Angular and chiral content of the ρ and ρ' mesons

Christian Rohrhofer, Markus Pak, Leonid Y. Glozman

based on 1603.04665 [hep-lat]

26 July 2016, Lattice 2016

Der Wissenschaftsfonds.

Mesons in the Isospin 1, 1^{--} channel

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Physical mesons

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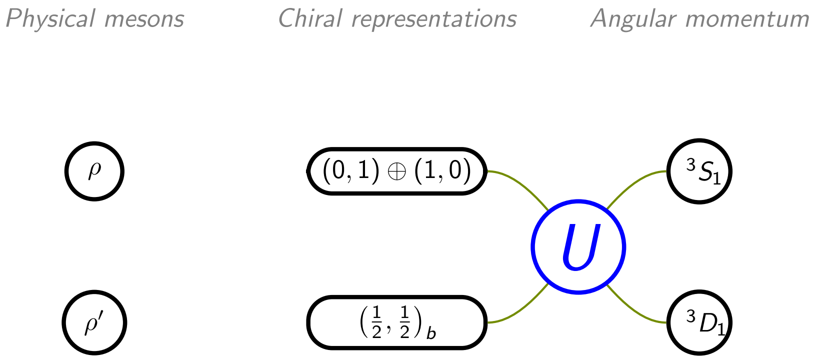

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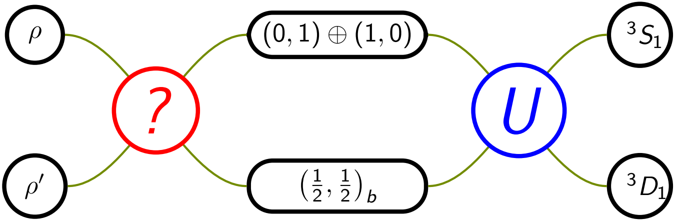

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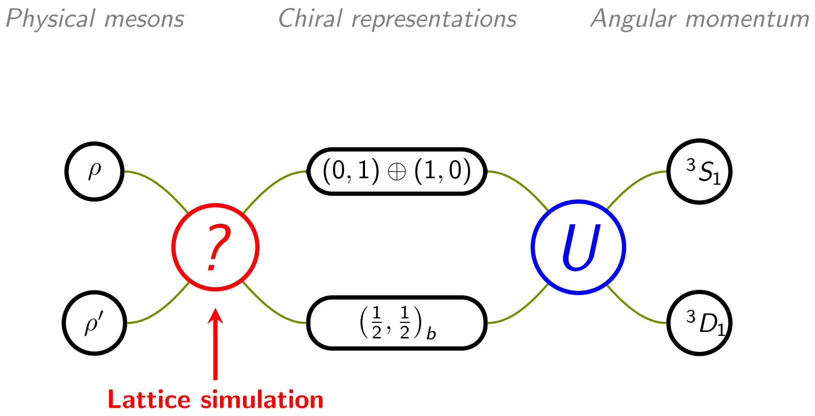

The quark model K.A. Olive et al. (PDG), Chin. Phys. C, 38, 090001 (2014)

$n\ ^{2s+1}\ell_J$	J^{PC}	type	I=1	I=$\frac{1}{2}$	I=0
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$1\ ^3P_2$	2^{++}	tensor	$a_2(1320)$	K_2^*	f_2
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"The physical vector mesons listed under $1\,^3D_1$ and $2\,^3S_1$ may be mixtures of $1\,^3D_1$ and $2\,^3S_1$, or even have hybrid components."

Angular momentum content of the ρ meson

$$|11^{--}\rangle = \mathbf{x}\ |(0,1)+(1,0)\rangle + \mathbf{y}\ |(\tfrac{1}{2},\tfrac{1}{2})_b\rangle$$

Glozman and Nefediev, 0704.2673 [hep-ph], *PhysRevD76,096004*

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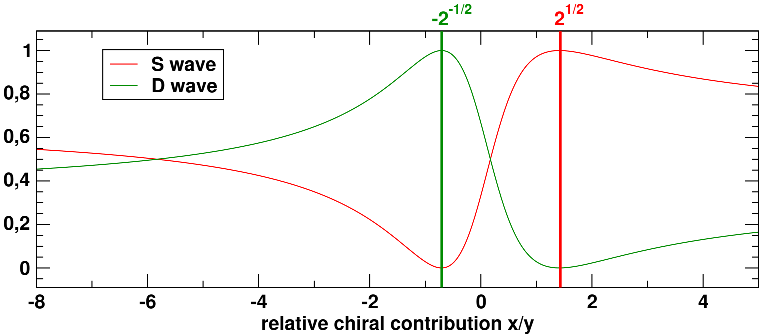

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Gauge configurations and lattice details

JLQCD $n_f = 2$ configurations

- Ensemble: 100 configurations
- Lattice size $16^3 x32$
- Spacing $a = 0.1184(3) fm$
- Topological charge $Q = 0$
- Dynamical overlap fermions
- Pion mass $m_\pi = (289 \pm 1.8) MeV$
- Lowest Energy for zero-momentum $\pi\pi$-state:

$$\approx 1400 MeV \longrightarrow \rho \text{ is stable particle}$$

JLQCD Collaboration, 0803.3197 [hep-lat], *10.1103/PhysRevD78,014508*

Interpolators for a 8x8 correlation matrix

	chiral representation	
	$(0,1)\oplus(1,0)$ **"vector"** $\bar{q}(\vec{\tau}\otimes\gamma^k)q$	$(\frac{1}{2},\frac{1}{2})_b$ **"tensor"** $\bar{q}(\vec{\tau}\otimes\gamma^0\gamma^k)q$

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super narrow	O^V_{sn}	O^T_{sn}
narrow	O^V_{n}	O^T_{n}
wide	O^V_{w}	O^T_{w}
ultra wide	O^V_{uw}	O^T_{uw}

↕ probes different resolutions

↔ probes chiral contribution

Variational analysis & interpolator contribution

?

Construct a cross correlation matrix

$$C_{ij}(t) = \langle O_i(t) O_j^\dagger(0) \rangle = \sum_n \langle 0 | O_i | n \rangle \langle n | O_j^\dagger | 0 \rangle e^{-E_n t}$$

where the overlap of interpolator O_i with physical state $|n\rangle$ is given by

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→ use ratio of overlaps as relative chiral contribution!

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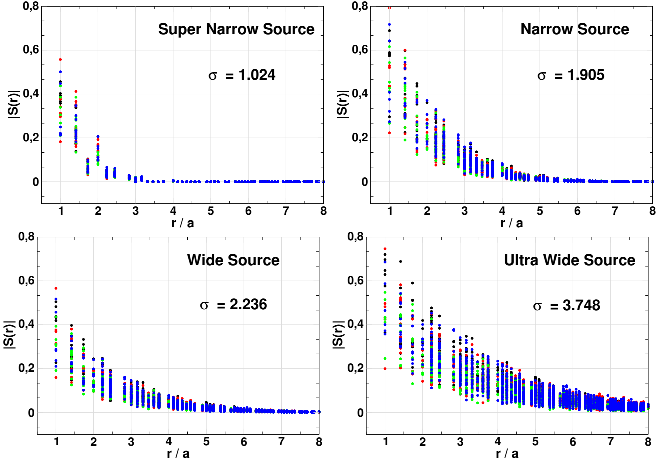

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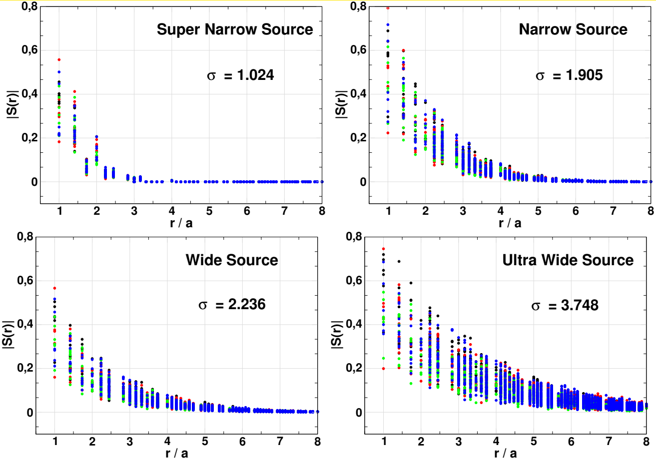
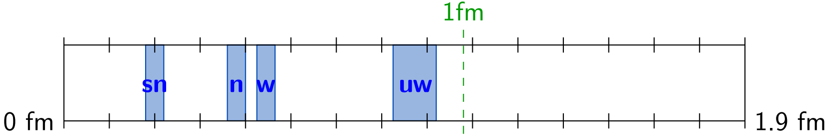
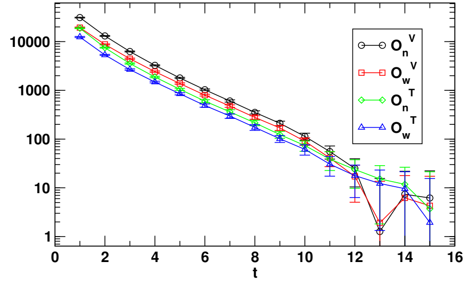
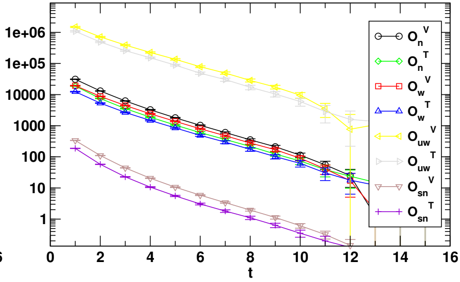




Correlators and Eigenvalues





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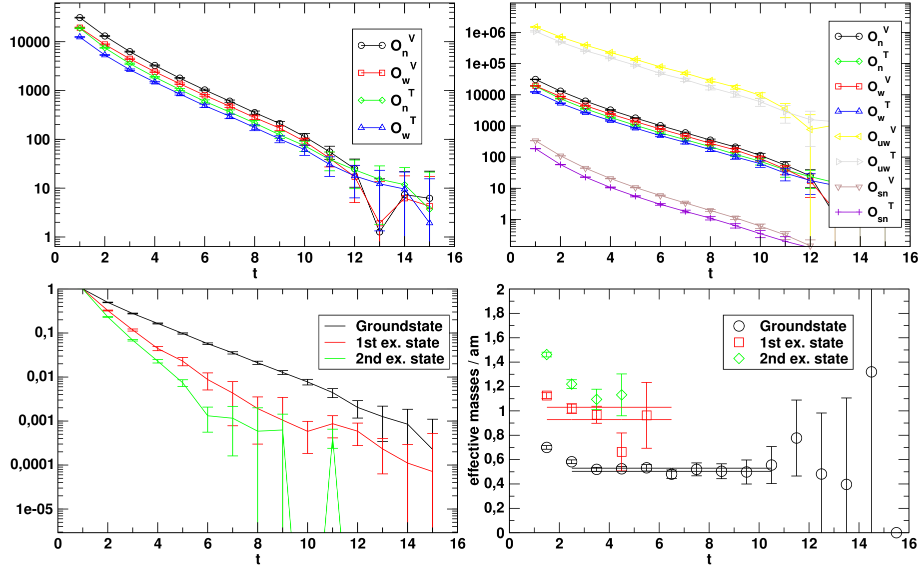

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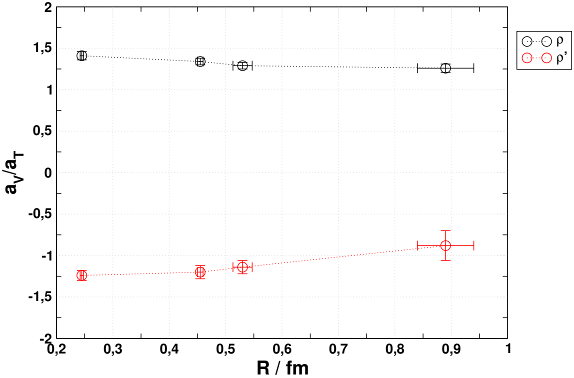

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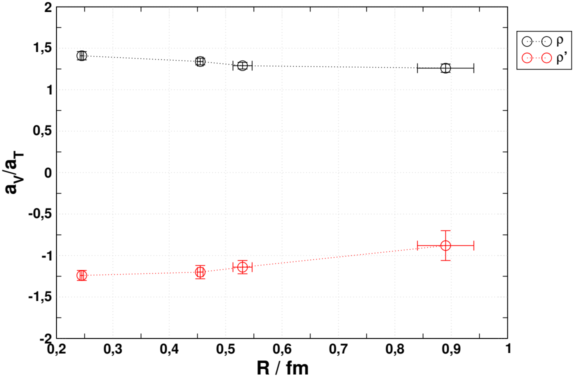

$$
\begin{aligned}
|\rho\rangle &= \quad (0.998 \pm 0.002)\, |^3S_1\rangle - \quad (0.05 \pm 0.025)\, |^3D_1\rangle \\
|\rho'\rangle &= -\,(0.106 \pm 0.09)\, |^3S_1\rangle - (0.994 \pm 0.005)\, |^3D_1\rangle \\
|\rho''\rangle &= \quad (0.99 \pm 0.1) \quad |^3S_1\rangle - \quad (0.01 \pm 0.12) \quad |^3D_1\rangle
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- k denotes the number of removed eigenmodes
- previous work e.g. (*PhysRevD*)
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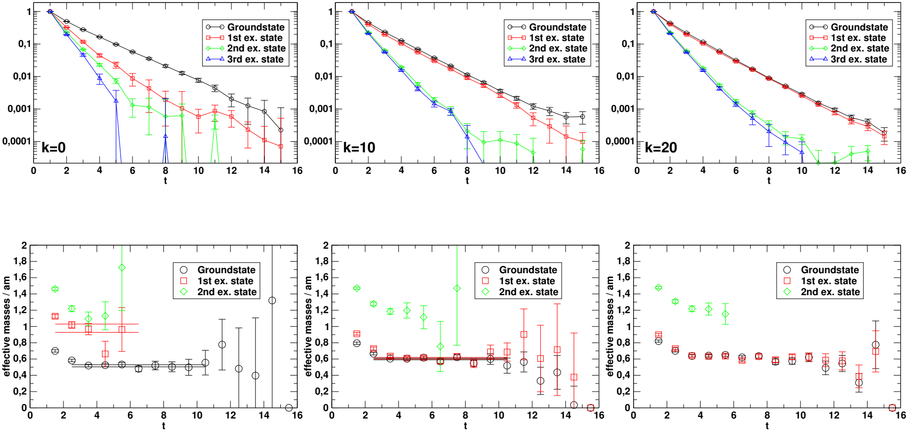
  → *chiral symmetry indeed gets restored in the hadron spectrum*

Effective masses after unbreaking $\mathcal{X}$ symmetry



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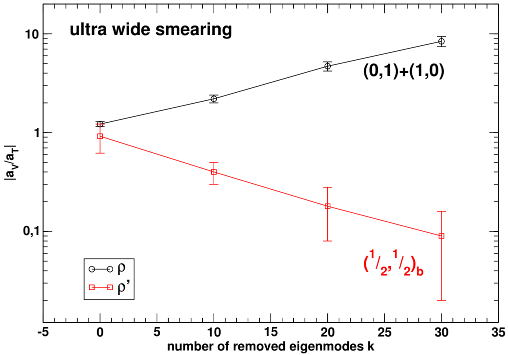

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CR *et al*, 1603.04665 [hep-lat]

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