Axion Phenomenology from Unquenched Lattice QCD

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• Axions
• LQCD & Axions
• Recent results & Outlook
Axions were originally proposed to deal with the strong CP problem

Massless QCD

\[ \mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \bar{\psi}_a^i \mathcal{D}_{ab} \psi_b^i - \frac{1}{4} G_A^{A \mu \nu} G_{A}^{\mu \nu} \]

Symmetries @ the classical level

\[ U(N_f)_L \times U(N_f)_R \sim SU(N_f)_L \times SU(N_f)_R \times U(1)_L \times U(1)_R \]

Non trivial vacuum (quark condensate \( \langle \bar{\psi} \psi \rangle \neq 0 \)) breaks spontaneously non singlet chiral symmetries

\[ SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V \]

\( U(1)_V \) is the conserved barion number
Diagonalized mass terms

$$\mathcal{L}_m = - \sum_{i=1}^{N_f} \left( m_i \bar{\psi}_L^i \psi_R^i + m_i^* \bar{\psi}_R^i \psi_L^i \right)$$

Explicitly broken symmetries:
if the masses are different from zero $SU(N_f)_A$ is broken
$SU(N_f)_V$ is broken if the masses are not equal

ANOMALY: we have to introduce a regularization
Two examples:
a) GW fermions: the action is invariant under a global chiral transformation but the fermion measure is not invariant (Fujikawa)
b) Wilson fermions: the action is not invariant but the measure is invariant
a) GW fermions: if we rotate the quark fields by a phase
\[ \psi_i \rightarrow e^{i\alpha_i} \psi_i \quad \bar{\psi}_i \rightarrow \bar{\psi}_i e^{i\alpha_i} \]

Then, because of the variation of the measure, the action is modified as
\[ \mathcal{L}_{QCD} + \mathcal{L}\{m_i\} \rightarrow \]
\[ \mathcal{L}_{QCD} + \mathcal{L}\{m_i e^{2i\alpha_i}\} + \left( \theta + \sum_{i=1}^{N_f} 2\alpha_i \right) \frac{N_f g^2}{32\pi^2} G^A_{\mu\nu} \tilde{G}^A_{\mu\nu} \]

Indeed the functional integral depends on the invariant combination
\[ \det[m_f] e^{-i\theta} \]

and if we apply a rotation to make the masses real (and positive)
\[ \theta \rightarrow \theta - \arg[\det[m_f]] \]
\[ \delta \int D\psi D\bar{\psi} e^{-S} = \]
\[ i \int d^4x \int D\psi D\bar{\psi} \alpha(x) \left[ \partial_\mu J_\mu^5(x) - 2m\bar{\psi}\gamma_5\psi(x) - \frac{N_f g^2}{32\pi^2} G^A_{\mu\nu} \tilde{G}_{A}^{\mu\nu}(x) \right] e^{-S} \]

From the rotation of the Action

From the rotation of the fermion measure

**TOPOLOGICAL CHARGE DENSITY AND SUSCEPTIBILITY**

\[ Q(x) = \frac{g^2}{32\pi^2} G^A_{\mu\nu} \tilde{G}_{A}^{\mu\nu}(x) \quad \int d^4 x Q(x) = n \]

\[ Q(x) = -\frac{1}{2} \text{Tr} \left[ \gamma_5 D(x, x) \right] \quad \chi_t = \int d^4 x \langle Q(x)Q(0) \rangle \]

b) Wilson Fermions: if we rotate the quark fields by a phase

\[ \psi_i \rightarrow e^{i\alpha_i} \psi_i \quad \bar{\psi}_i \rightarrow \bar{\psi}_i e^{i\alpha_i} \]

then the variation of the action is given by (Bochicchio & al.)

\[ \delta S = i\alpha(x) \left[ \partial_\mu J_\mu^5(x) - 2M \bar{\psi} \gamma_5 \psi(x) + X_5(x) \right] \]

where the last term is the chiral rotation of the Wilson term:

\[ \bar{X}_5(x) = \bar{Z}_5 \left[ X_5(x) - 2\bar{M} \bar{\psi} \gamma_5 \psi - (\bar{Z}_J - 1) \partial_\mu J_\mu^5(x) + Z_{G\tilde{G}} \frac{N_f g^2}{32\pi^2} G^A_{\mu\nu} \tilde{G}_A^{\mu\nu}(x) \right] \]

where the matrix elements of the operator on the l.h.s. are of O(a) (or a^2 with improved actions and operators)

\[ \bar{Z}_J \partial_\mu J_\mu^5(x) - 2\bar{m} \bar{\psi} \gamma_5 \psi(x) - Z_{G\tilde{G}} \frac{N_f g^2}{32\pi^2} G^A_{\mu\nu} \tilde{G}_A^{\mu\nu}(x) + O(a) = 0 \]
The $\theta$ term and the strong CP problem

- Because of the anomaly QCD depends, in general, on a parameter $\theta$
- A priori $\theta$ can have any value; physics invariant for $\theta \rightarrow \theta + 2\pi$
- This parameter gives rise to CP violation
- Neutron EDM $\leq 2.9 \times 10^{-26}$ e . cm implies $\theta \leq 10^{-9} - 10^{-10}$

Several possibilities among which:
- The mass of one quark is equal to zero
- Peccei-Quinn symmetry and the axion (Weinberg-Wilczek); see also M. A. Shifman, A. I. Vainshtein and V. I. Zakharov; A. R. Zhitnitsky, M. Dine, W. Fischler and M. Srednicki
<table>
<thead>
<tr>
<th>$N_f$</th>
<th>$m_u$</th>
<th>$m_d$</th>
<th>$m_u/m_d$</th>
<th>$R$</th>
<th>$Q$</th>
</tr>
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<tbody>
<tr>
<td>2+1+1</td>
<td>2.36(24)</td>
<td>5.03(26)</td>
<td>0.470(56)</td>
<td>35.6(5.1)</td>
<td>22.2 (1.6)</td>
</tr>
<tr>
<td>2+1</td>
<td>2.16(9)(7)</td>
<td>4.68(14)(7)</td>
<td>0.46(2)(2)</td>
<td>35.0(1.9)(1.8)</td>
<td>22.5(6)(6)</td>
</tr>
<tr>
<td>2</td>
<td>2.40(23)</td>
<td>4.80(23)</td>
<td>0.50(4)</td>
<td>40.7(3.7)(2.2)</td>
<td>24.3(1.4)(0.6)</td>
</tr>
</tbody>
</table>
The common lore

1) Implement a continuum $U(1)_{\text{PQ}}$ global chiral symmetry by adding extra particles (scalars, fermions);

2) The symmetry is spontaneously broken and this gives rise to a Goldstone boson, the axion;

3) Since the quark masses are different from zero the pseudo-Goldstone boson has a non zero mass which depends on the quark masses.
At low energy, neglecting heavy degrees of freedom, the effective action is given by

\[ \mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi + \mathcal{L}_{int} \left[ \bar{\psi}, \psi, \partial_{\mu} \phi \right] + \left( \theta + \frac{\phi}{f_{\phi}} \right) \frac{g^2}{32\pi^2} G^{A}_{\mu\nu} \tilde{G}^{\mu\nu}_{A} (x) \]

where \( f_{\phi} \) depends on the particular model.

This action, if not for the anomaly, is invariant under the non linear \( \text{U}(1)_{\text{PQ}} \) transformation \( \phi \rightarrow \phi + \alpha f_{\phi} \).

Let us define an effective axion action

\[ e^{-S_{\text{eff}}(\partial_{\mu} \phi, \phi)} = \int \mathcal{D} \left[ \psi, \bar{\psi}, G^{A}_{\mu} \ldots \right] e^{-S(\psi, \bar{\psi}, G^{A}_{\mu}, \ldots, \partial_{\mu} \phi, \phi)} \]
Minimizing the Axion Potential

\[ e^{-S_{\text{eff}}(\partial_\mu \phi, \phi)} = \int \mathcal{D} [\psi, \bar{\psi}, G^A_{\mu} \ldots] \ e^{-S(\psi, \bar{\psi}, G^A_{\mu}, \ldots, \partial_\mu \phi, \phi)} \]

The minimum of the effective potential

\[ \frac{\partial V_{\text{eff}}}{\partial \phi} = -\frac{1}{f_\phi} \frac{g^2}{32\pi^2} \langle G^A_{\mu\nu} \tilde{G}^{\mu\nu}_A(x) \rangle = 0 \]

corresponds to \( \phi/f_\phi = -\theta \) and solve the strong CP problem; from the second derivative of the effective potential we may compute the axion mass

\[ m^2_\phi = \frac{1}{f^2_\phi} \chi_t = \frac{1}{f^2_\phi} \int d^4x \langle Q(x)Q(0) \rangle \]
ZERO TEMPERATURE AND CHIRAL EXPANSION

2 flavours: the relevant (naïve) Ward identity

at non zero quark masses reads

\[ \chi_t = \frac{1}{4} \langle 0 | m_u \bar{u}u + m_d \bar{d}d | 0 \rangle - \]
\[ \frac{1}{4} \int d^4 x \langle 0 | T \left[ (m_u \bar{u} \gamma_5 u + m_d \bar{d} \gamma_5 d) [x] (m_u \bar{u} \gamma_5 u + m_d \bar{d} \gamma_5 d) [0] \right] | 0 \rangle \]

Expanding at small quark masses and saturating the T-product with Goldston boson intermediate states we get

\[ \chi_t = - \frac{1}{4} f_\pi^2 m_\pi^2 \frac{4m_u m_d}{(m_u + m_d)^2} \]

M. A. Shifman, A. I. Vainshtein and V. I. Zakharov; W. A. Bardeen and S.-H.H. Tye
The result depends on the number of flavours. For example if we add the strange quark and make an expansion at small quark masses we get

\[ x_t = -\frac{3}{4} f_\pi^2 \left( m_\pi^2 + m_\eta^2 \right) \frac{m_u m_d m_s}{(m_u + m_d + m_s)(m_u m_d + m_d m_s + m_u m_s)} \]

The topological susceptibility, and consequently the mass of the axion, vanishes whenever the mass of a quark is equal to zero. **AT ZERO T WE DO NOT NEED THE LATTICE !**

Similar results can be obtained in chiral perturbation theory, where the NLO corrections were recently computed and it was possible to extract the axion mass, self coupling and its full potential at the percent level

*P. Di Vecchia and G. Veneziano, Giovanni Grilli di Cortona, Edward Hardy, Javier Pardo Vega, Giovanni Villadoro*
Axion Bounds and Searches

[Graph showing the range of mass (m_a) in keV, eV, meV, μeV, neV and frequency (f_a) in GeV.

- Experiments
- Telescope
- CAST
- Direct searches
- ADMX (Seattle & Yale)

- Too much hot dark matter
- Too much cold dark matter (misalignment with Θ_i = 1)
- Too many events
- Too much energy loss
- White dwarf cooling?

Georg Raffelt, MPI Physics, Munich
THE AXION PHENOMENOLOGY

Basic Formula:

\[ \Omega_\phi = \frac{86}{33} \frac{\Omega_\gamma}{T_\gamma} \frac{n_\phi^*}{s^*} m_\phi, \]

where \( \Omega_\gamma \) and \( T_\gamma \) are the present abundance and temperature of photons while \( n_\phi^*/s^* \) is the ratio between the comoving axion number density \( n_\phi = m_\phi \phi^2 \) and the entropy density computed at a late time \( t^* \) such that the ratio \( n_\phi/s \) became constant.
The number density $n_\phi$ can be extracted by solving the axion equation of motion

$$\ddot{\phi} + 3H \dot{\phi} + \frac{dV(\phi)}{d\phi} = 0$$

The temperature (and time) dependence of the Hubble parameter $H$ is determined by the Friedmann equations and the QCD equation of state (measured in lattice QCD).

$$\frac{dV(\phi)}{d\phi} \sim m_\phi^2(T) \phi + \cdots \sim f_\phi m_\phi^2(T) \sin \left[ \frac{\phi}{f_\phi} \right]$$
At high temperatures the Hubble friction and the field is frozen to its initial value $\varphi_0$. As the Universe cools the pull from the potential wins over the friction (this happens when $T \approx T_{osc}$ $m_\varphi (T_{osc}) \approx 3 \ H(T_{osc})$) and the axion starts oscillating around the minimum.

Shortly after $H$ becomes negligible and the mass term is the leading scale in the evolution equation
\(m_a < 3H\)  
axion is frozen

\(m_a \approx 3H\)  
axion starts rolling, turns into pressureless matter.

axion number \(N_a\) is conserved
when \( H \) becomes negligible and the mass term is the leading scale, the approximate WKB solution has the form

\[
\phi(t) \sim A(t) \cos \left( \int_{t_0}^{t} dt' m_{\phi}(t') \right) = \\
\phi_0 \left( \frac{(m_{\phi})_0 R_0^3}{m_{\phi}(t) R^3(t)} \right)^{1/2} \cos \left( \int_{t_0}^{t} dt' m_{\phi}(t') \right)
\]

where \( R(t) \) is the cosmic scale factor. Since the energy density is given by \( \rho_{\phi} = m_{\phi}^2 A^2 / 2 \), the solution implies that what is conserved in the comoving volume is not the energy density but \( N_{\phi} = \rho_{\phi} R^3 / m_{\phi} \), which can be interpreted as the number of axions.

*Through the conservation of the comoving entropy \( S \), it follows that \( n_{\phi}^* / s^* \) becomes adiabatic invariant.*
The biggest uncertainty comes therefore from the temperature dependence of the axion potential $V(\phi)$

**THE AXION AT NON ZERO TEMPERATURE**

That is WHEN LQCD ENTERS THE GAME

Chiral Lagrangians allow to study the temperature dependence of the axion potential and its mass to finite temperatures below the crossover region $T_c \sim 150$ MeV.

Around $T_c$ there is no known reliable perturbative expansion under control and non-perturbative methods, such as lattice QCD are required.


A. Trunin, F. Burger, E.-M. Ilgenfritz, M. P. Lombardo and M. Muller-Preussker, arXiv:1510.02265 [hep-lat].

Axion phenomenology and θ-dependence
From $N_f = 2+1$ lattice QCD

C. Bonati, M. D’Elia, M. Mariti, G.M., M. Mesiti, F. Negro, F. Sanfilippo, G. Villadoro

We studied the topological properties and the θ-dependence $N_f = 2+1$ LQCD along a line of constant physics, corresponding to physical quark masses and for temperatures up to $4 T_c$, where $T_c = 155$ MeV is the pseudo-critical temperature at which chiral symmetry restoration takes place.

We explored several lattice spacings, in a range $0.05 - 0.1$ fm, in order to perform a continuum extrapolation of our results. Our investigation at even smaller lattice spacings has been hindered by a severe slowing down in the decorrelation of the topological charge.
Freezing of the topological charge at small lattice spacings

Topological charge time history and histogram for $a = 0.0824$ fm on a $32^4$ lattice, for $a = 0.0572$ fm on a $48^4$ lattice, for $a = 0.0397$ fm on a $40^4$ lattice
Large Cutoff Effects

\[ \chi_{tc}^{1/4}(a) = a \chi^{1/4}(a) \frac{m_{\pi}^{\text{phys}}}{a m_{n,gb}(a)} \]

\[ \chi^{1/4} = 73(9) \text{ MeV} \]

in reasonable agreement with \[ \chi_{\text{ChPT}}^{1/4} = 77.8(4) \text{ MeV} \]
Finite temperature susceptibility

\[ \chi(T) \propto \left(\frac{T}{T_c}\right)^{1/4} \]
Reducing cutoff effects

\[ \chi(T)/\chi(0) = D_0 (T/T_c)^{D_2} \]

Continuum ext.
DIGA
ChPT

\[ D_2^{DIGA} \sim -8 \]

\[ D_2^{our \ fit} \sim -3 \]

- Continuum ext.
- DIGA
- ChPT
- \( a = 0.0572 \text{ fm} \)
- \( a = 0.0707 \text{ fm} \)
- \( a = 0.0824 \text{ fm} \)
Comparison with EMTC (difficult with RBC)

ETMC results scaled according to

$$\chi(T) \sim m_q^2 \sim m_{\pi}^4$$
Higher Momenta

\[ b_2 = -\frac{\langle Q^4 \rangle_{\theta=0} - 3\langle Q^2 \rangle^2_{\theta=0}}{12\langle Q^2 \rangle_{\theta=0}} \]

\[ b_2^{DIGA} = -\frac{1}{12} \]

\[ b_2(T=0) \]

\[ a = 0.0824 \text{ fm} \]
\[ a = 0.0707 \text{ fm} \]
\[ a = 0.0572 \text{ fm} \]

ChPT
AXION RESULTS

Given that $b_2(T)$ converges relatively fast to the value predicted by a single cosine potential, we can assume $V(\phi) = -\chi_t(T) \cos(\phi/f_\phi)$ for $T \geq T_c$

Using the most conservative results for the fit

$$\chi(T)/\chi(0) = (1.8 \pm 1.5) (T_c/T)^{2.90 \pm 0.65}$$

we plot the prediction for the parameter $f_\phi$ as a function of the initial value of the axion field $\vartheta_0 = \phi_0/f_\phi$ assuming that the axion contribution make up for the whole observed dark matter abundance

$$\Omega_{DM} = 0.259(4)$$
Predictions for $f\phi$

(almost one order of magnitude larger than the instanton value)

$$f\phi(\theta_0) = \left(1.00^{+0.40+0.07}_{-0.26-0.18} \pm 0.06\right) \cdot 10^{12} \text{ GeV}$$
In particular for the value of $f_\phi \approx 10^{12}$ GeV the axion field starts oscillating around $T_{osc}=4.3$ GeV. An even longer extrapolation is required for $f_\phi \approx 1.67 \times 10^{11}$ GeV corresponding to $\Omega_\phi=0.1 \Omega_{DM}$, where the axion starts oscillating around $T_{osc}=7.2$ GeV.
The results however rely on the extrapolation of the axion mass fit formula up to few GeV

THE CONTROL OF THE LARGE $T$ REGION IS VERY IMPORTANT
Lattice QCD for Cosmology
Sz. Borsanyi1, Z. Fodor1;2;3, K.-H. Kampert1, S. D. Katz3;4, T. Kawanai2, T. G. Kovacs5, S. W. Mages2, A. Pasztor1, F. Pittler3;4, J. Redondo6;7, A. Ringwald8, K. K. Szabo1;2 1

See talks by Kalman SZABO and Dr. Szabolcs BORSANYI at this Conference

1) Upgraded and extended analysis of the equation of state (energy and entropy density) as a function of T within \( n_f = 2+1+1 \) staggered fermions
2) Study of the topological susceptibility up to large values of T
3) Detailed study of discretization errors

FIND MUCH LARGER EXPONENT CLOSER TO THE DILUITE INSTANTON GAS APPROXIMATION

However, several (reasonable) approximations are adopted

1) At high T calculation at fixed topological sector \( Q=0,1 \) only;
2) \( Z_Q/Z_0(T) \) computed via average action and condensate
3) Reweighting of the chiral condensate to reduce discretization errors

(MUCH) More investigation is needed.
CONCLUSIONS

- The deviations from the dilute instanton gas predictions a significant impact on axions, resulting in particular in a shift of the axion dark matter window by almost one order of magnitude.
- The softer temperature dependence of the topological susceptibility also changes the onset of the axion oscillations, which would now start at a higher temperatures ($T \sim 4$ GeV).
- A different power law behavior might set in at temperatures higher than 1 GeV. The main obstruction an extension of numerical simulations to this scale is represented by the freezing of the topological modes at smaller lattice spacings which would be necessary to investigate such temperatures ($a < 0.05$ fm).
- Such an obstruction could be overcome by the development of new Monte Carlo algorithms. In view of the exploration of higher temperatures, one should also consider the inclusion of dynamical charm quarks.
OUTLOOK: first essay at the physical point
Dark Energy 73% (Cosmological Constant)

Ordinary Matter 4% (of this only about 10% luminous)

Dark Matter 23%

Neutrinos 0.1–2%