Axion Phenomenology from Unquenched Lattice QCD

LATTICE 2016

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PRACE

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- ***Axions***
- ***LQCD & Axions***
- ***Recent results***

  & Outlook

Narcissus by Caravaggio

Caravaggio

Axions were originally proposed to deal with the strong CP problem

Massless QCD
$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \bar{\psi}_a^i \not{D}_{ab} \psi_b^i - \frac{1}{4} G_{\mu\nu}^A G_A^{\mu\nu}$$

Symmetries @ the classical level

$$U(N_f)_L \times U(N_f)_R \sim SU(N_f)_L \times SU(N_f)_R \times U(1)_L \times U(1)_R$$

Non trivial vacuum (quark condensate $\langle \bar{\psi}\psi \rangle \neq 0$) breaks spontaneously non singlet chiral symmetries

$$SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$$

$U(1)_V$ is the conserved barion number

Diagonalized mass terms

$$\mathcal{L}_m = -\sum_{i=1}^{N_f} \left(m_i \, \bar\psi^i_L \psi^i_R + m_i^* \, \bar\psi^i_R \psi^i_L \right)$$

Explicitely broken symmetries:

if the masses are different from zero $SU(N_f)_A$ is broken

$SU(N_f)_V$ is broken if the masses are not equal

ANOMALY: we have to introduce a regularization

Two examples:

a) GW fermions: the action is invariant under a global chiral transformation but the fermion measure is not invariant (Fujikawa)

b) Wilson fermions: the action is not invariant but the measure is invariant

a) GW fermions: if we rotate the quark fields by a phase

$$\psi_i \to e^{i\alpha_i}\,\psi_i \quad \bar{\psi}_i \to \bar{\psi}_i\, e^{i\alpha_i}$$

Then, because of the variation of the measure, the action is modified as $\mathcal{L}_{QCD} + \mathcal{L}_{\{m_i\}} \to$

$$\mathcal{L}_{QCD} + \mathcal{L}_{\{m_i e^{2i\alpha_i}\}} + \left(\theta + \sum_{i=1}^{N_f} 2\alpha_i\right) \frac{N_f g^2}{32\pi^2} G^A_{\mu\nu} \tilde{G}^{\mu\nu}_A$$

Indeed the functional integral depends on the invariant combination

$$det[m_f]e^{-i\theta}$$

and if we apply a rotation to make the masses real (and positive)

$$\theta \to \theta - arg[det[m_f]]$$

$$\delta \int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S} =$$

$$i \int d^4x \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \alpha(x) \left[\partial_\mu J^5_\mu(x) - 2m\bar{\psi}\gamma_5\psi(x) - \frac{N_f g^2}{32\pi^2} G^A_{\mu\nu} \tilde{G}^{\mu\nu}_A(x) \right] e^{-S}$$

From the rotation of the Action

From the rotation of the fermion measure

TOPOLOGICAL CHARGE DENSITY AND SUSCEPTIBILITY

$$Q(x) = \frac{g^2}{32\pi^2} G^A_{\mu\nu} \tilde{G}^{\mu\nu}_A(x) \qquad \int d^4x \, Q(x) = n$$

$$Q(x) = -\frac{1}{2} \mathrm{Tr} \left[\gamma_5 D(x,x) \right] \qquad \chi_t = \int d^4x \, \langle Q(x) Q(0) \rangle$$

M. Luscher, Phys. Lett. B428 (1998) 342

b) Wilson Fermions: if we rotate the quark fields by a phase

$$\psi_i \to e^{i\alpha_i}\,\psi_i \qquad \bar\psi_i \to \bar\psi_i\, e^{i\alpha_i}$$

then the variation of the action is given by (Bochicchio & al.)

$$\delta S = i\alpha(x)\ \left[\partial_\mu J^5_\mu(x) - 2M\bar\psi\gamma_5\psi(x) + X_5(x)\right]$$

where the last term is the chiral rotation of the Wilson term:

$$\bar X_5(x) = \bar Z_5\left[X_5(x) - 2\bar M\bar\psi\gamma_5\psi - (\bar Z_J - 1)\partial_\mu J^5_\mu(x) + Z_{G\tilde G}\frac{N_f g^2}{32\pi^2}G^A_{\mu\nu}\tilde G^{\mu\nu}_A(x)\right]$$

where the matrix elements of the operator on the l.h.s. are of O(a) (or a^2 with improved actions and operators)

$$\bar Z_J\partial_\mu J^5_\mu(x) - 2\bar m\bar\psi\gamma_5\psi(x) - Z_{G\tilde G}\frac{N_f g^2}{32\pi^2}G^A_{\mu\nu}\tilde G^{\mu\nu}_A(x) + O(a) = 0$$

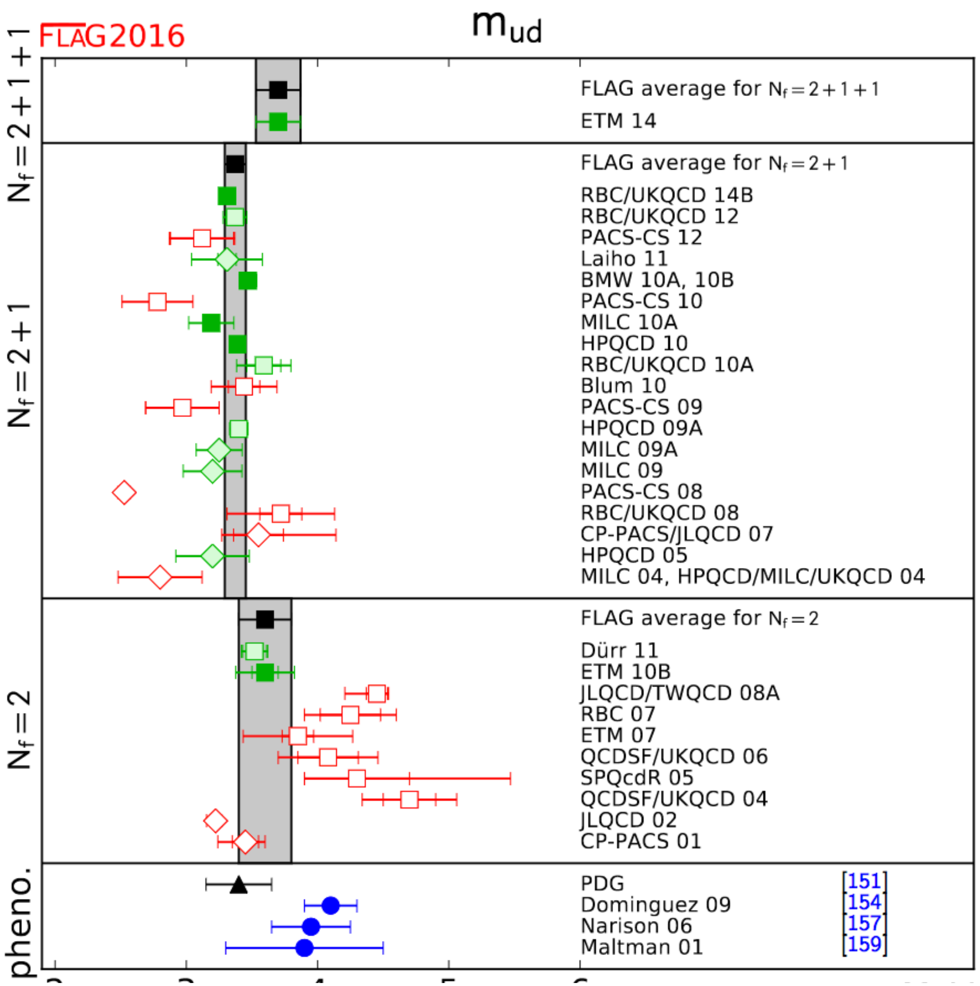
The θ term and the strong CP problem

- Because of the anomaly QCD depends, in general, on a parameter θ
- A priori θ can have any value; physics invariant for θ -> θ + 2 π
- This parameter gives rise to CP violation
- Neutron EDM ≤ 2.9 10^{-26} e . cm implies θ ≤ 10^{-9} -10^{-10}

Several possibilities among which:

-The mass of one quark is equal to zero

-Peccei-Quinn symmetry and the axion (Weinberg-Wilczek); see also M. A. Shifman, A. I. Vainshtein and V. I. Zakharov; A. R. Zhitnitsky, M. Dine, W. Fischler and M. Srednicki



N_f	m_u	m_d	m_u/m_d	R	Q
2+1+1	2.36(24)	5.03(26)	0.470(56)	35.6(5.1)	22.2 (1.6)
2+1	2.16(9)(7)	4.68(14)(7)	0.46(2)(2)	35.0(1.9)(1.8)	22.5(6)(6)
2	2.40(23)	4.80(23)	0.50(4)	40.7(3.7)(2.2)	24.3(1.4)(0.6)

The common lore

1) Implement a continuum $U(1)_{PQ}$ global chiral symmetry by adding extra particles (scalars, fermions);

2) The symmetry is spontaneously broken and this gives rise to a Goldstone boson, **the axion;**

3) Since the quark masses are different from zero the pseudo-Goldstone boson has a non zero mass which depends on the quark masses.

At low energy, neglecting heavy degrees of freedom, the effective action is given by

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi + \mathcal{L}_{int}\left[\bar{\psi}, \psi, \partial_\mu \phi\right] +$$
$$\left(\theta + \frac{\phi}{f_\phi}\right) \frac{g^2}{32\pi^2} G^A_{\mu\nu} \tilde{G}^{\mu\nu}_A(x)$$

where f_ϕ depends on the particular model.

This action, if not for the anomaly, is invariant under the non linear $U(1)_{PQ}$ transformation $\phi \to \phi + \alpha f_\phi$.

Let us define an effective axion action

$$e^{-S_{eff}(\partial_\mu \phi, \phi)} = \int \mathcal{D}\left[\psi, \bar{\psi}, G^A_\mu \ldots\right] e^{-S\left(\psi, \bar{\psi}, G^A_\mu, \ldots, \partial_\mu \phi, \phi\right)}$$

Minimizing the Axion Potential

$$e^{-S_{eff}(\partial_\mu \phi, \phi)} = \int \mathcal{D}\left[\psi, \bar{\psi}, G^A_\mu \dots\right] e^{-S\left(\psi, \bar{\psi}, G^A_\mu, \dots, \partial_\mu \phi, \phi\right)}$$

The minimum of the effective potential

$$\frac{\partial V_{eff}}{\partial \phi} = -\frac{1}{f_\phi} \frac{g^2}{32\pi^2} \langle G^A_{\mu\nu} \tilde{G}^{\mu\nu}_A(x) \rangle = 0$$

corresponds to $\varphi / f_\phi = -\theta$ and solve the strong CP problem; from the second derivative of the effective potential we may compute the axion mass

$$m^2_\phi = \frac{1}{f^2_\phi} \chi_t = \frac{1}{f^2_\phi} \int d^4x \, \langle Q(x) Q(0) \rangle$$

ZERO TEMPERARTURE AND CHIRAL EXPANSION

2 flavours: the relevant (naïve) Ward identity
at non zero quark masses reads

$$\chi_t = \frac{1}{4}\langle 0|m_u \bar{u}u + m_d \bar{d}d|0\rangle -$$
$$\frac{1}{4}\int d^4x\, \langle 0|T\left[\left(m_u\bar{u}\gamma_5 u + m_d\bar{d}\gamma_5 d\right)[x]\left(m_u\bar{u}\gamma_5 u + m_d\bar{d}\gamma_5 d\right)[0]\right]|0$$

Expanding at small quark masses and saturating the T-product with Goldston boson intermediate states we get

$$\chi_t = -\frac{1}{4} f_\pi^2 m_\pi^2 \frac{4 m_u m_d}{(m_u + m_d)^2}$$

M. A. Shifman, A. I. Vainshtein and V. I. Zakharov ; W. A. Bardeen and S.-H.H. Tye

The result depends on the number of flavours. For example if we add the strange quark and make an expansion at small quark masses we get

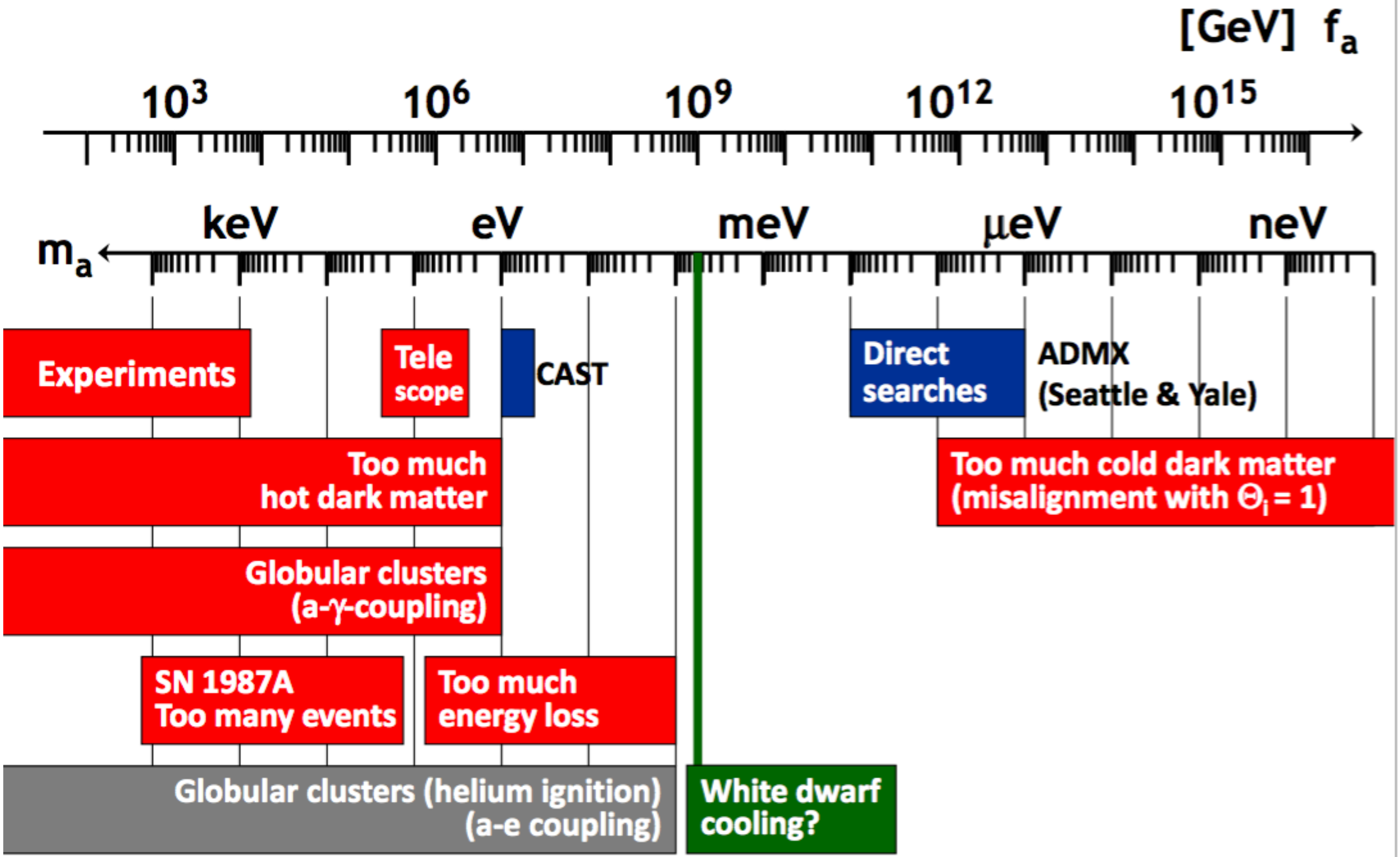
$$\boxed{\chi_t = -\frac{3}{4} f_\pi^2 \left(m_\pi^2 + m_\eta^2\right) \frac{m_u m_d m_s}{(m_u + m_d + m_s)(m_u m_d + m_d m_s + m_u m_s)}}$$

The topological susceptibility, and consequently the mass of the axion, vanishes whenever the mass of a quark is equal to zero. AT ZERO T WE DO NOT NEED THE LATTICE !

Similar results can be obtained in chiral perturbation theory, where the NLO corrections were recently computed and it was possible to extract the axion mass, self coupling and its full potential at the percent level

P. Di Vecchia and G. Veneziano, Giovanni Grilli di Cortona, Edward Hardy, Javier Pardo Vega, Giovanni Villadoro

Axion Bounds and Searches



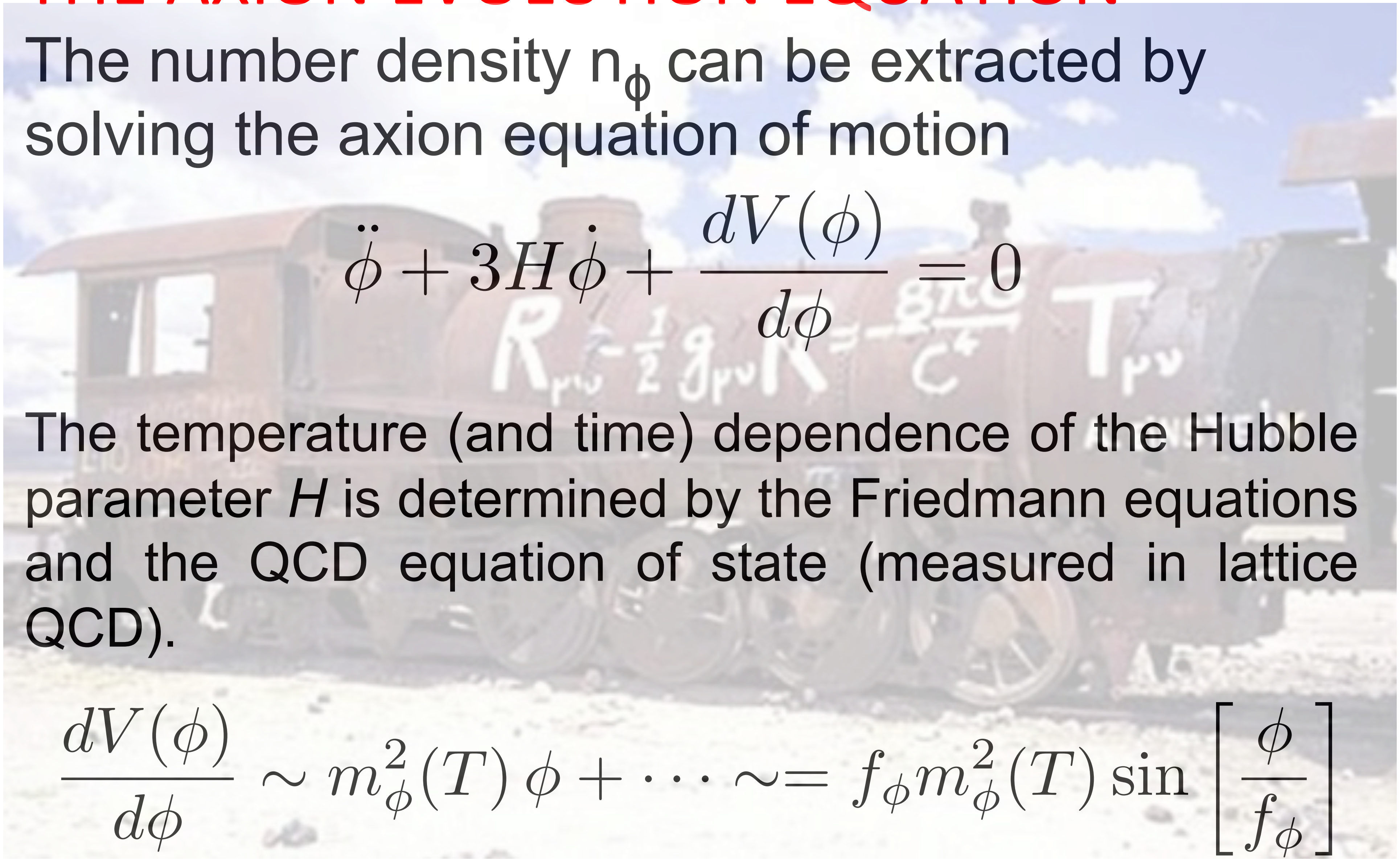
THE AXION PHENOMENOLOGY

Basic Formula:

$$\Omega_\phi = \frac{86}{33}\frac{\Omega_\gamma}{T_\gamma}\frac{n^\star_\phi}{s^\star}m_\phi \,,$$

where Ω_γ and T_γ are the present abundance and temperature of photons while n_ϕ*/s* is the ratio between the comoving axion number density $n_\phi=m_\phi\, \phi$^2 and the entropy density computed at a late time t* such that the ratio n_ϕ/s became constant

THE AXION EVOLUTION EQUATION



The number density n_ϕ can be extracted by solving the axion equation of motion

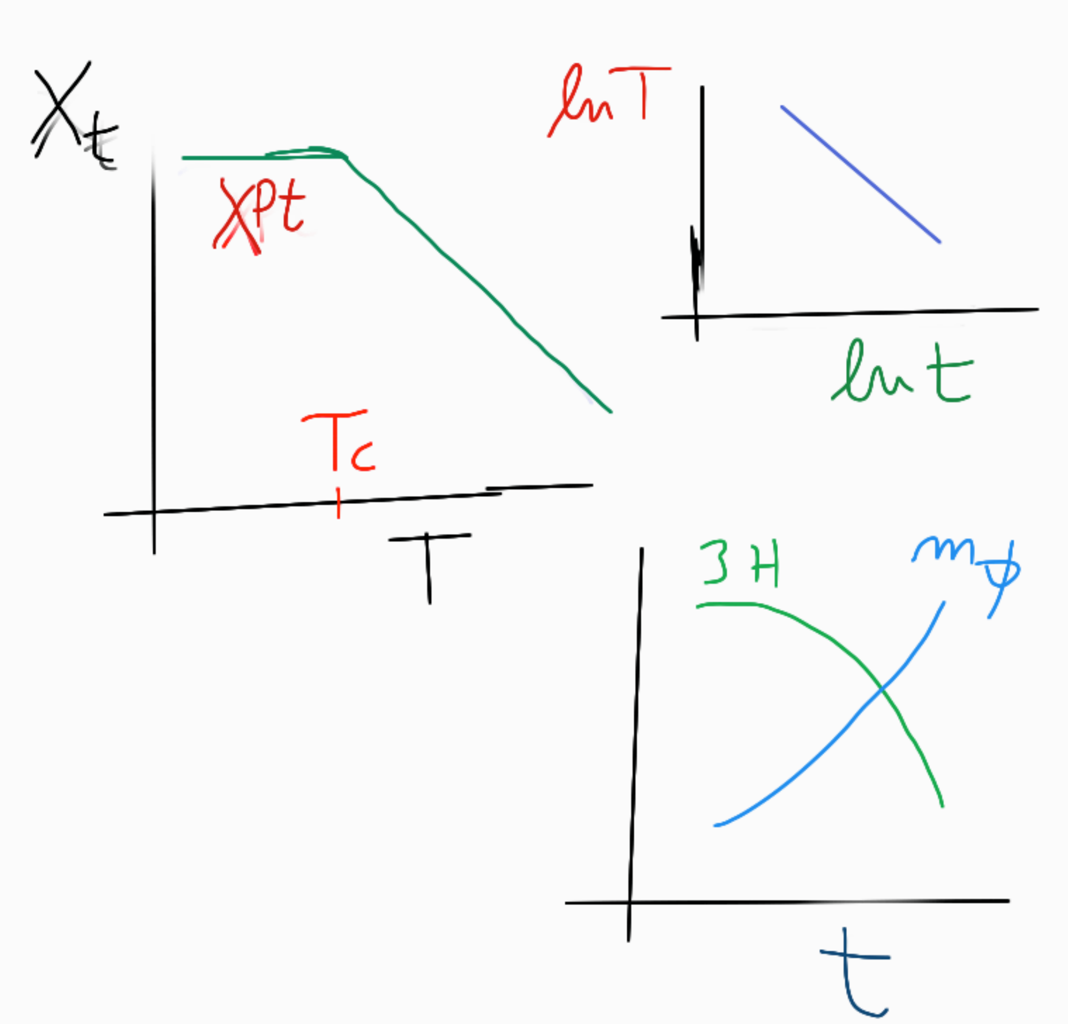
$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0$$

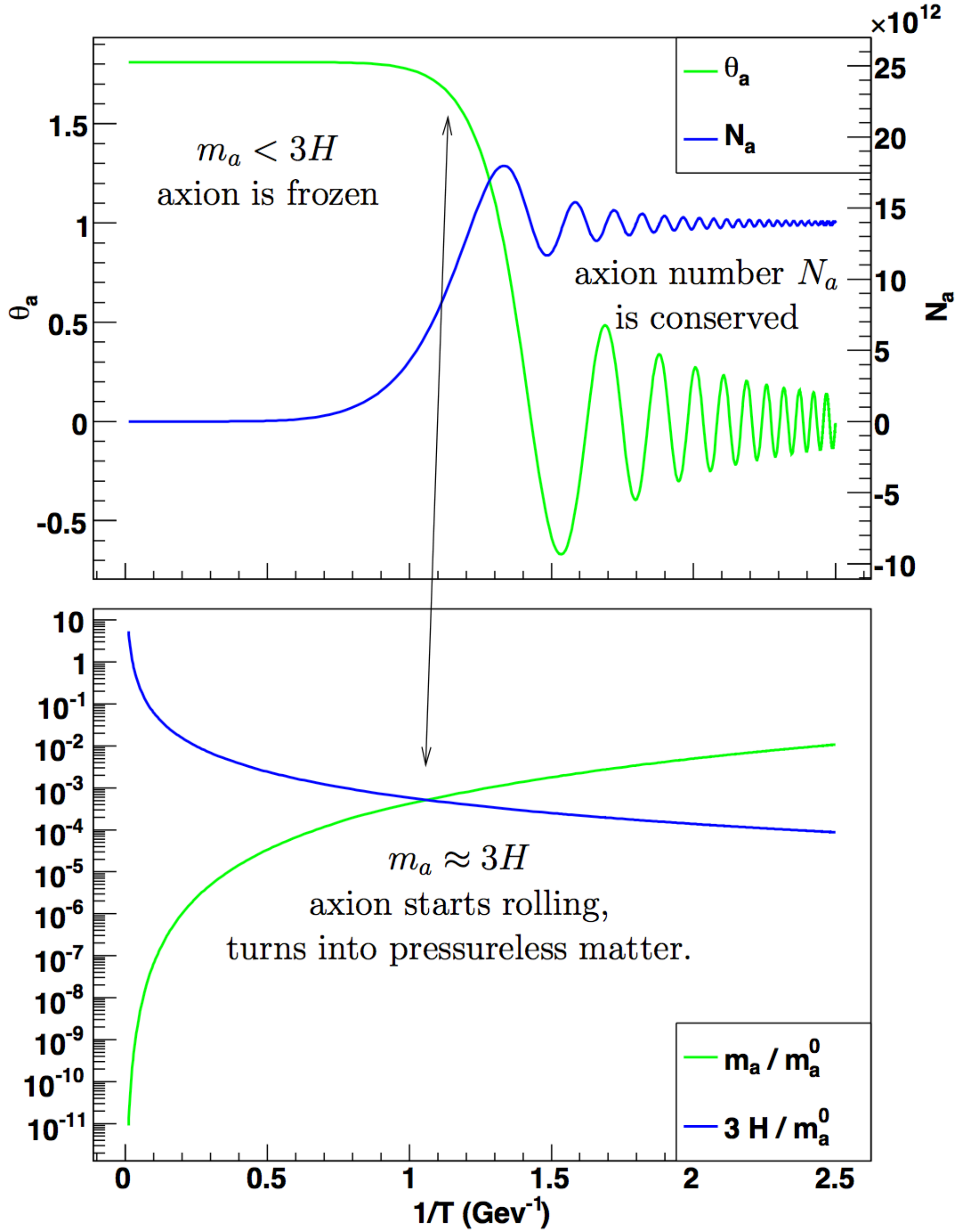
The temperature (and time) dependence of the Hubble parameter *H* is determined by the Friedmann equations and the QCD equation of state (measured in lattice QCD).

$$\frac{dV(\phi)}{d\phi} \sim m_\phi^2(T)\,\phi + \cdots \sim= f_\phi m_\phi^2(T) \sin\left[\frac{\phi}{f_\phi}\right]$$

At high temperatures the Hubble friction and the field is frozen to its initial value φ_0. As the Universe cools the pull from the potential wins over the friction (this happens when $T \approx T_{osc}$ $m_\varphi (T_{osc}) \approx 3 H(T_{osc})$) and the axion starts oscillating around the minimum.

Shortly after H becomes negligible and the mass term is the leading scale in the evolution equation





Wantz and Shellard

when H becomes negligible and the mass term is the leading scale the approximate WKB solution has the form

$$\phi(t) \sim A(t) \cos\left(\int_{t_0}^{t} dt' m_\phi(t')\right) =$$

$$\phi_0 \left(\frac{(m_\phi)_0 R_0^3}{m_\phi(t) R^3(t)}\right)^{1/2} \cos\left(\int_{t_0}^{t} dt' m_\phi(t')\right)$$

where R(t) is the cosmic scale factor. Since the energy density is given by $\rho_\phi = m^2_\phi A^2/2$, the solution implies that what is conserved in the comoving volume is not the energy density but $N_\phi = \rho_\phi R^3/m_\phi$, which can be interpreted as the number of axions.

Through the conservation of the comoving entropy S, it follows that n^*_ϕ / s^* *becomes adiabatic invariant.*

The biggest uncertainty comes therefore from the temperature dependence of the axion potential $V(\phi)$

THE AXION AT NON ZERO TEMPERATURE

That is WHEN LQCD ENTERS THE GAME

Chiral Lagrangians allow to study the temperature dependence of the axion potential and its mass to finite temperatures below the crossover region Tc ~150 MeV.

Around Tc there is no known reliable perturbative expansion under control and non-perturbative methods, such as lattice QCD are required.

S. Borsanyi et al., Phys. Lett. B 752, 175 (2016) [arXiv:1508.06917 [hep-lat]]
C. Bonati, M. D'Elia, H. Panagopoulos and E. Vicari, Phys. Rev. Lett. 110, 252003 (2013) [arXiv:1301.7640 [hep-lat]].
C. Bonati, JHEP 1503, 006 (2015) [arXiv:1501.01172 [hep-lat]]
G. Y. Xiong, J. B. Zhang, Y. Chen, C. Liu, Y. B. Liu and J. P. Ma, Phys. Lett. B 752, 34
(2016) [arXiv:1508.07704
A. Trunin, F. Burger, E.-M. Ilgenfritz, M. P. Lombardo and M. Muller-Preussker, arXiv:1510.02265 [hep-lat].
M. I. Buchoff et al., Phys. Rev. D 89, 054514 (2014) [arXiv:1309.4149 [hep-lat]].

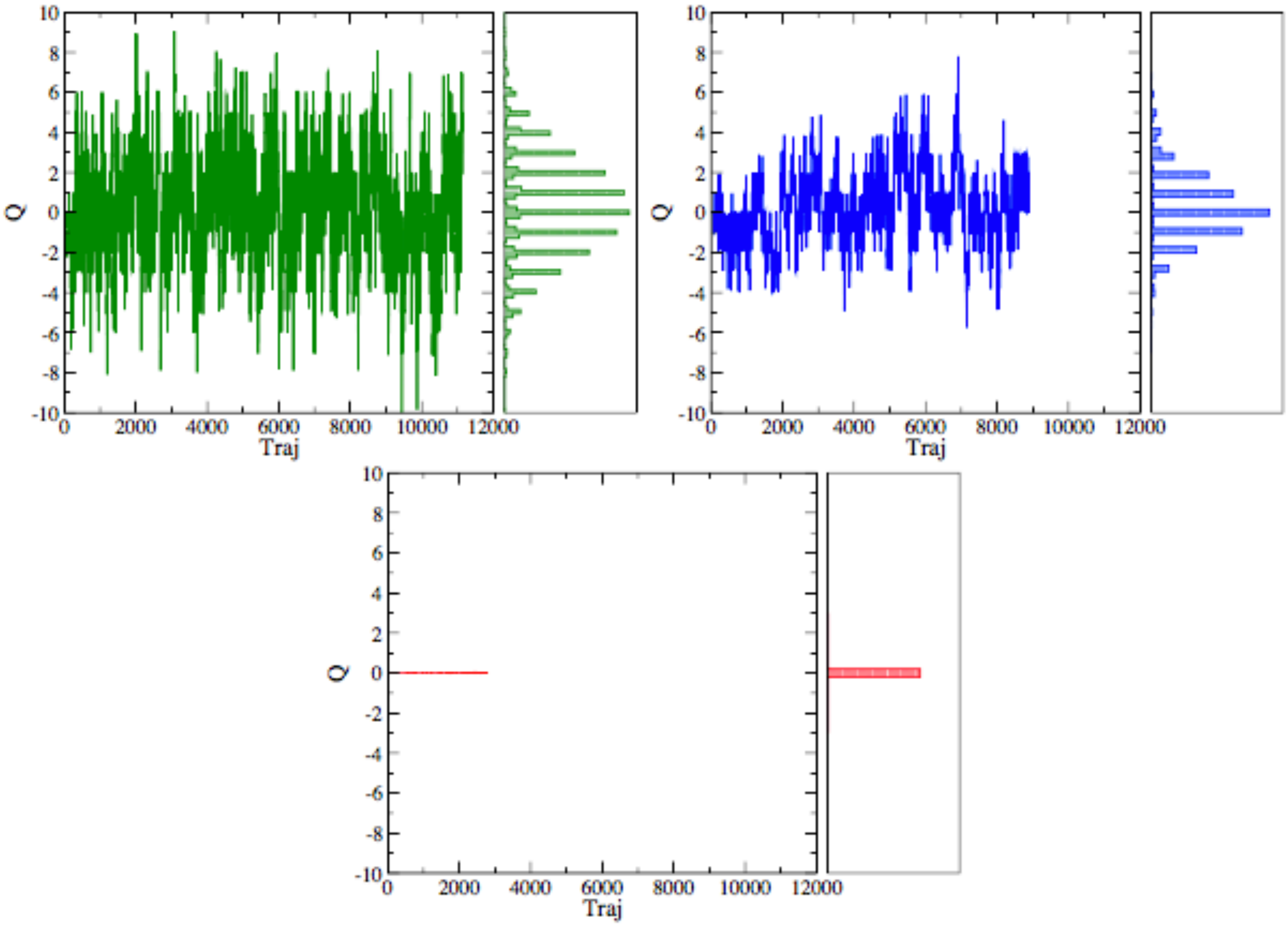
Axion phenomenology and θ-dependence From N_f =2+1 lattice QCD

C. Bonati, M. D'Elia, M. Mariti, G.M., M. Mesiti,
F. Negro, F. Sanfilippo, G. Villadoro

We studied the topological properties and the θ-dependence N_f =2+1 LQCD along a line of constant physics, corresponding to physical quark masses and for temperatures up to 4 T_c, where T_c = 155 MeV is the pseudo-critical temperature at which chiral symmetry restoration takes place.

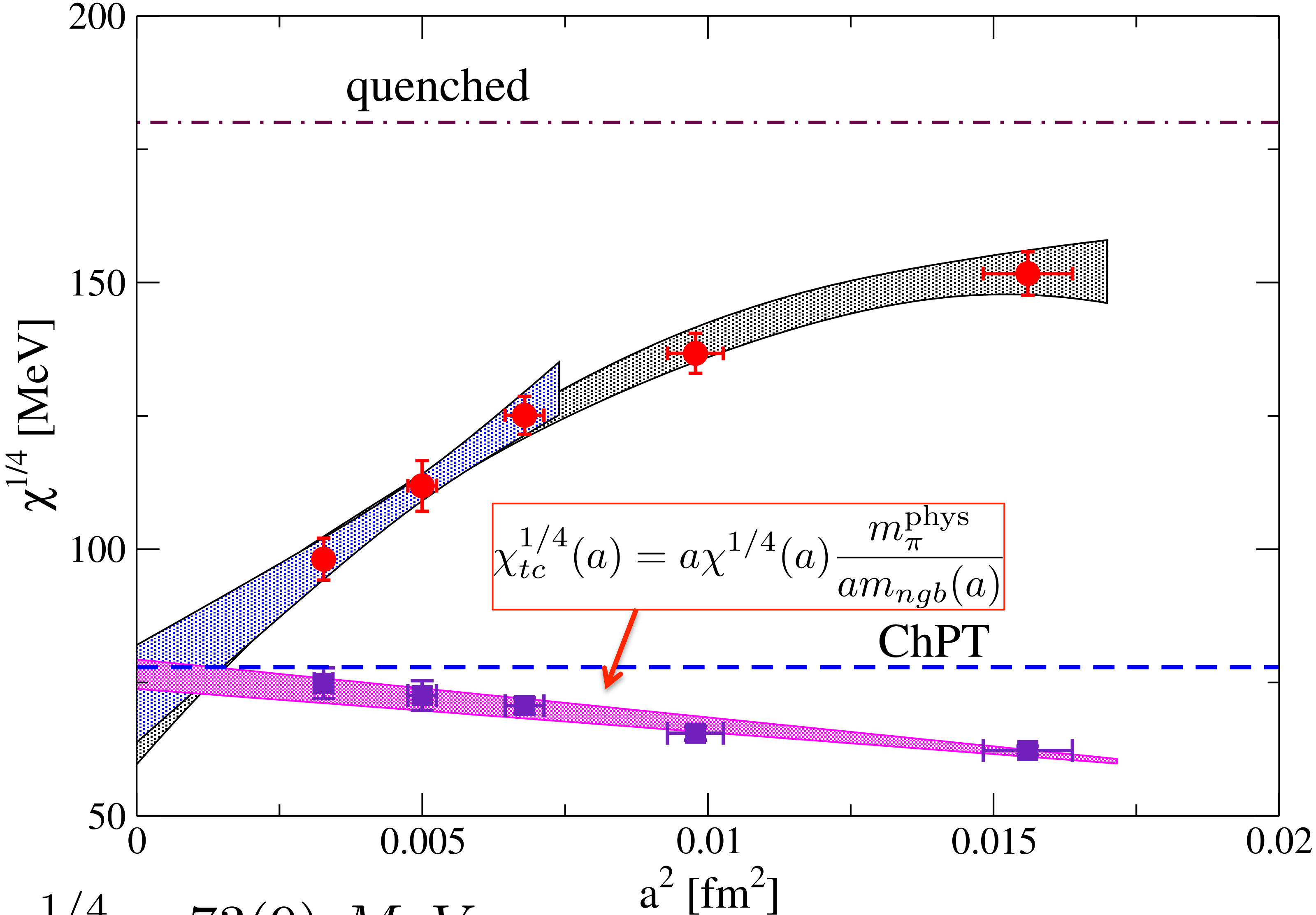
We explored several lattice spacings,
in a range 0.05 - 0.1 fm, in order to perform a continuum extrapolation of our results. *Our investigation at even smaller lattice spacings has been hindered by a severe slowing down in the decorrelation of the topological charge.*

Freezing of the topological charge at small lattice spacings



Topological charge time history and histogram for a = 0.0824 fm on a 32^4 lattice, for a = 0.0572 fm on a 48^4 lattice, for a = 0.0397 fm on a 40^4 lattice

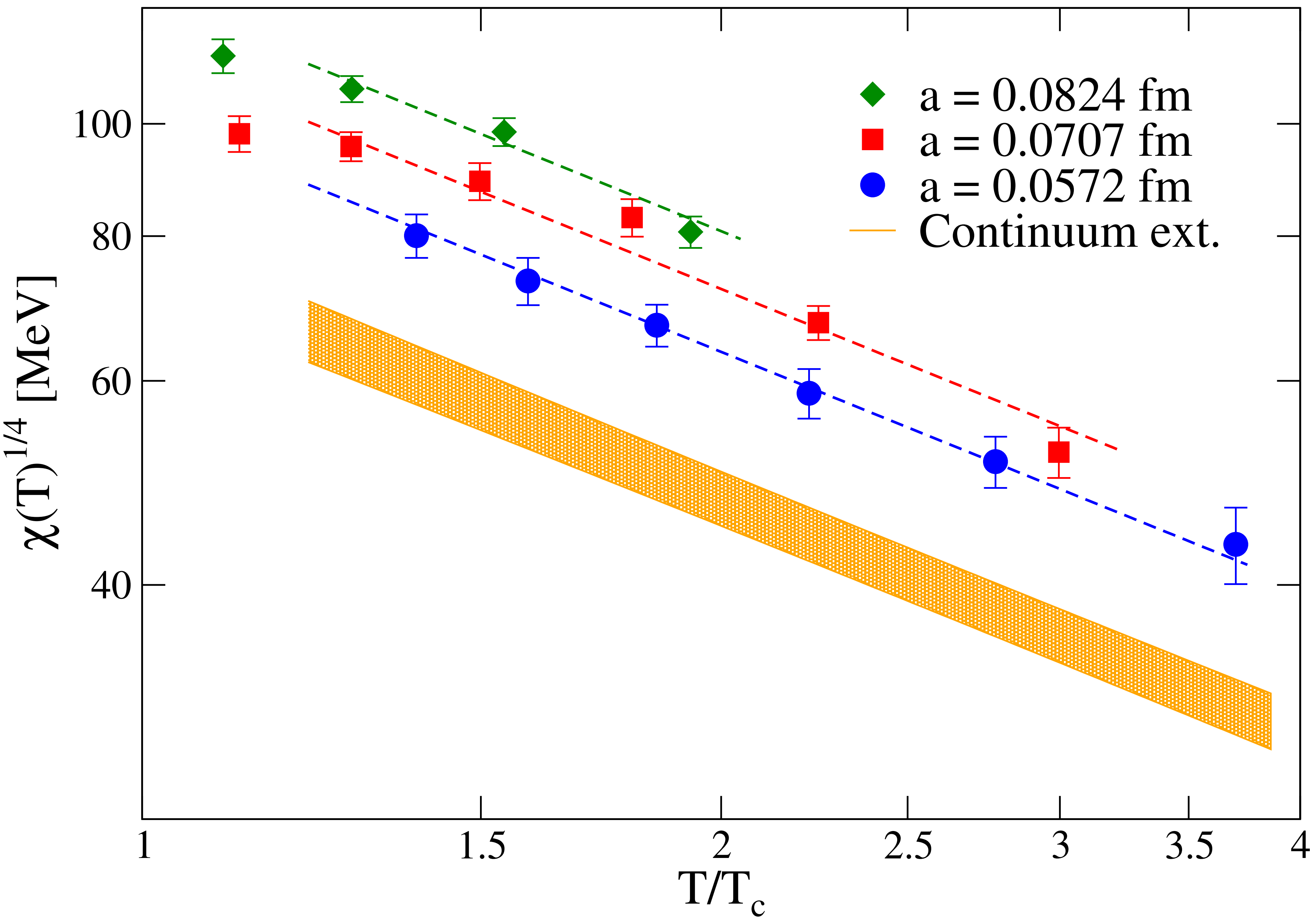
Large Cutoff Effects



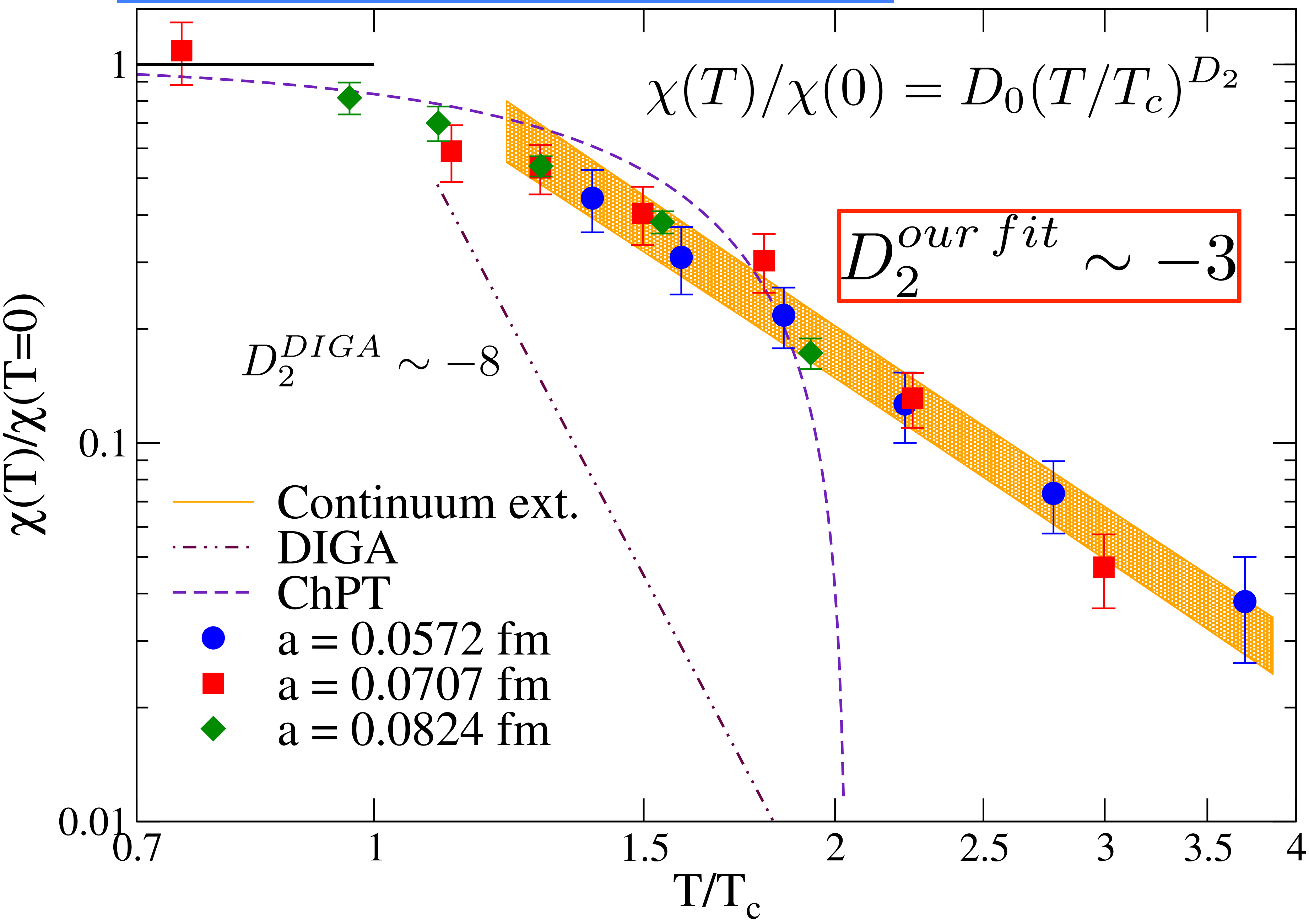
$$\chi^{1/4} = 73(9)\ MeV$$

in reasonable agreement with $\chi_{\text{ChPT}}^{1/4} = 77.8(4)\ MeV$

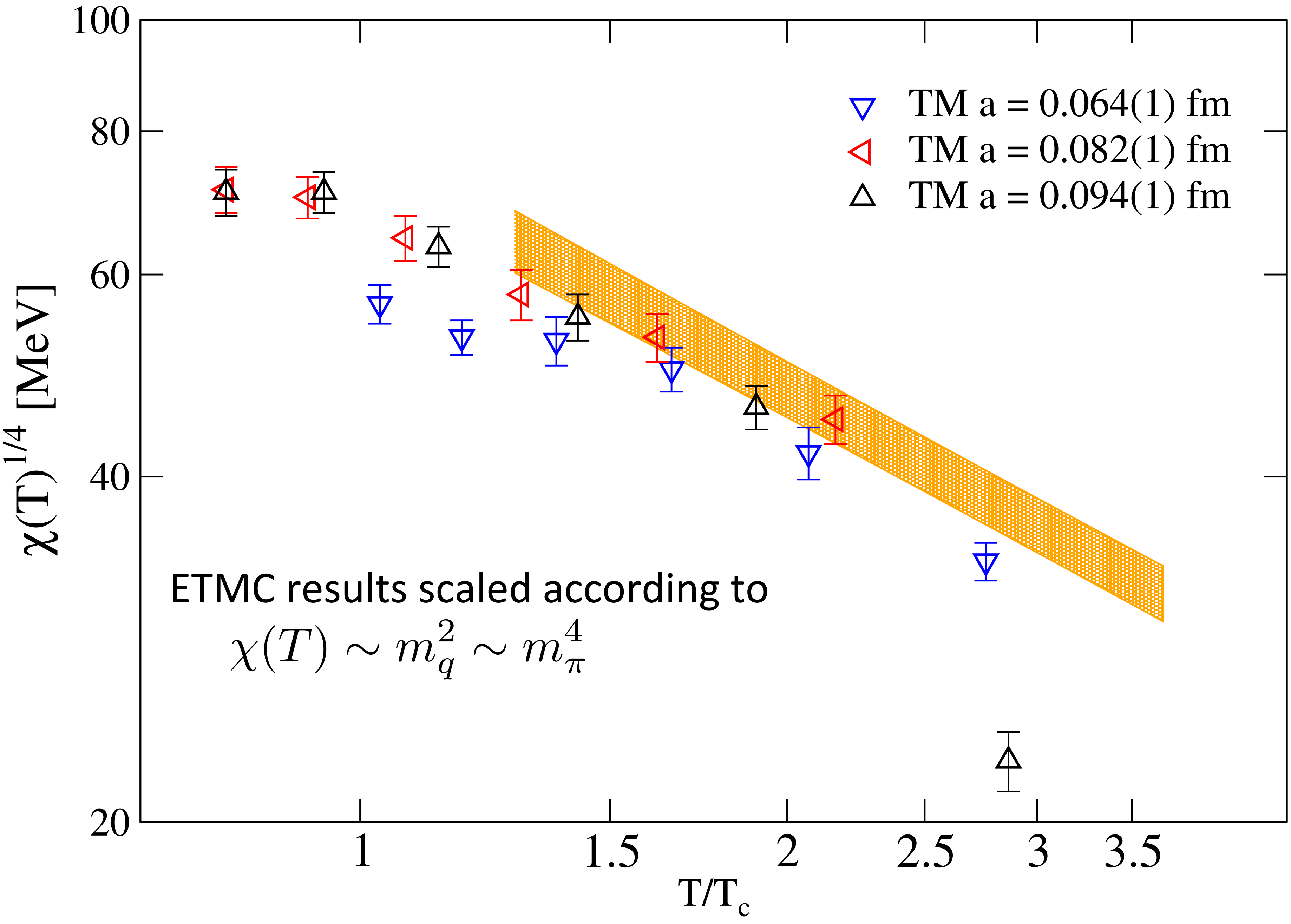
Finite temperature susceptibility



Reducing cutoff effects



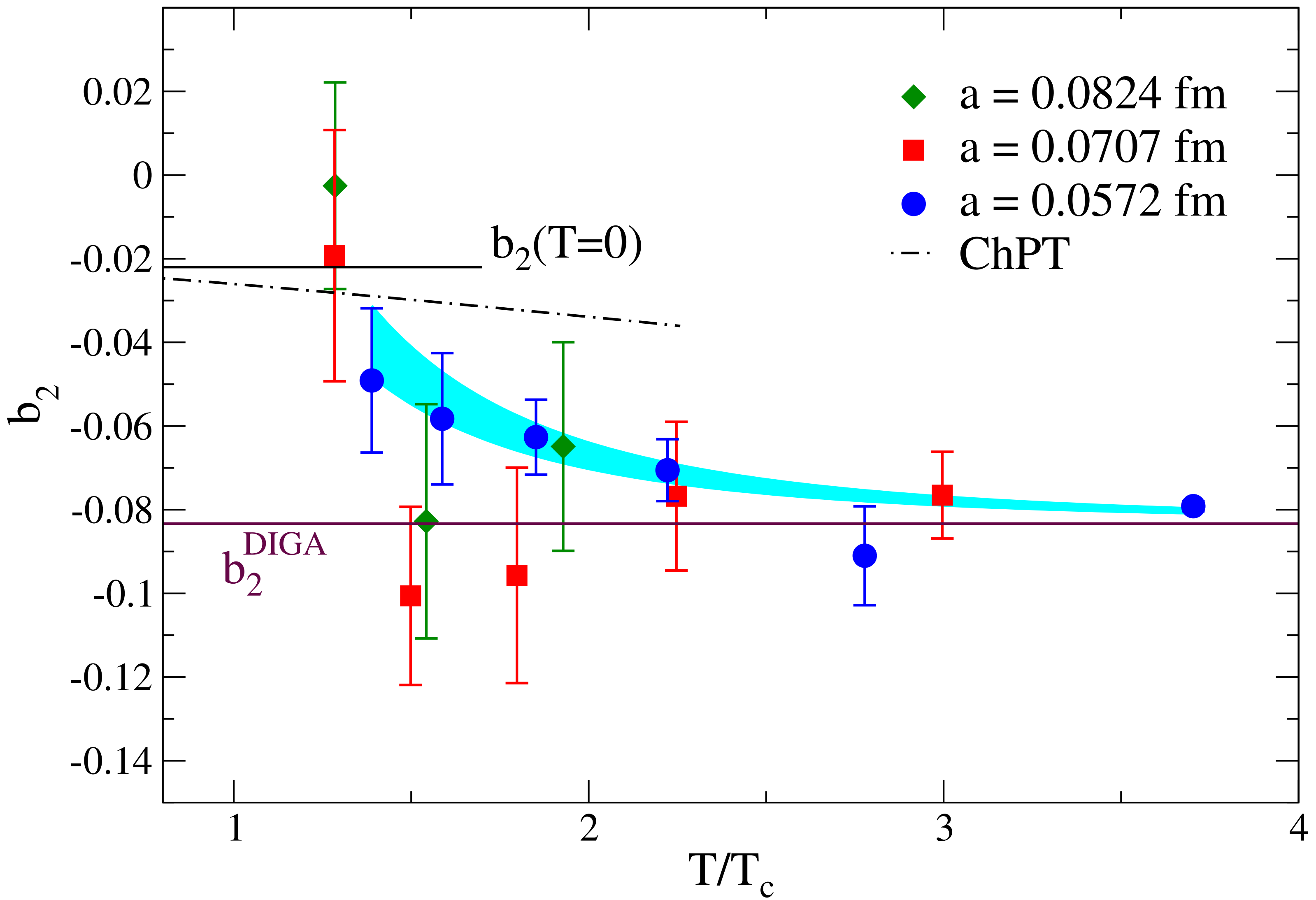
Comparison with EMTC (difficult with RBC)



Higher Momenta

$$b_2 = -\frac{\langle Q^4\rangle_{\theta=0} - 3\langle Q^2\rangle^2_{\theta=0}}{12\langle Q^2\rangle_{\theta=0}}$$

$$b_2^{DIGA} = -1/12$$



AXION RESULTS

Given that $b_2(T)$ converges relatively fast to the value predicted by a single cosine potential, we can assume $V(\phi)=-\chi_t(T)\cos(\phi/f_\phi)$ for $T \geq Tc$

Using the most conservative results for the fit

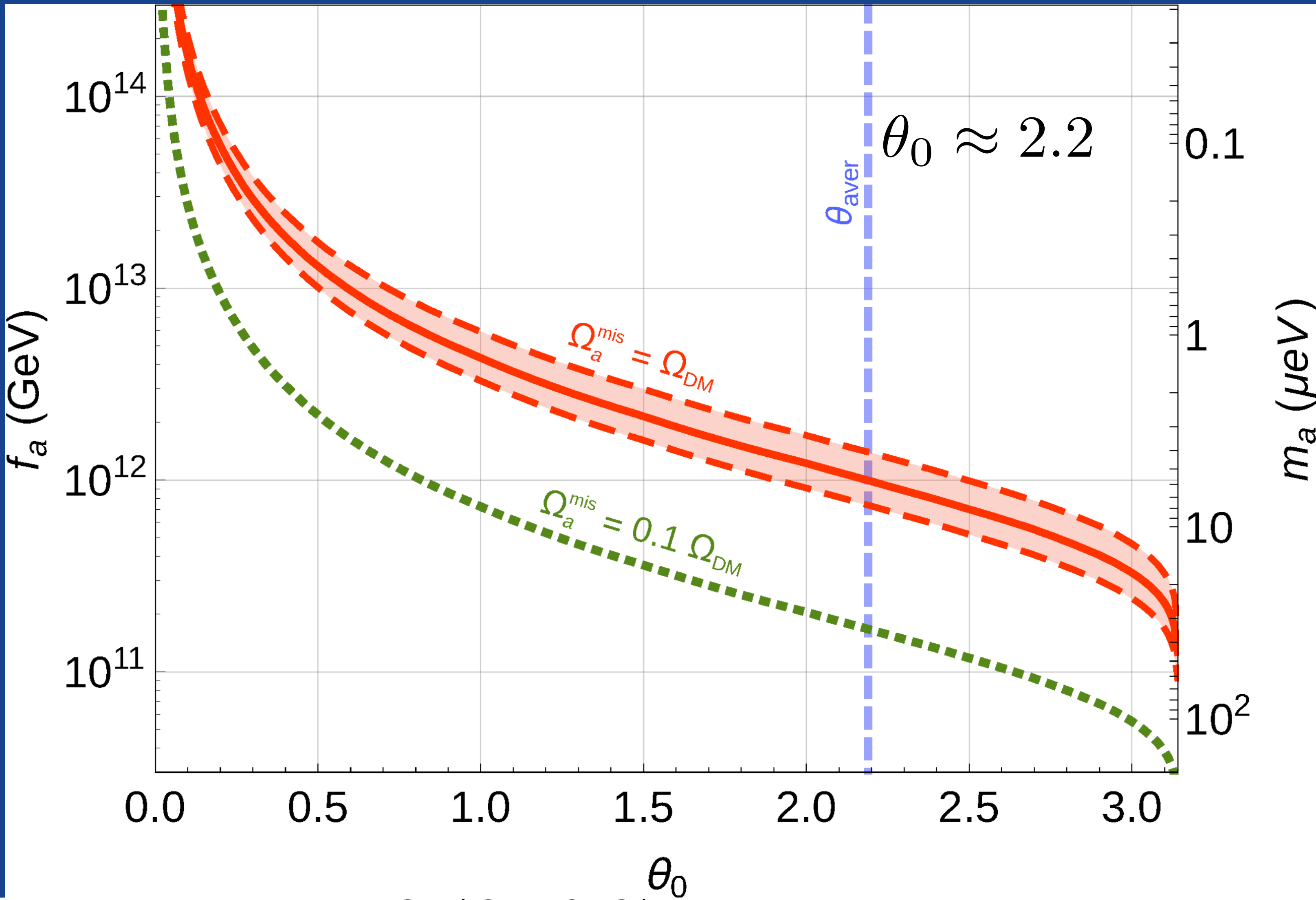
$$\chi(T)/\chi(0) = (1.8 \pm 1.5)(T_c/T)^{2.90\pm0.65}$$

we plot the prediction for the parameter f_ϕ as a function of the initial value of the axion field $\vartheta_0=\phi_0/f_\phi$ assuming that the axion contribution make up for the whole observed dark matter abundance

$$\Omega_{\rm DM} = 0.259(4)$$

Predictions for fϕ

(almost one order of magnitude larger than the instanton value)



$$f_\phi(\theta_0) = (1.00^{+0.40+0.07}_{-0.26-0.18} \pm 0.06) \cdot 10^{12}\ \mathrm{GeV}$$

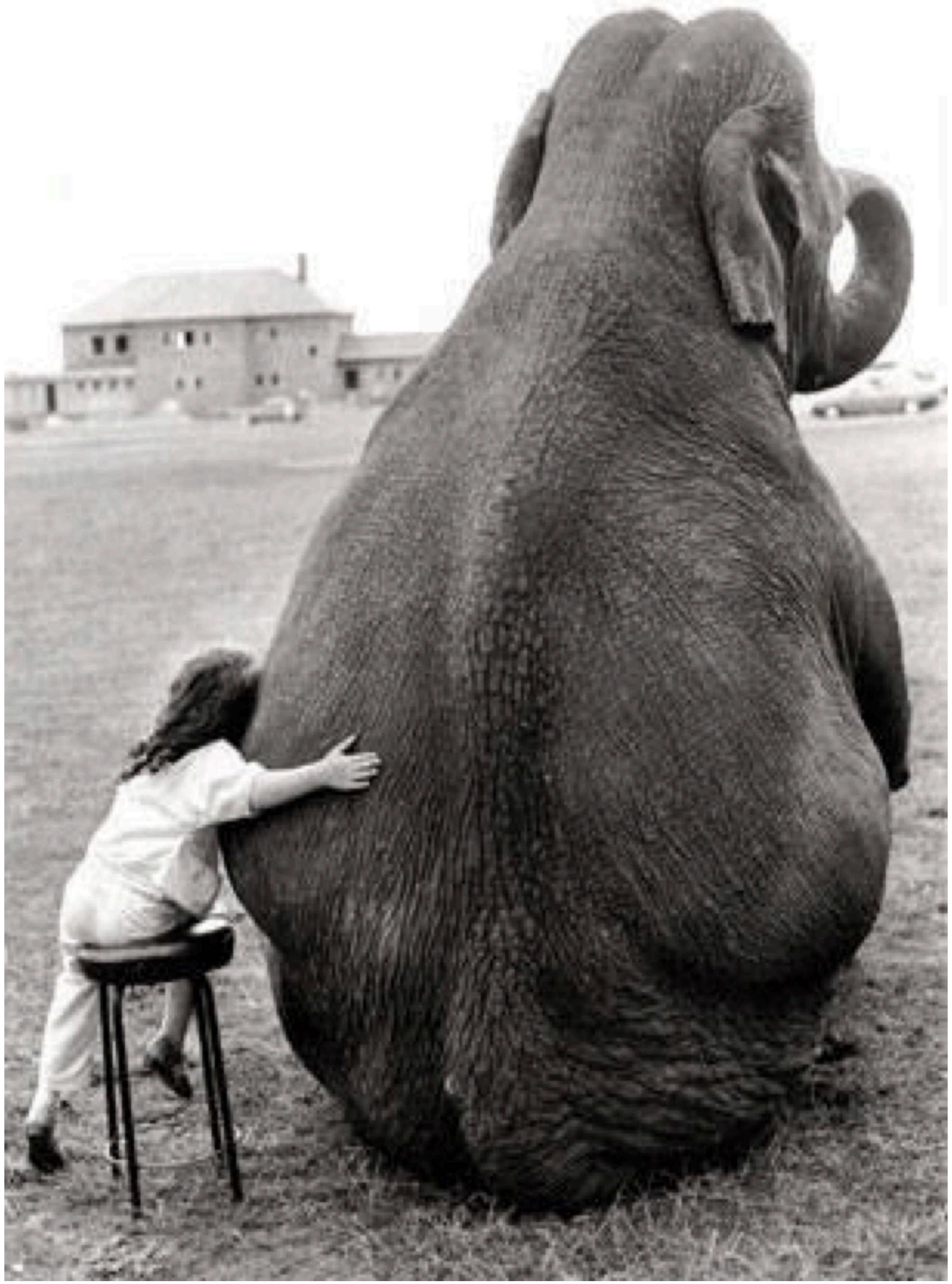
MAIN MESSAGE

In particular for the value of $f_{\phi} \approx 10^{12}$ GeV the axion field starts oscillating around Tosc=4.3 GeV. An even longer extrapolation is required for $f_{\phi} \approx 1.67\ 10^{11}$ GeV corresponding to Ω_{ϕ}=0.1 Ω_{DM}, where the axion starts oscillating around Tosc=7.2 GeV.

MAIN MESSAGE

The results however rely on the extrapolation of the axion mass fit formula up to few GeV

THE CONTROL OF THE LARGE T REGION IS VERY IMPORTANT



Lattice QCD for Cosmology

Sz. Borsanyi1, Z. Fodor1;2;3, K.-H. Kampert1, S. D. Katz3;4, T. Kawanai2, T. G. Kovacs5, S. W. Mages2, A. Pasztor1, F. Pittler3;4, J. Redondo6;7, A. Ringwald8, K. K. Szabo1;2 1

See talks by Kalman SZABO and Dr. Szabolcs BORSANYI at this Conference

1) Upgraded and extended analysis of the equation of state (energy and entropy density) as a function of T within nf=2+1+1 staggered fermions
2) Study of the topological susceptibility up to large values of T
3) Detailed study of discretization errors

FIND MUCH LARGER EXPONENT CLOSER TO THE DILUITE INSTANTON GAS APPROXIMATION

However several (reasonable) approximations are adopted

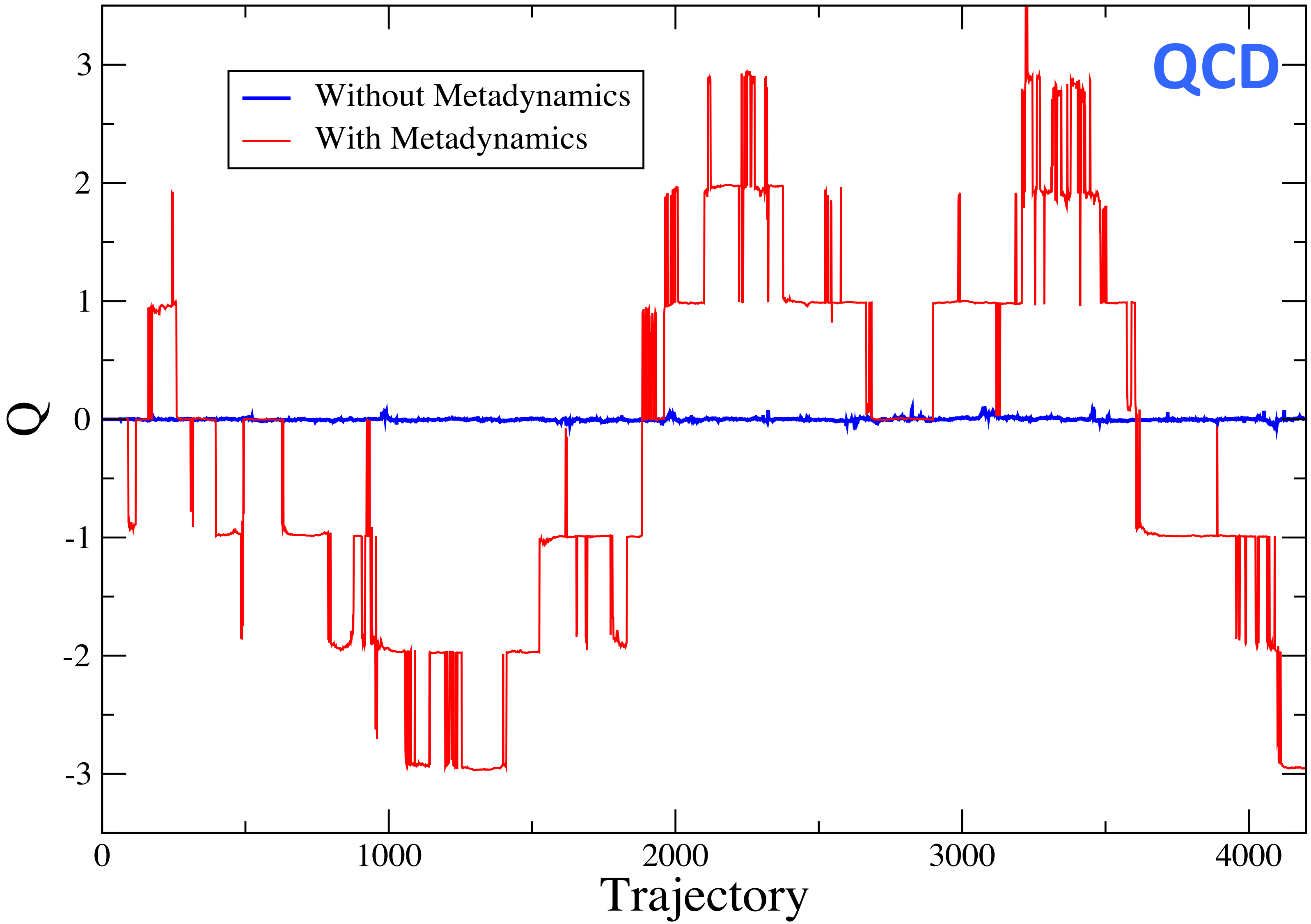
1) At high T calculation at fixed topological sector Q=0,1 only;
2) ZQ/Z0(T) computed via average action and condensate
3) Reweighting of the chiral condensate to reduce discretization errors

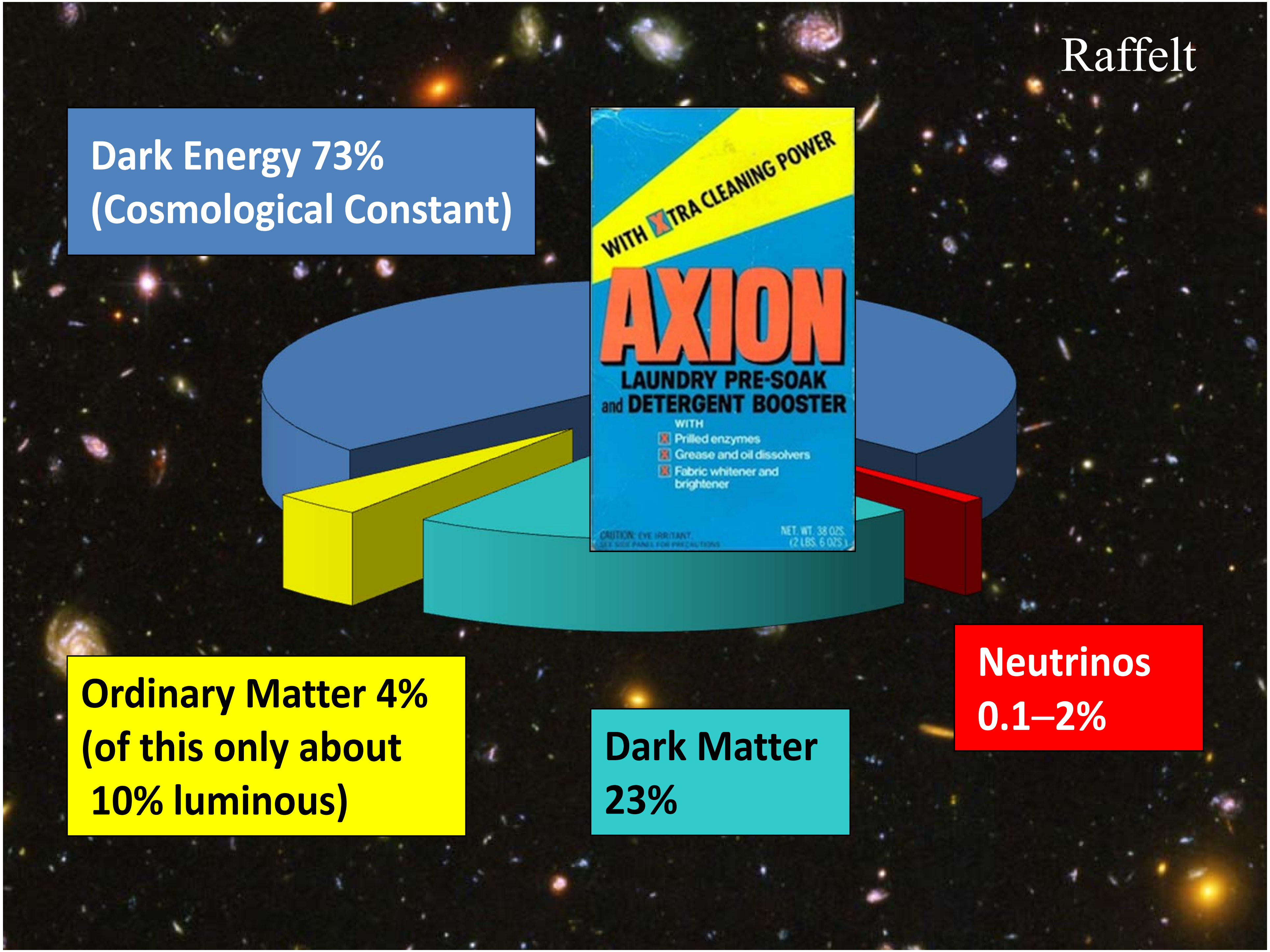
(MUCH) More investigation is needed.

CONCLUSIONS

- The deviations from the dilute instanton gas predictions a significant impact on axions, resulting in particular in a shift of the axion dark matter window by almost one order of magnitude
- The softer temperature dependence of the topological susceptibility also changes the onset of the axion oscillations, which would now start at a higher temperatures (T ~ 4 GeV)
- A different power law behavior might set in at temperatures higher than 1 GeV The main obstruction an extension of numerical simulations to this scale is represented by the freezing of the topological modes at smaller lattice spacings which would be necessary to investigate such temperatures (a < 0.05 fm)
- Such an obstruction could be overcome by the development of new Monte Carlo algorithms In view of the exploration of higher temperatures, one should also consider the inclusion of dynamical charm quarks.

OUTLOOK: first essay at the physical point




Raffelt
Dark Energy 73%
(Cosmological Constant)
WITH XTRA CLEANING POWER
AXION
LAUNDRY PRE-SOAK
and DETERGENT BOOSTER
WITH
Prilled enzymes
Grease and oil dissolvers
Fabric whitener and brightener
Ordinary Matter 4%
(of this only about
10% luminous)
Dark Matter
23%
Neutrinos
0.1–2%