A high-statistics lattice QCD study of nucleon sigma terms

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Budapest-Marseille-Wuppertal collaboration (BMWc)

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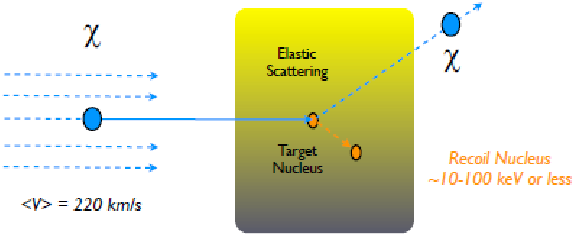
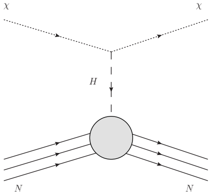
(Preliminary results)

Direct WIMP dark matter detection



$$\downarrow$$



$$\mathcal{L}_{q\chi} = \sum_q \lambda_q^\Gamma [\bar{q}\Gamma q][\bar{\chi}\Gamma\chi] \longrightarrow \mathcal{L}_{N\chi} = \lambda_N^\Gamma [\bar{N}\Gamma N][\bar{\chi}\Gamma\chi]$$

Quark are confined within nucleons
$\rightarrow$ nonperturbative QCD tool

WIMP-nucleus spin-independent cross section

In low-E limit

$$\frac{d\sigma^{\rm SI}_{\chi\,{}^A_Z X}}{dq^2} = \frac{1}{\pi v^2}\left[Z f_p + (A-Z) f_n\right]^2 \; |F_X(q^2)|^2$$

w/ $F_X(\vec{q}=0)=1$ nuclear FF and χN couplings $(N=p,n)$

$$\frac{f_N}{M_N} = \sum_{q=u,d,s} f_{qN}\frac{\lambda_q}{m_q} + \sum_{Q=c,b,t} f_{QN}\frac{\lambda_Q}{m_Q}$$

such that ($f=u,\ldots,t$ and $\langle N(\vec{p}')|N(\vec{p})\rangle = (2\pi)^3\delta^{(3)}(\vec{p}'-\vec{p})$)

$$f_{udN}M_N = \sigma_{\pi N} = m_{ud}\langle N|\bar{u}u+\bar{d}d|N\rangle, \qquad f_{fN}M_N = \sigma_{fN} = m_f\langle N|\bar{f}f|N\rangle$$

For heavy $Q=c,b,t$, can use (Shifman et al '78)

$$f_{QN} \approx \frac{2}{27}\Big(1-\sum_{q=u,d,s} f_{qN}\Big)$$

Feynman-Hellmann method

- Use Feynman-Hellmann (FH) theorem

$$\langle N|m_q\bar{q}q|N\rangle = m_q \left.\frac{\partial M_N}{\partial m_q}\right|_{m_q^\Phi}$$

- FH vs direct matrix element (ME) method

  ✓ Only simpler and less noisy 2pt-fn is needed
  ✓ No difficult quark-disconnected contributions
  ✓ No difficult singlet renormalization w/ Wilson fermions

  ✗ m_{ud}-dependence not very large

$$M_N \simeq 939\,\mathrm{MeV} \simeq 850\,\mathrm{MeV} + O(m_{ud})$$

  $\rightarrow$ must extract smallish correction
  ✗ m_s dependence even smaller

Lattice details

- $N_f = 1 + 1 + 1 + 1$
- 3HEX clover-improved Wilson fermions on tree-level improved Symanzik gluons
- 29 ensembles w/ total $\sim$ 155000 trajectories
- $\sim$ 500 measurements per configuration
- 4 $a \in [0.064, 0.102]\,\mathrm{fm}$;
- $M_\pi \in [195, 450]\,\mathrm{MeV}$ w/ $LM_\pi > 4$

Improvements over BMWc, PRL '16

✓ Charm in sea

✓ $\gtrsim \times 100$ in statistics (reduced to $\gtrsim 35$ by later plateaux)

✓ $\gtrsim \times 2$ lever arm in m_s

✓ Like PRL '16 FH in terms of quark and not meson masses

✗ No physical m_{ud}, but small enough and know M_N

Analysis strategy

- Dependence of 4 hadron masses analyzed simultaneously, vs m_{ud}, m_s and $m_{d-u} = m_d - m_u$:
  - M_Ω, $M^2_{\pi^+}$ and $M^2_{K\chi} = (M^2_{K^+} + M^2_{K^0} - M^2_{\pi^+})/2$ to fix a, m^Φ_{ud} and m^Φ_s
  - M_N to determine sigma terms (and sometimes scale)

- Quark masses obtained w/ *ratio-difference method* (BMWc, JHEP 1108)

- Since $Z_S(\beta)$ is not computed, write functional dependence in terms of ($q = ud, s,$ or $, d-u$)

$$c_q\left[\frac{(am_q)}{aZ_s(\beta)} - m^\Phi_q\right] \longrightarrow \tilde{c}_q\left[\frac{(am_q)}{aZ_s(\beta)m^\Phi_q} - 1\right] \longrightarrow \tilde{c}_q\left[\frac{(am_q)}{a\cdot\tilde{m}^\Phi_q(\beta)} - 1\right]$$

- Cross checks:
  - $\tilde{m}^\Phi_s/\tilde{m}^\Phi_{ud} = m^\phi_s/m^\phi_{ud}$?
  - Values of $Z_S(\beta)$?

Analysis details (cont'd)

- Minimize $\chi^2 = V^T C^{-1} V$, with C full covariance matrix,

$$V^T = \Big[aM_{[\Omega,1]} - f_\Omega(c, X_{[q,1]}), \ldots, aM_{[N,N_{\rm ens}]} - f_N(c, X_{[q,N_{\rm ens}]}),$$
$$m_{[q,1]} - X_{[q,1]}, \ldots, m_{[q,N_{\rm ens}]} - X_{[q,N_{\rm ens}]}\Big]$$

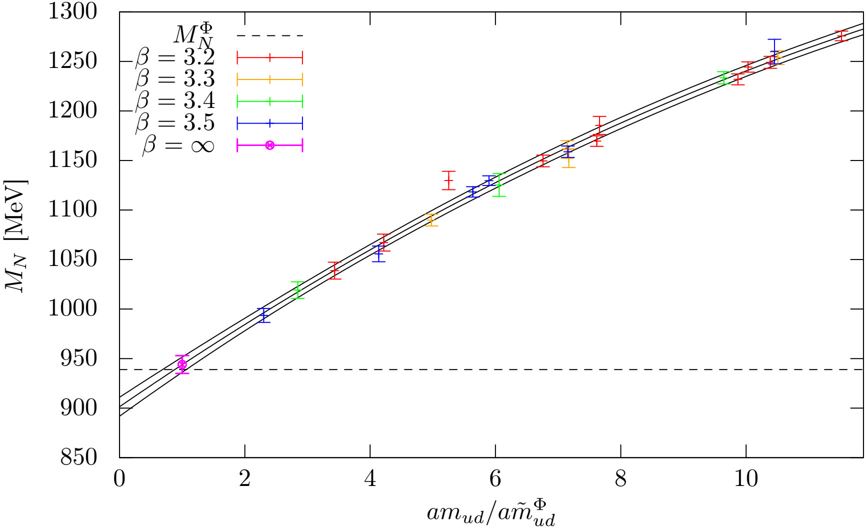
  and e.g.

$$f_N(c,X) = a(\beta) M_N^\Phi [1+\mathrm{FV}(aM_\pi, L/a)] \left\{ 1 + \sum_i^2 c_{ud,i} \left[\frac{(am_{ud})}{a \cdot \tilde{m}_{ud}^\Phi(\beta)} - 1 \right]^i + c_{s,1} \left[\frac{(am_s)}{a \cdot \tilde{m}_s^\Phi(\beta)} - 1 \right] \right\}$$

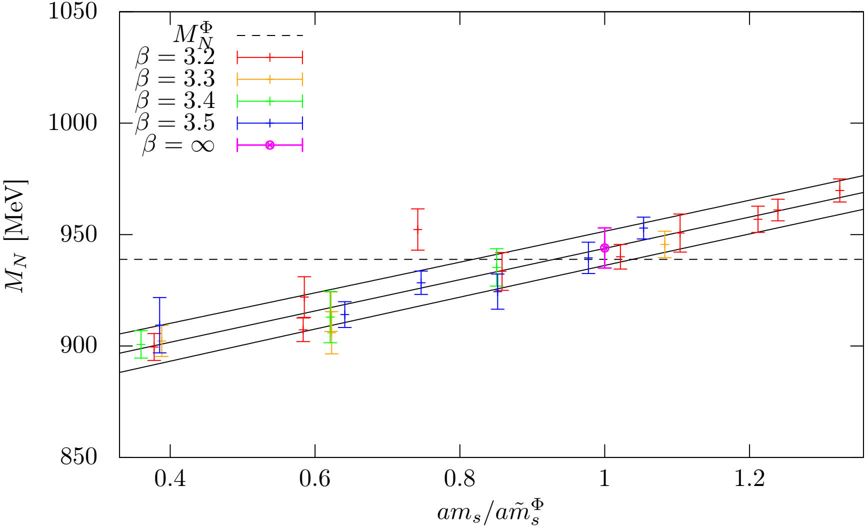
- Statistical error with $N_{\rm boot} = 1000$

- To determine fit ranges, use Kolmogorov-Smirnov test with different $t_{\rm min}$ (BMWc, Science '15)

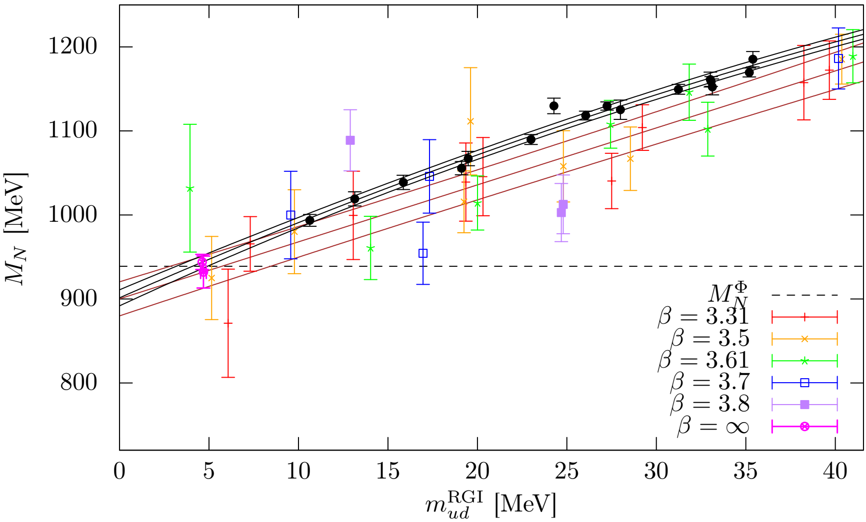
Example fits (preliminary)



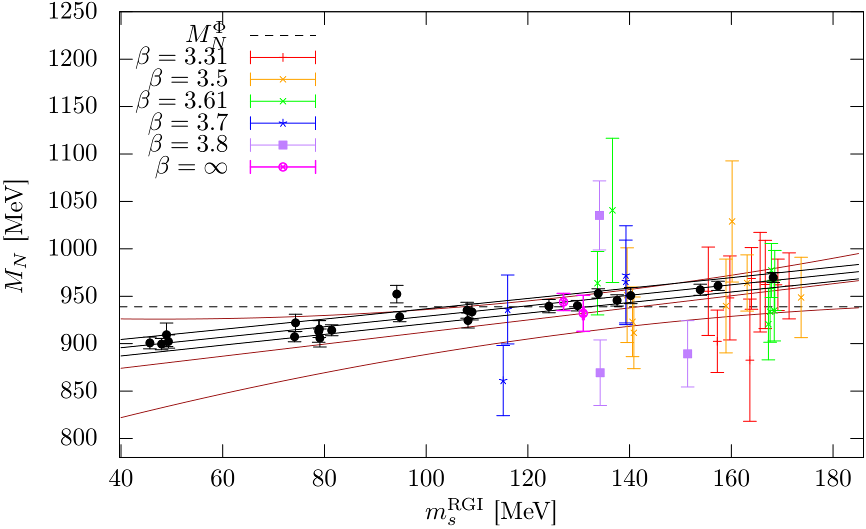
Example fits (preliminary)



Comparison with BMWc, PRL '16 (preliminary)



Comparison with BMWc, PRL '16 (preliminary)



New method for obtaining $f_{u/d}^{p/n}$

[BMWc, PRL 116 (2016)]

- Input: f_{ud}^N and $\Delta_{\text{QCD}}M_N = M_n - M_p$ (from BMWc, Science '15)
- SU(2) relations w/ $\delta m = m_d - m_u$

$$H = H_{\text{iso}} + H_{\delta m}\,, \quad H_{\delta m} = \frac{\delta m}{2}\int d^3x\,(\bar{d}d - \bar{u}u)$$

$$\Delta_{\text{QCD}}M_N = \delta m\langle p|\bar{u}u - \bar{d}d|p\rangle$$

lead to, w/ $r = m_u/m_d$,

$$\begin{aligned} f_u^{p/n} &= \left(\frac{r}{1+r}\right) f_{ud}^N \pm \frac{1}{2}\left(\frac{r}{1-r}\right)\frac{\Delta_{\text{QCD}}M_N}{M_N} \\ f_d^{p/n} &= \left(\frac{1}{1+r}\right) f_{ud}^N \mp \frac{1}{2}\left(\frac{1}{1-r}\right)\frac{\Delta_{\text{QCD}}M_N}{M_N} \end{aligned}$$

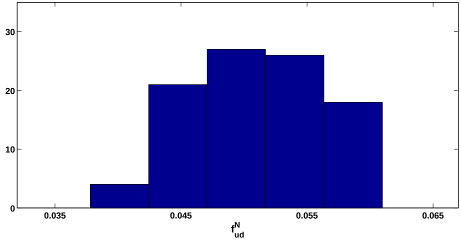
- Huge improvement on usual $SU(3)$-flavour approach

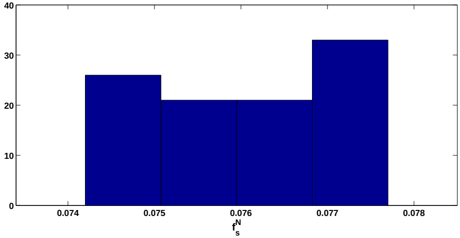
systematic: $$\left(\frac{m_s - m_{ud}}{\Lambda_{QCD}}\right)^2 \approx 10\% \quad \longrightarrow \quad \left(\frac{m_d - m_u}{\Lambda_{QCD}}\right)^2 \approx 0.01\%\,.$$

Systematic error assessment (preliminary)

Estimated using extended frequentist approach (BMWc, Science '08)

- Excited state contamination: 2 time intervals ($t_{min} = 1.3$ or $1.4\,\mathrm{fm}$)
- Mass interpolation/extrapolation errors
  - $M_\pi \leq 400\,\mathrm{MeV}$ & $450\,\mathrm{MeV}$
  - different m_q dependences (polynomials & Padés)
- lattice spacing uncertainty: M_Ω vs M_N
- continuum extrapolation: $O(a^2)$ vs $O(\alpha_S a)$
- AIC weight (BMWc, Science '15)





Preliminary results

- Checks
  - $Z_S(\beta) \sim 0.7$ and expected β-dependence
  - Compare

$$M_N = 940(9)(6)\,\mathrm{MeV} \qquad m_s/m_{ud} = 26.33(37)(2)$$

w/ $M_N = 939\,\mathrm{MeV}$ (PDG '15) and $m_s/m_{ud} = 27.53(20)(08)$ (BMWc, JHEP 08 (2011))

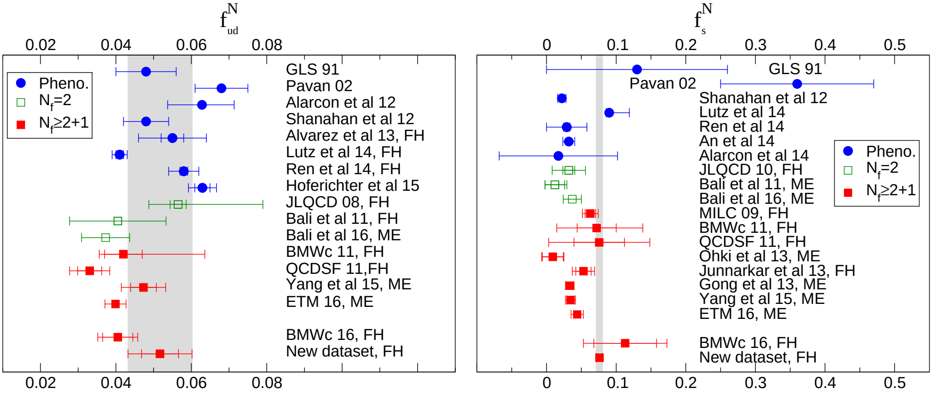
- Predictions

$$f_{ud}^N = 0.0517(49)(70) \qquad f_s^N = 0.0760(43)(13)$$

$$f_u^p = 0.0174(16)(23) \qquad f_d^p = 0.0329(34)(48)$$

$$f_u^n = 0.0151(16)(22) \qquad f_d^n = 0.0379(34)(48)$$

Comparison



f_{ud}^N
0.02
0.04
0.06
0.08
Pheno.
$N_f=2$
$N_f\geq2+1$
GLS 91
Pavan 02
Alarcon et al 12
Shanahan et al 12
Alvarez et al 13, FH
Lutz et al 14, FH
Ren et al 14, FH
Hoferichter et al 15
JLQCD 08, FH
Bali et al 11, FH
Bali et al 16, ME
BMWc 11, FH
QCDSF 11,FH
Yang et al 15, ME
ETM 16, ME
BMWc 16, FH
New dataset, FH
f_s^N
0
0.1
0.2
0.3
0.4
0.5
GLS 91
Pavan 02
Shanahan et al 12
Lutz et al 14
Ren et al 14
An et al 14
Alarcon et al 14
JLQCD 10, FH
Bali et al 11, ME
Bali et al 16, ME
MILC 09, FH
BMWc 11, FH
QCDSF 11, FH
Ohki et al 13, ME
Junnarkar et al 13, FH
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