A high-statistics lattice QCD study of nucleon sigma terms

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(Preliminary results)
Direct WIMP dark matter detection

\[ <v> = 220 \text{ km/s} \]

\[ \chi \bar{\chi} \rightarrow L \bar{N} = \lambda \Gamma \bar{N} \Gamma L \bar{\chi} \chi \]

Quark are confined within nucleons

\[ \mathcal{L}_{q\chi} = \sum_q \lambda_q^\Gamma [\bar{q} \Gamma q][\bar{\chi} \Gamma \chi] \rightarrow \mathcal{L}_{N\chi} = \lambda_N^\Gamma [\bar{N} \Gamma N][\bar{\chi} \Gamma \chi] \]

\[ \rightarrow \] nonperturbative QCD tool
In low-\(E\) limit

\[
\frac{d\sigma_{\chi A/X}^{\text{SI}}}{dq^2} = \frac{1}{\pi v^2} \left[ Z f_p + (A - Z) f_n \right]^2 |F_X(q^2)|^2
\]

w/ \(F_X(\vec{q} = 0) = 1\) nuclear FF and \(\chi N\) couplings \((N = p, n)\)

\[
\frac{f_N}{M_N} = \sum_{q=u,d,s} f_{qN} \frac{\lambda_q}{m_q} + \sum_{Q=c,b,t} f_{QN} \frac{\lambda_Q}{m_Q}
\]

such that \((f = u, \ldots, t\) and \(\langle N(\vec{p}')|N(\vec{p})\rangle = (2\pi)^3 \delta^3(\vec{p}' - \vec{p})\))

\[
f_{udN} M_N = \sigma_{\pi N} = m_{ud} \langle N|\bar{u}u + \bar{d}d|N\rangle, \quad f_{fN} M_N = \sigma_{fN} = m_f \langle N|\bar{f}f|N\rangle
\]

For heavy \(Q = c, b, t\), can use \(\text{(Shifman et al '78)}\)

\[
f_{QN} \approx \frac{2}{27} \left(1 - \sum_{q=u,d,s} f_{qN}\right)
\]
Feynman-Hellmann method

- Use Feynman-Hellmann (FH) theorem

\[ \langle N | m_q \bar{q} q | N \rangle = m_q \left. \frac{\partial M_N}{\partial m_q} \right|_{m_q^\Phi} \]

- FH vs direct matrix element (ME) method

  ✓ Only simpler and less noisy 2pt-fn is needed
  ✓ No difficult quark-disconnected contributions
  ✓ No difficult singlet renormalization w/ Wilson fermions

  ✗ \( m_{ud} \)-dependence not very large

\[ M_N \approx 939 \text{ MeV} \approx 850 \text{ MeV} + O(m_{ud}) \]

  → must extract smallish correction

  ✗ \( m_s \) dependence even smaller
Lattice details

- $N_f = 1 + 1 + 1 + 1$
- 3HEX clover-improved Wilson fermions on tree-level improved Symanzik gluons
- 29 ensembles w/ total $\sim 155000$ trajectories
- $\sim$ 500 measurements per configuration
- $4a \in [0.064, 0.102]$ fm;
- $M_\pi \in [195, 450]$ MeV w/ $LM_\pi > 4$

Improvements over BMWc, PRL ’16

- ✔ Charm in sea
- ✔ $\geq \times 100$ in statistics (reduced to $\geq 35$ by later plateaux)
- ✔ $\geq \times 2$ lever arm in $m_s$
- ✔ Like PRL ’16 FH in terms of quark and not meson masses
- ✗ No physical $m_{ud}$, but small enough and know $M_N$
Analysis strategy

- Dependence of 4 hadron masses analyzed simultaneously, vs $m_{ud}$, $m_s$ and $m_d - m_u$:
  - $M_\Omega$, $M^2_{\pi}$ and $M^2_{K\chi} = (M^2_{K+} + M^2_{K0} - M^2_{\pi+})/2$ to fix $a$, $m_{ud}^\phi$ and $m_s^\phi$
  - $M_N$ to determine sigma terms (and sometimes scale)

- Quark masses obtained w/ *ratio-difference method* (BMWc, JHEP 1108)

- Since $Z_S(\beta)$ is not computed, write functional dependence in terms of $(q = ud, s, or, d - u)$

\[
c_q \left[ \frac{(am_q)}{aZ_s(\beta)} - m^\phi_q \right] \rightarrow \tilde{c}_q \left[ \frac{(am_q)}{aZ_s(\beta)m^\phi_q} - 1 \right] \rightarrow \tilde{c}_q \left[ \frac{(am_q)}{a \cdot \tilde{m}^\phi_q(\beta)} - 1 \right]
\]

- Cross checks:
  - $\tilde{m}_s^\phi / \tilde{m}_{ud}^\phi = m_s^\phi / m_{ud}^\phi$?
  - Values of $Z_S(\beta)$?
Minimize $\chi^2 = V^T C^{-1} V$, with $C$ full covariance matrix,

$$V^T = \left[ aM_{[\Omega,1]} - f_{\Omega}(c, X[q,1]), \ldots, aM_{[N,N_{\text{ens}}]} - f_{N}(c, X[q,N_{\text{ens}}]), \\ m[q,1] - X[q,1], \ldots, m[q,N_{\text{ens}}] - X[q,N_{\text{ens}}] \right]$$

and e.g.

$$f_{N}(c, X) = a(\beta)M_{N}^{\Phi}[1 + FV(a M_{\pi}, L/a)]\left\{ 1 + \sum_{i}^{2} c_{ud,i} \left[ \frac{(am_{ud})}{a \cdot \tilde{m}_{ud}(\beta)} - 1 \right]^i + c_{s,1} \left[ \frac{(am_{s})}{a \cdot \tilde{m}_{s}(\beta)} - 1 \right] \right\}$$

Statistical error with $N_{\text{boot}} = 1000$

To determine fit ranges, use Kolmogorov-Smirnov test with different $t_{\text{min}}$

(BMWc, Science ’15)
Example fits (preliminary)

\[
M_N^\Phi \quad \beta = 3.2 \\
\beta = 3.3 \\
\beta = 3.4 \\
\beta = 3.5 \\
\beta = \infty
\]

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Lattice 2016, University of Southampton, 24-30 July 2016
Example fits (preliminary)

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Comparison with BMWc, PRL ’16 (preliminary)

\[ M_N^{\Phi} \]

\[ \beta = 3.31 \]
\[ \beta = 3.5 \]
\[ \beta = 3.61 \]
\[ \beta = 3.7 \]
\[ \beta = 3.8 \]
\[ \beta = \infty \]

\[ M_N \ [\text{MeV}] \]

\[ m_{s \text{RGI}} \ [\text{MeV}] \]
New method for obtaining $f_{u/d}^{p/n}$

[BMWc, PRL 116 (2016)]

- Input: $f_{ud}^N$ and $\Delta_{QCD}M_N = M_n - M_p$ (from BMWc, Science ’15)

- SU(2) relations w/ $\delta m = m_d - m_u$

$$H = H_{iso} + H_{\delta m}, \quad H_{\delta m} = \frac{\delta m}{2} \int d^3 x (\bar{d}d - \bar{u}u)$$

$$\Delta_{QCD}M_N = \delta m \langle p|\bar{u}u - \bar{d}d|p\rangle$$

lead to, w/ $r = m_u/m_d$,

$$f_{u}^{p/n} = \left(\frac{r}{1+r}\right) f_{ud}^N \pm \frac{1}{2} \left(\frac{r}{1-r}\right) \frac{\Delta_{QCD}M_N}{M_N}$$

$$f_{d}^{p/n} = \left(\frac{1}{1+r}\right) f_{ud}^N \pm \frac{1}{2} \left(\frac{1}{1-r}\right) \frac{\Delta_{QCD}M_N}{M_N}$$

- Huge improvement on usual SU(3)-flavour approach

$$\text{systematic: } \left(\frac{m_s - m_{ud}}{\Lambda_{QCD}}\right)^2 \approx 10\% \rightarrow \left(\frac{m_d - m_u}{\Lambda_{QCD}}\right)^2 \approx 0.01\%.$$
Systematic error assessment (preliminary)

Estimated using extended frequentist approach (BMWc, Science '08)

- Excited state contamination: 2 time intervals ($t_{\text{min}} = 1.3$ or $1.4 \text{ fm}$)
- Mass interpolation/extrapolation errors
  - $M_\pi \leq 400 \text{ MeV } \& \text{ 450 MeV}$
  - different $m_q$ dependences (polynomials & Padés)
- Lattice spacing uncertainty: $M_\Omega$ vs $M_N$
- Continuum extrapolation: $O(a^2)$ vs $O(\alpha_S a)$
- AIC weight (BMWc, Science ’15)
Preliminary results

Checks

\[ Z_S(\beta) \sim 0.7 \] and expected \( \beta \)-dependence

Compare

\[ M_N = 940(9)(6) \text{ MeV} \quad m_s/m_{ud} = 26.33(37)(2) \]

w/ \( M_N = 939 \text{ MeV} \) (PDG '15) and \( m_s/m_{ud} = 27.53(20)(08) \) (BMWc, JHEP 08 (2011))

Predictions

\[ f^N_{ud} = 0.0517(49)(70) \quad f^N_s = 0.0760(43)(13) \]

\[ f^p_u = 0.0174(16)(23) \quad f^p_d = 0.0329(34)(48) \]

\[ f^n_u = 0.0151(16)(22) \quad f^n_d = 0.0379(34)(48) \]
Comparison

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