Estimating excited-state contamination using experimental data

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July 29th, 2016

MTH and Harvey B. Meyer, to appear

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JG|U

The nucleon axial charge is a basic ingredient in describing neutron beta decay

$$g_{A,\mathrm{expt}} = 1.2723 \pm 0.0023$$

Particle data group (2015 update)

In addition, g_A ...

parametrizes the nucleon-pion coupling in ChPT

reveals how quark spin contributes to nucleon spin

determines how nuclear properties vary with quark mass

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$$\langle N, \mathbf{p}, \sigma' | \overline{\mathcal{Q}} \tau^a \gamma^\mu \gamma_5 \mathcal{Q} | N, \mathbf{p}, \sigma \rangle = g_A \overline{u}_{\sigma'}(\mathbf{p}) \tau^a \gamma^\mu \gamma_5 u_\sigma(\mathbf{p})$$

Determining the QCD prediction for this benchmark quantity will improve our understanding of how nuclear structure emerges from the underlying theory

Nuclear charges are typically accessed from Lattice QCD (LQCD) by constructing ratios of correlators

$$R(T,\tau) \equiv \frac{\langle \mathcal{O}(T) A(\tau) \overline{\mathcal{O}}(0) \rangle}{\langle \mathcal{O}(T) \overline{\mathcal{O}}(0) \rangle} = g_A + b_1 \left(e^{-\Delta E_1 (T-\tau)} + e^{-\Delta E_1 \tau} \right) + \cdots$$

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Can excited state contamination explain why lattice values of g_A tend to be below the experimental value?

At leading order in Chiral Perturbation Theory (ChPT) the value of excited state contamination is universal (interpolator-independent) and positive.

Brian Tiburzi, *Phys. Rev.* D91, 094510 (2015)
Oliver Bär, arXiv:1606.09385

Is excited state contamination well-described by LO ChPT?

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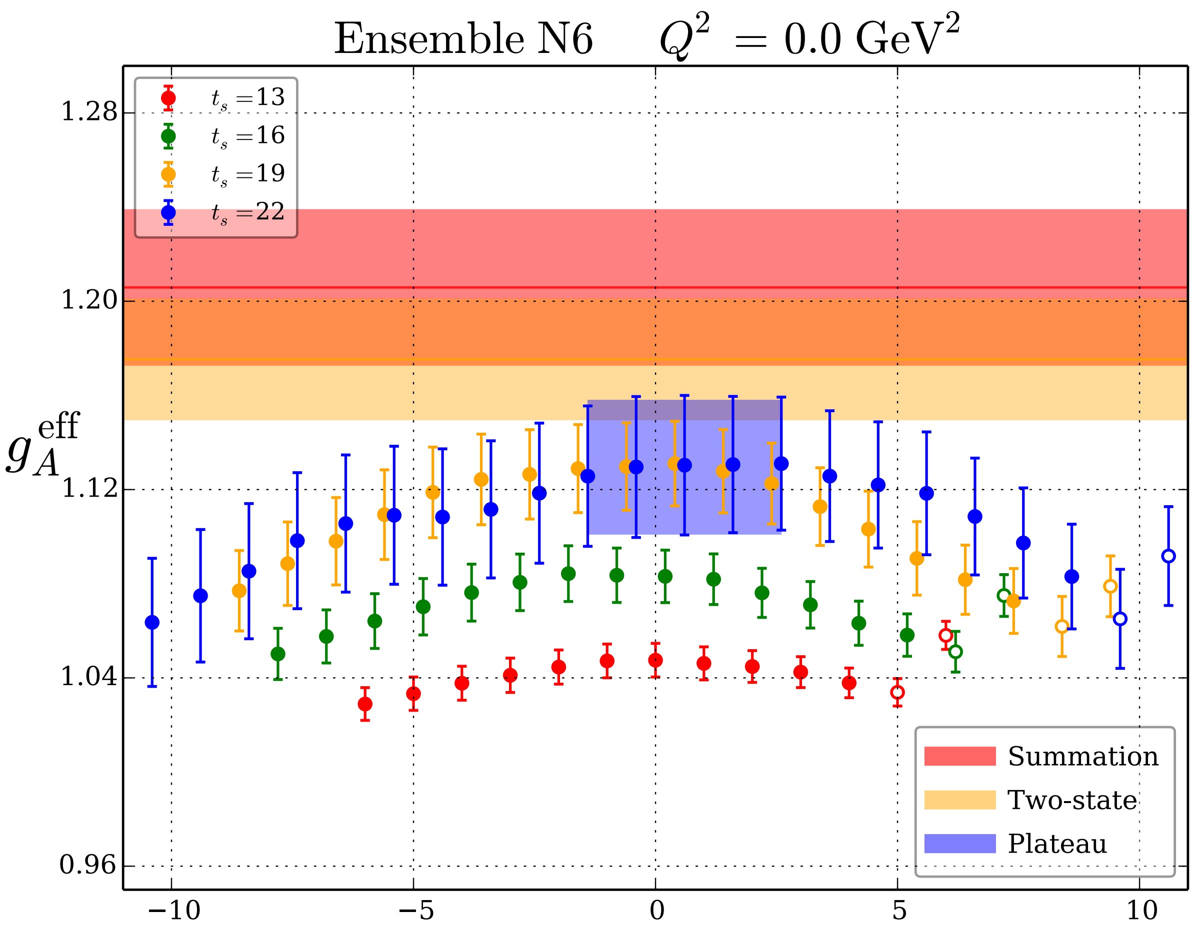
LO ChPT predicts positive excited state contamination



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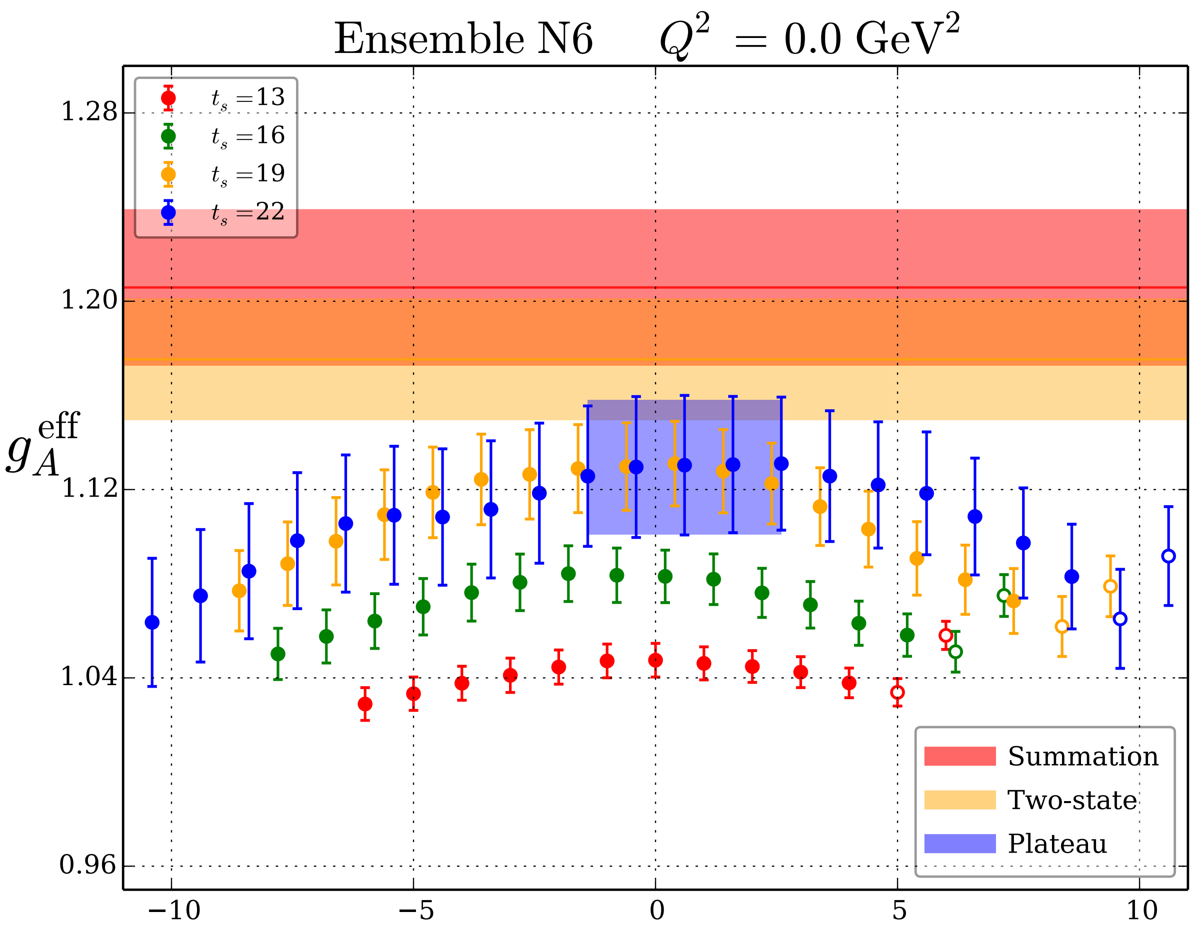

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and physical pion masses…

How many states are needed to get an accurate estimate of

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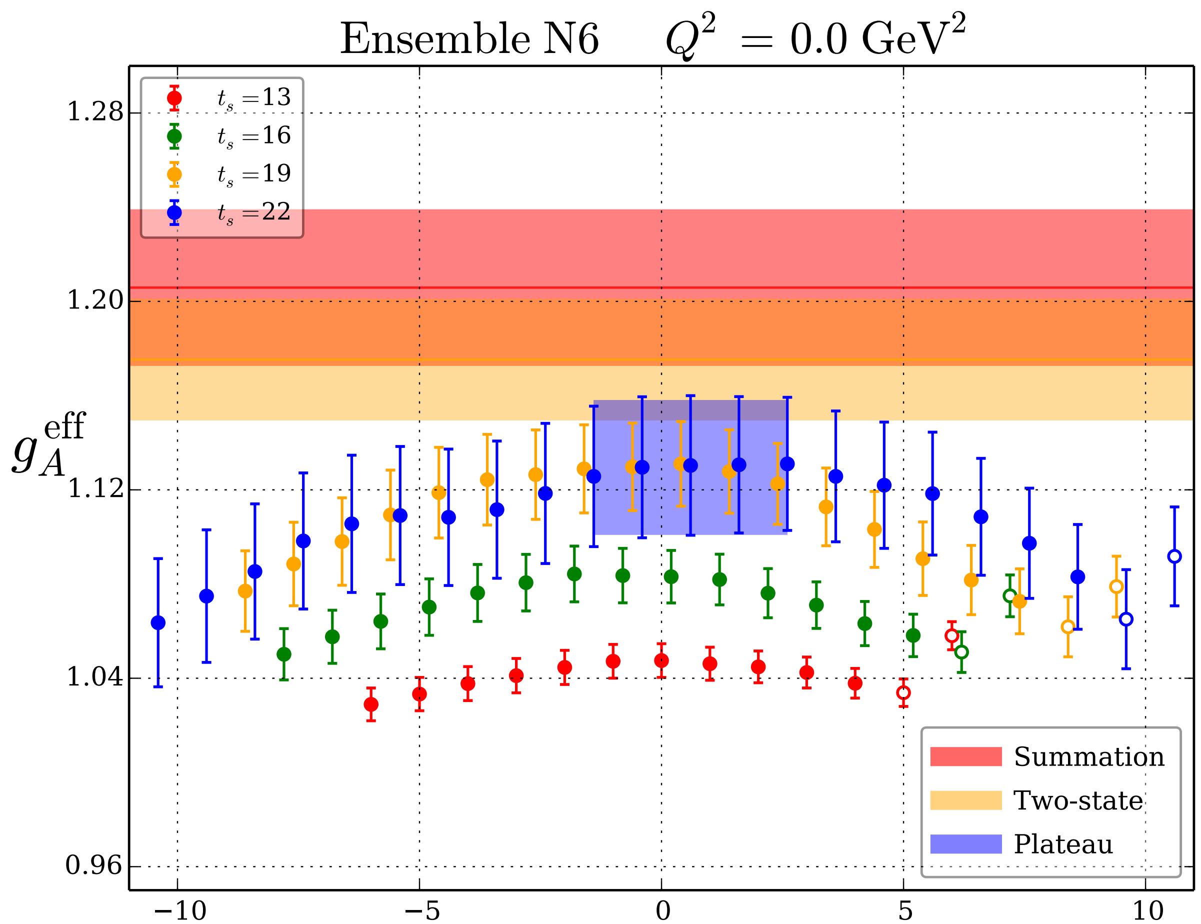

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Using the LO ChPT prediction

$T(\text{fm})$	n_{states} (5%)	n_{states} (3%)
2	2	0
1.5	2	5
1	7	10

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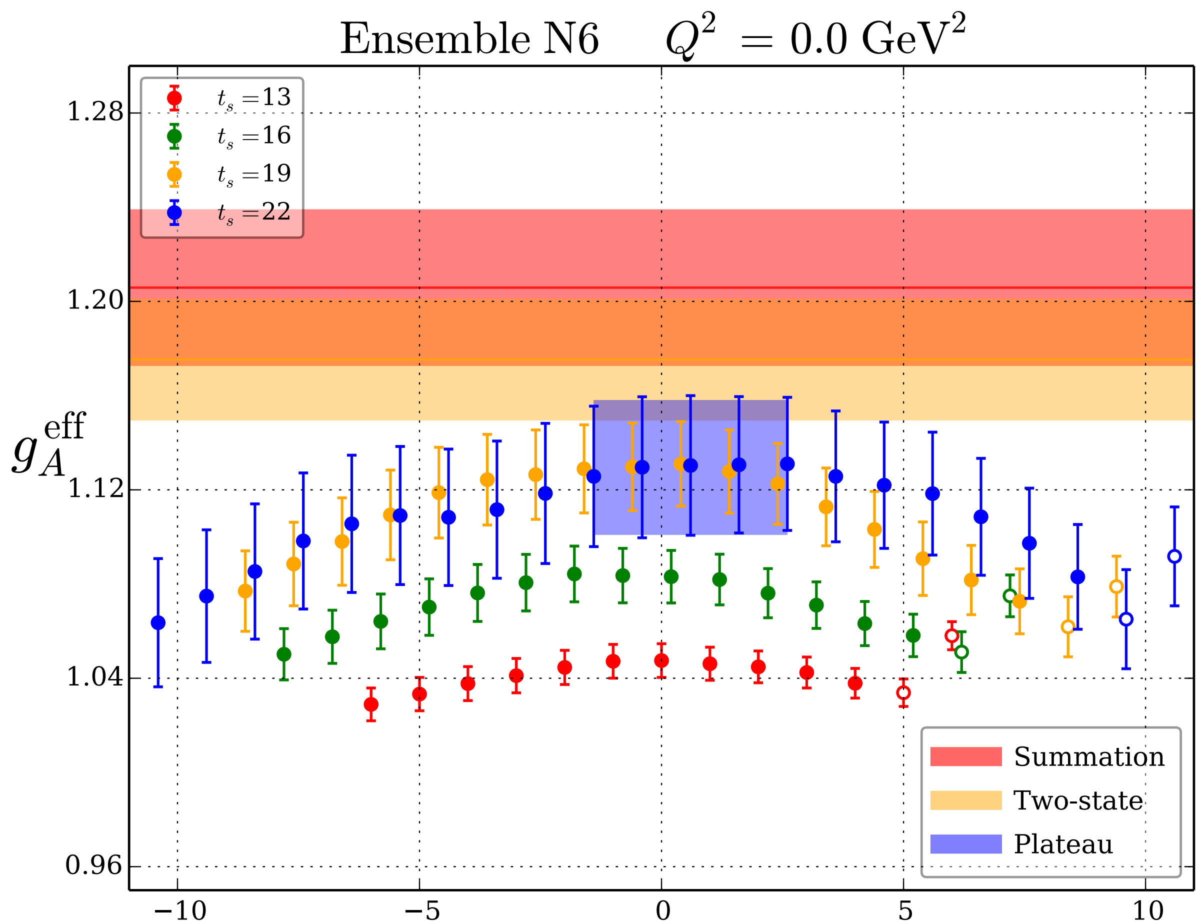

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If higher finite-volume states are important, then the Roper could also play an important role.

To precisely define the excited state contamination we introduce

$$R(T,\tau) = \frac{\langle \mathcal{O}(T) A(\tau) \overline{\mathcal{O}}(0) \rangle}{\langle \mathcal{O}(T) \overline{\mathcal{O}}(0) \rangle} = g_A + \mathcal{E}(T,\tau)$$

$$A(\tau) \equiv 2i \int d\mathbf{x}\ A_z^3(\tau,\mathbf{x})$$

$$\mathcal{O}(\tau) \equiv \int d\mathbf{x}\ \frac{\overline{u}_\uparrow(0)}{\sqrt{m_N}} \cdot N(\tau,\mathbf{x})$$

$$\mathcal{E}(T,\tau) = \sum_{n=1}^{\infty} b_n \left[e^{-\Delta E_n (T-\tau)} + e^{-\Delta E_n \tau} \right] + c_n e^{-\Delta E_n T} + \cdots$$

$$\Delta E_n = E_n(L) - m_N + \mathcal{O}(e^{-M_\pi L})$$

$$b_n = \frac{\langle 0 | \mathcal{O}(0) | n, L \rangle \langle n, L | A(0) | N, L \rangle}{\langle 0 | \mathcal{O}(0) | N, L \rangle}$$

finite-volume excited states

finite-volume one-particle state

$c_n =$ **more complicated form and smaller contribution**

We aim to estimate finite-volume energies and matrix elements using experimental scattering data

Finite-volume energies

$$\Delta E_n = E_n(L) - m_N + \mathcal{O}(e^{-M_\pi L})$$

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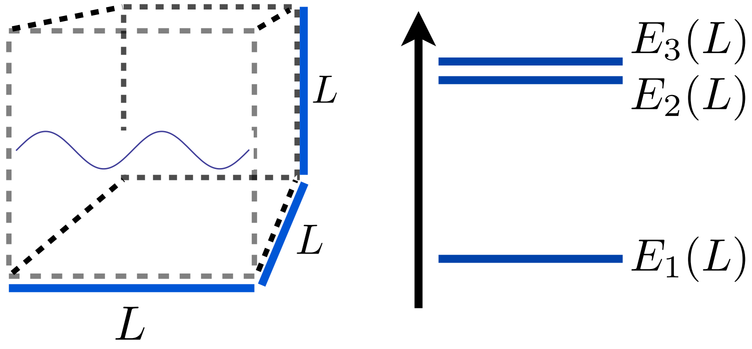

cubic, spatial volume (extent L)

periodic boundary conditions

$$\vec{p} \in (2\pi/L)\mathbb{Z}^3$$

L large enough to drop $e^{-M_\pi L}$

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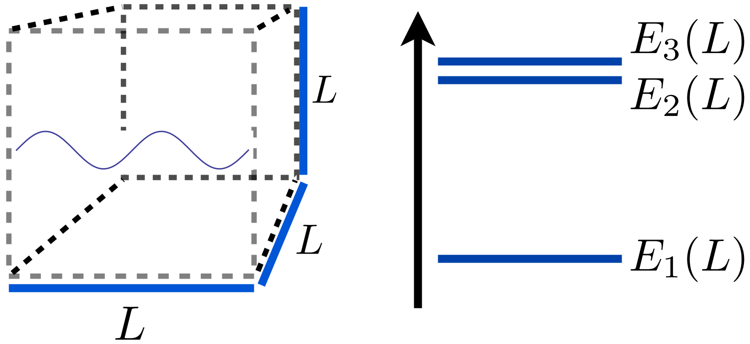

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Isospin and parity are good quantum numbers in finite-volume

$$I = 1/2\,, \quad P = +$$

Can also project to a finite-volume irrep

$$G_1^+ = \left(J = \frac{1}{2}\right) \oplus \left(J = \frac{5}{2}\right) \oplus \cdots$$

neglecting $\ell \geq 3 \rightarrow J \geq 5/2$

we find…

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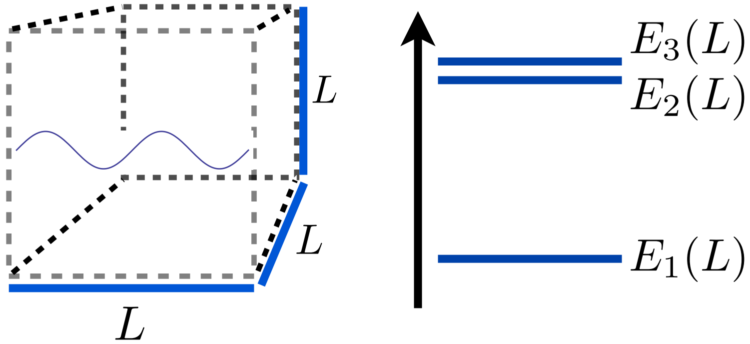

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Single channel quantization condition

$I(J^P) = {}^1/_2\,({}^1/_2{}^+)$

scattering phase shift

$$\delta(E_n) + \phi(E_n, L) = 0$$

known geometric function

Lüscher, M. *Nucl. Phys* B354, 531-578 (1991)

Beane et al., *Nucl. Phys.* A747, 55 (2005)

Li and Liu, Phys. Rev. D87, 014502 (2013)

Briceño, *Phys. Rev.* D 89, 074507 (2014)

M. Göckeler et al., *Phys. Rev.* D86 094513 (2012)

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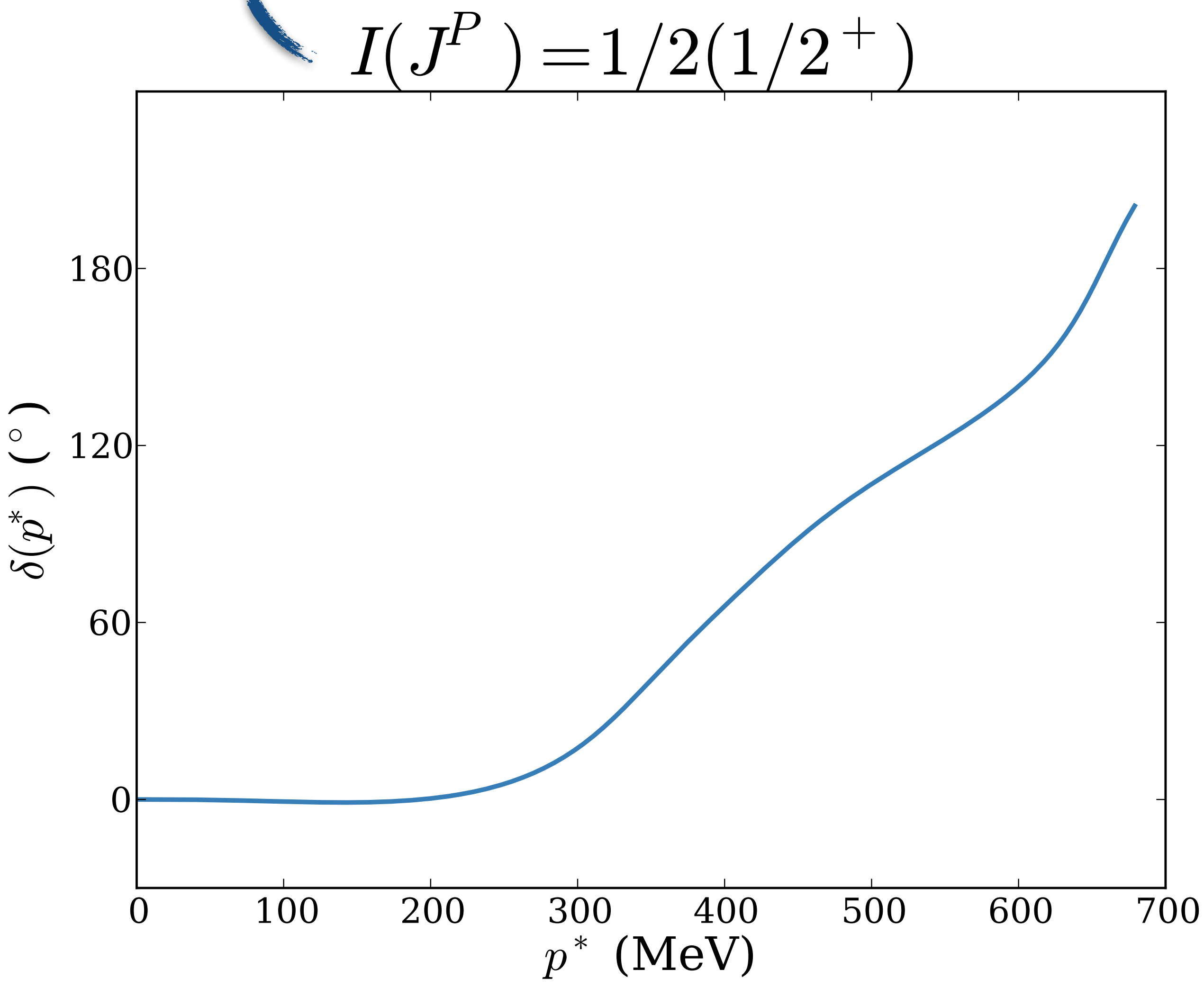


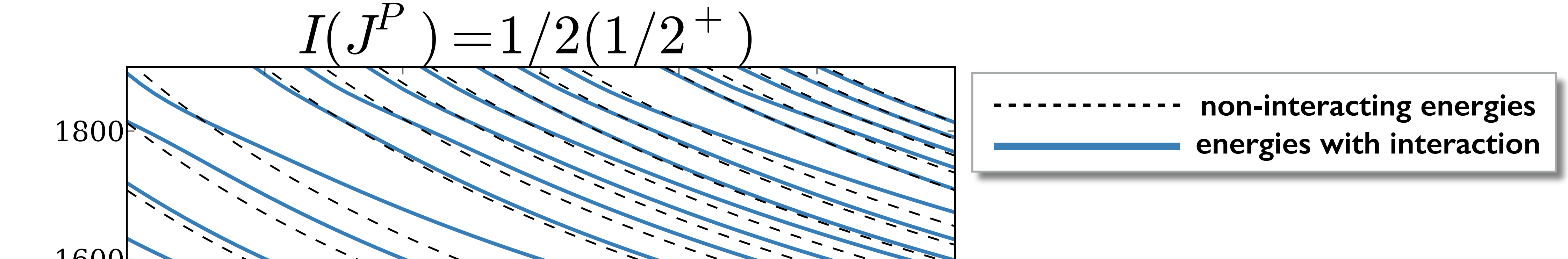
Phase shift data is determined using experimental data from CERN, JLab, LAMPF, TRIUMF, PSI

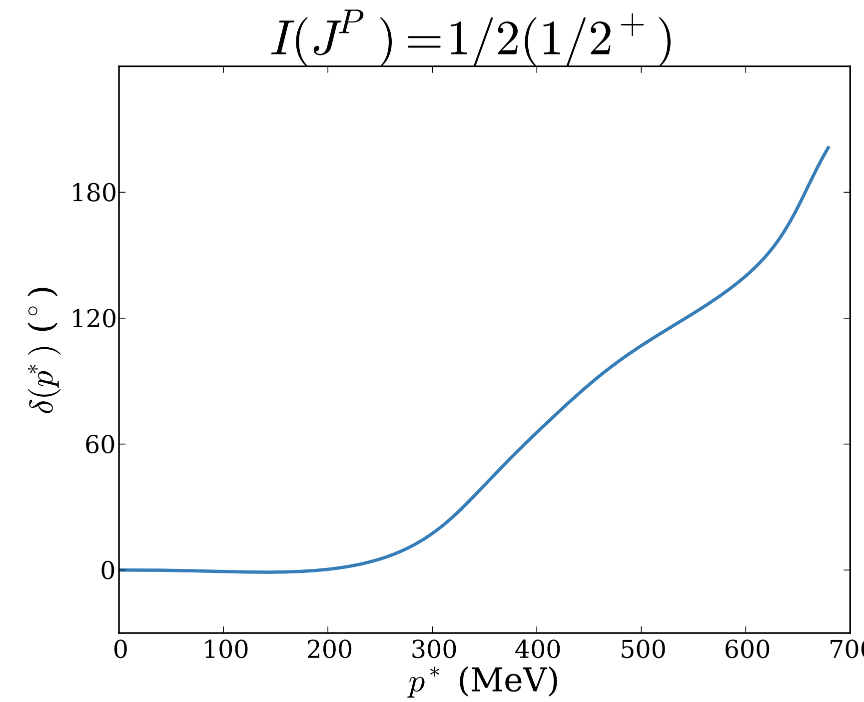
The data base is described in
Arndt et. al., *Phys. Rev.* C74, 045205 (2006)

The solution used here (WI08) is described in
Workman et. al., *Phys. Rev.* C86, 035202 (2012)

The fits were performed to eigenvalues of the S matrix

$N\pi$ finite-volume states





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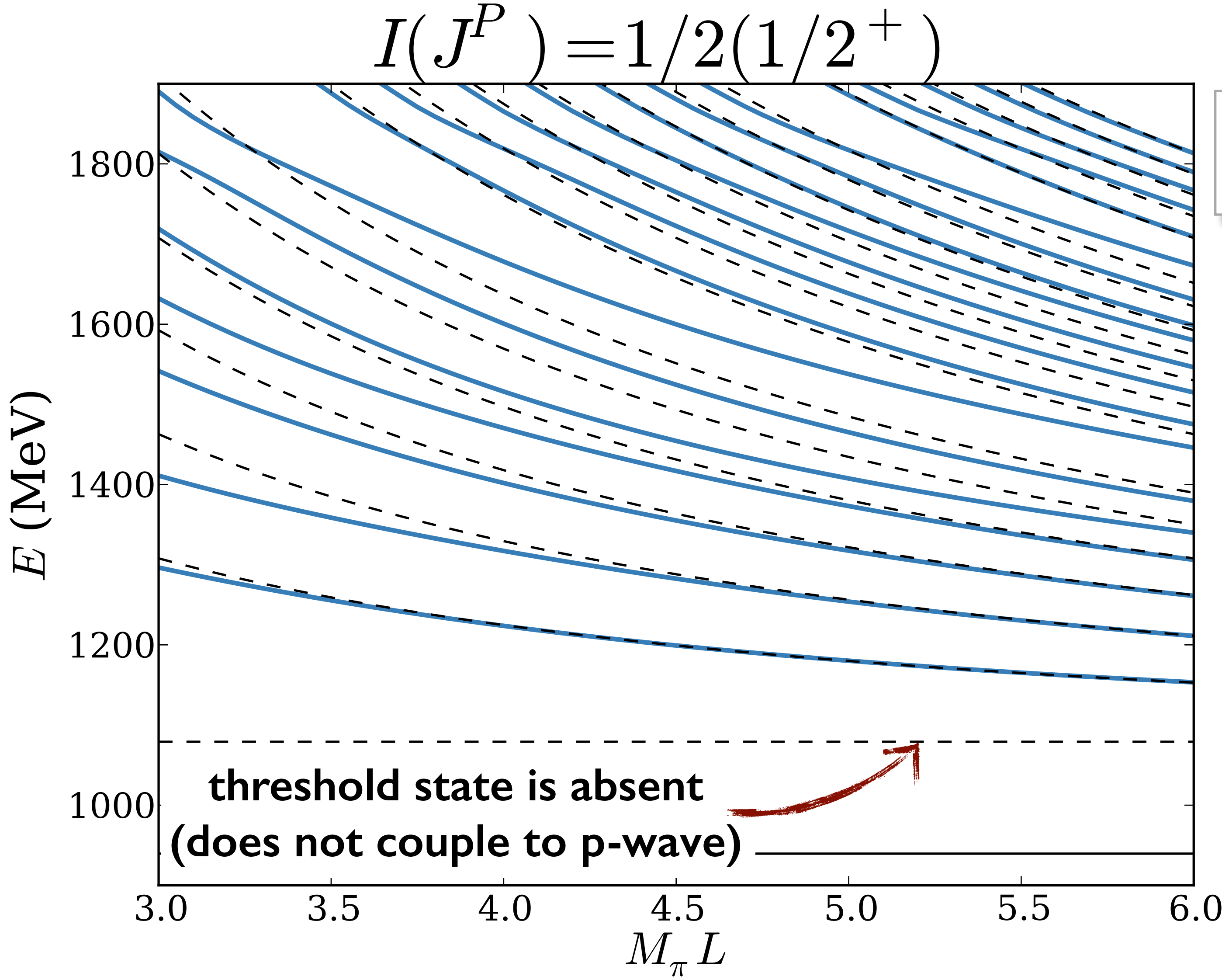

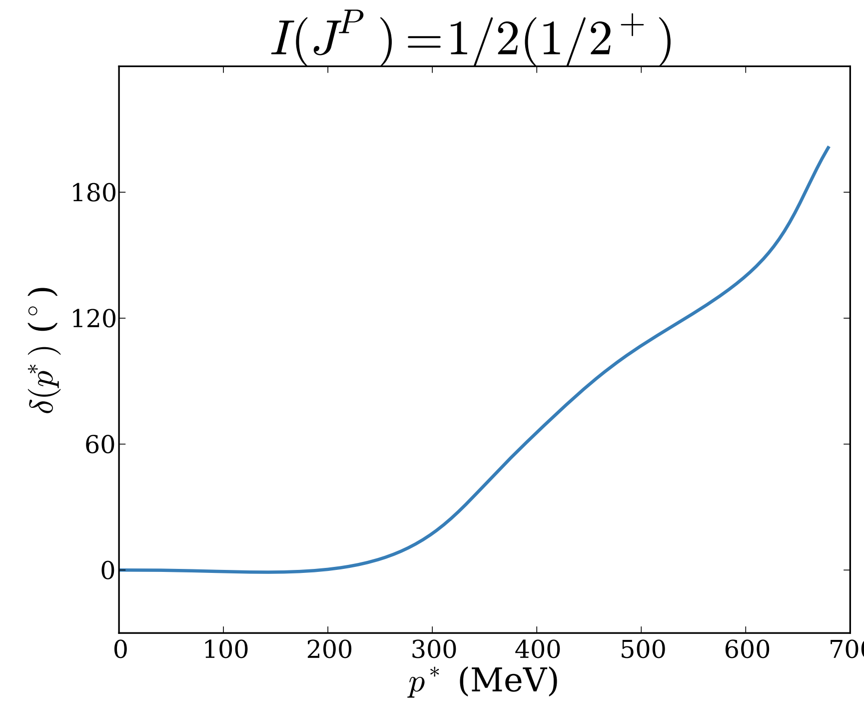

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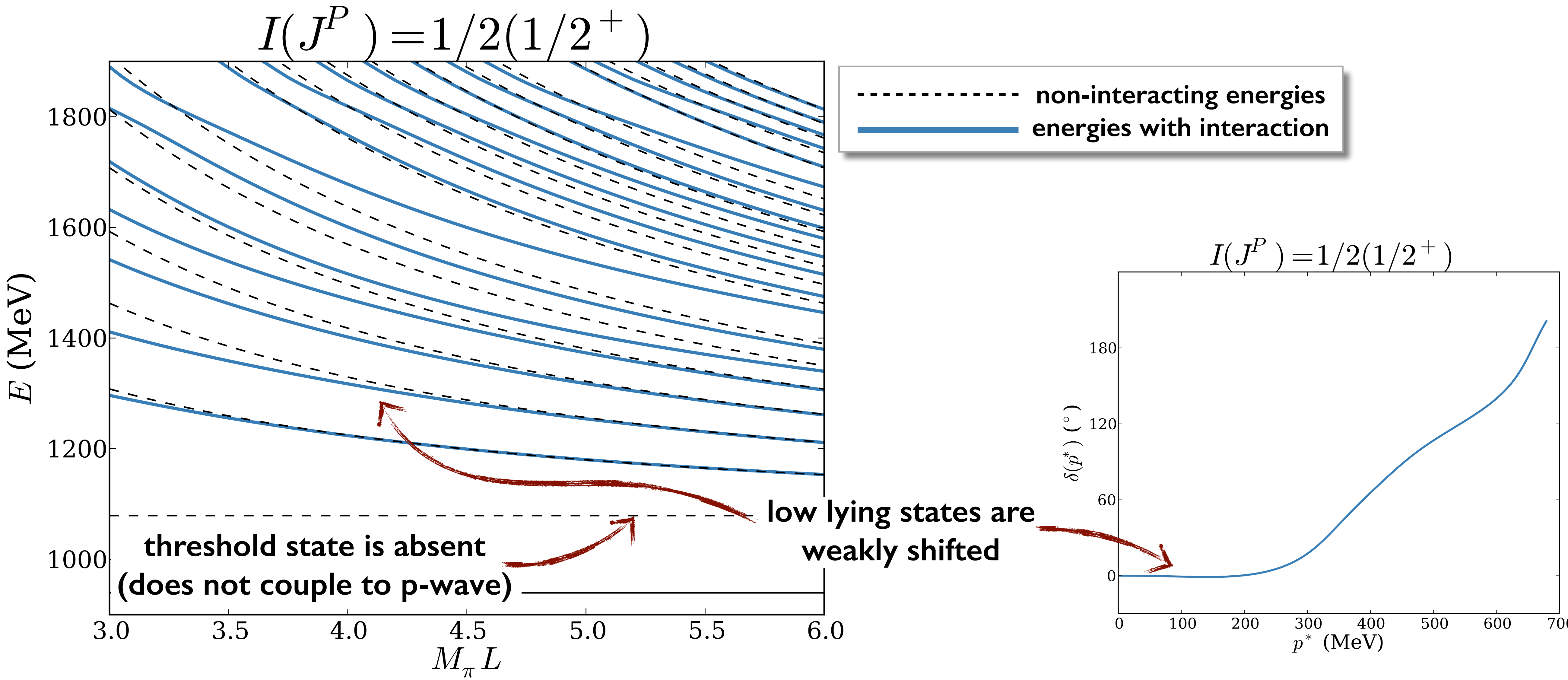

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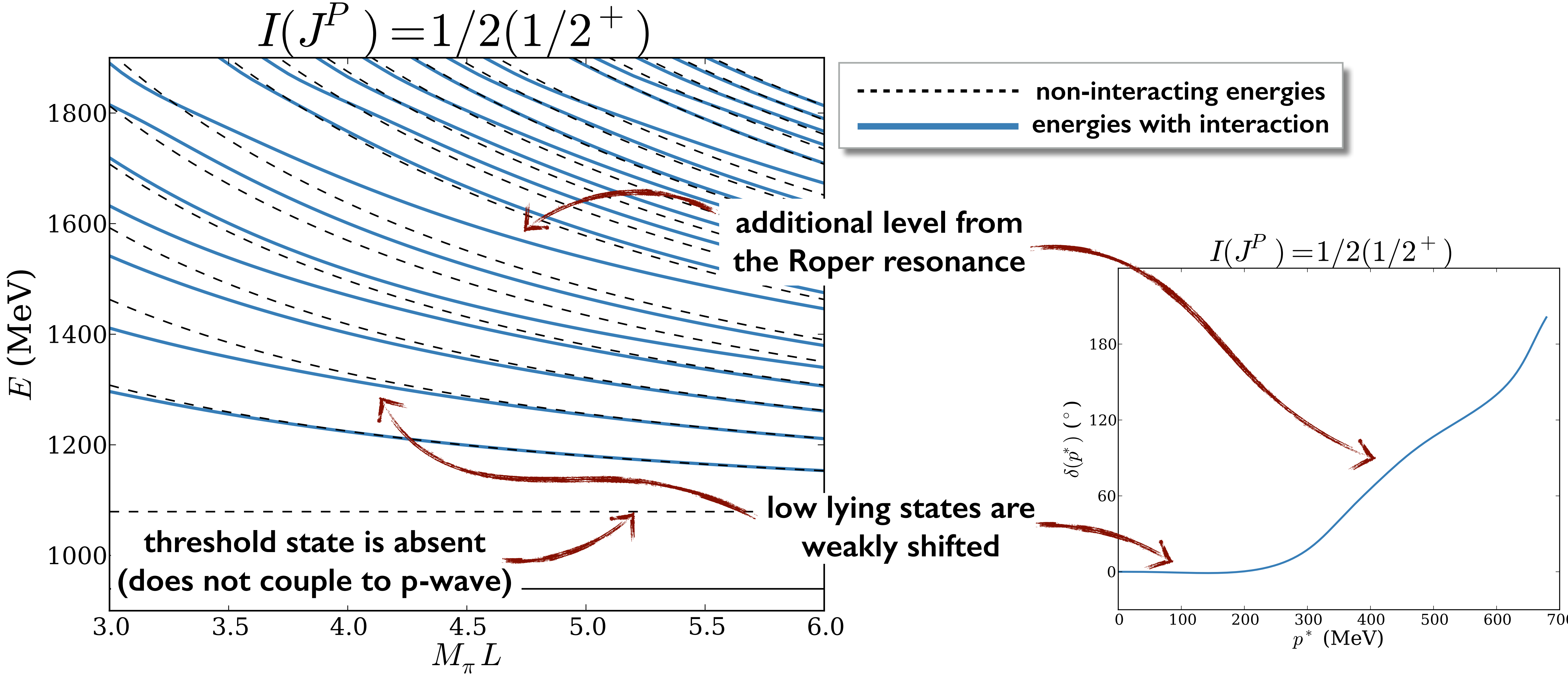

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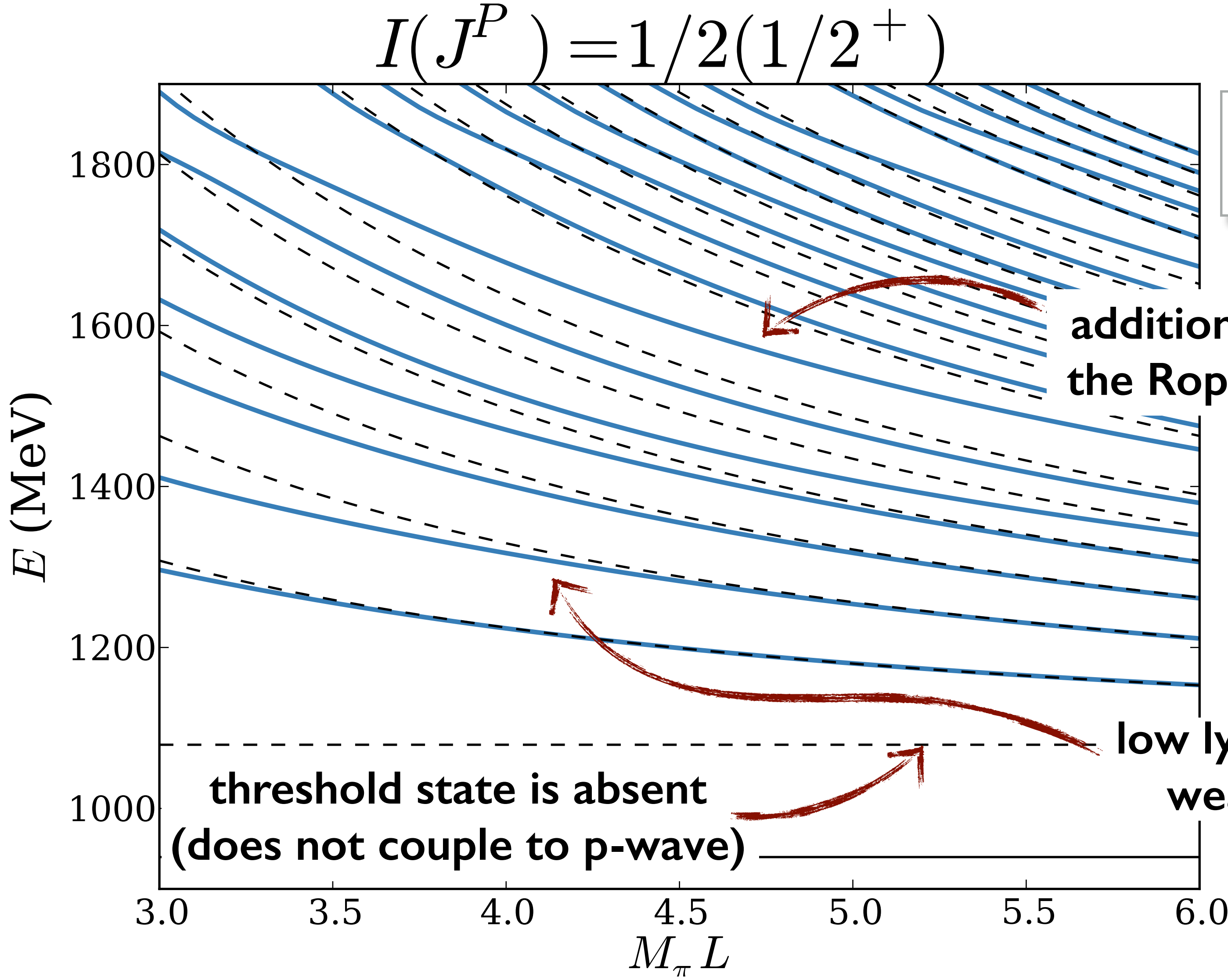
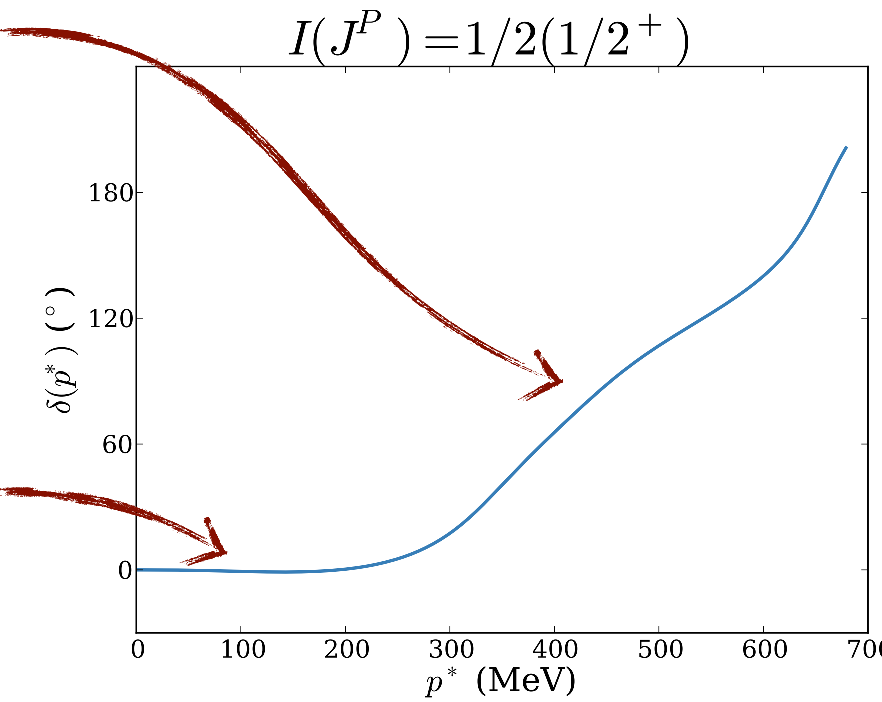


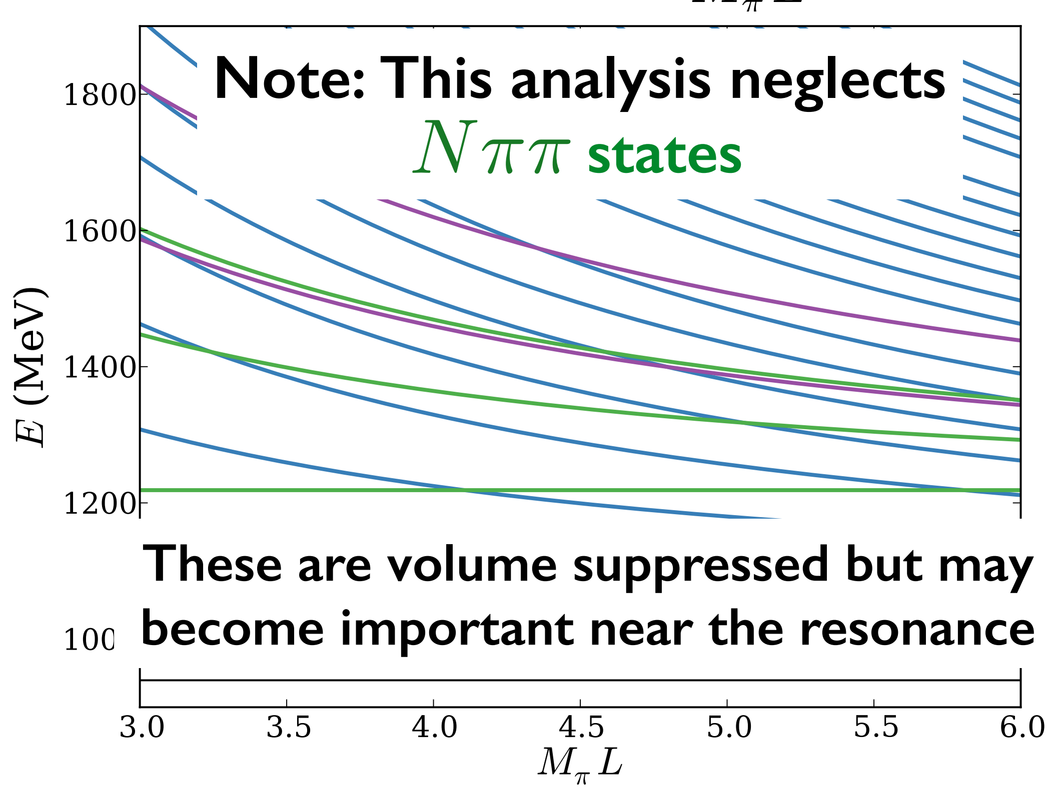
non-interacting energies
energies with interaction

additional level from the Roper resonance

low lying states are weakly shifted

threshold state is absent (does not couple to p-wave)





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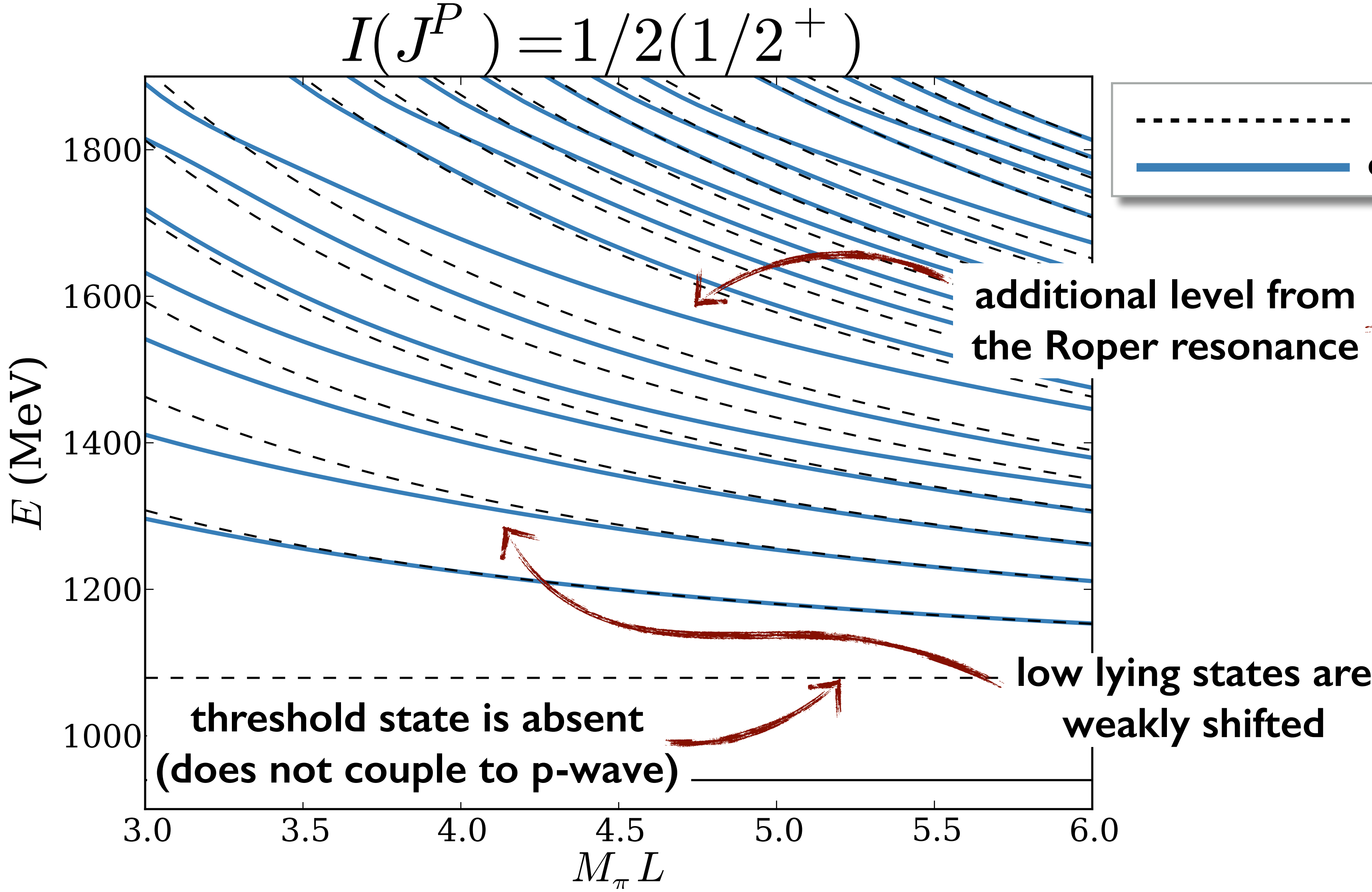

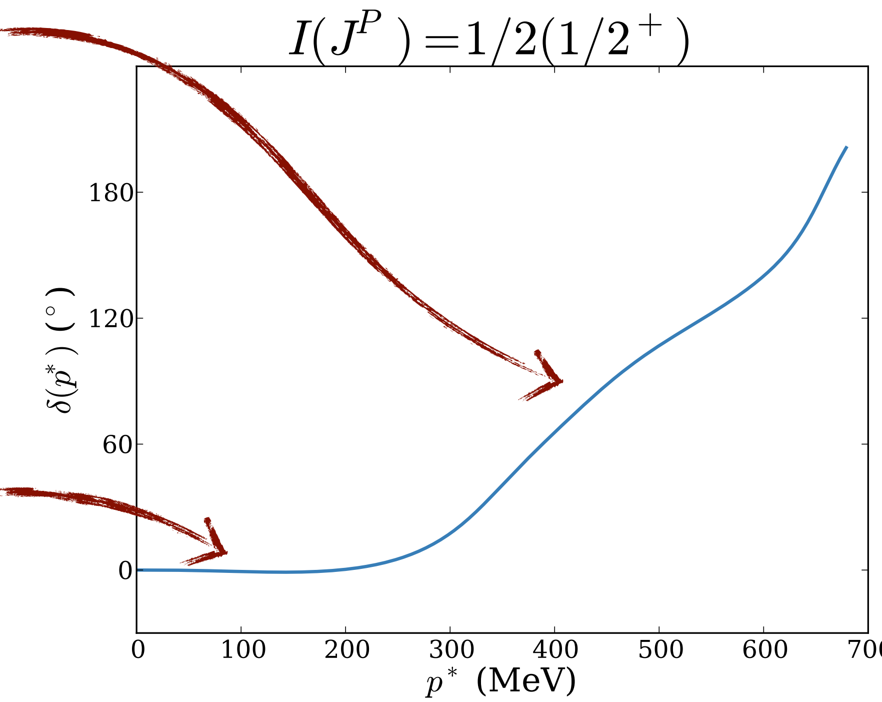

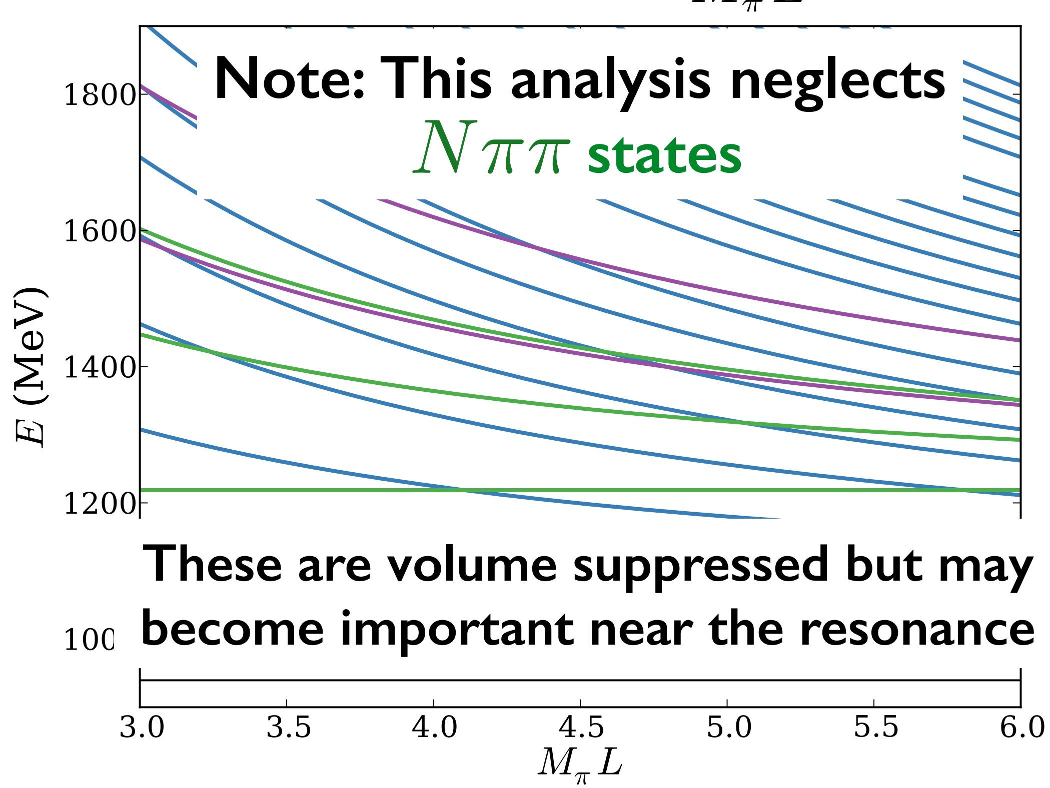

Three-particle states can be included with generalized three-particle formalism

MTH and Sharpe, 1408.5933, 1504.04248

Briceño, MTH, Sharpe, underway

Note: LO ChPT uses non-interacting two-particle energies

$$R(T,\tau) = \frac{\langle \mathcal{O}(T) A(\tau) \overline{\mathcal{O}}(0) \rangle}{\langle \mathcal{O}(T) \overline{\mathcal{O}}(0) \rangle} = g_A + \mathcal{E}(T,\tau)$$

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✓

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We now turn to the coefficients of excited state exponentials

$$b_n = \frac{\langle 0|\mathcal{O}|n,L\rangle \langle n,L|A|N,L\rangle}{\langle 0|\mathcal{O}|N,L\rangle}$$

$$c_n = -\frac{\langle 0|\mathcal{O}|n,L\rangle \langle n,L|\overline{\mathcal{O}}|0\rangle}{\langle 0|\mathcal{O}|N,L\rangle \langle N,L|\overline{\mathcal{O}}|0\rangle} + \frac{\langle 0|\mathcal{O}|n,L\rangle \langle n,L|A|n,L\rangle \langle n,L|\overline{\mathcal{O}}|0\rangle}{\langle 0|\mathcal{O}|N,L\rangle \langle N,L|\overline{\mathcal{O}}|0\rangle}$$

Finite-volume matrix elements

One can rewrite b_n using extensions of the Lellouch-Lüscher formalism

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Lellouch-Lüscher factor

$$\mathcal{C}(E_n, L) = 4\pi^2 q^3 e^{-2i\delta}\left(q\frac{\partial\phi}{\partial q} + p^*\frac{\partial\delta}{\partial p^*}\right)^{-1} \quad \left(q \equiv \frac{p^* L}{2\pi}\right)$$

(Neglects higher angular momenta and three-particle states)

Lellouch and Lüscher, *Commun. Math. Phys.* 219, 31 (2001)
Briceño, MTH, Walker-Loud, *Phys. Rev.* D91, 034501 (2015)
Agadjanov et al., (2014), Nucl.Phys. B886, 1199 (2014).
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In the free theory this counts the degeneracy of finite-volume states

(Neglects higher angular momenta and three-particle states)

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Can use this result to evaluate b_n in ChPT

$$\mathcal{C}(E_n, L) = \nu_n + \cdots$$

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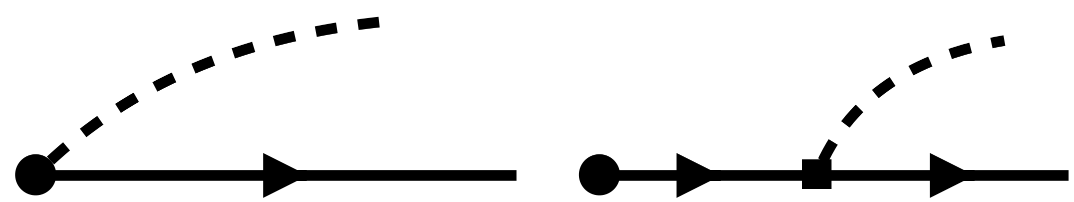
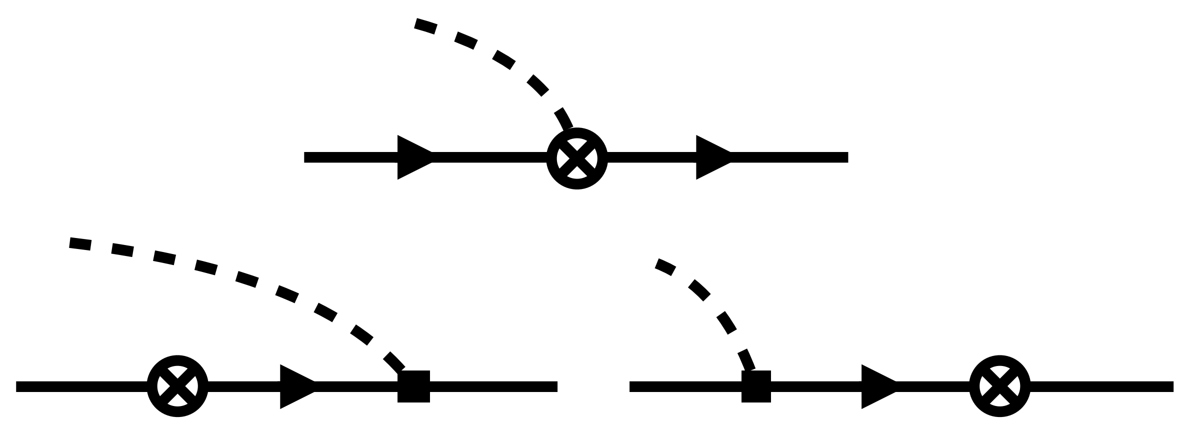
$\langle 0|\mathcal{O}|N\pi, \mathrm{in}\rangle =$  $+ \cdots$

$\langle N\pi, \mathrm{out}|A_z^3|N\rangle =$  $+ \cdots$

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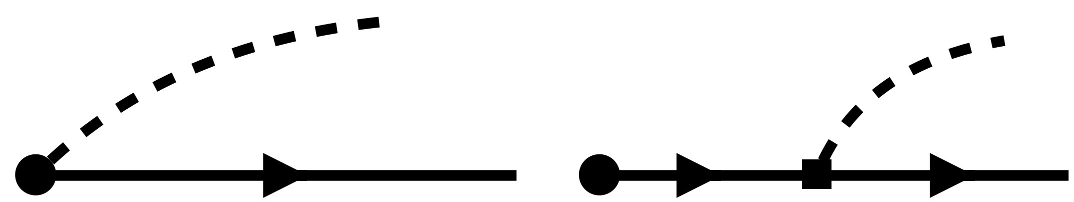

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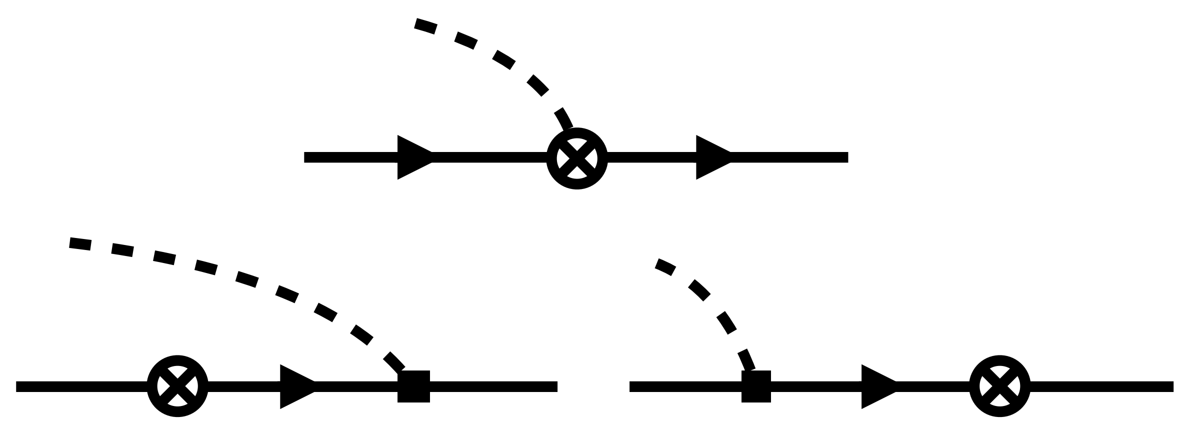

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Can one use experimental scattering data to go beyond the ChPT prediction?

$\mathcal{C}(E_n, L)$ beyond LO ChPT

non-interacting result

$$4\pi^2 q^2 \left(\frac{\partial \phi}{\partial q} \right)^{-1}$$

interacting result

$$4\pi^2 q^3 \left(q \frac{\partial \phi}{\partial q} + p^* \frac{\partial \delta}{\partial p^*} \right)^{-1}$$

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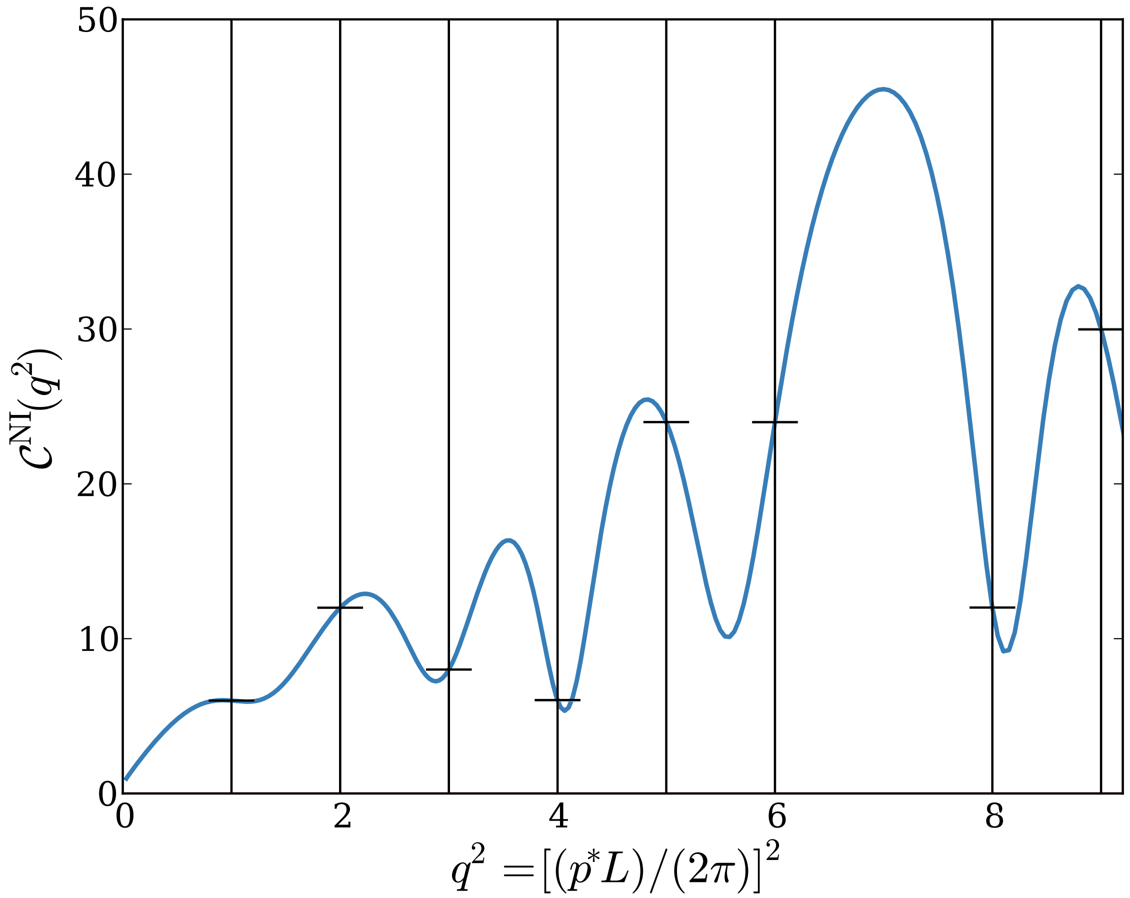

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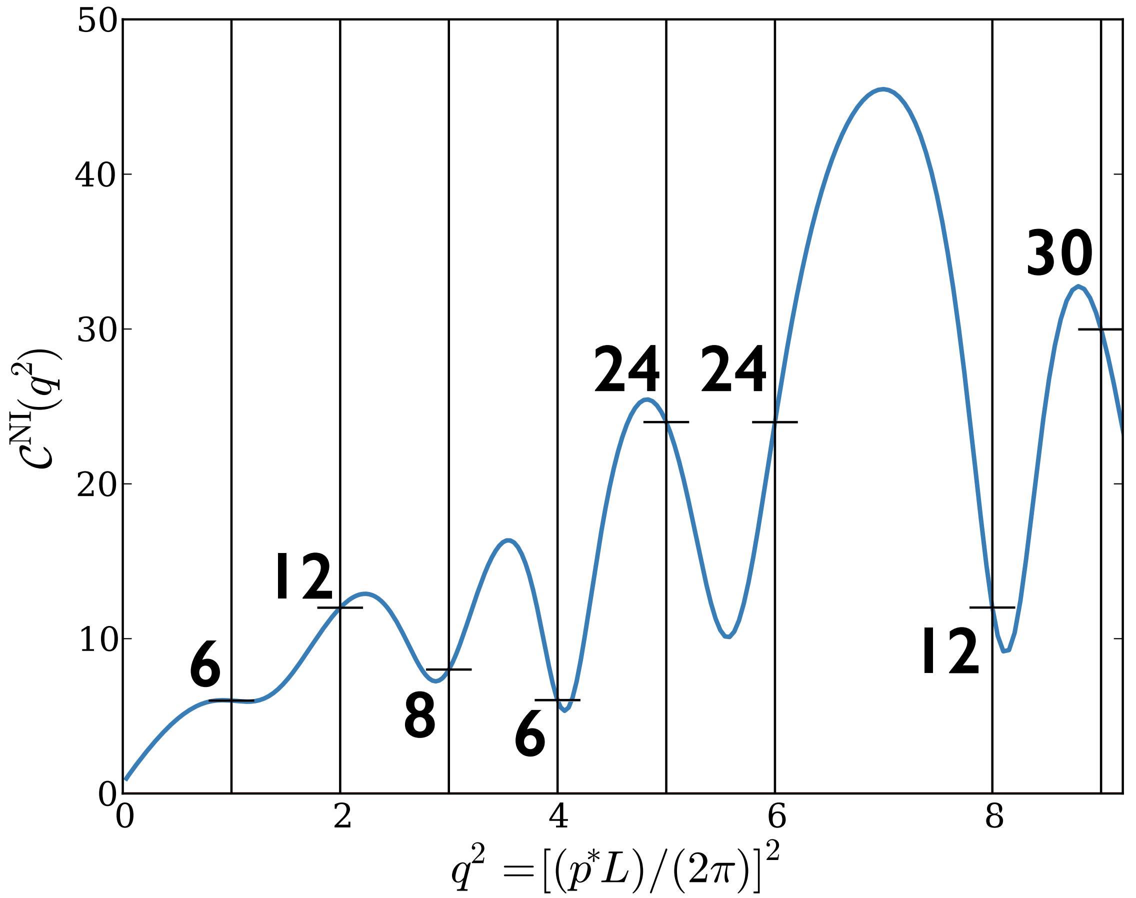

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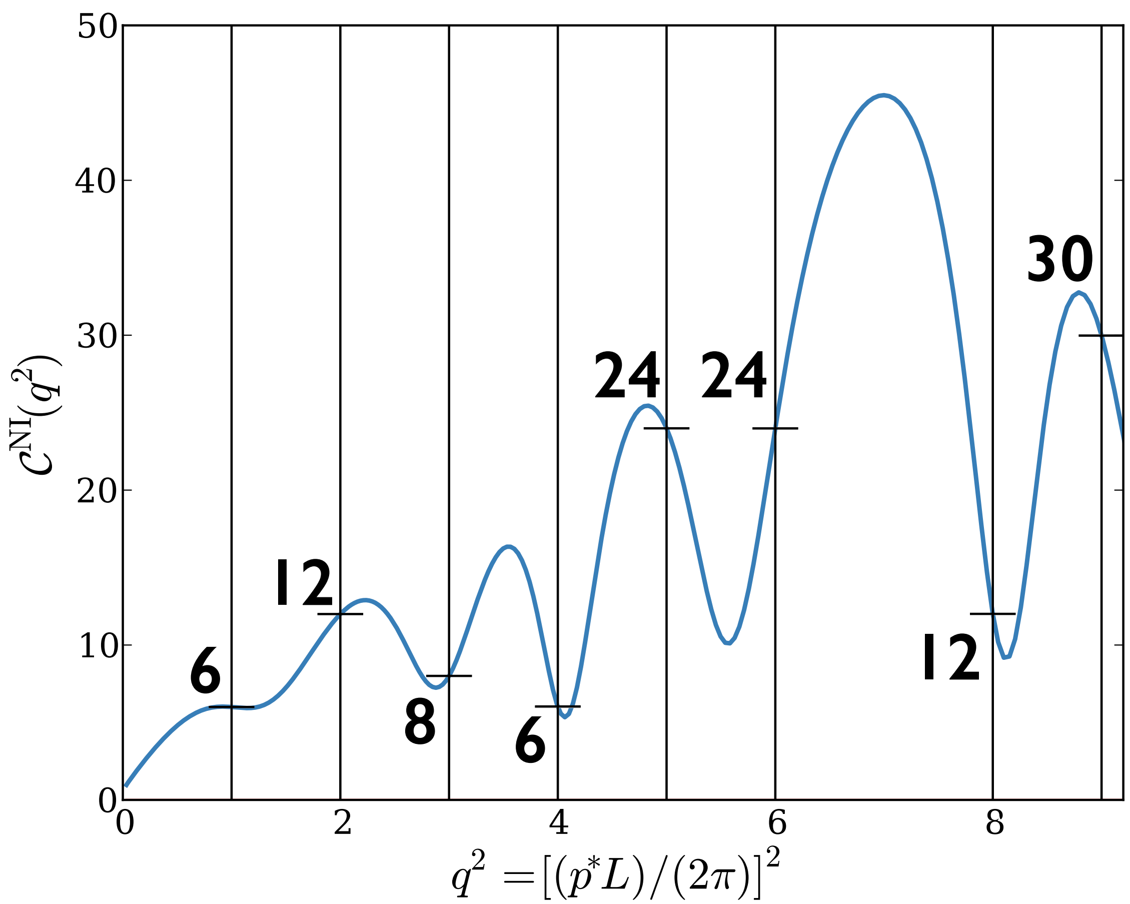

interacting result

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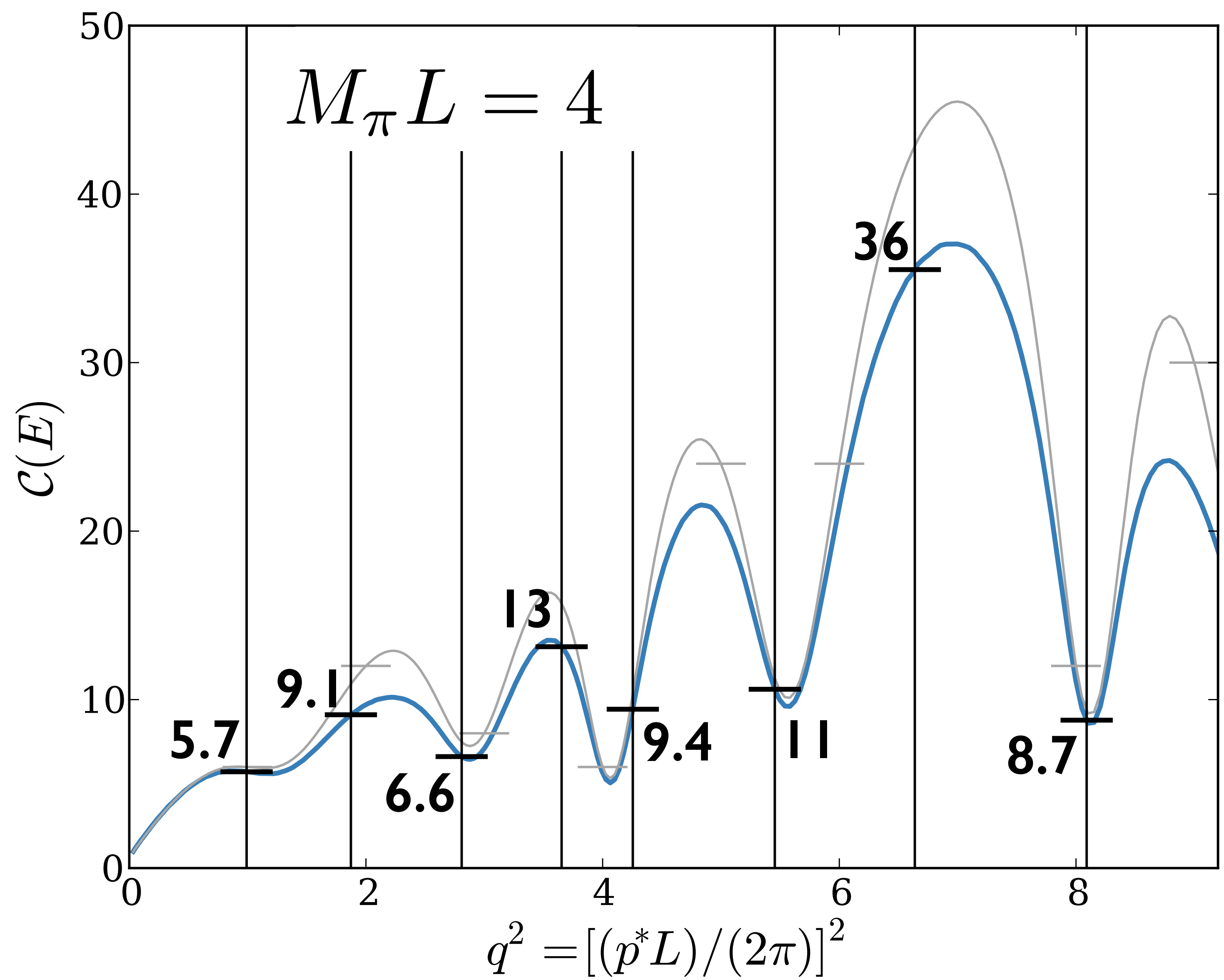

$\mathcal{C}(E_n, L)$ beyond LO ChPT

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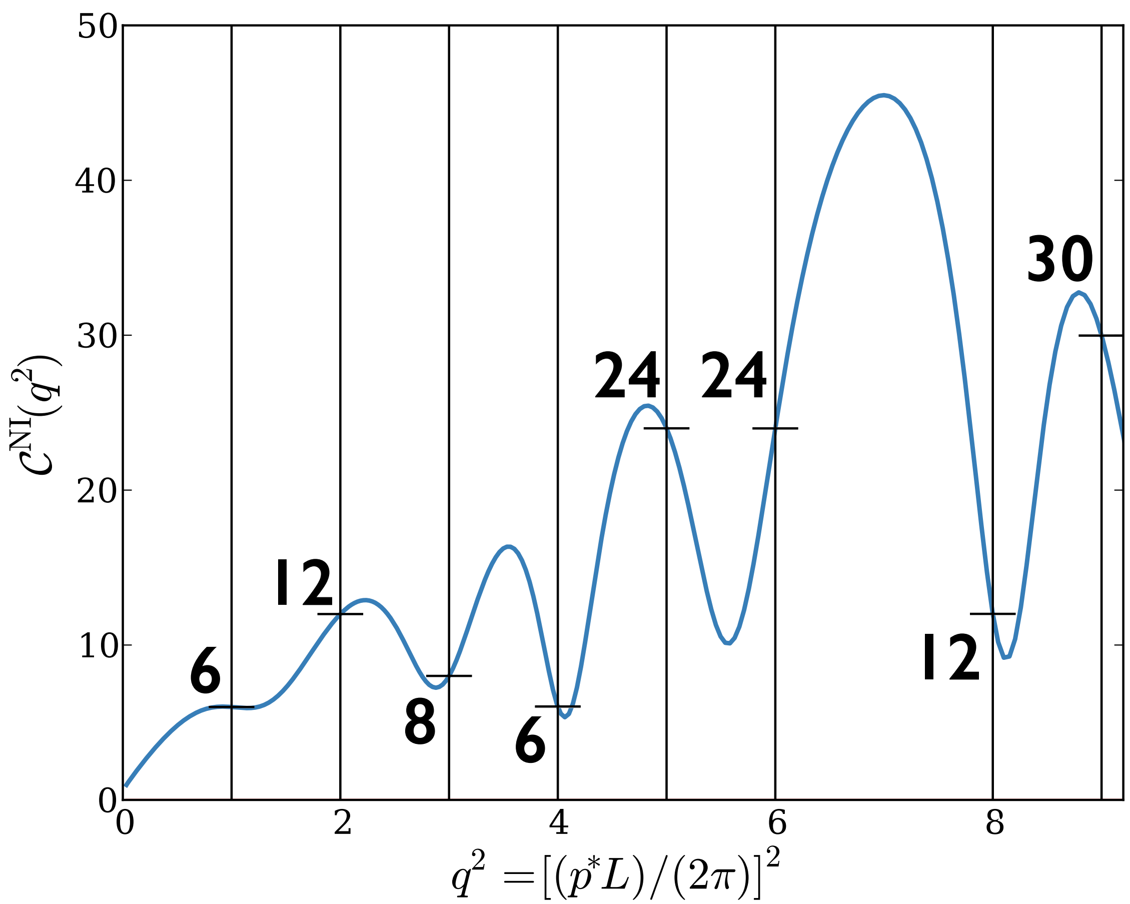

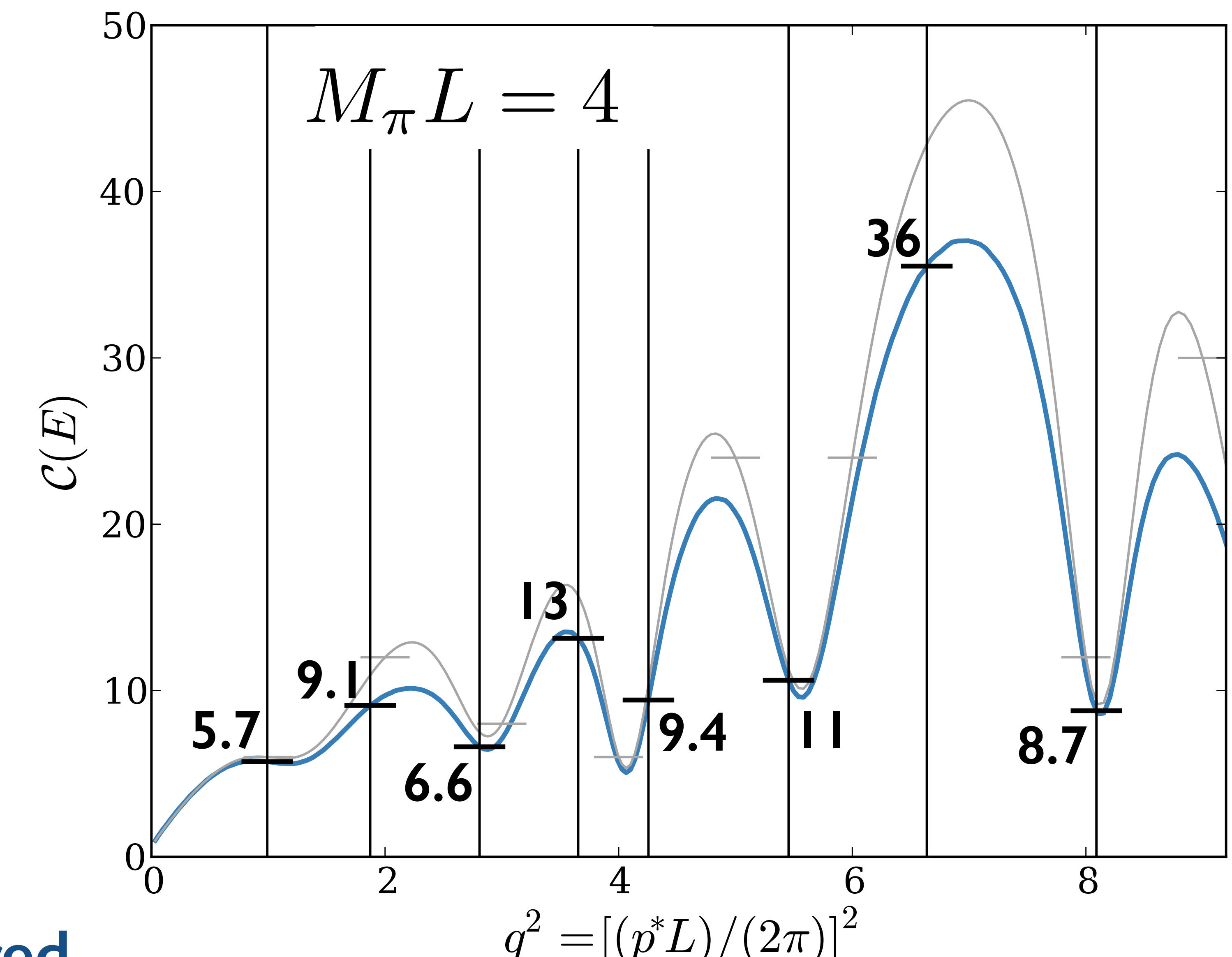

Two distinct effects:

Lellouch-Lüscher curve is lowered

Places on the curve with physical meaning change

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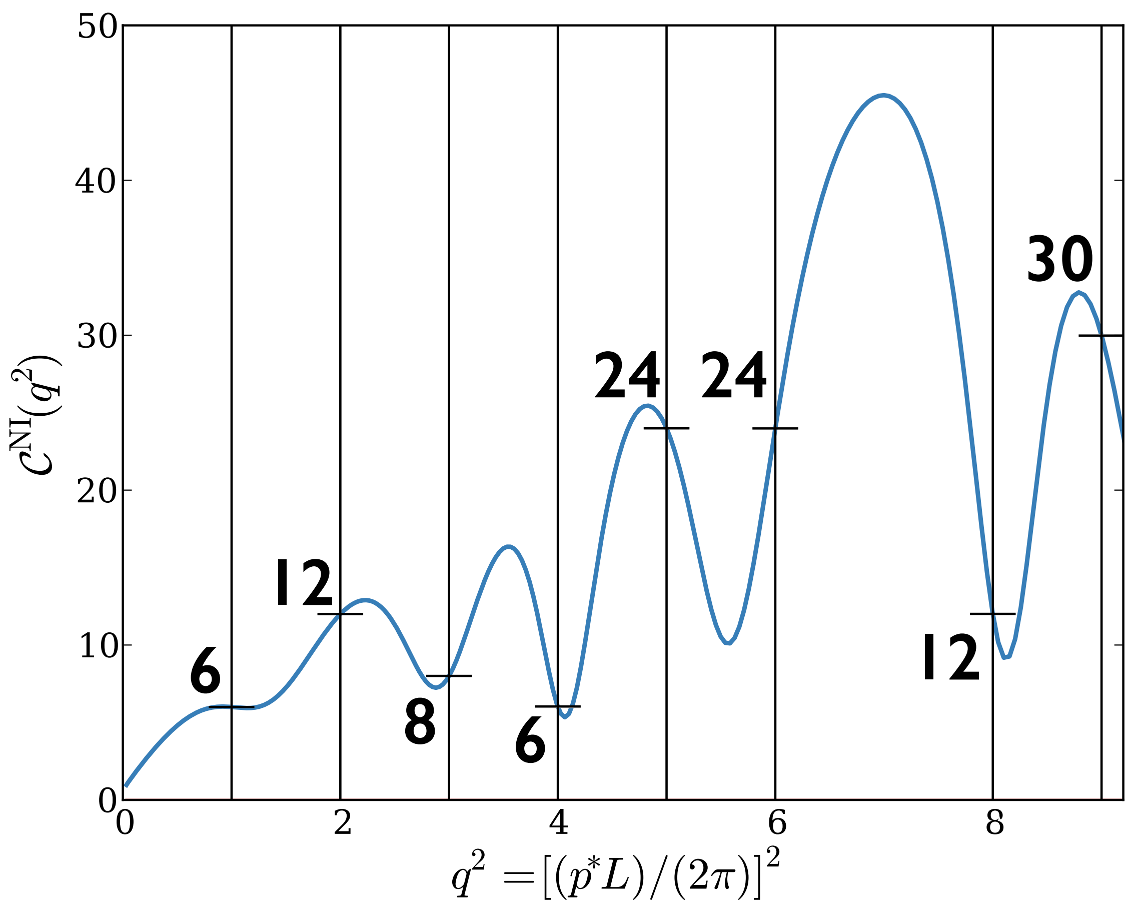

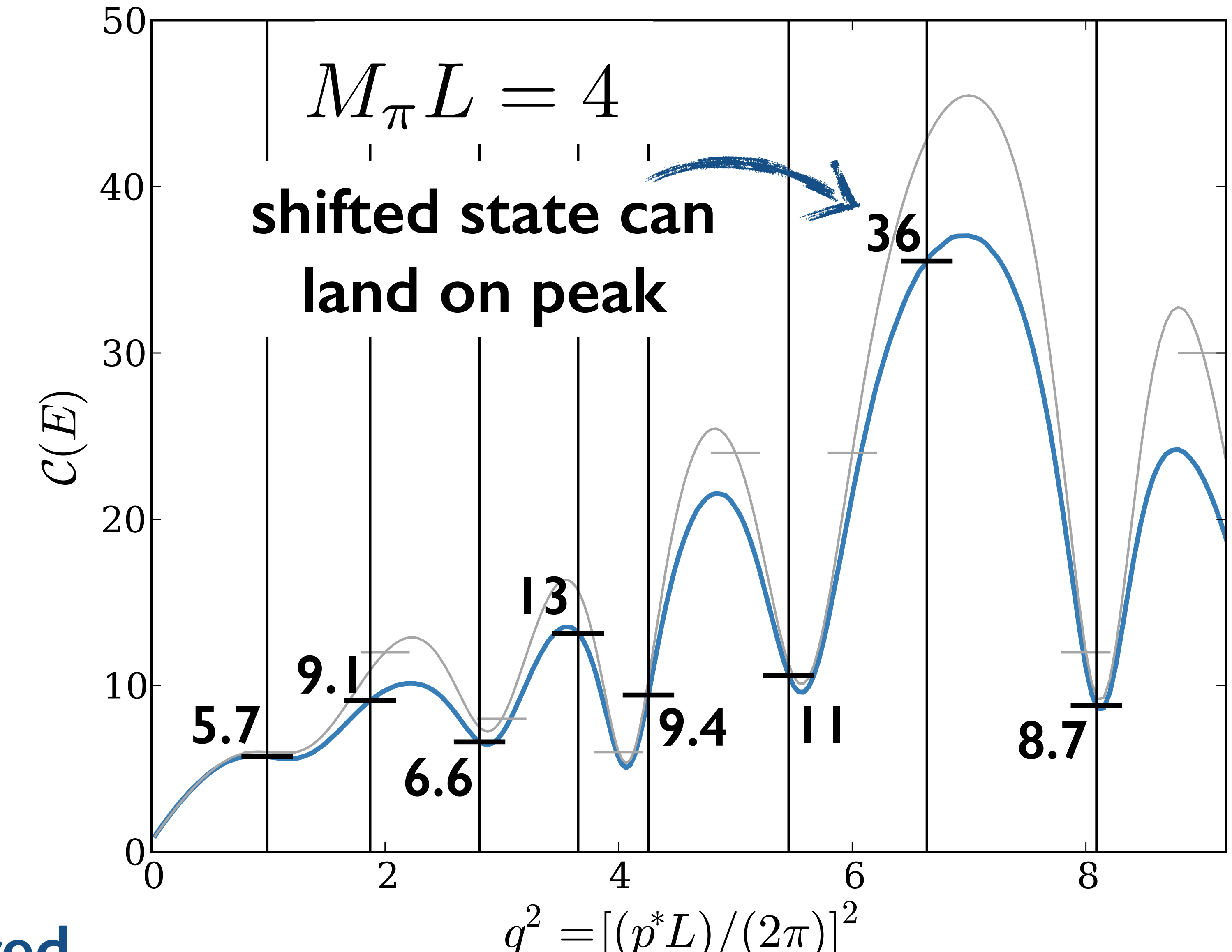

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In principle this can be extracted from experiment but present data is insufficient $N\nu \to N\pi$

Here we consider a (primitive) model of the matrix element

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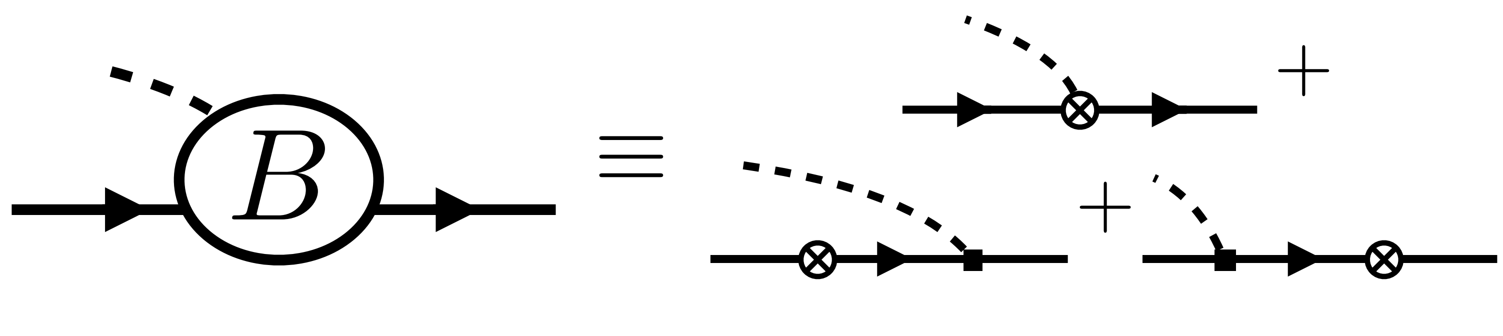

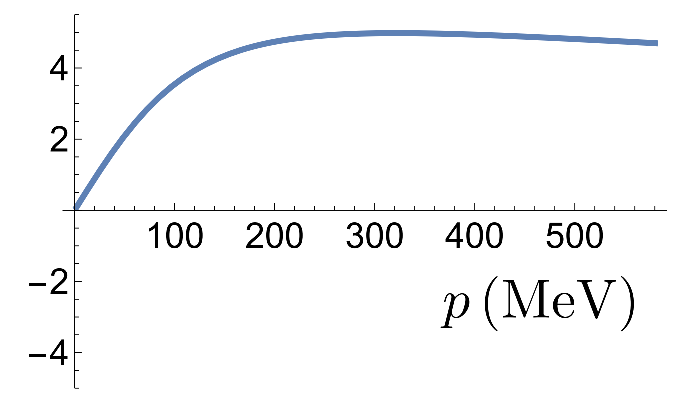

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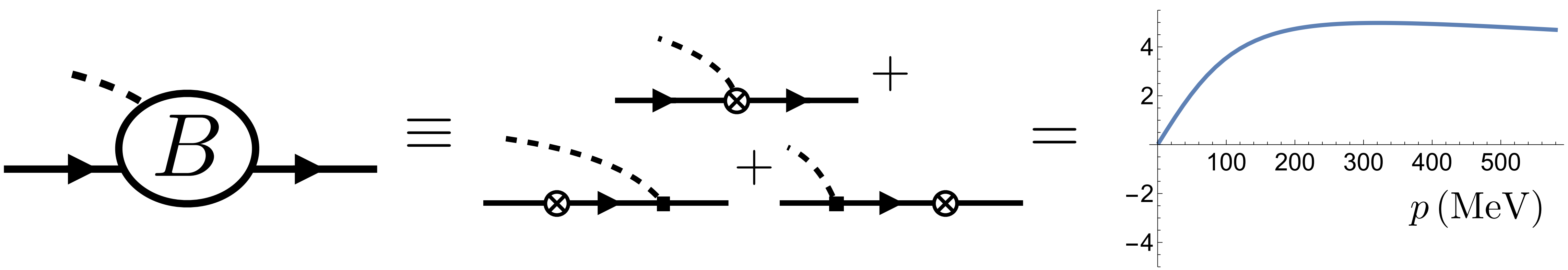
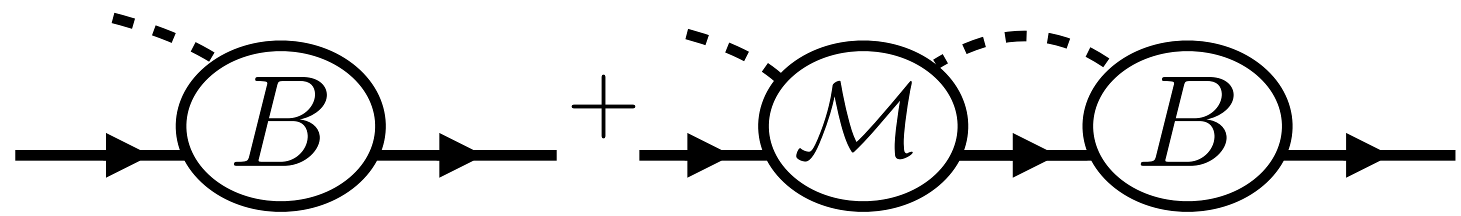


$$\langle N\pi, \mathrm{out}|A_z^3|N\rangle =$$



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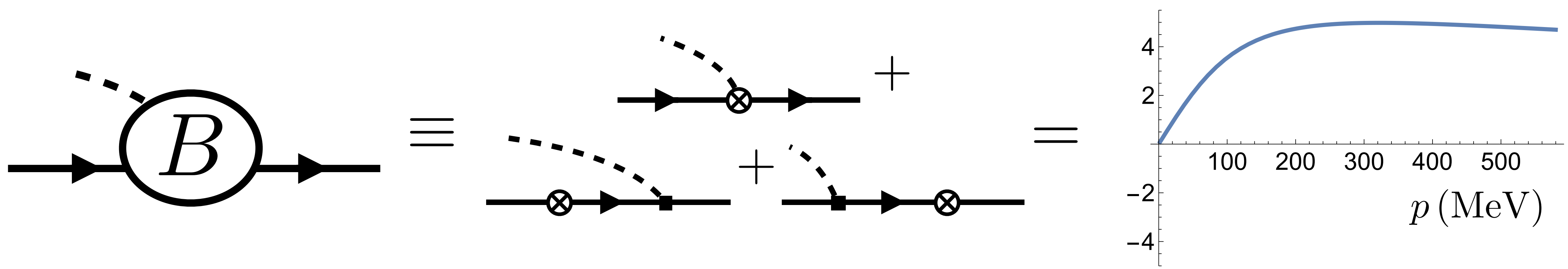
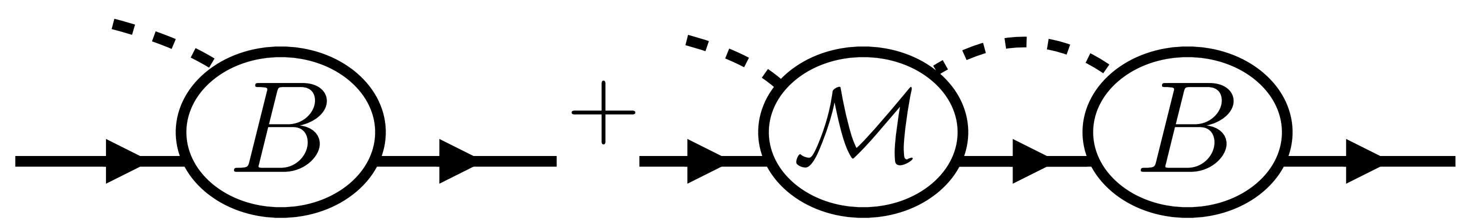
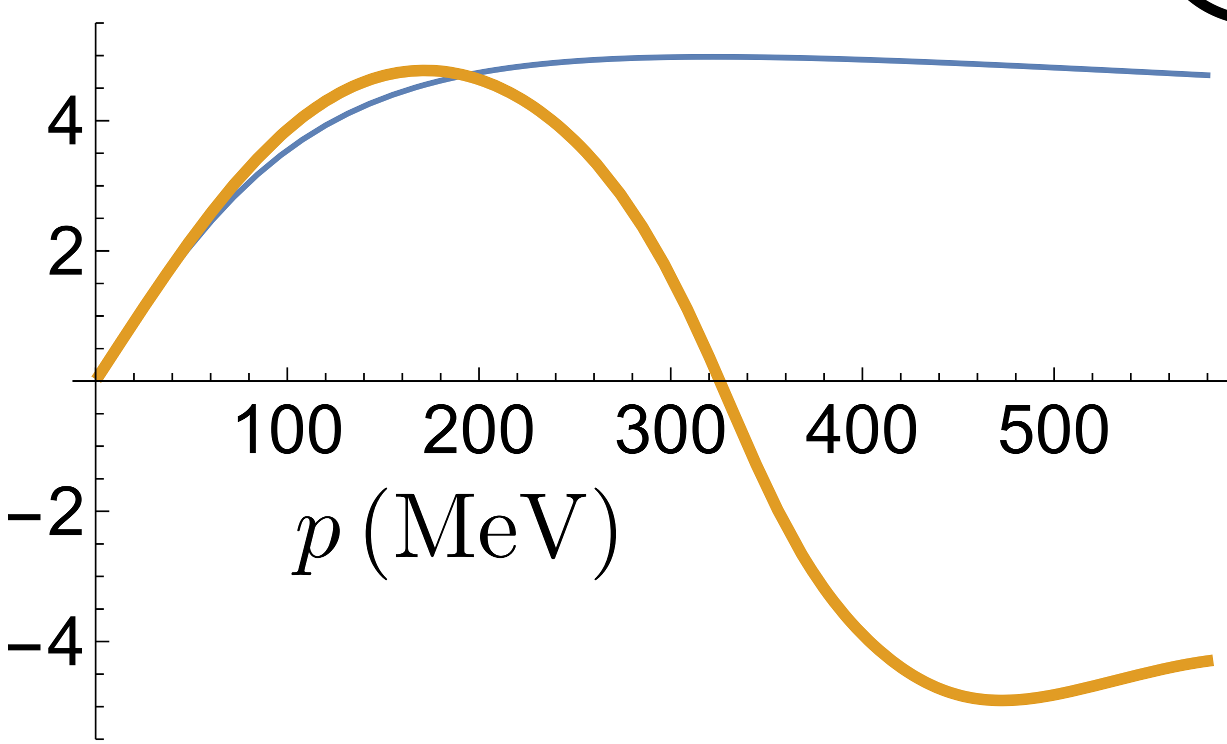


$\langle N\pi, \mathrm{out}|A_z^3|N\rangle =$ 



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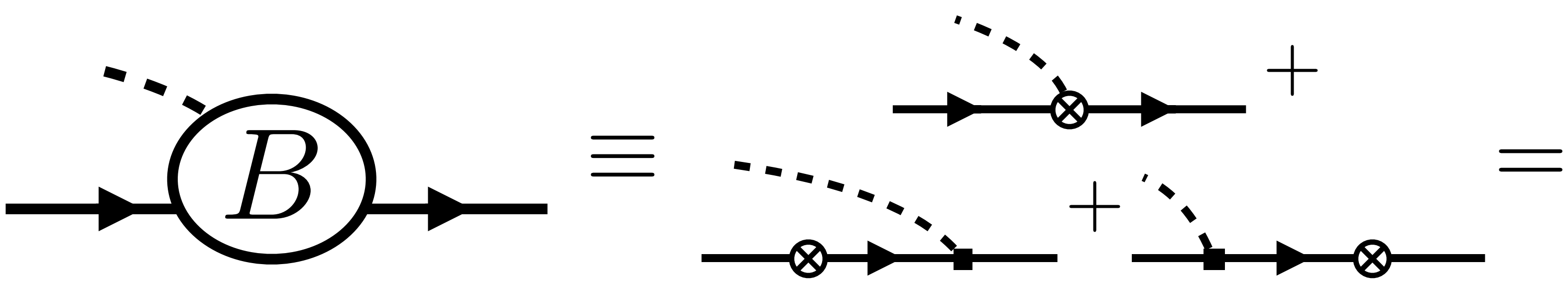
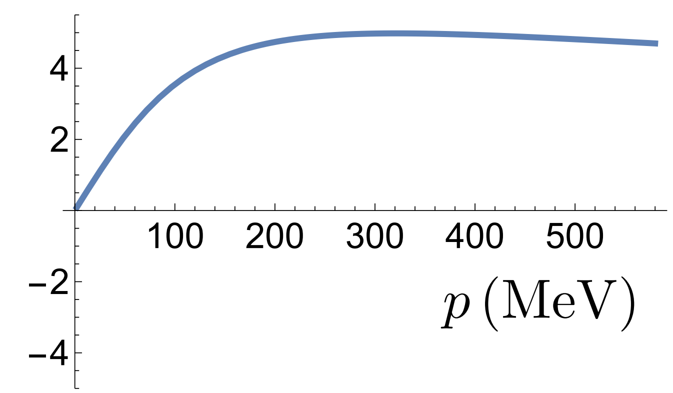
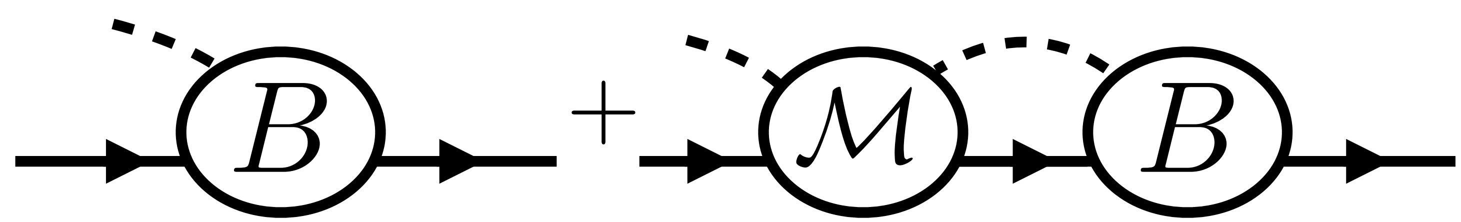
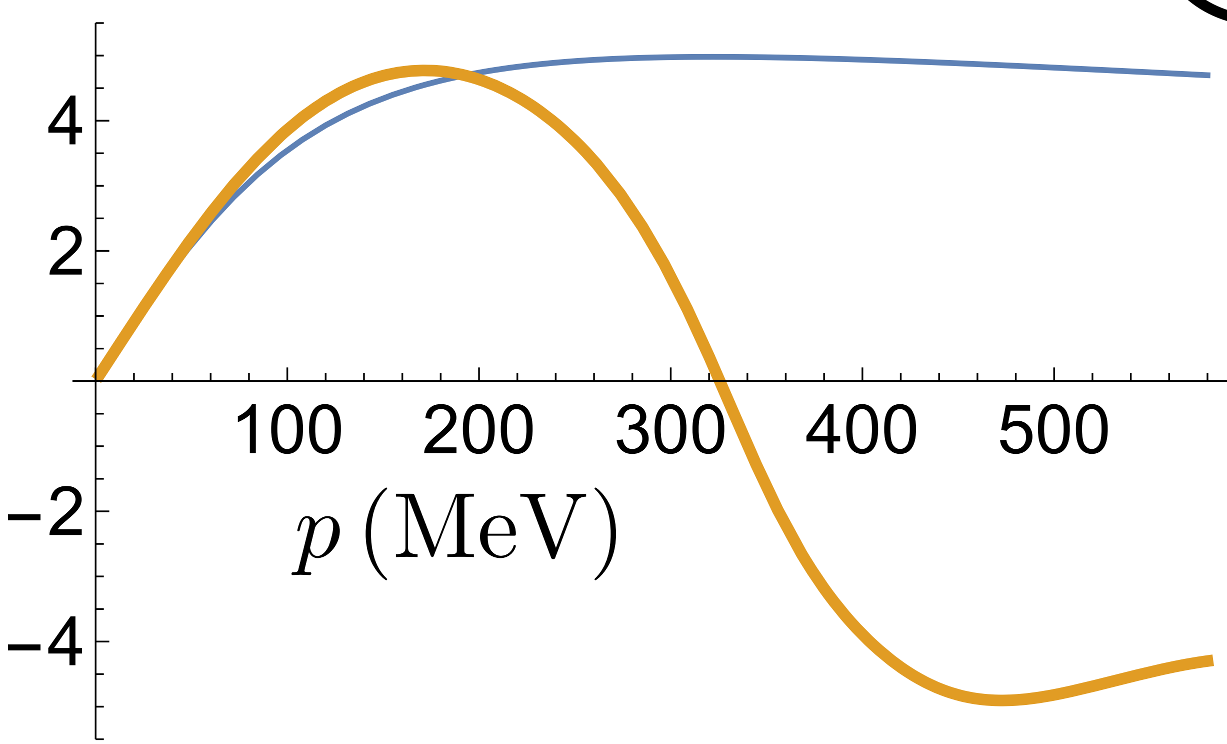




$\langle N\pi, \mathrm{out}|A_z^3|N\rangle =$ 



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This model produces the correct phase (Watson's theorem)

The loop must be regulated, here we choose a cutoff at $p = 800\mathrm{MeV}$

Not a rigorous determination…
But a plausible value that gives insight into the problem of excited state contamination

$$R(T,\tau) = \frac{\langle \mathcal{O}(T) A(\tau) \overline{\mathcal{O}}(0) \rangle}{\langle \mathcal{O}(T) \overline{\mathcal{O}}(0) \rangle} = g_A + \mathcal{E}(T,\tau)$$

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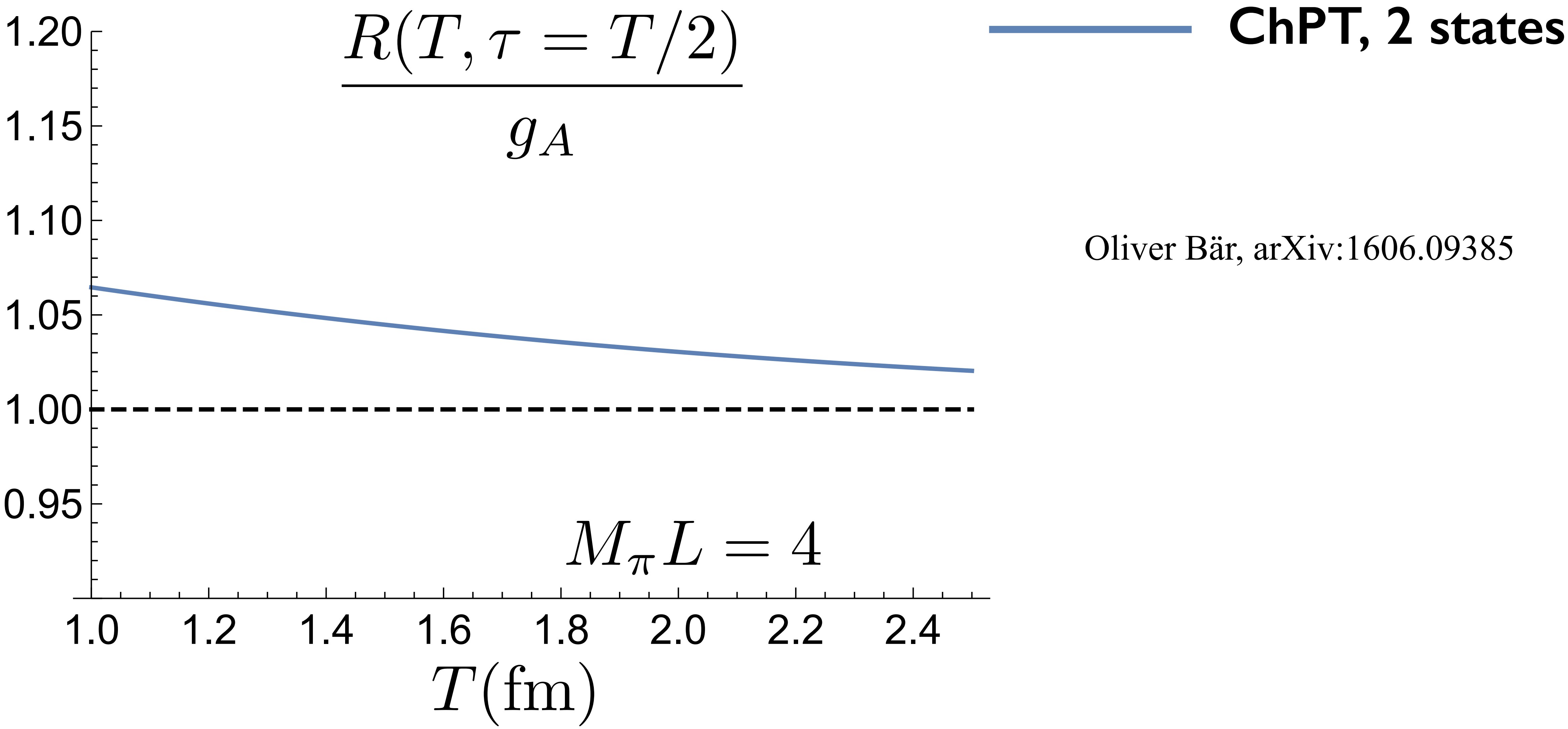
Effect of modifying ΔE_n, $\mathcal{C}(E_n, L)$**, and** $\langle N\pi, \mathrm{out} | A_z^3 | N \rangle$



Oliver Bär, arXiv:1606.09385

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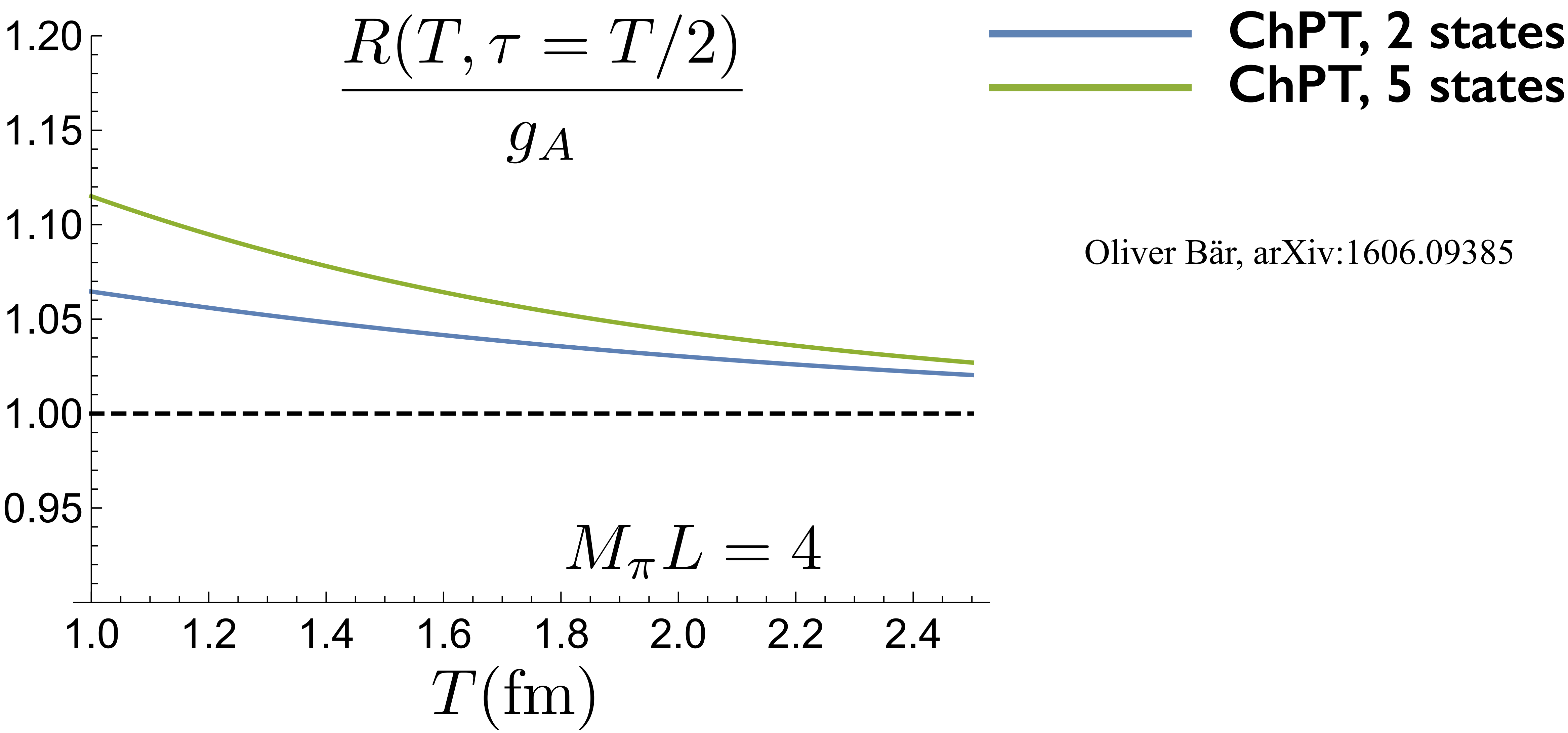

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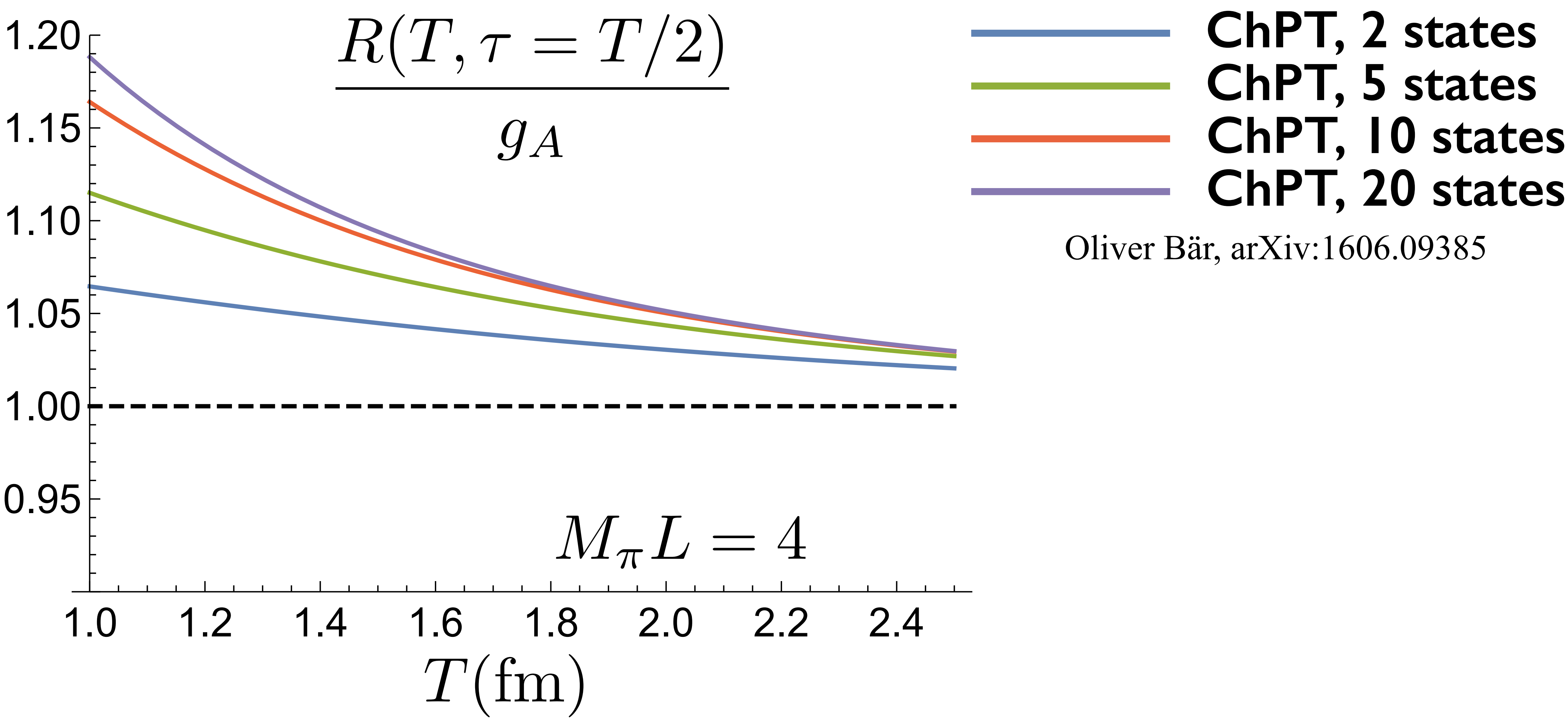

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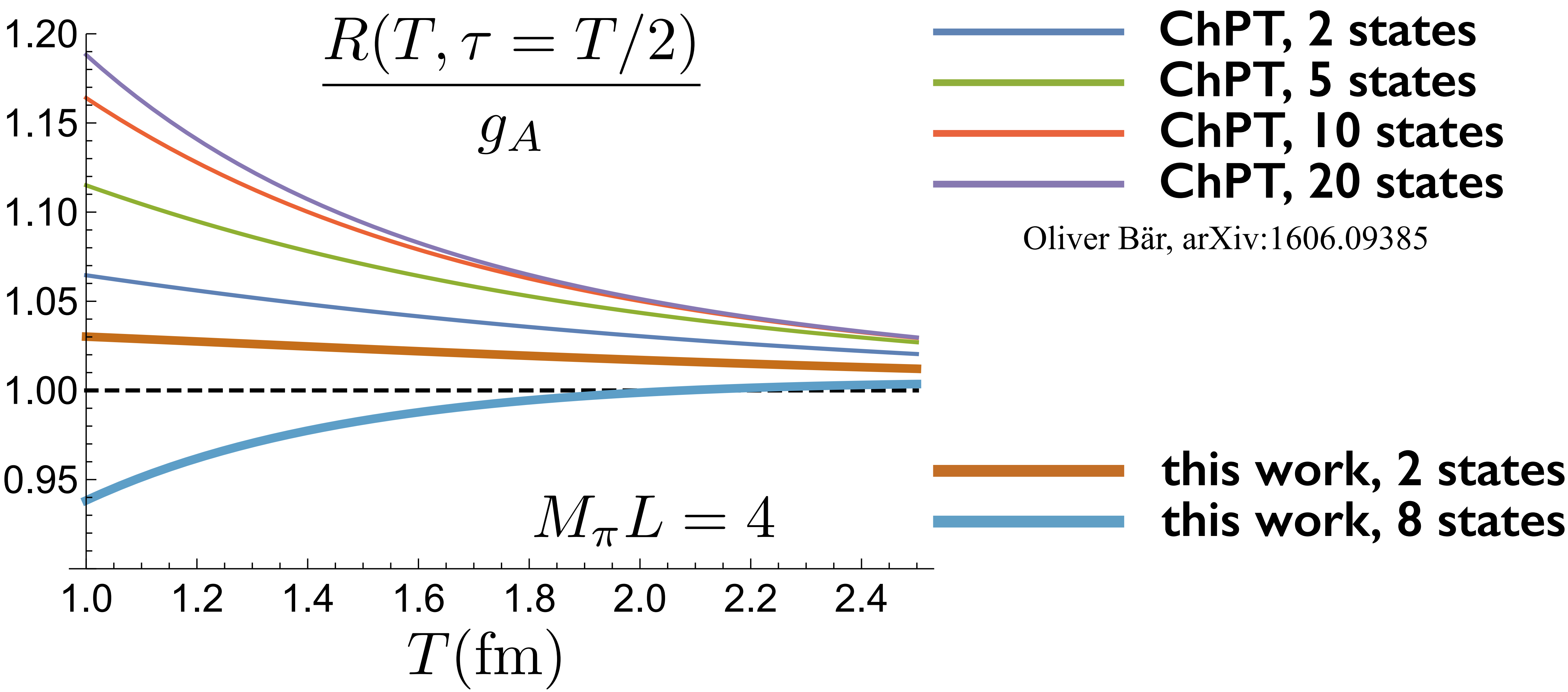

Summary

The Lüscher and Lellouch-Lüscher formalisms allow one to use experimental inputs to estimate excited state contamination

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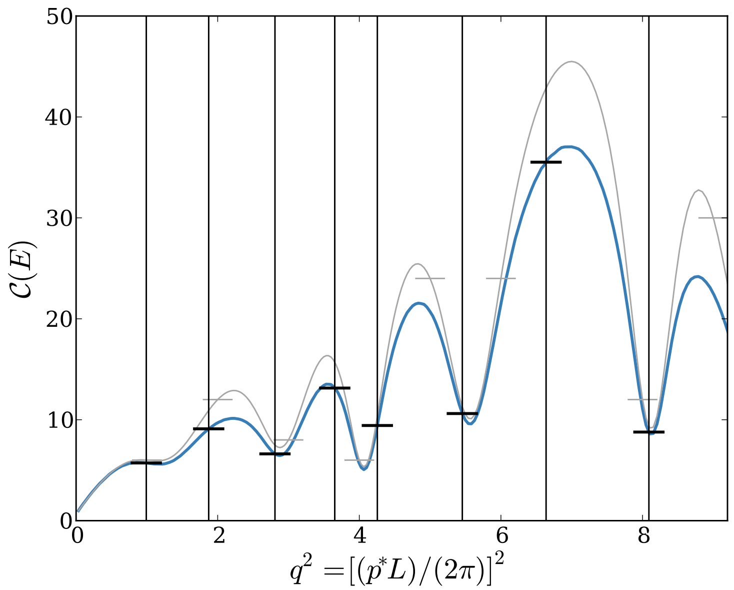
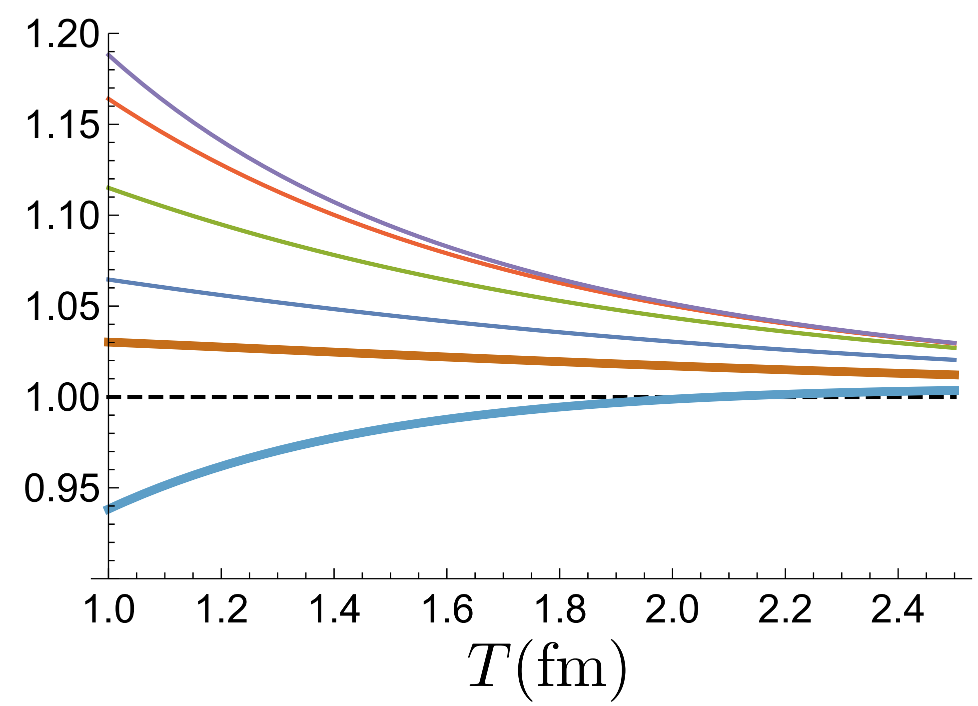
The Lellouch-Lüscher relation gives a great deal of insight about the coefficients including some surprises:

Amputated tree-level diagrams can be used to extract the same LO ChPT predictions given by standard one-loop diagrams

Interactions can shift the Lellouch-Lüscher factors dramatically due to the highly oscillatory function



A sign flip in the axial matrix element could lead to the observed sign of the excited state contamination



Conclusions

Many finite-volume states can be important in excited state contamination

It may be that the positive ChPT result is flipped by higher states

This emphasizes the importance of well-known techniques to reduce excited state contamination: e.g. variational method

Future work

Better estimate the infinite-volume matrix element

Include the effects of heavier than physical pions

Include the effects of three-particle states

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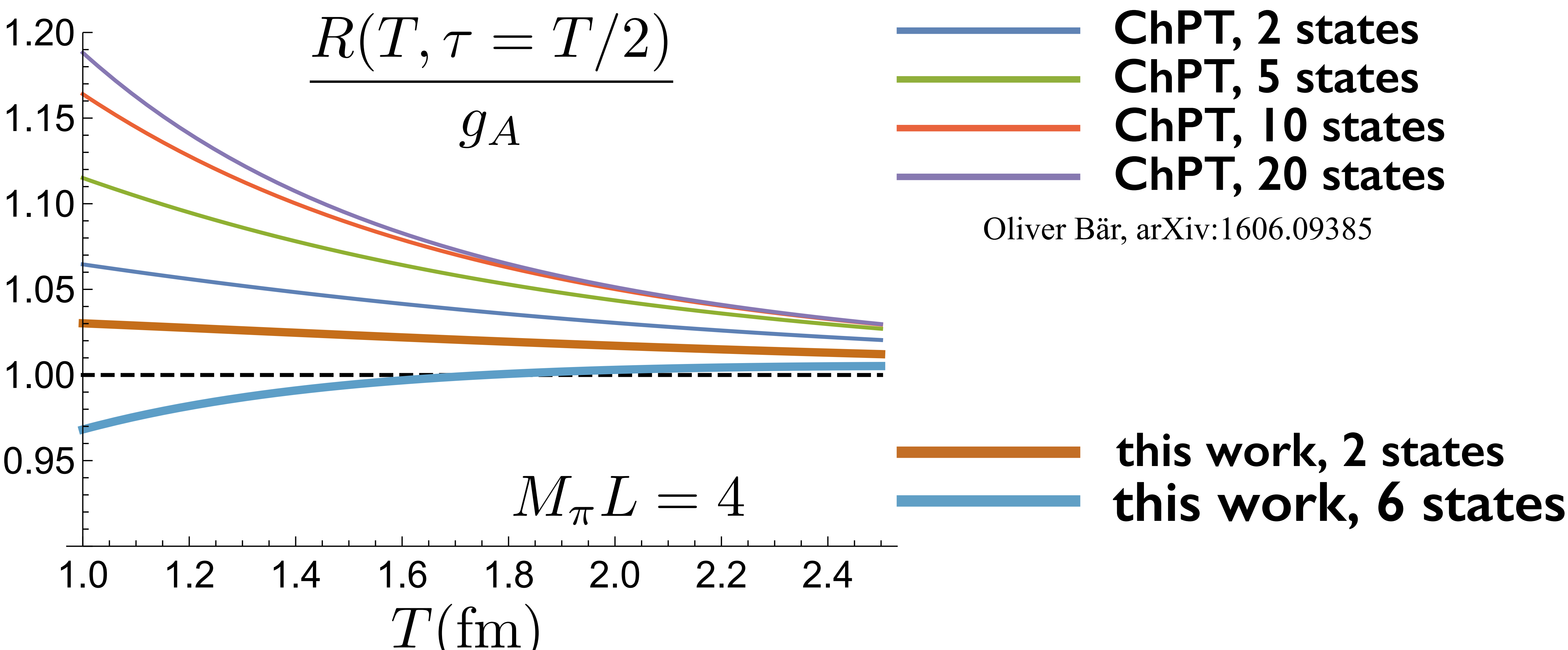

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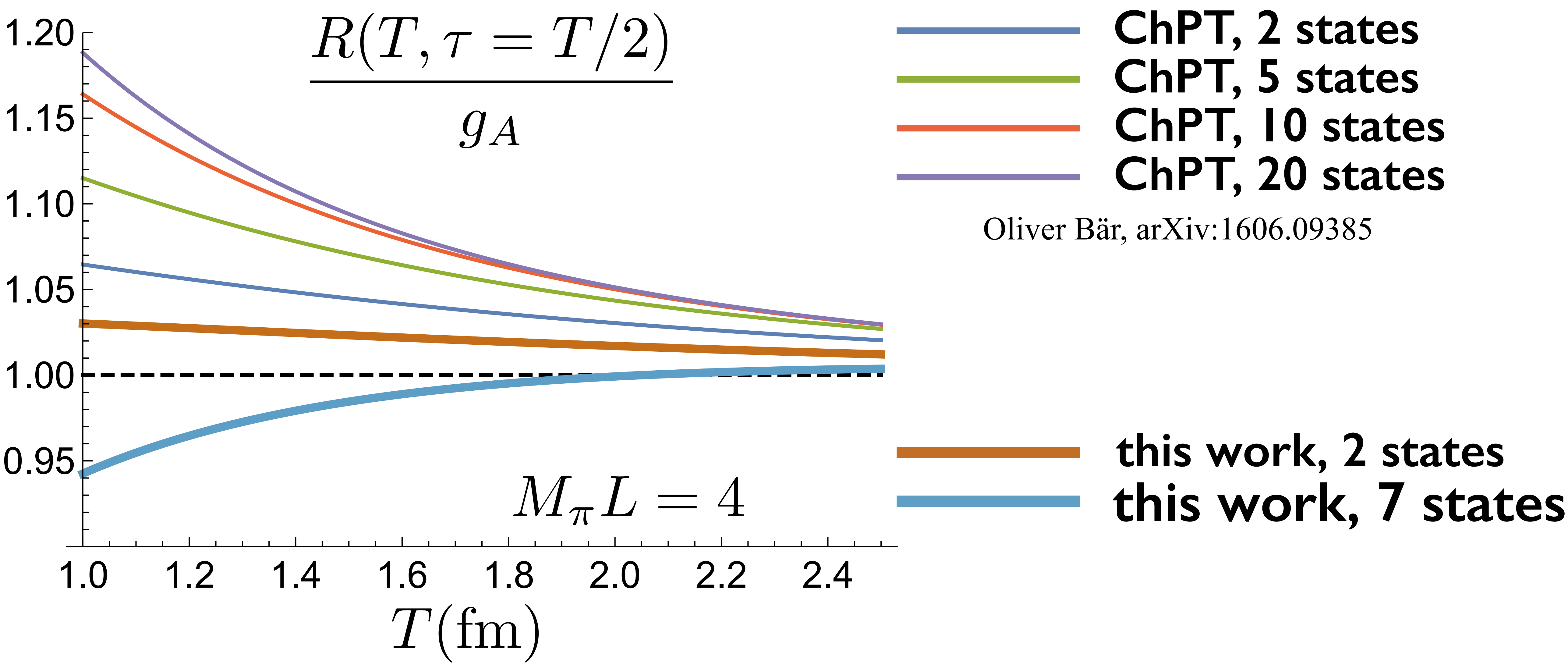

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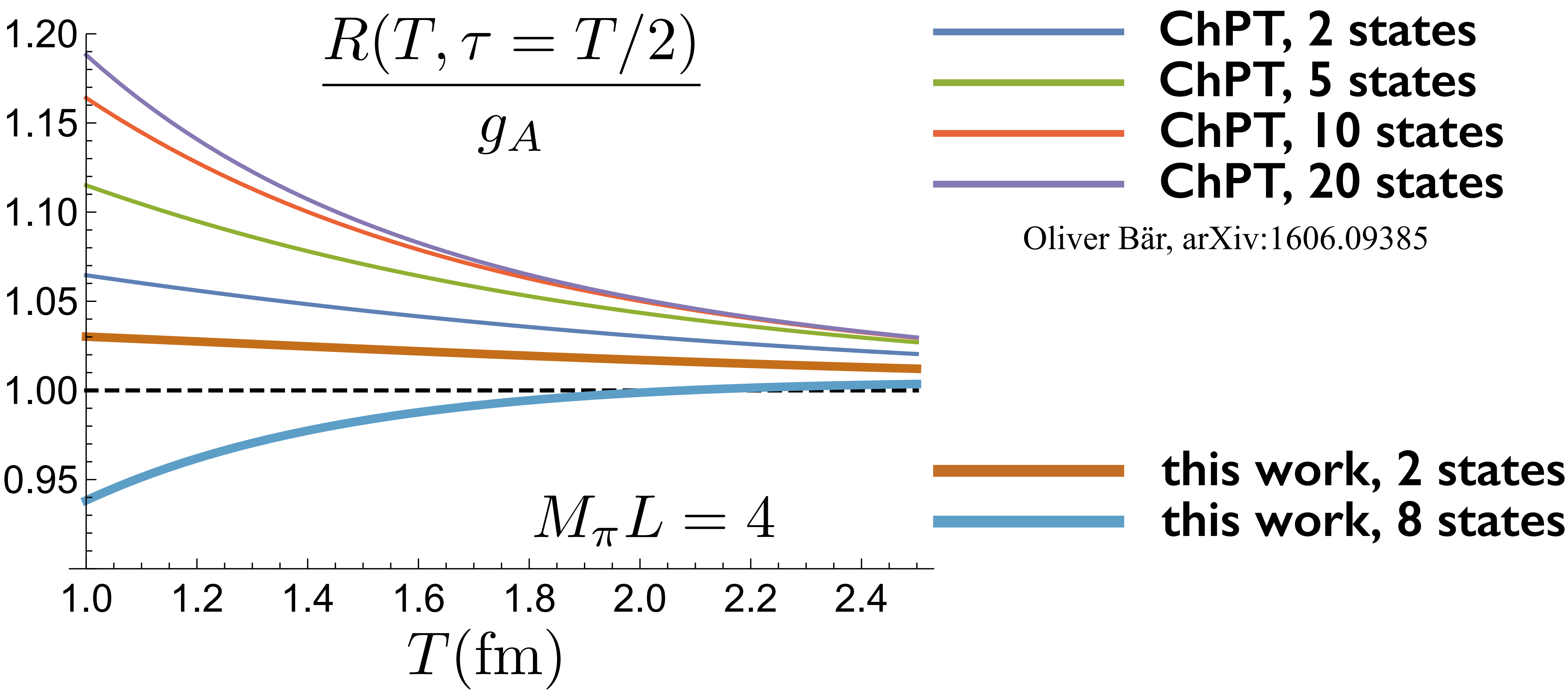

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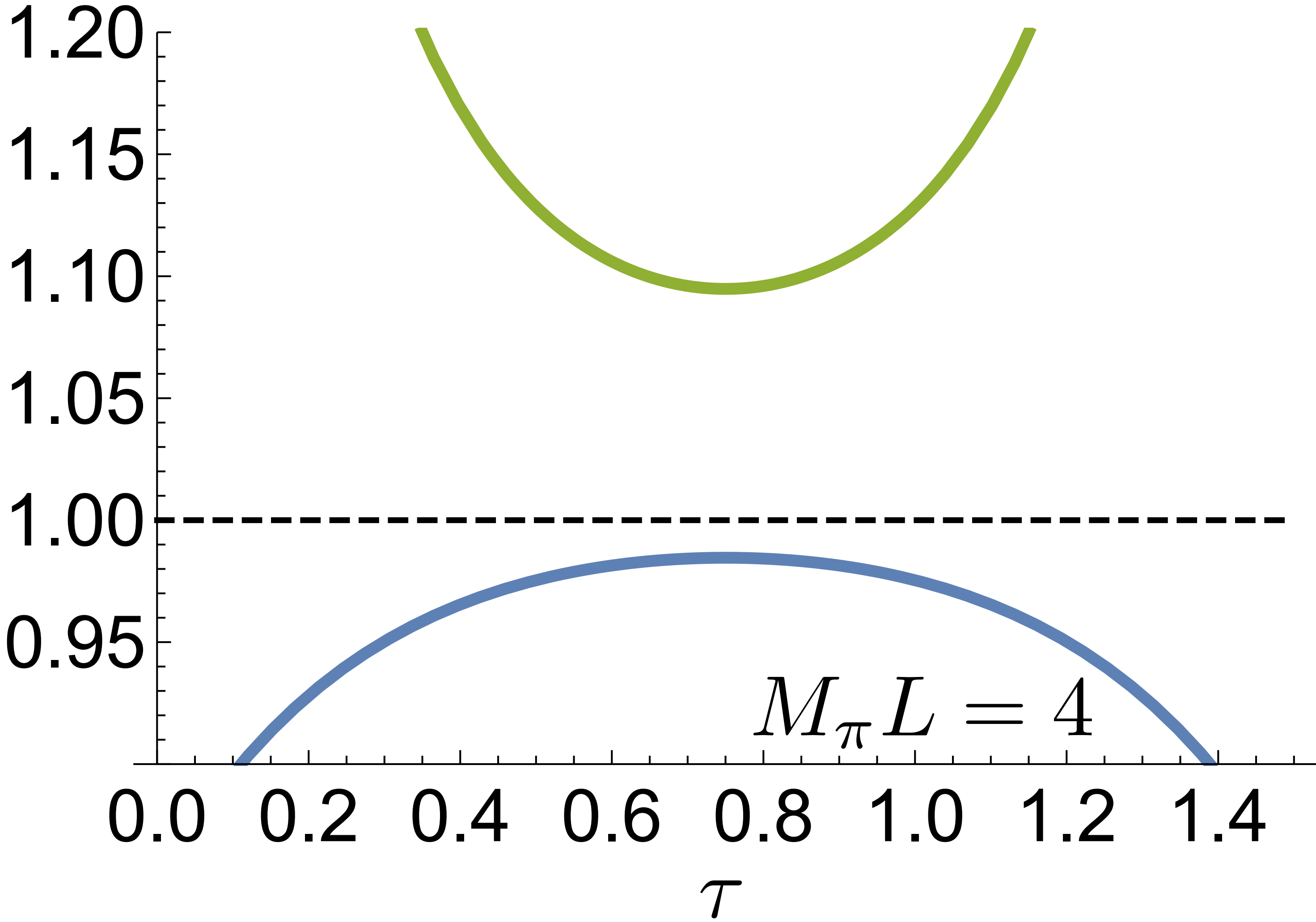

$$\frac{R(T = 1.5\mathrm{fm}, \tau)}{g_A}$$

ChPT, 20 states

Oliver Bär, arXiv:1606.09385

this work, 8 states