Progress in the calculation of ε' on the lattice

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Motivation for studying K→ππ Decays

- Likely explanation for matter/antimatter asymmetry in Universe, baryogenesis, requires violation of CP.
- Amount of CPV in Standard Model appears too low to describe measured M/AM asymmetry: tantalizing hint of new physics.
- Direct CPV first observed in late 90s at CERN (NA31/NA48) and Fermilab (KTeV) in $K^0\to\pi\pi$:

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}, \qquad \eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}.$$

$$\mathrm{Re}(\epsilon'/\epsilon) \approx \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{\pm}}\right|^2\right) = 16.6(2.3)\times 10^{-4}$$ (experiment)

measure of direct CPV — measure of indirect CPV

- In terms of isospin states: $\Delta I=3/2$ decay to $I=2$ final state, amplitude A_2
  $\Delta I=1/2$ decay to $I=0$ final state, amplitude A_0

$$\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$$

$$A(K^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2 e^{i\delta_2},$$
$$A(K^0 \to \pi^0\pi^0) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} - 2\sqrt{\frac{1}{3}}A_2 e^{i\delta_2}.$$

→

$$\epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

(δ_I are strong scattering phase shifts.)

- Small size of ε' makes it particularly sensitive to new direct-CPV introduced by most BSM models.

Results for ε'

[Phys.Rev.Lett. 115 (2015) 21, 212001]

- $Re(A_0)$ and $Re(A_2)$ from expt.
- Lattice values for $Im(A_0)$, $Im(A_2)$ and the phase shifts,

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$= 1.38(5.15)(4.43)\times 10^{-4}, \quad \text{(this work)}$$

$$16.6(2.3)\times 10^{-4} \quad \text{(experiment)}$$

- Total error on Re(ε'/ε) is ~3x the experimental error.
- Find reasonable consistency with Standard Model (at 2.1σ level).
- Tantalizing hint of discrepancy strong motivation for continued study!
- Error is dominated by those on A_0.

Our main focuses are therefore to:

- Increase statistics on A_0 calculation, enabling improved precision and better systematic error estimation.
- Reduce dominant NPR systematic error. Although not as vital as the above, this error is difficult to estimate and yet is relatively straightforward to improve.

Statistics increase

- Aim for 4x increase in # of measurements from present 216 within a year.
- This includes replacing existing data affected by a recently discovered RNG seeding error, although we believe the effects of this error to be small.

Resource	Million BG/Q equiv core-hours	Independent cfgs.
USQCD (BNL 512 BG/Q nodes)	50	220
RBRC/BNL (BNL 512 BG/Q nodes)	17	50
UKQCD (DIRAC 512 BG/Q nodes)	17	50
NCSA (Blue Waters)	108	380
KEK (present allocation)	25	80
KEK (512 BG/Q nodes - proposed)	72	315
Total	289	1095

Table 1: A breakdown of the various resources we intend to utilize. Note that we require 4 molecular dynamics time units per independent configuration.

- Aside from continued BG/Q running via USQCD, UKQCD and RBRC/BNL allocations, we are also planning to run on Blue Waters (Cray XE6 CPU) using CPS code built on the high-performance Grid library.

[arXiv:1512.0348 https://github.com/paboyle/Grid]

- Much additional work undergoing in optimizing measurement code and porting to Intel architectures (including KNL, KNH, etc) using Grid.
- Aim to perform all new measurements using Cori I (Haswell) and II (KNL).

NPR technique

- We use the Rome-Southampton RI-SMOM technique for the 7 independent 4-quark operators entering the calculation.

$$\mathcal{O}_i^{\mathrm{RI}}(\mu) = Z_{ij}^{\mathrm{lat}\to\mathrm{RI}}(\mu, a)\mathcal{O}_j(a)$$

$$Z_q^{-2}\mathcal{P}_{ab}\left\langle E_{ab}\mathcal{O}_i^{\mathrm{RI}}(2q)\right\rangle_{\mathrm{amputated}} = \text{tree level val.}$$

field strength renorm

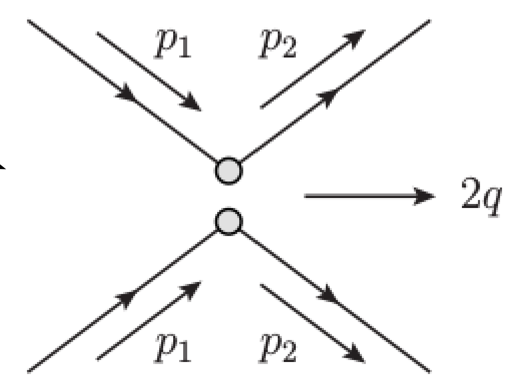
chosen projector

chosen external state
(4 quark fields with momentum injected)

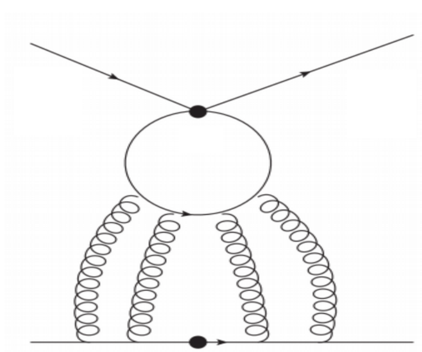
- 2 schemes: $\mathrm{SMOM}(\gamma^\mu, \gamma^\mu)$ $\mathrm{SMOM}(\not{q}, \not{q})$

(Matrices in brackets differentiate the projectors used for O_i and Z_q resp.)

- SMOM scheme defines $q^2 = p_1^2 = p_2^2 = (p_1 - p_2)^2 = \mu^2$



NB: Disconnected diagrams enter here too



statistically quite noisy (for NPR)

Inclusion of G_1

[McGlynn arXiv:1605.08807]

- Aside from traditional 7 independent 4-quark operators Q'_i, for off-shell Green's functions (e.g. in NPR) also mixing with dim-6 two-quark operator

$$G_1 = -\frac{4}{g^2}\bar{s}\gamma_\mu(1-\gamma_5)(D_\nu G_{\nu\mu})d \qquad (8,1)$$

- G_1 included in perturbative RI->MSbar matching and Wilson coeffs.
- Has not heretofore been included in lattice→RI NPR because mixing enters at one-loop and therefore can be assumed small.
- Nevertheless we can remove this potential systematic uncertainty by including G_1 explicitly in our NPR:

$$Q_i'^{\,RI} = Z_{ij}^{\text{lat}\to RI} Q_j'^{\,lat} + c_i^{\text{lat}\to RI} G_1^{\text{lat}}$$

$$G_1'^{\,RI} = d_i^{\text{lat}\to RI} Q_i'^{\,lat} + Z_{G_1}^{\text{lat}\to RI} G_1^{\text{lat}}$$

- G_1 also enters into the on-shell K→ππ amplitudes, but...

For on-shell matrix elements, *continuum* EOM for bare operators implies

$$G_1 = Q_2' + \frac{7}{3}Q_3' - \frac{1}{3}Q_5' + Q_6'$$

For renormalized Green's functions this implies

$$\langle f|G_1^{\text{RI}}(\mu)|i\rangle = s_i(\mu)\langle f|Q_i'^{\,\text{RI}}(\mu)|i\rangle$$

$$\langle f|G_1^{\mathrm{RI}}(\mu)|i\rangle = s_i(\mu)\langle f|Q_i'^{\mathrm{RI}}(\mu)|i\rangle$$

$$\begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_5 \\ s_6 \\ s_7 \\ s_8 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 7/3 \\ -1/3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + O(\alpha_s(\mu))$$

corrections from PT renorm.

(these are actually 2-loop corrections and can therefore be neglected in this work)

Note that this is a perturbative result and so should be applied at sufficiently large μ.

- Plugging this into our definitions of renormalized operators and rearranging we obtain a formula relating on-shell matrix elements of *unrenormalized lattice operators:*

$$\langle G_1^{lat}(a)\rangle = k_j(\mu,a)\langle Q_j'^{lat}(a)\rangle \;\; ; \quad k_i(\mu,a) \equiv \frac{s_i Z_{ij}^{lat\to RI}(\mu,a) - d_j^{lat\to RI}(\mu,a)}{Z_{G_1}^{lat\to RI}(\mu,a) - s_k c_k^{lat\to RI}(\mu,a)}$$

This means we never actually have to compute matrix elements of G_1!

(NB: We could have computed this with the lattice EOM instead but this would force us to use a very specific and complicated discretization of G_1)

- Plug back into definition of renormalized operators Q':

$$\langle f|Q_i'^{\,\mathrm{RI}}(\mu)|i\rangle = R_{ij}^{\mathrm{lat}\to\overline{\mathrm{MS}}}(\mu,a)\langle f|Q_i'^{\,\mathrm{lat}}|i\rangle$$

$$R_{ij}^{lat\to RI}(\mu,a) \equiv Z_{ij}^{lat\to RI}(\mu,a) + c_i^{lat\to RI}(\mu,a)k_j(\mu,a)$$

- So we can include G_1 in renormalization without needing to compute matrix elements of G_1. Only have to compute G_1 NPR coefficients $c_i^{\mathrm{lat}\to RI}$

- Given freedom of discretization, choose

staple

$$G_1^{lat}(x) = \frac{1}{2}\bar{s}_x\gamma_\mu(1-\gamma_5)[U_{x,\mu}L_{x,\mu} + L_{x-\hat{\mu},\mu}U_{x-\hat{\mu},\mu}]_{TA}d_x$$

- Now we just have to choose a projection operator and external state, and wok out the tree-level value to complete the RI-scheme definition.
- (Also in practise need to define subtracted G_1 with power-divergent mixing with lower-dimensional operators removed)

Results

On 24^3x64 DWF+I 1.78 GeV lattice, μ=2.29 GeV (γ_μ-scheme):

(G$_1$ eliminated using PT)

$$Z_q^{-2} R^{lat\to RI} = \begin{pmatrix} 0.846179(42) & & & & & & \\ & 0.9367(45) & -0.0942(51) & -0.0015(16) & -0.0026(15) & & \\ & -0.0661(16) & 0.9867(24) & -0.00738(66) & 0.01855(83) & & \\ & -0.029(14) & -0.024(15) & 0.9665(46) & -0.1440(45) & & \\ & 0.0506(53) & 0.1379(76) & -0.0700(18) & 0.7577(24) & & \\ & & & & & 0.959102(32) & -0.142791(16) \\ & & & & & -0.052603(11) & 0.703316(50) \end{pmatrix}$$

Absolute difference wrt Z without G$_1$

$$Z_q^{-2} \Delta R^{lat\to RI} = \begin{pmatrix} 0 & & & & & & \\ & -0.00090(41) & -0.0025(12) & 0.00044(20) & -0.00090(42) & & \\ & 0.00510(24) & 0.01423(65) & -0.00248(11) & 0.00512(23) & & \\ & -0.0005(13) & -0.0013(36) & 0.00023(63) & -0.0005(13) & & \\ & 0.01470(76) & 0.0410(20) & -0.00717(35) & 0.01476(73) & & \\ & & & & & 0 & 0 \\ & & & & & 0 & 0 \end{pmatrix}$$

- Effect resolvable but small as expected

NPR improvement

- PRL calculation μ=1.53 GeV renormalization scale somewhat low for reliable application of PT.
- Inverse lattice spacing a^{-1} = 1.38 GeV too small to push μ much further.
- Solution is **step-scaling**. Idea is that non-perturbative running is universal up to discretization effects. Then

$$[\alpha_s^2 = 0.0774] \qquad\qquad [\alpha_s^2 = 0.1546]$$

$$Z(2.28\ \text{GeV},\ step-scaled) = \frac{Z(2.28\ \text{GeV},\ 24\text{I})}{Z(1.33\ \text{GeV},\ 24\text{I})} Z(1.33\ \text{GeV},\ 32\text{ID})$$

Finer 24^3x64 DWF+I a^{-1}=1.78 GeV lattice

lowered coarse-lattice scale to reduce potential discretization errors

- For this analysis:
  - Include G_1 explicitly (8x8 matrix) and remove using PT only at high scale.
  - Do not take the continuum limit of the Z-factors on either lattice
  - Do not extrapolate to the chiral limit but mass dependence typically tiny for SMOM schemes

Scheme for estimating sys. error

- Dominant NPR error arises from use of 1-loop PT for RI→MSbar matching
- To estimate size of truncation errors look at spread of differences between

$$R_{ij}^{\mathrm{lat}\to\overline{\mathrm{MS}}} = R_{ik}^{\mathrm{RI}\to\overline{\mathrm{MS}}} Z_{kj}^{\mathrm{lat}\to\mathrm{RI}}$$

which would be independent of the RI-scheme if PT were 100% accurate.

- If we assume missing NNLO contributions roughly the same size for the two schemes but with potentially opposite signs then

$$0 \le |R_{ij}^{lat\to\overline{\mathrm{MS}},\ \mathrm{via}\,\not{q},\not{q}} - R_{ij}^{lat\to\overline{\mathrm{MS}},\ \mathrm{via}\,\gamma^\mu,\gamma^\mu}| \lesssim 2\Delta R^{\mathrm{NNLO}}$$

- To estimate fractional error on renormalized operators

$$(R^{lat\to\overline{\mathrm{MS}},\ \mathrm{via}\,\not{q},\not{q}} - R^{lat\to\overline{\mathrm{MS}},\ \mathrm{via}\,\gamma^\mu,\gamma^\mu})\vec{Q}'_{\mathrm{lat}}$$

$$= \left(1 - \frac{R^{lat\to\overline{\mathrm{MS}},\ \mathrm{via}\,\gamma^\mu,\gamma^\mu}}{R^{lat\to\overline{\mathrm{MS}},\ \mathrm{via}\,\not{q},\not{q}}}\right) R^{lat\to\overline{\mathrm{MS}},\ \mathrm{via}\,\not{q},\not{q}}\vec{Q}'_{\mathrm{lat}}$$

Ratio of matrices can be compared to unity

PRELIMINARY

Op idx.	$\mu = 1.32$ GeV 24I	$\mu = 1.33$ GeV 32ID	$\mu = 1.53$ GeV 32ID	$\mu = 2.29$ GeV 24I	$\mu = 2.29$ GeV stepscaled no G_1	$\mu = 2.29$ GeV stepscaled with G_1
(1,1)	0.06076(15)	0.063454(63)	0.05978(13)	0.036954(41)	0.03948(16)	0.03948(16)
(2,2)	0.203(19)	0.204(15)	0.300(68)	0.0795(70)	0.080(33)	0.092(35)
(2,3)	0.310(21)	0.317(16)	0.363(76)	0.1486(59)	0.153(36)	0.161(42)
(2,5)	0.0120(48)	0.0076(42)	0.030(22)	0.0033(23)	0.0083(89)	0.0086(93)
(2,6)	0.0120(42)	0.0005(31)	0.015(20)	0.0039(15)	0.0074(53)	0.0081(77)
(3,2)	0.283(22)	0.268(15)	0.264(87)	0.1547(42)	0.143(23)	0.174(26)
(3,3)	0.391(25)	0.414(17)	0.44(11)	0.2207(39)	0.238(25)	0.297(32)
(3,5)	0.0012(59)	0.0002(34)	0.019(27)	0.0077(13)	0.0057(59)	0.0017(66)
(3,6)	0.0128(62)	0.0264(43)	0.008(27)	0.0190(11)	0.0247(46)	0.0090(68)
(5,2)	0.118(70)	0.094(53)	0.24(25)	0.037(24)	0.07(10)	0.06(11)
(5,3)	0.113(76)	0.073(62)	0.26(30)	0.026(20)	0.08(11)	0.09(13)
(5,5)	0.006(20)	0.006(16)	0.076(88)	0.0318(80)	0.024(30)	0.023(31)
(5,6)	0.023(18)	0.019(12)	0.046(79)	0.0014(50)	0.033(19)	0.027(26)
(6,2)	0.239(28)	0.205(28)	0.19(17)	0.0957(84)	0.048(63)	0.033(74)
(6,3)	0.404(34)	0.347(37)	0.25(20)	0.1885(90)	0.101(77)	0.075(99)
(6,5)	0.0106(80)	0.0174(98)	0.039(62)	0.0028(26)	0.044(20)	0.031(23)
(6,6)	0.0810(100)	0.0740(93)	0.016(56)	0.0303(23)	0.012(15)	0.062(21)
(7,7)	0.00461(21)	0.005040(92)	0.006154(96)	0.001631(63)	0.00185(24)	0.00185(24)
(7,8)	0.002132(46)	0.003396(25)	0.002073(59)	0.001567(12)	0.002002(43)	0.002002(43)
(8,7)	0.02221(12)	0.024916(60)	0.024131(91)	0.018826(21)	0.02260(20)	0.02260(20)
(8,8)	0.127052(87)	0.133903(52)	0.12284(25)	0.080743(12)	0.08705(11)	0.08705(11)

Table 7: The non-zero elements of $|(1 - Z/Z')|$ where Z and Z' are the two lattice$\rightarrow \overline{\mathrm{MS}}$ conversion matrices

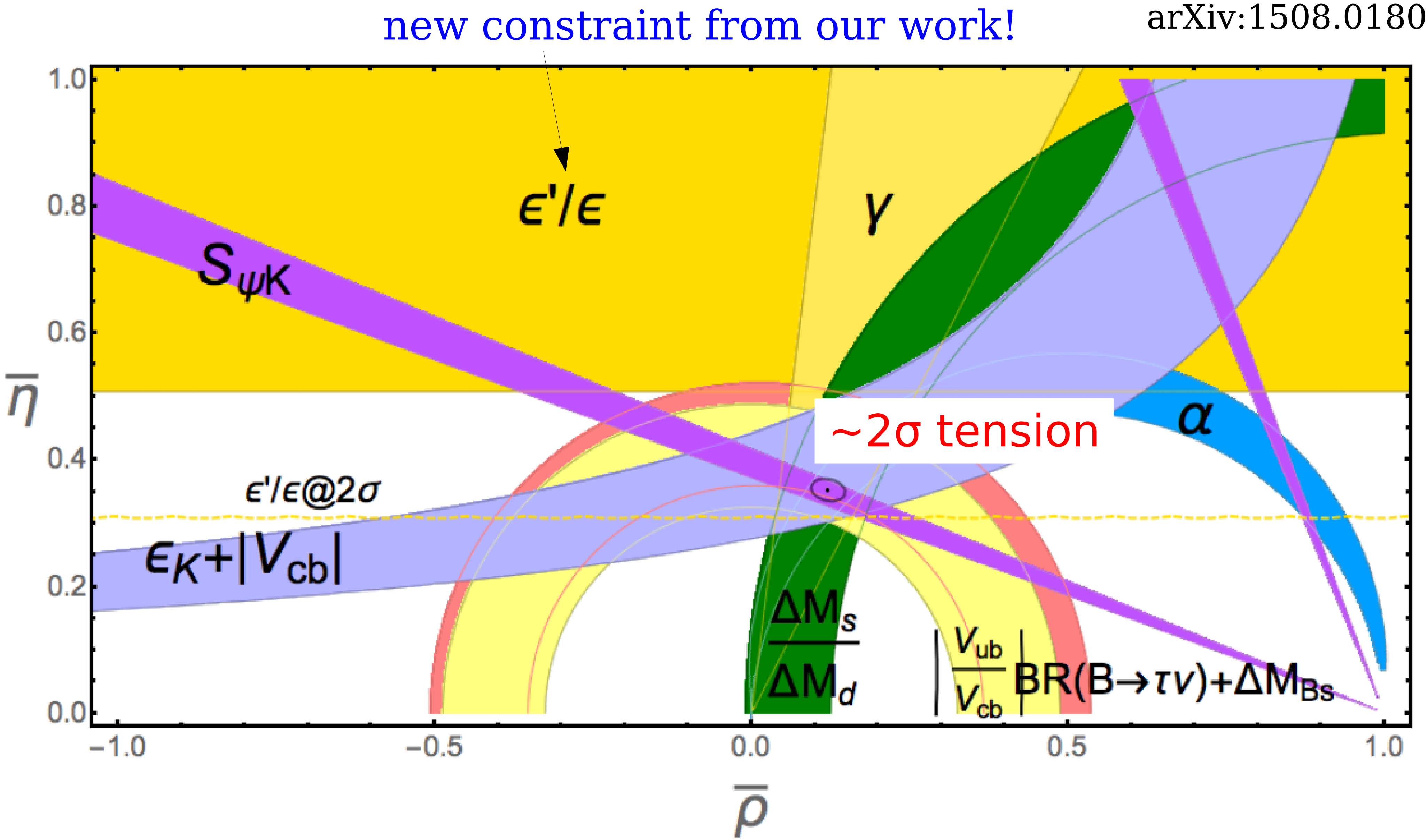
- 3,3 qq-scheme 1-loop term O(25%) vs O(<5%) for majority of other elements.
  - Suggests NNLO correction will also be abnormally large as suggested by data.
  - As Q_3' contribution to result small, ignore this outlier.
- Largest remaining elements suggest O(15%) truncation error at 1.53 GeV reduces to O(8%), roughly consistent with change in α_s^2 and with 7% truncation error we assigned to continuum A_2 for which μ=3.0 GeV.

Conclusions and Outlook

- Error on ε'/ε dominated by error on A_0.
- Expect 4x increase in number of configurations on ~1 year timescale leading to ~2x decrease in dominant, stat. error on A_0.
- Intend to replace data affected by (seemingly minor) RNG error.
- Significant effort to optimize code and port to upcoming HPC architectures (KNL, KNH).
- G_1 operator that mixes with four-quark operators at 1-loop now fully included in calculation. Effect seems to be %-level as expected.
- Step-scaled renormalization factors increase μ from 1.53 GeV to 2.29 GeV while simultaneously lowering low scale to 1.33 GeV, reducing potential discretization errors in NPR.
- Preliminary analysis suggests 15% scheme-matching PT truncation systematic will be reduced to O(8%), roughly in line with the scaling of α_s^2.
- With increase of μ we might expect similar reduction in O(12%) PT truncation error on Wilson coefficients; analysis forthcoming.

- ε' also provides a new horizontal band constraint on CKM matrix:

[Lehner et al arXiv:1508.01801]



Overview of calculation

- At energy scales μ«M_W, K→ππ decays use weak EFT:

10 effective four-quark operators

$$H_W^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{j=1}^{10} [z_j(\mu) + \tau y_j(\mu)] Q_j$$

perturbative Wilson coeffs.

$$\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} = 0.0014606 + 0.00060408i$$

Imaginary part solely responsible for CPV
(everything else is pure-real)

LL finite-volume correction

renormalization matrix (mixing)

$$A^I = F \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \sum_{i=1}^{10} \sum_{j=1}^{7} \left[(z_i(\mu) + \tau y_i(\mu))\, Z_{ij}^{\mathrm{lat}\to\overline{\mathrm{MS}}} M_j^{I,\,\mathrm{lat}} \right]$$

$M_j^{I,lat} = \langle (\pi\pi)_I | Q_j | K \rangle$ (lattice)

- Operators must be renormalized into same scheme as Wilson coeffs: Use RI-(S)MOM NPR and perturbatively match to MSbar at high scale.

Summary of RBC/UKQCD calculations

[Phys.Rev. D91 (2015) no.7, 074502]

- A_2 computed on RBC/UKQCD 64^3x128 and 48^3x96 2+1f Mobius DWF ensembles with the Iwasaki gauge action.
- a^{-1}= 2.36 GeV and 1.73 GeV resp - continuum limit taken.
- 10% and 12% total errors on Re(A_2) and Im(A_2) resp.
- Statistical errors sub-percent, dominant systematic errors due to truncation of PT series in computation of renormalization and Wilson coefficients.

[Phys.Rev.Lett. 115 (2015) 21, 212001]

- A_0 computed on 32^3x64 Mobius DWF ensemble with Iwasaki+DSDR gauge action. G-parity BCs in 3 directions to give physical kinematics.
- Single, coarse lattice with a^{-1}= 1.38 GeV but large physical volume to control FV errors.
- 21% and 65% stat errors on Re(A_0) and Im(A_0) due to disconn. diagrams and, for Im(A_0) a strong cancellation between Q_4 and Q_6.
- Dominant, 15% systematic error is due again to PT truncation errors exacerbated by low renormalization scale 1.53 GeV.