Complete Monopole Dominance of the Static Quark Potential

Nigel Cundy
Weonjong Lee, YongMin Cho

July 29, 2016

arXiv:1503.07033, arXiv:1603.03470v2

Introduction

- We wish to identify the topological objects that lead to quark confinement.
- Our tool is the gauge-independent Abelian decomposition first proposed by Cho, Duan and Ge.
- This decomposes the non-Abelian gauge field A_μ into an Abelian (restricted) part, $\hat{A}$ and a coloured field $\hat{X}$,

$$A_\mu(x) = \hat{A}_\mu(x) + X_\mu(x) \qquad U_{\mu,x} = \hat{U}_{\mu,x}\hat{X}_{\mu,x+\hat{\mu}},$$

where U_μ is the lattice gauge link $P[e^{ig\int A_\mu dx_\mu}]$
$\hat{U}$ and $\hat{X}$ the links corresponding to the $\hat{A}$ and X fields.

- The decomposition is performed independent of the gauge by choosing:
  1. an SU(N) field $\theta(x)$,
  2. a subsequent colour direction $n_j = \theta\lambda_j\theta^\dagger$, $j = 3, 8, \ldots$

$$D_\mu[\hat{A}]n_j = 0 \qquad \text{tr } n_j X_\mu = 0$$

$$\hat{A}_\mu = \frac{1}{2} n_j \text{tr}\left(n_j A_\mu + \frac{i}{g}\lambda_j \theta^\dagger \partial_\mu \theta\right).$$

- The two parts of $\hat{A}$ are known as the Maxwell part and θ-part.
- The field strength $F_{\mu\nu}[\hat{A}]$ is a function of n_j
- There are $N-1$ redundant d.o.f. in θ: ($\theta \to \theta e^{id^j\lambda_j}$)

$$\theta = \begin{pmatrix} \cos a & i \sin a e^{ic} \\ i \sin a e^{-ic} & \cos a \end{pmatrix} e^{id\lambda_3},$$

$0 \leq a \leq \frac{\pi}{2}$ and $c, d \in \mathbb{R}$

- We can choose a θ field and perform this decomposition regardless of the gauge.
- If under a gauge transformation $\Lambda \in SU(2)$

$$U_{\mu,x} \to \Lambda_x U_{\mu,x} \Lambda^\dagger_{x+\hat{\mu}} \qquad \theta_x \to \Lambda_x \theta_x,$$

then $\hat{U}$ and $\hat{X}$ will transform gauge covariantly,

$$\hat{U}_{\mu,x} \to \Lambda_x \hat{U}_{\mu,x} \Lambda^\dagger_{x+\hat{\mu}} \qquad \hat{X}_{\mu,x} \to \Lambda_x \hat{X}_{\mu,x} \Lambda^\dagger_x.$$

- $\hat{A}$ transforms as a U(1) field. In SU(2),

$$\frac{1}{2}\text{tr}\left(n_3 A_\mu + \frac{i}{g}\lambda_3 \theta^\dagger \partial_\mu \theta\right) \to \frac{1}{2}\text{tr}\left(n_3 A_\mu + \frac{i}{g}\lambda_3 \theta^\dagger \partial_\mu \theta\right) + \frac{1}{g}\partial_\mu (d' - d)$$

- We can construct gauge invariant observables from the restricted field alone.
- $\hat{U}$ is Abelian, so is much easier for analytical studies.
- We choose θ so that it contains the eigenvalues of the Wilson Loop operator
- Along the Wilson loop (in xt plane), this implies $\hat{U} = U$.

$$\text{tr}\, W_L = \text{tr} \prod U_{\mu,x} = \text{tr} \prod \hat{U}_{\mu,x} = \text{tr}\, e^{ig\lambda_3 \oint dx_\mu \frac{1}{2}\text{tr}\left(n_3 A_\mu + \frac{i}{g}\lambda_3 \theta^\dagger \partial_\mu \theta\right)}.$$

In SU(2),

$$\theta^\dagger \partial_\mu \theta = i\phi\partial_\mu a + i\bar{\phi}\sin a \cos a \partial_\mu c - i\lambda_3 \sin^2 a \partial_\mu c,$$

with

$$\phi = \begin{pmatrix} 0 & e^{ic} \\ e^{-ic} & 0 \end{pmatrix} \qquad \bar{\phi} = \begin{pmatrix} 0 & ie^{ic} \\ -ie^{-ic} & 0 \end{pmatrix}.$$

- The expression for the Wilson Loop contains the factor

$$\text{tr}W_L = \text{tr}\prod U_{\mu,x} = \text{tr}e^{\cdots + i\lambda_3 \oint dx_\mu \sin^2 a \partial_\mu c}.$$

- The parameter c is ill-defined at $a = 0$ or $\pi/2$
- We can have non-zero winding number around these points.

$$(t, x, y, z) = \\ r(\cos\varphi_1, \sin\varphi_1\cos\varphi_2, \sin\varphi_1\sin\varphi_2\cos\varphi_3, \sin\varphi_1\sin\varphi_2\sin\varphi_3)$$

- Then write $c = \nu\varphi_1$ for winding number ν.
- $\oint dx_\mu \sin^2 a \partial_\mu c$ will contain a contribution proportional to the sum of the winding number within the loop
- It should scale with the number of monopoles within the loop
- This will be proportional to the area of the loop.

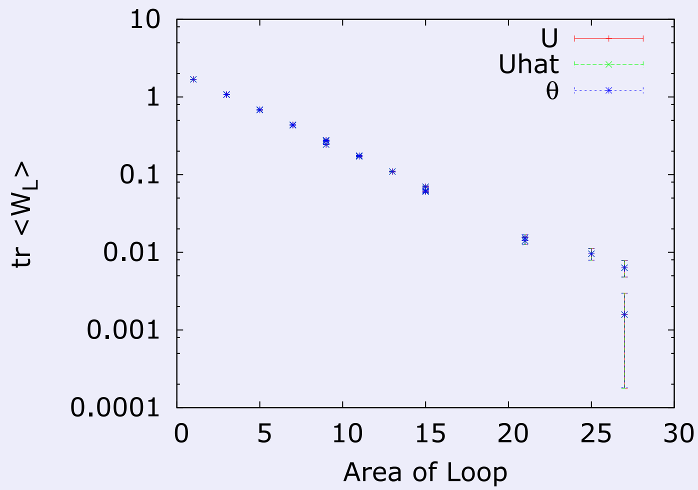
- My goal is to show that the topological objects discussed above are
  - Present in the configuration
  - Contribute to quark confinement.
- θ is gauge dependent – we need to fix the gauge.
- But whatever causes confinement should not be gauge dependent.
- Under a gauge transformation:

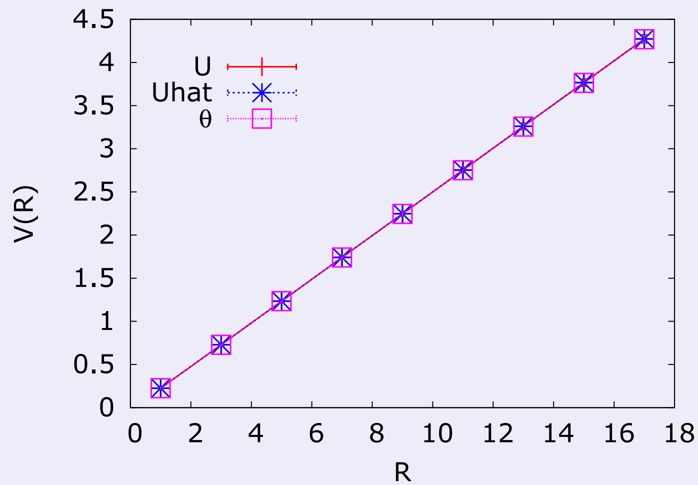
$$\text{tr}(n_3(gA_\mu)) \to \text{tr}(\lambda_3(g\theta^\dagger A_\mu\theta + i\theta^\dagger\partial_\mu(\Lambda^\dagger)\Lambda\theta))$$

$$\text{tr}(i\lambda_3(\theta^\dagger\partial_\mu\theta)) \to \text{tr}(i\lambda_3(\theta^\dagger\partial_\mu\theta - \theta^\dagger\partial_\mu(\Lambda^\dagger)\Lambda\theta)) + i\partial_\mu(d' - d).$$

- Λ has the same structure as θ
- A gauge transformation moves features from one part of $\hat{A}$ to the other part
- The objects themselves are not gauge dependent

- We can search for the objects within the $\theta^\dagger\partial_\mu\theta$ part of $\hat{A}$
- This is harder in the Maxwell part (tr nA_μ).
- Our proposal is to fix the gauge so that the θ term alone accounts for the whole Wilson Loop.
- We can then isolate the topological objects and study them.





- So far, we just have numerical results in SU(2), $\beta = 3.4$, stout smeared Lüscher-Weisz gauge action, $16^3 \times 32$ lattice.

Fixing the Gauge

- We used a set of nested Wilson Loops in the xt plane to construct θ and to measure the string tension
- These loops are then stacked on top of each other in the y and z directions.
- θ is loop-dependent
- The topological objects contributing to confinement depend on the quarks we are attempting to confine.

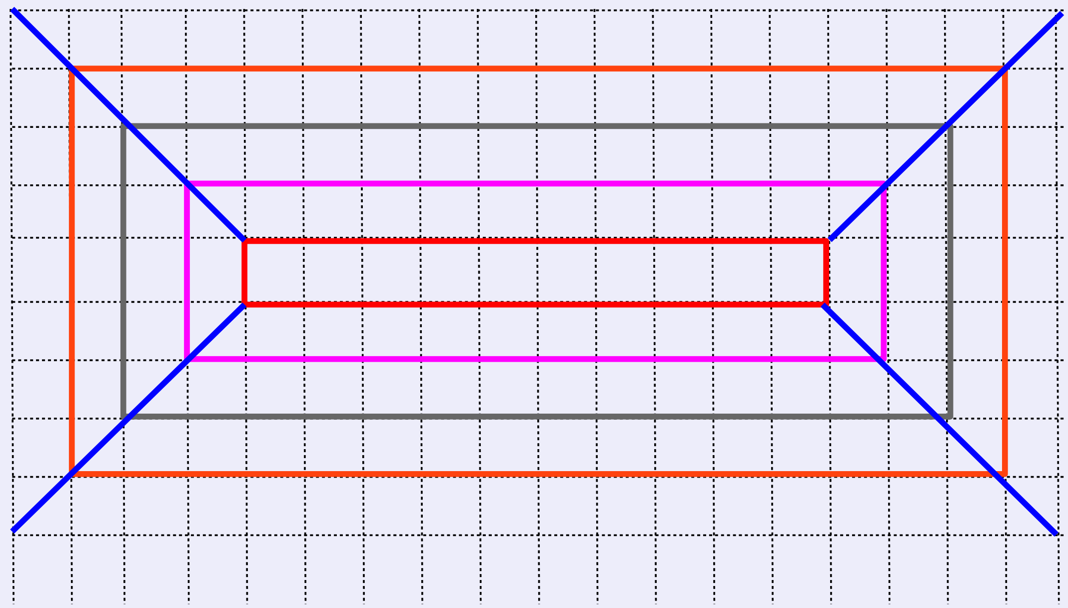
- The continuum version of the gauge fixing needs to satisfy three requirements:
  1. θ should be smooth and differentiable;
  2. The θ term should dominate the Wilson Loop;
  3. We should avoid a Fadeev-Poppov ghost determinant
- The lattice equivalent of this gauge fixing procedure is to
  1. Choose d so that $\theta^\dagger_x \hat{U}_{x,\mu}\theta_{x+\hat{\mu}} \sim 1 + iO(\epsilon)$.
  2. Fix the gauge so that

$$e^{i\epsilon\lambda_3 \text{tr} i\lambda_3\theta^\dagger\partial_\mu\theta} = \theta^\dagger \hat{U}_\mu \theta \qquad\qquad d = 0$$

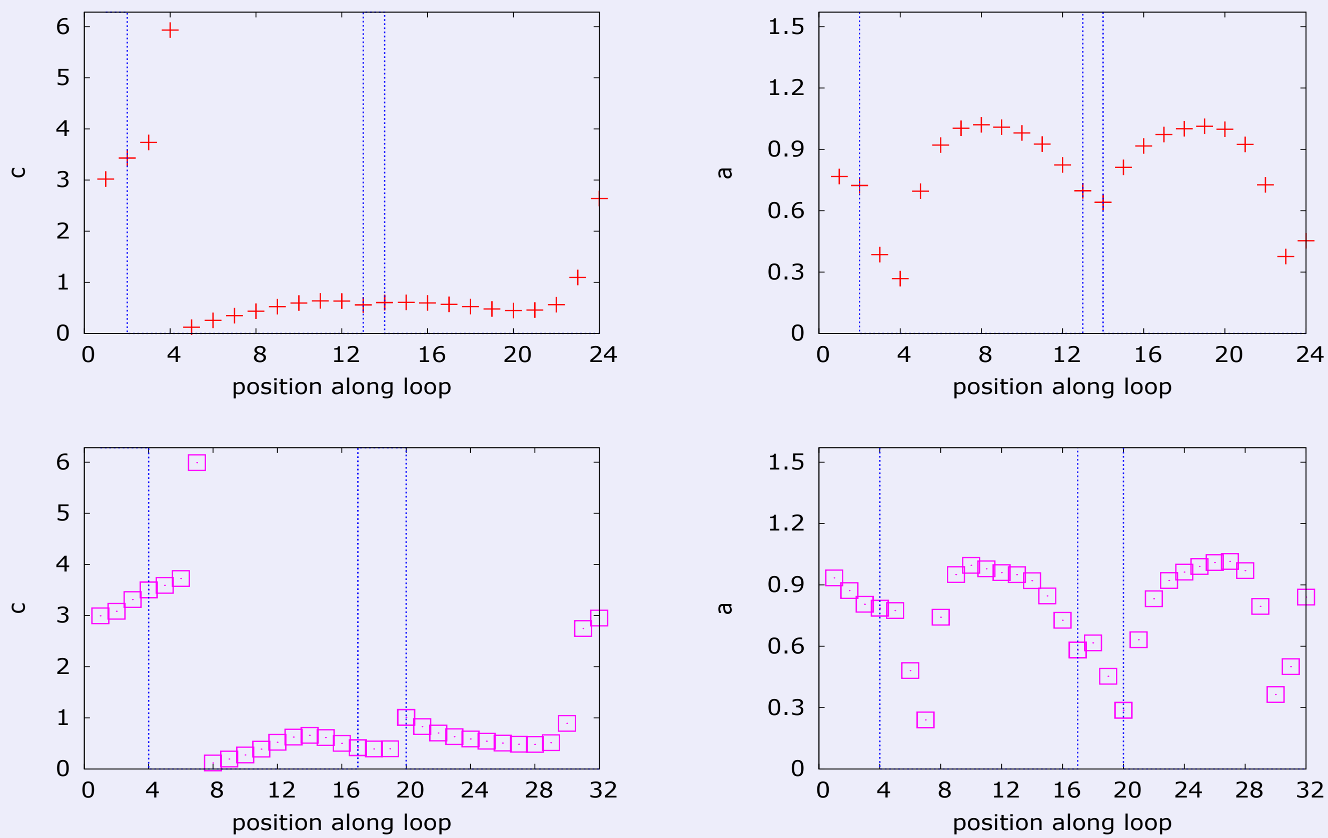
  where the direction $\mu = x, t$, ϵ is the lattice spacing.
- We apply the gauge fixing condition exactly along the Wilson Loop, and as close as we can for the other links.

Finding the Monopoles

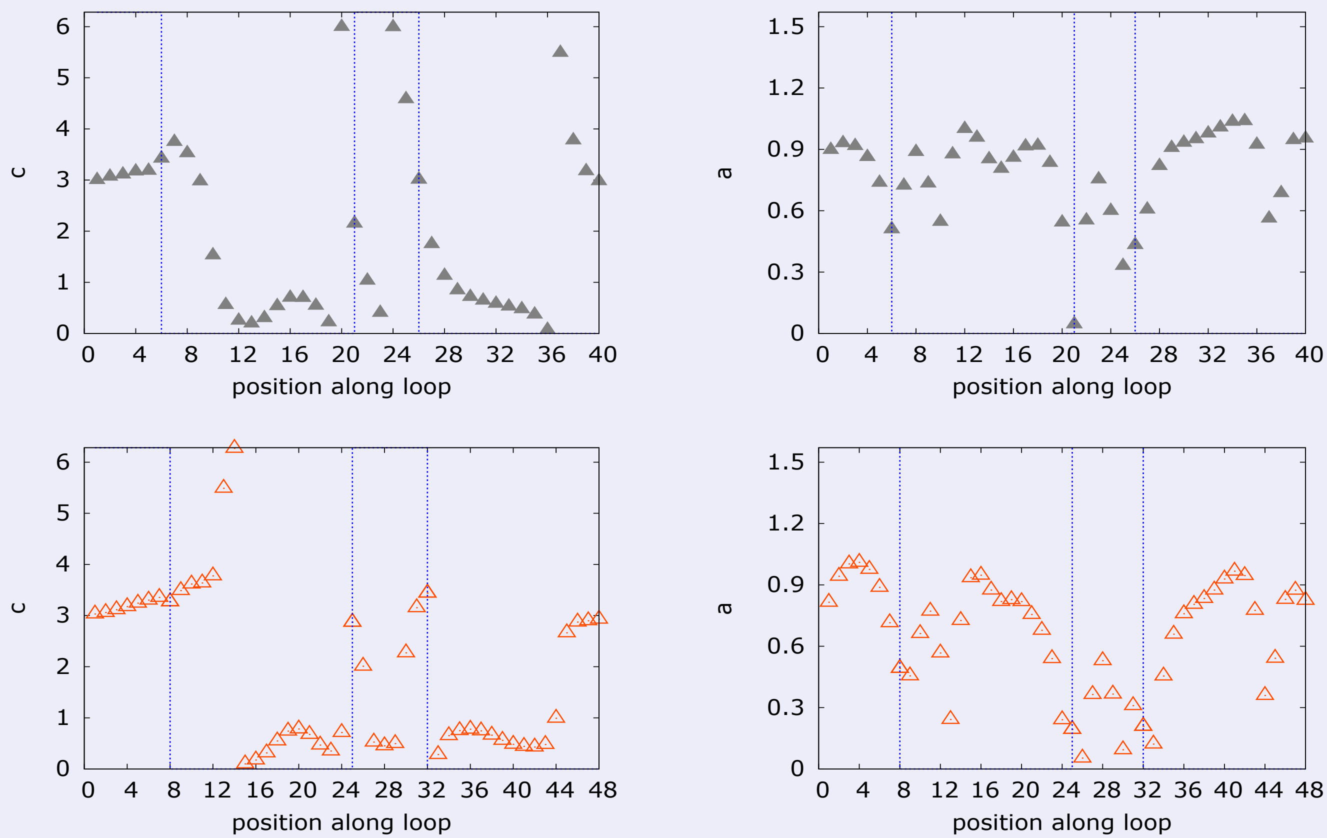
$$\text{tr}\, W_L \sim \text{tr}\, e^{i \oint \lambda_3 \sin^2 a \partial_\mu c dx_\mu}$$



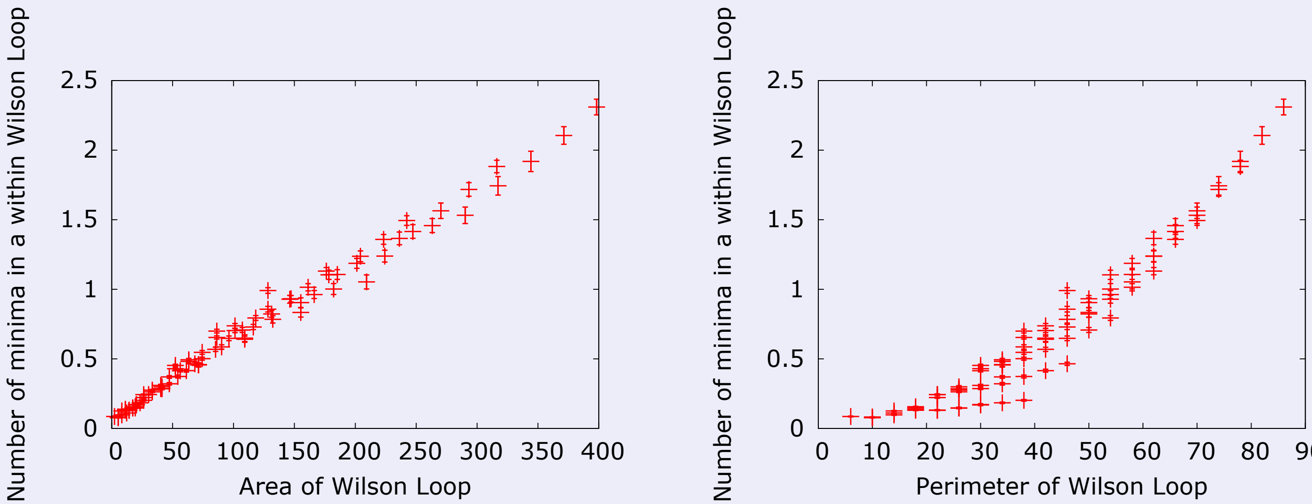
$$\text{tr}\, W_L \sim \text{tr}\, e^{i \oint \lambda_3 \sin^2 a \partial_\mu c dx_\mu}$$



$$\text{tr}\, W_L \sim \text{tr}\, e^{i \oint \lambda_3 \sin^2 a \partial_\mu c dx_\mu}$$

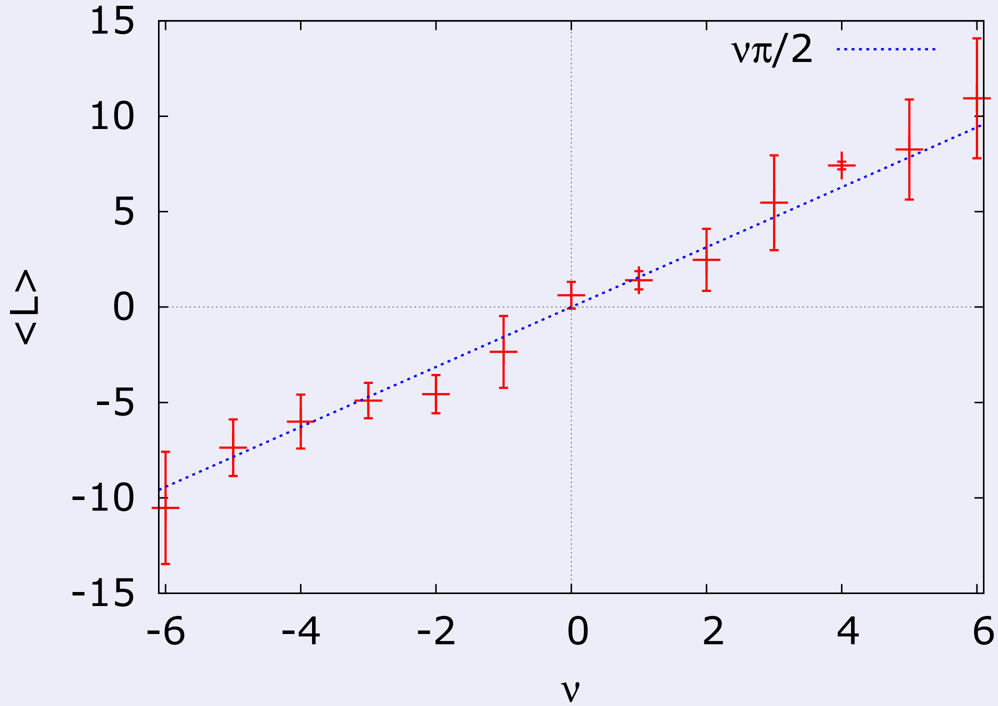


- Are these objects (minimum in a, winding in c) distributed according to an area law?



- Is the winding number responsible for the Wilson Loop?

$$L = \oint dx_\mu \frac{1}{2}\text{tr}\left(\lambda_j \theta^\dagger A_\mu \theta + \frac{i}{g}\lambda_j \theta^\dagger \partial_\mu \theta\right)$$
$$= \oint dx_\mu \sin^2 a \partial_\mu c \sim \left(\frac{1}{2}\right)(2\pi\nu)$$



Conclusion

- Through a combination of an Abelian decomposition and gauge fixing, we can isolate the topological objects responsible for confinement in SU(2) pure Yang Mills gauge theory.
- The θ term is wholly responsible for the static quark potential.
- We are able to see the emergence of a localised topological winding number at minimum values of a
- These objects are distributed according to an area law.
- The value of the Wilson Loop is proportional to the winding number, and changes where there is winding.
- These topological objects are present in the Yang Mills gauge fields, and a good candidate for studies into the cause of confinement in SU(2) Yang-Mills theory.