Continuing the Saga of Fluffy Mirror Fermions

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INSTITUTE for NUCLEAR THEORY

work with David B. Kaplan
Phys.Rev.Lett. 116 211602 (2016) [arXiv:1511.03649]
arXiv:1608.xxxx

Lattice Regularization of Chiral Gauge Theories

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Motivation: Well-Defined Chiral Gauge Theories

Big Question 1: Basic ingredients necessary for self-consistent chiral gauge theories (χGT)

- Perturbative regulator provides controlled theoretical description of perturbative phenomena
- Electroweak experiments probe weakly coupled χGT
- Currently no experimental access to nonperturbative behavior

Big Question 2: Properties of strongly coupled χGT

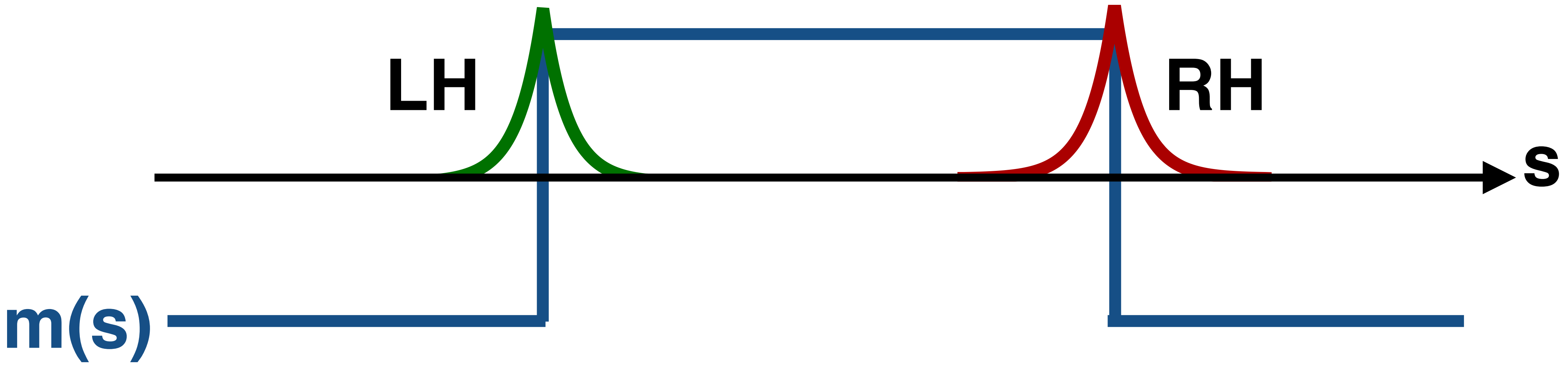
- Lattice methods allow for numerical simulation of nonperturbative systems

To address these questions, must find a lattice regulator

Requirements for Lattice Regulated χGT

- Complex fermion representation with decoupling of doublers
- Road to failure for anomalous fermion representations
  - Continuum: Only anomaly-free χGT are well-defined
  - Lattice: Symmetries cannot be anomalous
- Gauge invariant in the continuum limit
- Unambiguous definition of fermion determinant phase
  - Continuum: Kinetic operator for LH Weyl fermion maps between two different spaces
    - Ill-defined eigenvalue problem

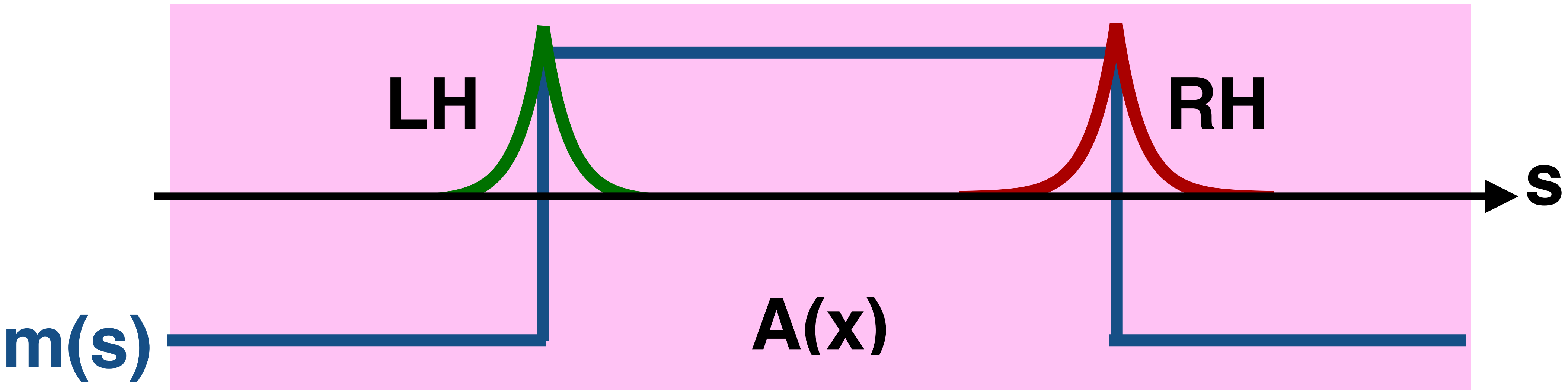
Domain Wall Fermions



- Light modes localized on domain wall
    - Exactly massless in limit of infinite extra dimension

*Kaplan '92

Domain Wall Fermions



- Light modes localized on domain wall
    - Exactly massless in limit of infinite extra dimension
- Constant s-independent gauge field A(x) throughout the bulk
- U(1)$_A$ anomaly due to Callan-Harvey Mechanism: incomplete decoupling of heavy bulk modes (Callan and Harvey, 84)
- Gives rise to solution of Ginsparg-Wilson equation

*Kaplan '92

Ginsparg-Wilson Equation

- Derived by spin-blocking continuum theory

$$\gamma_5 D^{-1} + D^{-1}\gamma_5 = a\gamma_5$$

- Operator that satisfies Ginsparg-Wilson equation
    - Preserves all chiral symmetries except $U(1)_A$
    - Violates $U(1)_A$ by amount required to reproduce continuum anomaly

Ginsparg & Wilson '85

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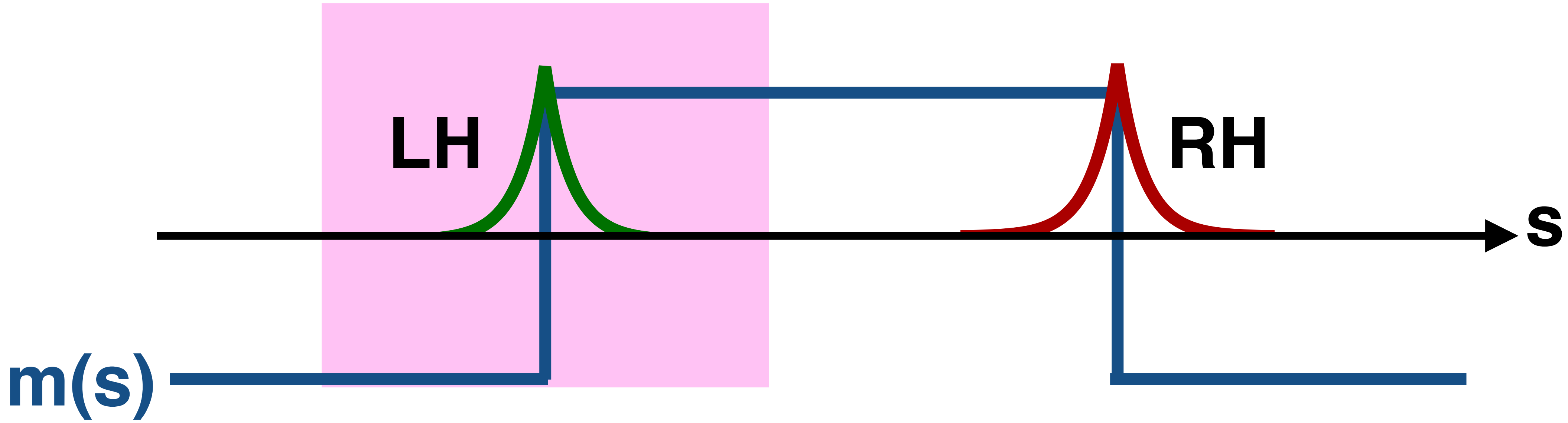
- DWF on compact extra dimension in limit of infinite wall separation give rise to overlap operator **(Narayanan & Neuberger '94, '95; Neuberger, '98)**

$$D_{NN} = \frac{1 + \gamma_5 \epsilon(H)}{2}$$

$$\begin{aligned} H &= \gamma_5 (D_w - m) \\ \epsilon(H) &= \frac{H}{\sqrt{H^\dagger H}} \end{aligned}$$

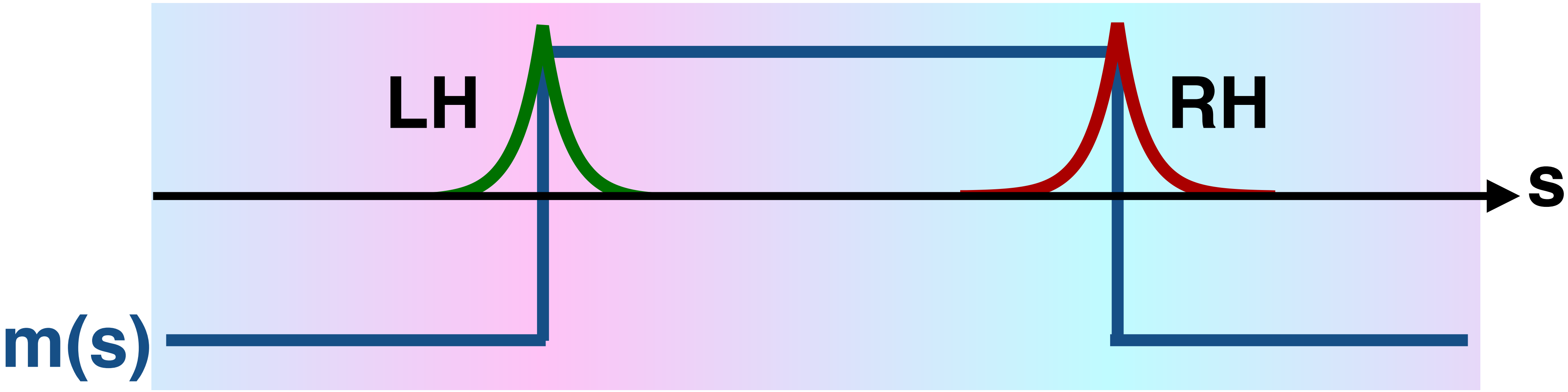
Ginsparg & Wilson '85

Proposal for Lattice Regulated χGT



- LH and RH modes have different gauge field interactions if gauge field is localized around one wall
- Previous attempt: Waveguide Model
    - Gauge field only nonzero around one wall
    - Gauge invariance broken at interface
    - Introduction of charged scalars at interface leads to Dirac spectrum (Golterman Jansen and Vink '93)

Proposal for Lattice Regulated χGT



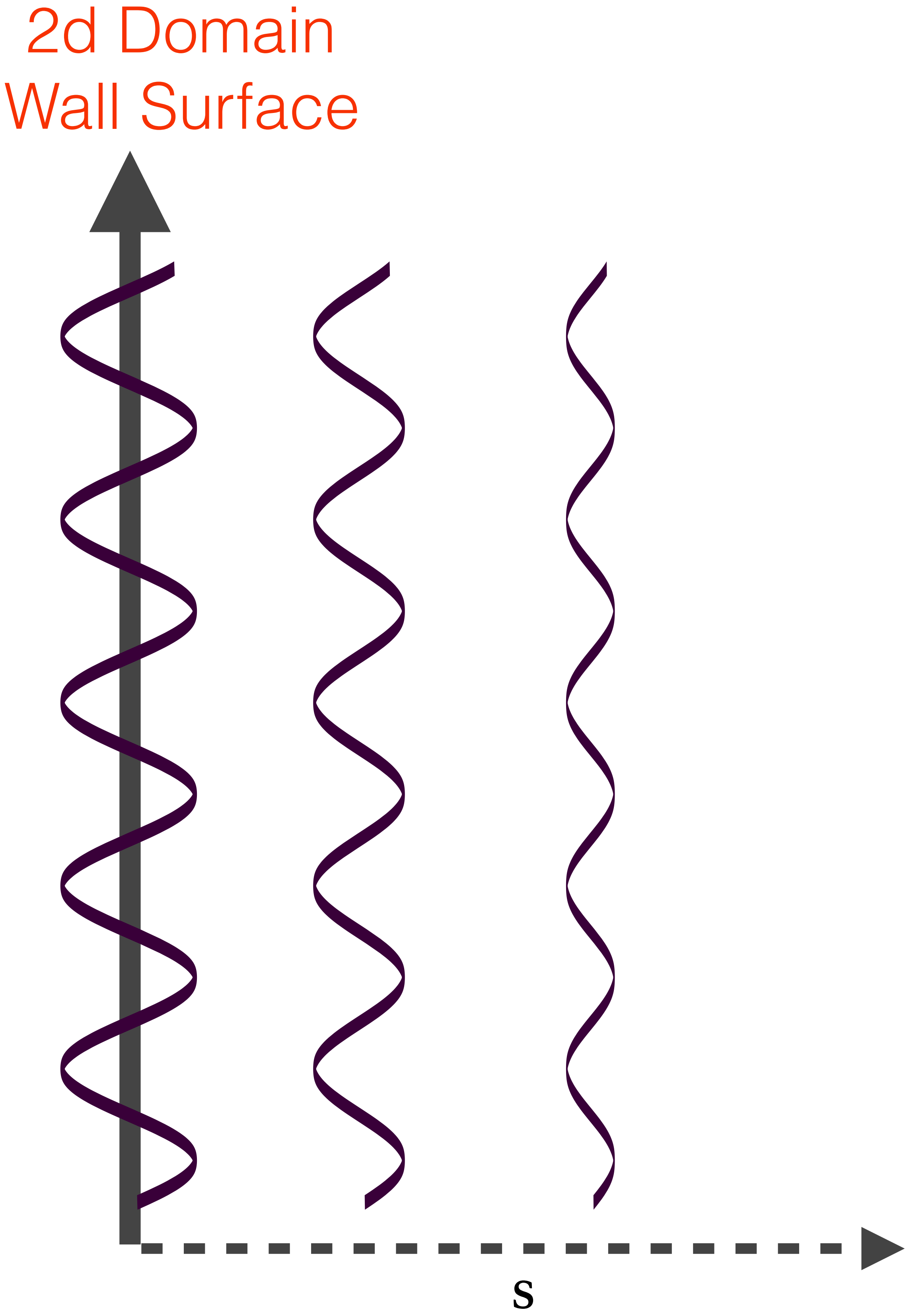
- Utilize Domain Wall Construction for localization of light modes
- Gauge field obeys Gradient Flow equation in extra dimension

$$\partial_s \bar{A}_\mu(x,s) = \frac{\epsilon(s)}{\Lambda} D_\mu \bar{F}_{\mu\nu} \qquad \text{BC: } \bar{A}_\mu(x,0) = A_\mu(x)$$

  - Damps out high momentum modes in gauge invariant way
  - Localizes gauge field around one wall

*DMG & Kaplan '15

Example: 2d/3d QED



- Write field in terms of gauge and physical degree of freedom

$$\bar{A}_\mu = \partial_\mu \bar{\omega} + \epsilon_{\mu\nu}\partial_\nu \bar{\lambda}$$

- Each DoF has its own flow equation

$$\partial_s \bar{\lambda} = \Box \bar{\lambda} \qquad \partial_s \bar{\omega} = 0$$

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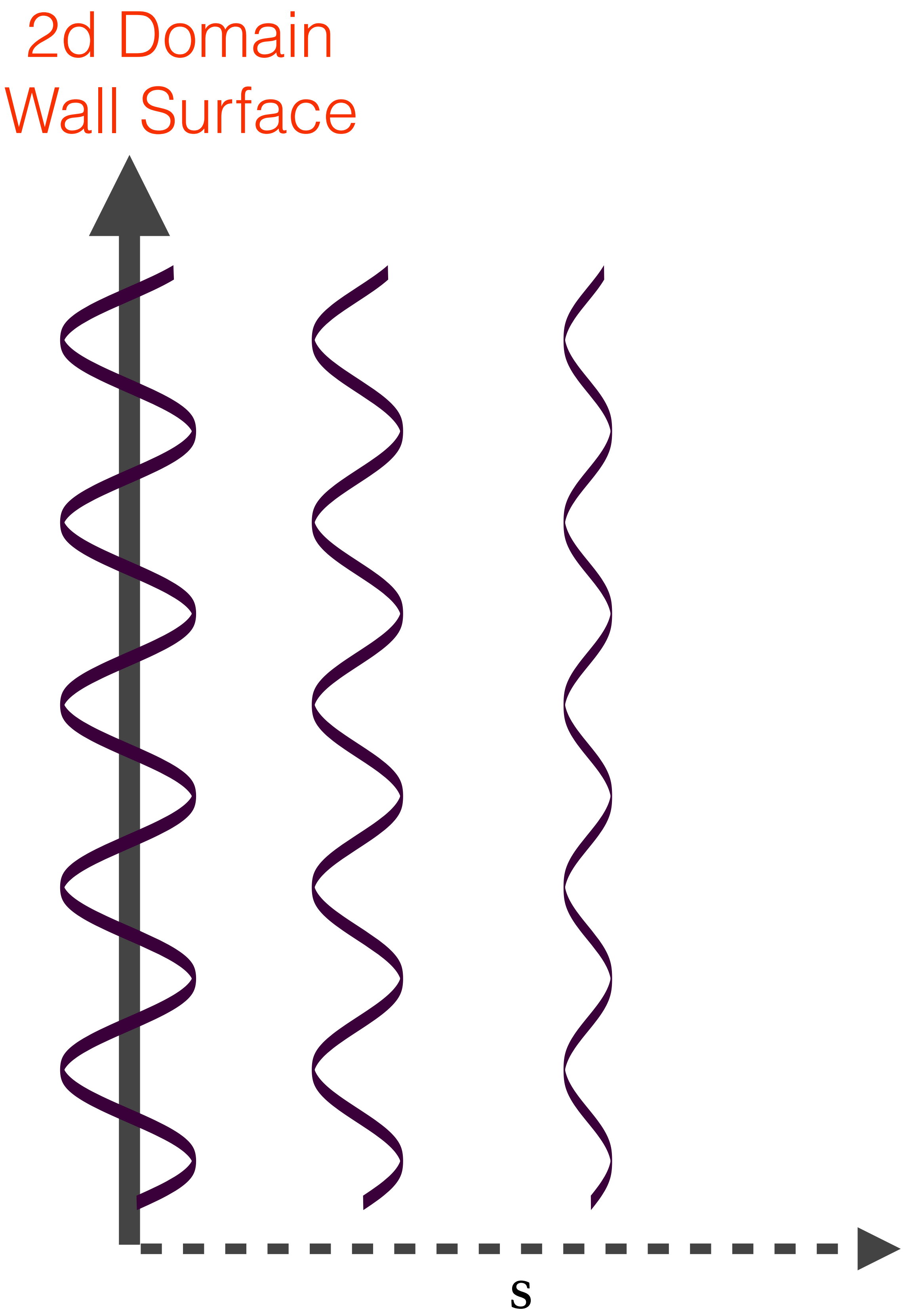

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- Flow only damps out high momentum modes of physical DoF

$$\bar{\lambda}(p,s) = e^{-p^2 s/\Lambda}\lambda(p)$$

Anomalies and Callan-Harvey Mechanism

- Integrating out bulk fermions generates a Chern-Simons term
- In 3 dimensions, the Chern Simons action is

$$S_3^{\text{bulk}} = c_3 \frac{\Lambda}{|\Lambda|} \int (\epsilon(s) - 1) \, \mathrm{Tr} \left(\bar{F} \bar{A} - \tfrac{1}{3} \bar{A}^3 \right)$$

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Fermion Contribution

CS only depends on sign
of domain wall mass

Pauli Villars Contribution

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Pauli Villars Contribution

- This approximation is only valid far away from domain wall

Anomaly Cancellation and Nonlocality

- DWF with flowed gauge fields gives rise to a nonlocal 2d theory

$$S_3^{\text{bulk}} = 2e^2 c_3 \frac{\Lambda}{|\Lambda|} \int dx^2 dy^2 \left(\frac{\partial_\mu \partial_\alpha}{\Box} A_\alpha(x)\right) \Gamma(x-y) \left(\frac{\partial_\mu \partial_\beta}{\Box} \epsilon_{\beta\gamma} A_\gamma(y)\right)$$

- For multiple fields, Chern Simons prefactor is

$$\sum_i e_i^2 \frac{\Lambda_i}{|\Lambda_i|}$$

- The theory is not local if this prefactor does not vanishes

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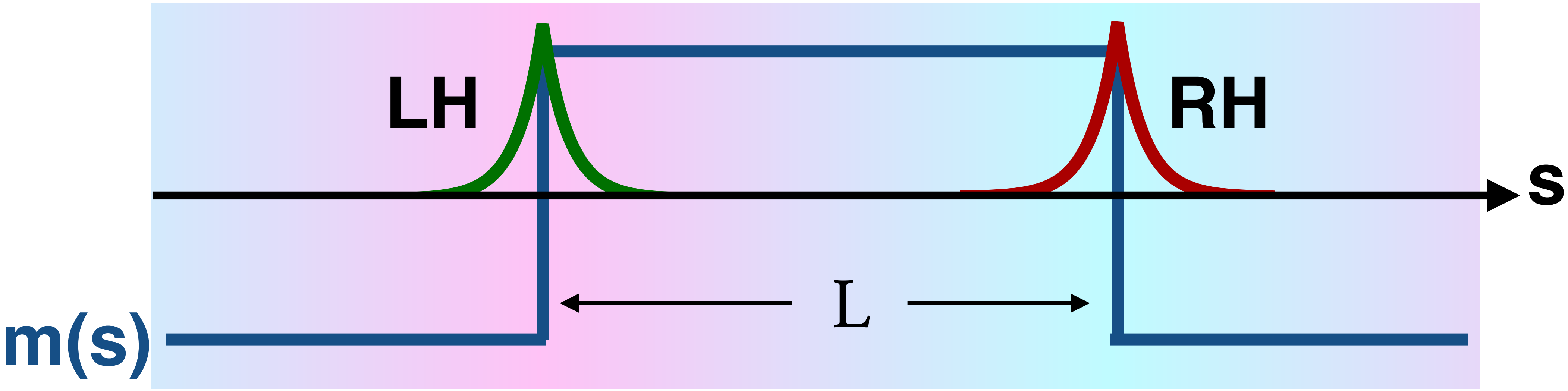
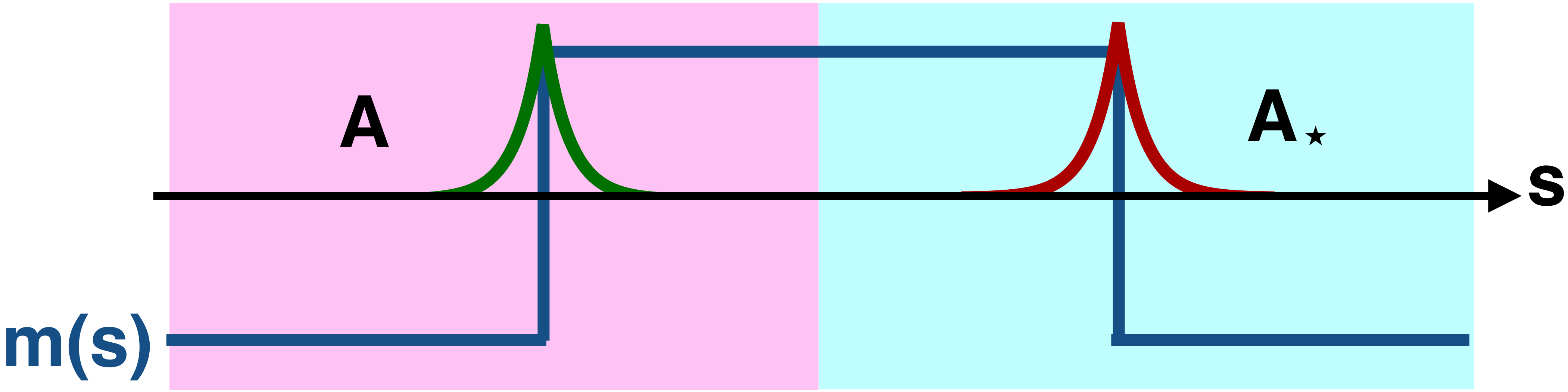
This is exactly equivalent to the requirement that the chiral fermions be in an anomaly free representation

Infinite Wall Separation



- Exactly massless modes require infinite extra dimension
- Wick rotation of theory at finite L is ill-defined due to incomplete decoupling of mirror fermions
    - RH mode sees gauge field with $e^{-p^2 L/\Lambda}$ form factor
    - Wick rotation changes sign of p^2
- Well-defined chiral gauge theory requires working at infinite L

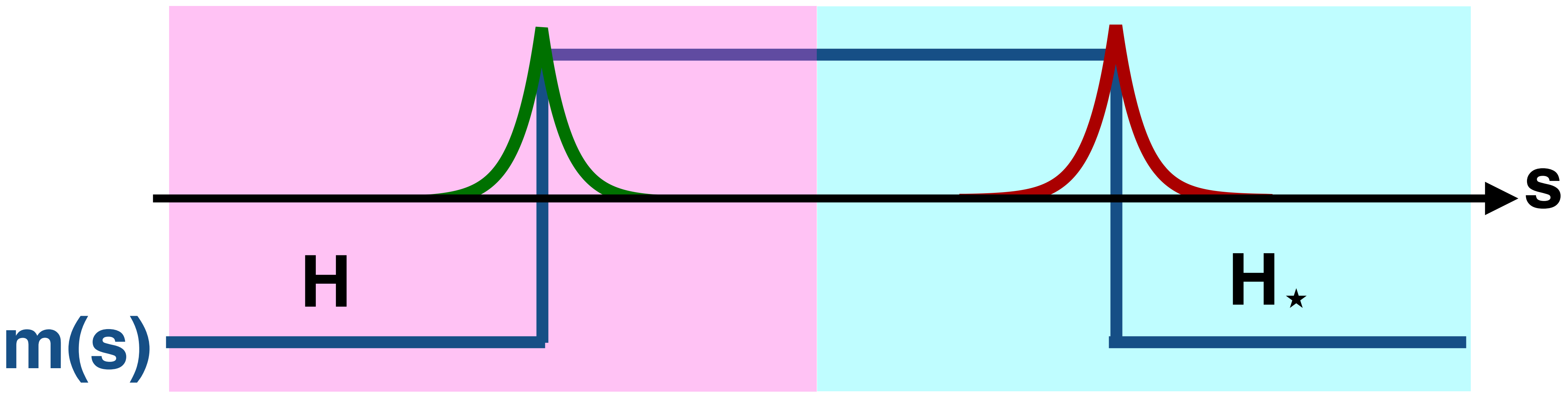
Infinite Wall Separation



$$A_\star(x) = \lim_{s\to\infty} \bar{A}(x,s)$$

- Working at infinite L_s allows for simplified picture
    - All flow is localized to a very narrow region between walls
- Looks similar to waveguide model, but theory is gauge-invariant at interface between A and $A_\star$
    - Do not expect new domain wall to appear

Chiral Overlap Operator



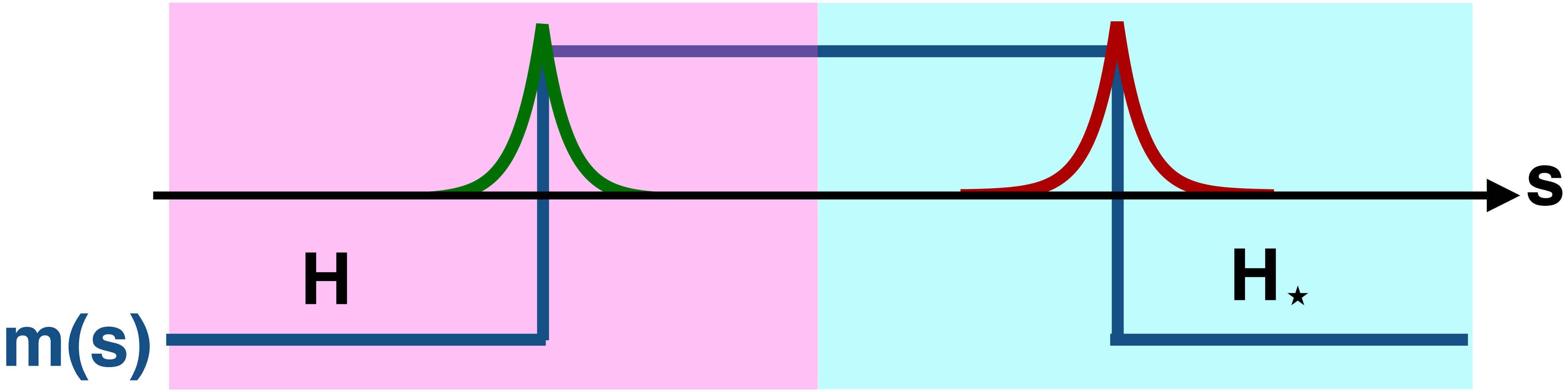
- In limit of infinite wall separation, theory is amenable to an overlap treatment

- The chiral overlap operator is

$$\mathcal{D}_\chi = 1 + \gamma_5 \left[1 - (1 - \epsilon_\star) \frac{1}{\epsilon\,\epsilon_\star + 1} (1 - \epsilon) \right]$$

$$\epsilon = \epsilon(H) \qquad \epsilon_\star = \epsilon(H_\star)$$

Chiral Overlap Operator



$$\mathcal{D}_\chi = 1 + \gamma_5 \left[1 - (1 - \epsilon_\star) \frac{1}{\epsilon\,\epsilon_\star + 1} (1 - \epsilon)\right]$$

- Chiral overlap operator has several key properties
  - Satisfies Ginsparg-Wilson equation
  - Correct continuum limit: $\lim_{a \to 0} D_\chi = D^\mu \gamma_\mu P_L + D_\star^\mu \gamma_\mu P_R$
  - No phase ambiguity

Summary

- New proposal for lattice regularization of chiral gauge theories
    - Combines domain wall fermions and gradient flow
    - Road to failure for anomalous theories is nonlocality
- In the limit of infinite extra dimensions, the fermion determinant is described by a chiral version of the Overlap operator

$$\mathcal{D}_\chi = 1 + \gamma_5 \left[1 - (1 - \epsilon_\star) \frac{1}{\epsilon\,\epsilon_\star + 1} (1 - \epsilon) \right]$$

-
    - Operator satisfies Ginsparg-Wilson relation
    - Determinant of operator has unambiguous phase and correct continuum limit for a chiral theory