Neuberge '94, '95; Neuberger, '98)
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- DWF on compact extra dimension in limit of infinite wall separation give rise to overlap operator (Narayanan & Neuberger ’94, ’95; Neuberger, ’98)
  \[ D_{NN} = \frac{1 + \gamma_5 \epsilon(H)}{2} \]
  \[ H = \frac{\gamma_5 (D_w - m)}{\sqrt{H^\dagger H}} \]
  \[ \epsilon(H) = \frac{H}{\sqrt{H^\dagger H}} \]
Proposition for Lattice Regulated $\chi$GT

- LH and RH modes have different gauge field interactions if gauge field is localized around one wall.

- Previous attempt: Waveguide Model
  - Gauge field only nonzero around one wall
  - Gauge invariance broken at interface
  - Introduction of charged scalars at interface leads to Dirac spectrum (Goltermann Jansen and Vink ’93)
Proposal for Lattice Regulated $\chi$GT

- Utilize Domain Wall Construction for localization of light modes
- Gauge field obeys Gradient Flow equation in extra dimension
  \[
  \partial_s \tilde{A}_\mu(x, s) = \frac{\epsilon(s)}{\Lambda} D_\mu \bar{F}_{\mu\nu} \quad \text{BC: } \tilde{A}_\mu(x, 0) = A_\mu(x)
  \]
- Damps out high momentum modes in gauge invariant way
- Localizes gauge field around one wall

*DMG & Kaplan ’15
Example: 2d/3d QED

- Write field in terms of gauge and physical degree of freedom
  \[ \tilde{A}_\mu = \partial_\mu \tilde{\omega} + \epsilon_{\mu \nu} \partial_\nu \tilde{\lambda} \]
- Each DoF has its own flow equation
  \[ \partial_s \tilde{\lambda} = \Box \tilde{\lambda} \quad \partial_s \tilde{\omega} = 0 \]
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- Each DoF has its own flow equation

\[ \partial_s \bar{\lambda} = \Box \bar{\lambda} \quad \partial_s \bar{\omega} = 0 \]

- Flow only damps out high momentum modes of physical DoF

\[ \bar{\lambda}(p, s) = e^{-p^2 s / \Lambda} \lambda(p) \]
Anomalies and Callan-Harvey Mechanism

- Integrating out bulk fermions generates a Chern-Simons term
- In 3 dimensions, the Chern Simons action is

\[
S_3^{\text{bulk}} = c_3 \frac{\Lambda}{|\Lambda|} \int (\epsilon(s) - 1) \text{Tr} (\bar{F} \bar{A} - \frac{1}{3} \bar{A}^3)
\]
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Pauli Villars Contribution
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  - CS only depends on sign of domain wall mass
  - Fermion Contribution
  - Pauli Villars Contribution

- This approximation is only valid far away from domain wall
Anomaly Cancellation and Nonlocality

- DWF with flowed gauge fields gives rise to a nonlocal 2d theory

\[
S_{3}^{\text{bulk}} = 2e^2 c_3 \frac{\Lambda}{|\Lambda|} \int dx^2 dy^2 \left( \frac{\partial_\mu \partial_\alpha}{\Box} A_\alpha(x) \right) \Gamma(x - y) \left( \frac{\partial_\mu \partial_\beta}{\Box} \epsilon_{\beta\gamma} A_\gamma(y) \right)
\]

- For multiple fields, Chern Simons prefactor is

\[
\sum_i e_i^2 \frac{\Lambda_i}{|\Lambda_i|}
\]

- The theory is not local if this prefactor does not vanishes
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This is exactly equivalent to the requirement that the chiral fermions be in an anomaly free representation
Infinite Wall Separation

- Exactly massless modes require infinite extra dimension

- Wick rotation of theory at finite L is ill-defined due to incomplete decoupling of mirror fermions
  - RH mode sees gauge field with $e^{-p^2 L/\Lambda}$ form factor
  - Wick rotation changes sign of $p^2$

- Well-defined chiral gauge theory requires working at infinite L
Infinite Wall Separation

\[ A_*(x) = \lim_{s \to \infty} \bar{A}(x, s) \]

- Working at infinite \( L_s \) allows for simplified picture
  
  - All flow is localized to a very narrow region between walls
  
  - Looks similar to waveguide model, but theory is gauge-invariant at interface between \( A \) and \( A_* \)

  - Do not expect new domain wall to appear
Chiral Overlap Operator

- In limit of infinite wall separation, theory is amenable to an overlap treatment

- The chiral overlap operator is

\[ D_\chi = 1 + \gamma_5 \left[ 1 - (1 - \epsilon_\star) \frac{1}{\epsilon_\star + 1} (1 - \epsilon) \right] \]

\[ \epsilon = \epsilon(H) \quad \epsilon_\star = \epsilon(H_\star) \]
Chiral overlap operator has several key properties

• Satisfies Ginsparg-Wilson equation

• Correct continuum limit: \( \lim_{a \to 0} D^\chi = D^{\mu} \gamma_\mu P_L + D^{\mu}_* \gamma_\mu P_R \)

• No phase ambiguity

\[ D^\chi = 1 + \gamma_5 \left[ 1 - (1 - \varepsilon_*) \frac{1}{\varepsilon \varepsilon_* + 1} (1 - \varepsilon) \right] \]
Summary

- New proposal for lattice regularization of chiral gauge theories
  - Combines domain wall fermions and gradient flow
  - Road to failure for anomalous theories is nonlocality

- In the limit of infinite extra dimensions, the fermion determinant is described by a chiral version of the Overlap operator

\[
D_\chi = 1 + \gamma_5 \left[ 1 - (1 - \epsilon_\star) \frac{1}{\epsilon \epsilon_\star + 1} (1 - \epsilon) \right]
\]

- Operator satisfies Ginsparg-Wilson relation
- Determinant of operator has unambiguous phase and correct continuum limit for a chiral theory