Nucleon Matrix Elements at the Physical Point and Cost Comparison

- Z_3 smeared grid source and low mode substitution (LMS)
- Stochastic sandwich 3-pt correlators with LMS
- Comparison of cost of overlap fermions with twisted mass, clover, and domain-wall fermions.
- Isovector g_A

χ QCD Collaboration

Lattice 2016

Southampton, July 25, 2016

Overlap valence on 2+1 flavor DWF Configurations (RBC-UKQCD)

La ~ 4.6 fm
m_π ~ 170 MeV

32^3 x 64, a =0.143 fm

($O(a^2)$ extrapolation)

La ~ 2.7 fm
m_π ~ 330 MeV

24^3 x 64, a =0.111 fm

La ~ 5.5 fm
m_π ~ 140 MeV

48^3 x 96, a =0.114 fm

La ~ 2.7 fm
m_π ~ 300 MeV

32^3 x 64, a =0.0828 fm

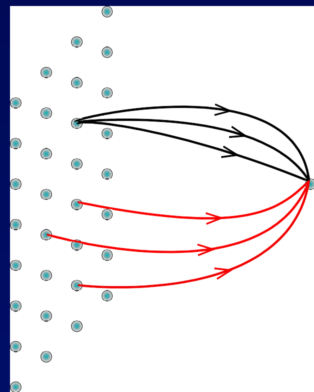
La ~ 5.3 fm
m_π ~ 140 MeV

64^3 x 128, a =0.083 fm

Correlator improvement by exploiting special features of overlap fermion

- Eigenvectors of D_{ov} (not $\gamma_5 D_{ov}$) are for all m so that one do multi-mass deflation in calculating quark propagators.
- Storage: save only half of eignevectors ($\Psi_2 = \gamma_5 \Psi_1$) and zero modes. This can be a big issue for large lattices.
- Smeared grid source with Z_3 noise ($\eta_i^3 = 1$)
- Low mode substitution to reverse the worse SNR

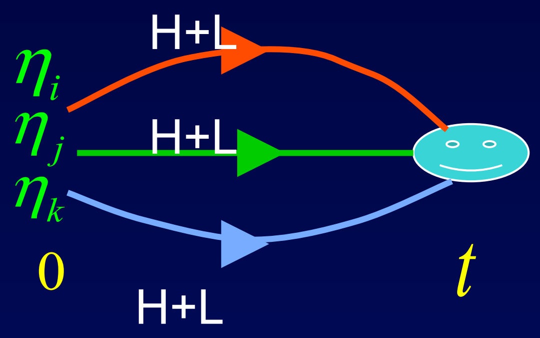
$$\frac{C_N(t)}{\sigma_N(t)} \approx \sqrt{\frac{n}{V_3}}\, e^{-(m_N - 3/2 m_\pi)t}$$

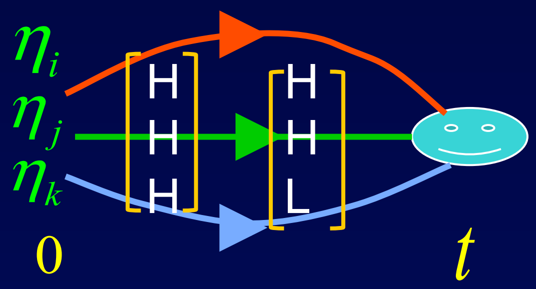


- Low mode average for the quark loop is very efficient for scalar and pseudoscalar densities (> 90% saturated).
- Stochastic sandwich method for connected 3-point correlators with noise source and sink with LMS.

Nucleon with LLL and HLL substitution

1005.5424

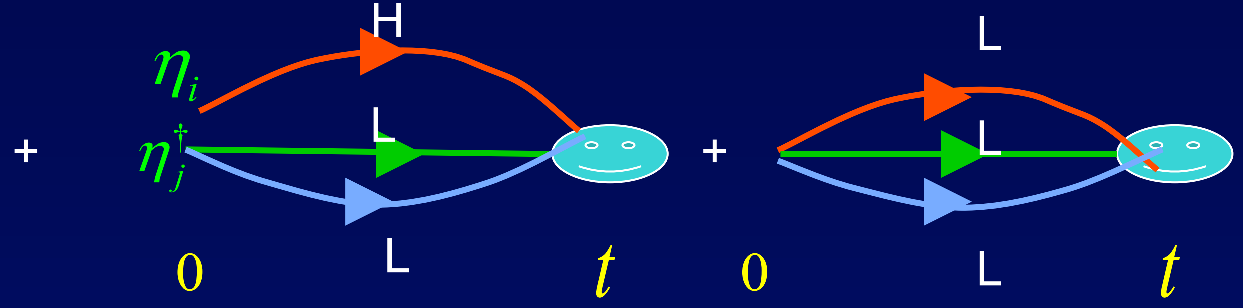




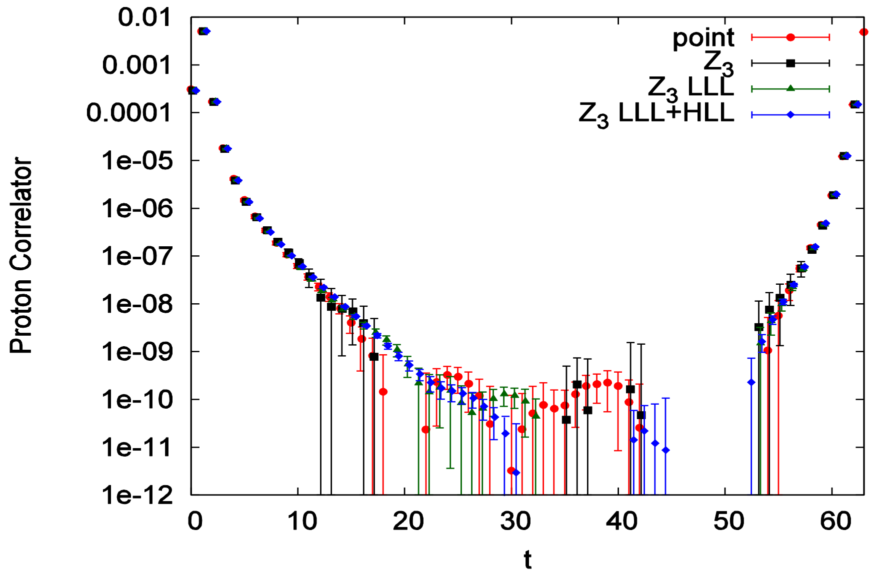
$$\frac{C_N(t)}{\sigma_N(t)} \approx \sqrt{\frac{n}{V_3}}\, e^{-(m_N - 3/2 m_\pi)t}$$

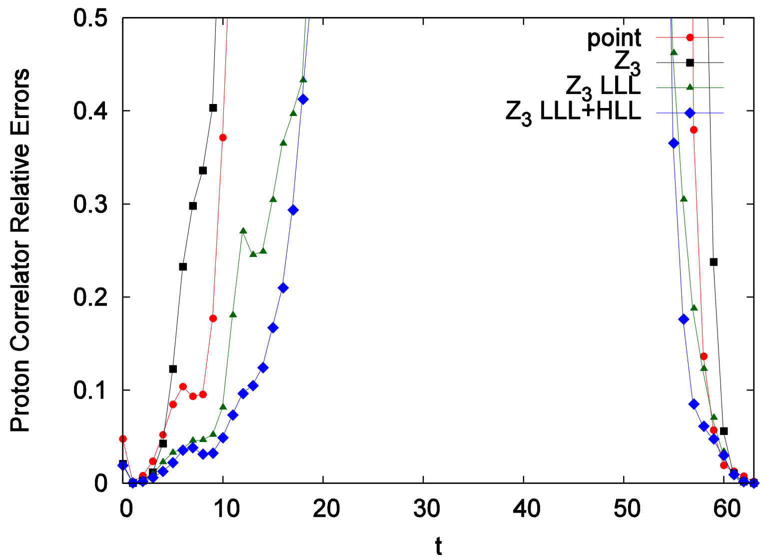
$$\langle \eta_i \eta_j \eta_k \rangle = \delta_{ij}\delta_{jk}$$

$$\langle \eta_i^\dagger \eta_j \rangle = \delta_{ij}$$



m_π ~ 300 MeV

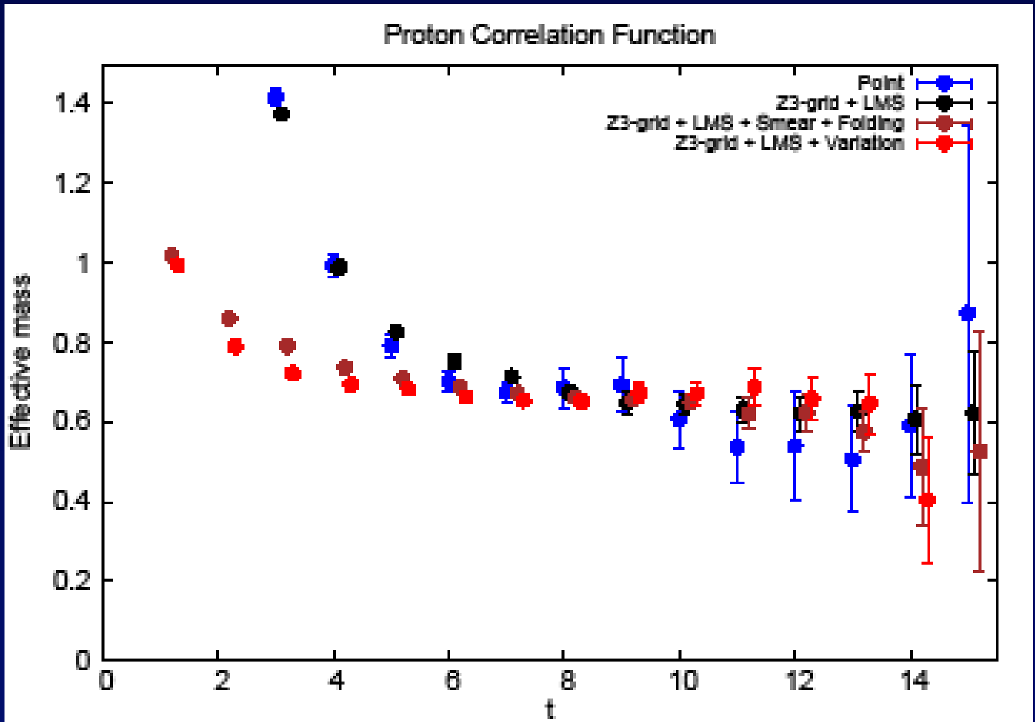


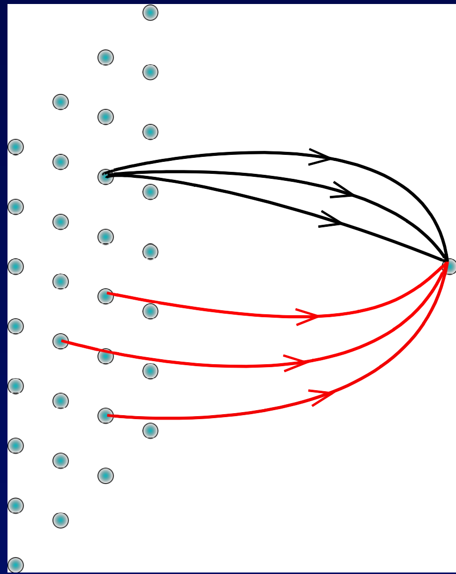


Nucleon Correlator

1304.1194

- Improvement of nucleon correlator with low-mode substitution





$24^3 \times 64$ lattice with $m_\pi = 331$ MeV, a = 1.73 GeV^{-1}

47 configurations

Point source: $m_N = 1.13(14)$ GeV;

Z_3 grid source: $m_N = 1.08(5)$ GeV;

Point smeared: $m_N = 1.11(5)$ GeV;

Z_3 grid smeared source: $m_N = 1.14(2)$ GeV;

Variation: $m_N = 1.16(1)$ GeV

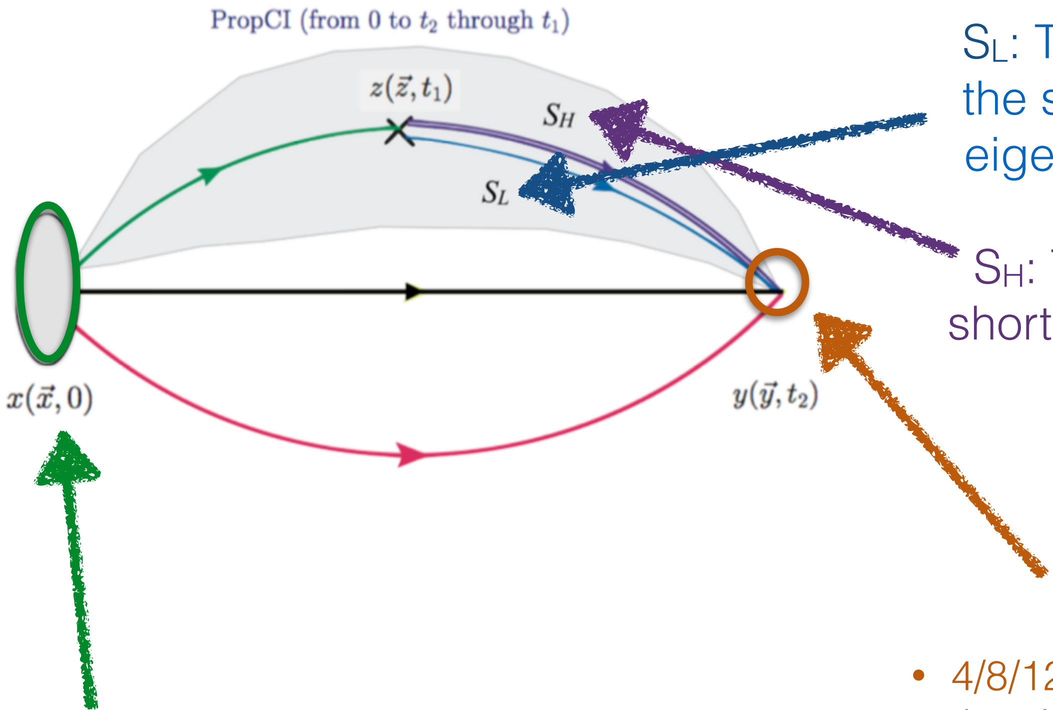
Stochastic Sandwich Method for CI

1509.04616

- Grid smeared source with Z_3 noise (4^3x3 for 48I lattice) with LMS.
- Grid noise sink for the high modes of the propagator connecting the sink and the current.
- Being a sandwich method, one sink inversion of the high modes is good for all flavor, polarization, and momentum transfer or different hadrons. This is independent of the source unlike sink sequential method.
- Has a choice of current sequential or sink sequential.
- Multi-mass is accommodated.
- Can become expensive if there are many currents and many low modes.

Simulation setup

On the 48^3x96

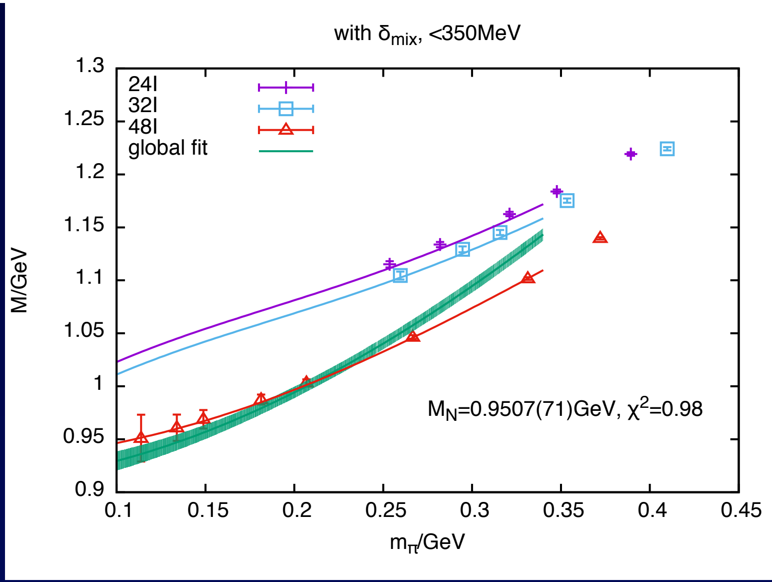


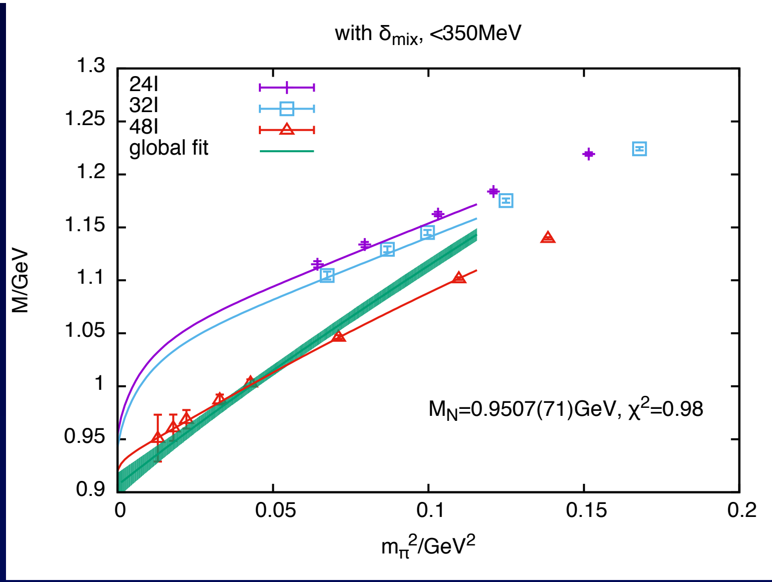
S_L: The exact long distance part of the sequential propagator from the eigensystem of the Dirac operator.

S_H: The stochastic estimate of the short distance part of the sequential propagator.

5 Z-3 noise grid sources: 4^3x3 points on 48^3x96 lattice, each point is a block smear source with size=5

- 4/8/12 noise grid sources (only for $\mathbf{S_H}$) at the six time slices separated to **the source** by 8a, 10a, 12a.
- No smear on the propagator while the block smear with size=5 is applied on the sink for the contraction.





m_N (134) = 961(12) MeV, m_N(global) = 950.7(7.1) MeV

Cost comparison

Clover on HISQ (1606.07049)

$$\frac{\text{Clover (a09m130)}}{\text{Overlap (48I)}} = \frac{(84768 + 7064 \times 3)}{(81 \times 32)}(\text{inver}) \times \frac{1}{44}(\text{time}) \times \begin{cases} (\frac{0.011}{0.013})^2(\text{var})(\text{unitary}) = 0.66 \\ (\frac{0.011}{0.007})^2(\text{var})(\text{multi-mass}) = 2.3 \end{cases}$$

$$\frac{\text{DWF (48I)}}{\text{Overlap (48I)}} = \frac{(640 + 20 \times 3)}{(81 \times 32)}(\text{inver}) \times \frac{1}{5.7}(\text{time}) \times (\frac{0.075}{0.013})^2(\text{var}) = 1.7$$

Results:

Compare to the SNR of Twistedmass+Clover

48^3x96

The results of TM+C come from arXiv: 1507.04936

	m_π	N_cfg	Inverstions	Measurements	Method
Overlap	133 MeV	81	5+4+8+12=29	0.4k(80k)	SSM+LMSS
TM+C	131 MeV	96	16*(1+3*8)=400	1.5k	Sequential

With the factor 4^3x3 (points in the volume), the measurements would be 80k.

	OV, 0.91 fm	**OV, 1.14 fm**	**OV, 1.37 fm**	**TMC, 0.9fm**	**TMC, 1.08 fm**	**TMC 1.26 fm**
g_A^3	1.133(15)	1.150(25)	1.233(66)	1.158(16)	1.162(30)	1.242(57)
g_S^3	0.72(8)	0.93(17)	0.782(41)	0.55(18)	1.18(34)	2.20(54)
g_S^0**(CI)**	6.80(15)	7.23(33)	7.77(70)	6.46(27)	7.84(48)	8.93(86)
$\langle x \rangle_{u-d}$	0.214(9)	0.194(11)	0.208(27)	0.248(9)	0.218(15)	0.208(24)
$\langle x \rangle_{u+d(CI)}$	0.519(11)	0.456(15)	0.400(36)	0.645(13)	0.587(18)	0.555(63)

Cost Comparison with TM (1601.01624)

Fermion	a (fm)	Conf.	Source	Inversion	time	LMS	Error
TM (CI)	0.093	96	16	16*(8*3)	1	0	x%
Overlap (CI)	0.112	81	5	5+4+8+12	~10	25%	x%

Fermion	a (fm)	Conf.	Source	Loop	time	Error
TM (strange)	0.093	1800	100	300	1	12%
Overlap (strange)	0.112	81	32	32	~10	40%/17% (global)

CI

$$\frac{\text{TM (48)}}{\text{Overlap (48I)}} = \frac{96}{81}(\text{conf}) \times \frac{384}{29}(\text{inver}) \times \frac{1}{10}(\text{time}) \times \frac{1}{1.25}(\text{LMS}) \times \frac{x}{x}(\text{var}) = 1.3$$

DI (scalar)

$$\frac{\text{TM (48)}}{\text{Overlap (48I)}} = \frac{1800}{81}(\text{conf}) \times \frac{100+300}{32+32}(\text{inver}) \times \frac{1}{10}(\text{time}) \times \begin{cases} (\frac{0.12}{0.40})^2 (\text{var})(\text{unitary}) = 1.3 \\ (\frac{0.12}{0.17})^2 (\text{var})(\text{global}) = 6.9 \end{cases}$$

Cost Comparison with Clover for g_s^3 (1602.07737)

Fermion	a (fm)	Conf.	Source	Sink	tsep	time	LMS	Error
Clover (CI)	0.081	400	100	2	4	1	0	10%
Overlap (CI)	0.083	300	1	3	3	~ 44	8%	6%

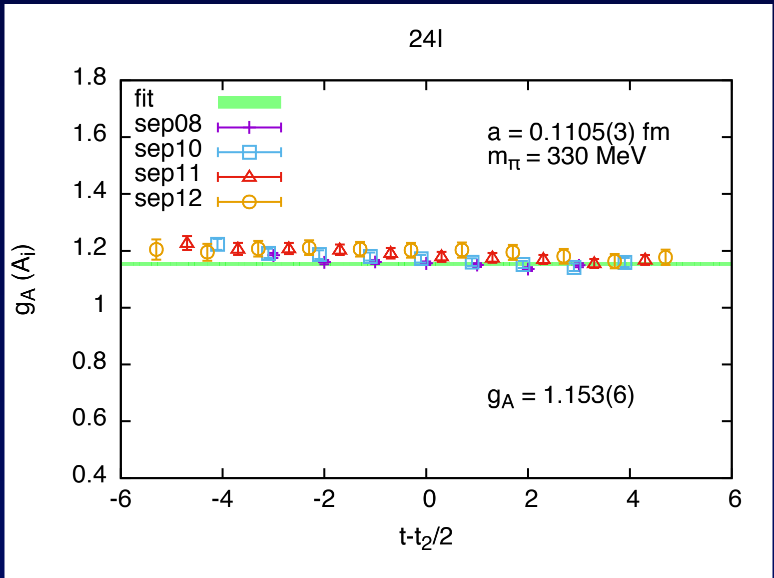
$$\frac{\text{Clover (32)}}{\text{Overlap (32I)}} = \frac{400}{300}(\text{conf})\times\frac{(49(\text{L})+4.5(\text{H}))\ (\text{total inv})}{1\times 16+(3\times 3)\times 13\ (\text{total inv})}\times\frac{1}{1.08}(\text{LMS})\times(\frac{0.10}{0.06})^2(\text{var}) = 1.4$$

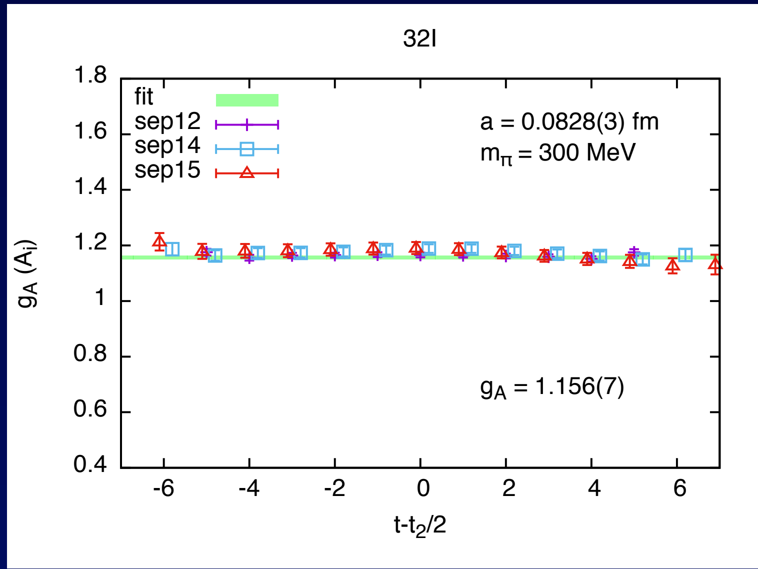
Cost Comparison with DWF for g_A (C14-08-11.3)

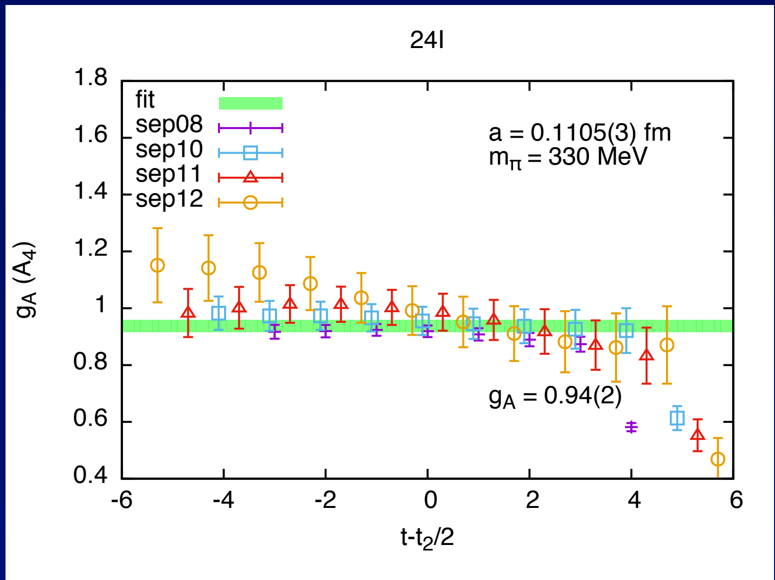
Fermion	a (fm)	Conf.	Source	Sink	tsep	time	LMS	Error
DWF (CI)	0.114	20	32+1	2	4	1	0	9.7%
Overlap (CI)	0.114	81	5	4/8/12	3	~ 5.7	25%	2.2%

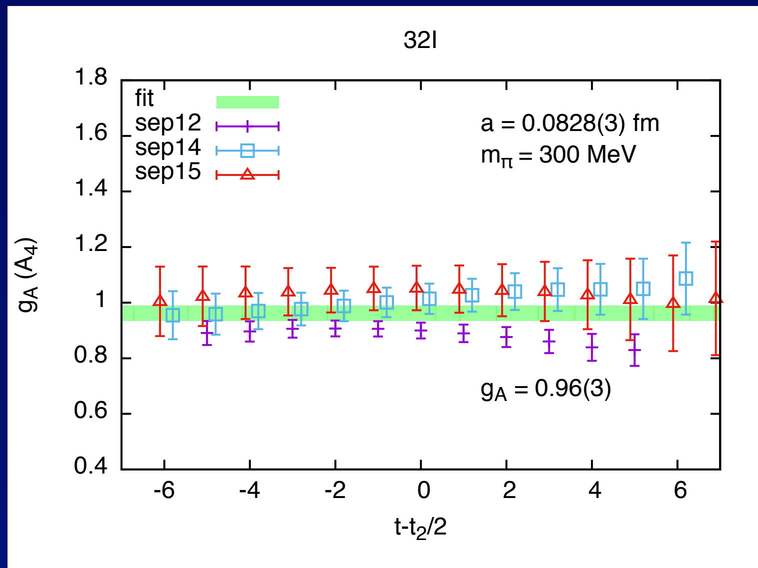
$$\frac{\text{DWF (48I)}}{\text{Overlap (48I)}} = \frac{20}{81}(\text{conf})\times\frac{(32+1*3)\ (\text{source})\times(2*4)\ (\text{sink-tsep})}{5\ (\text{source}) + (4{+}8{+}12)\ (\text{sink-tsep})}\times\frac{1}{5.7}\ (\text{time})\times\frac{1}{1.25}(\text{LMS})\times(\frac{0.097}{0.022})^2(\text{var}) = 6.5$$

An issue with isovector g_A

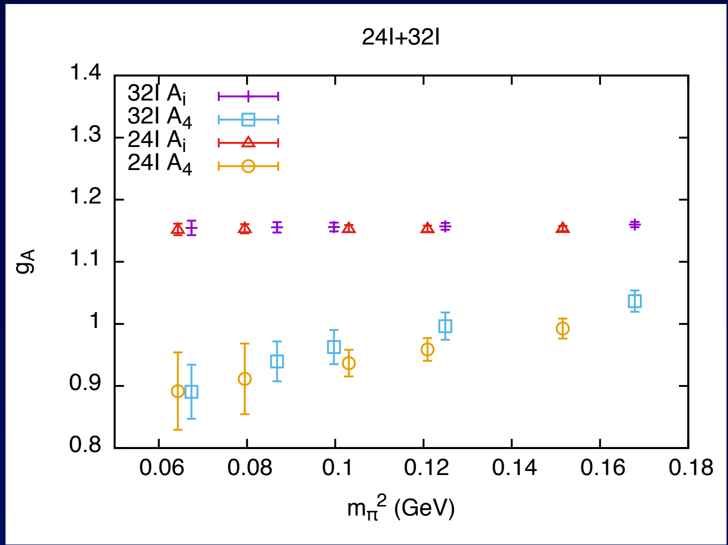


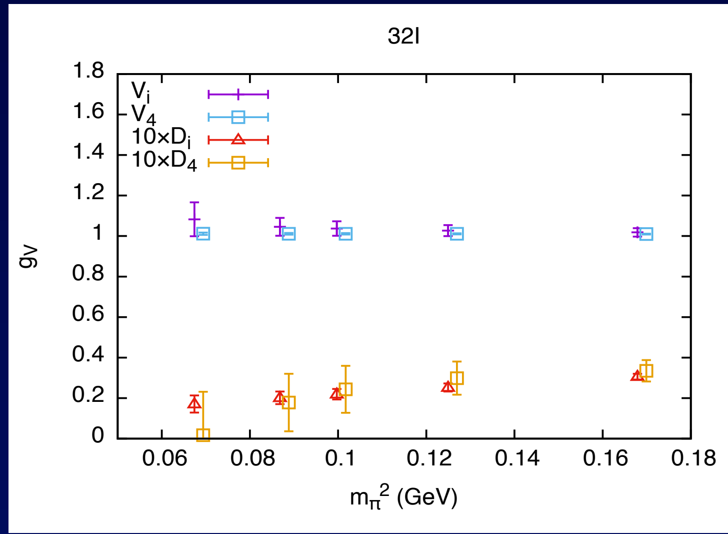


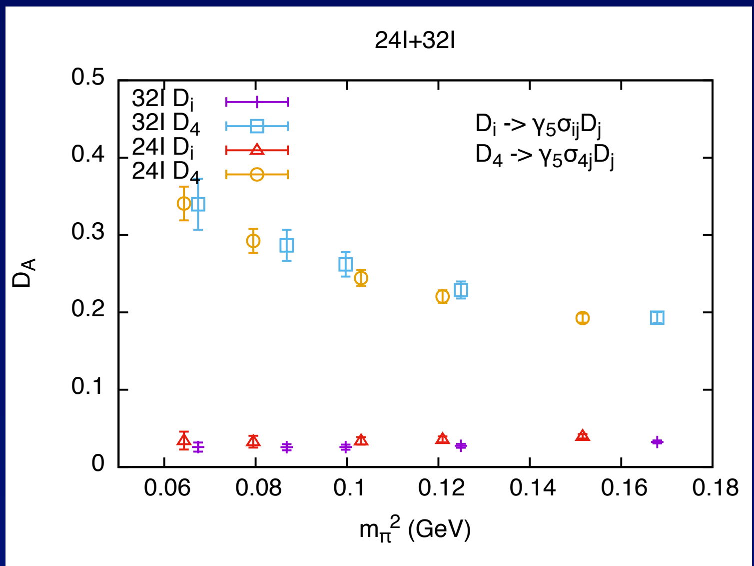




Operator improvement for g_A (J. Liang poster)







Improment with diension 4 op.

$$A_i + f\ \bar{\psi}\gamma_5\sigma_{i\mu}\overleftrightarrow{D}_\mu\psi$$

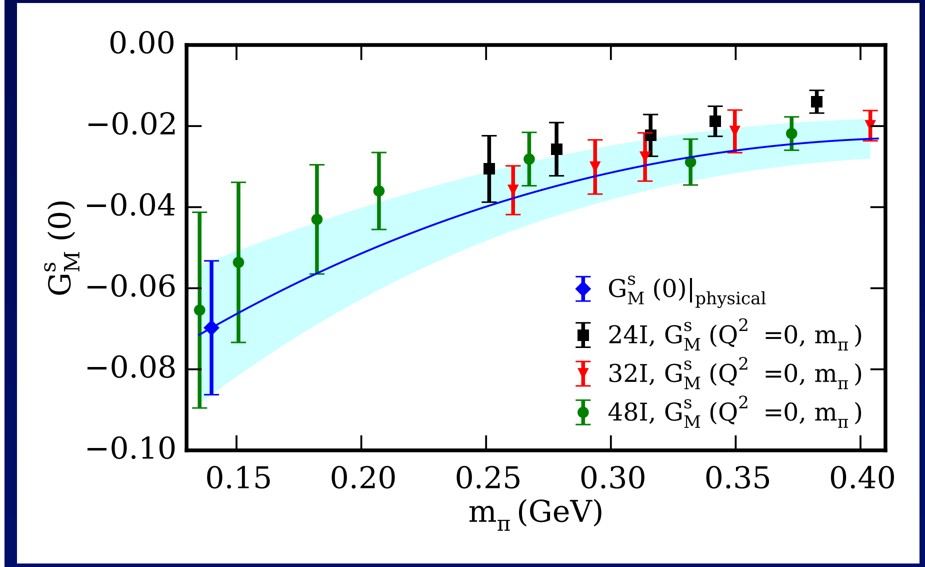
$$A_4 + f\ \bar{\psi}\sigma_{4j}\overleftrightarrow{D}_j\psi$$

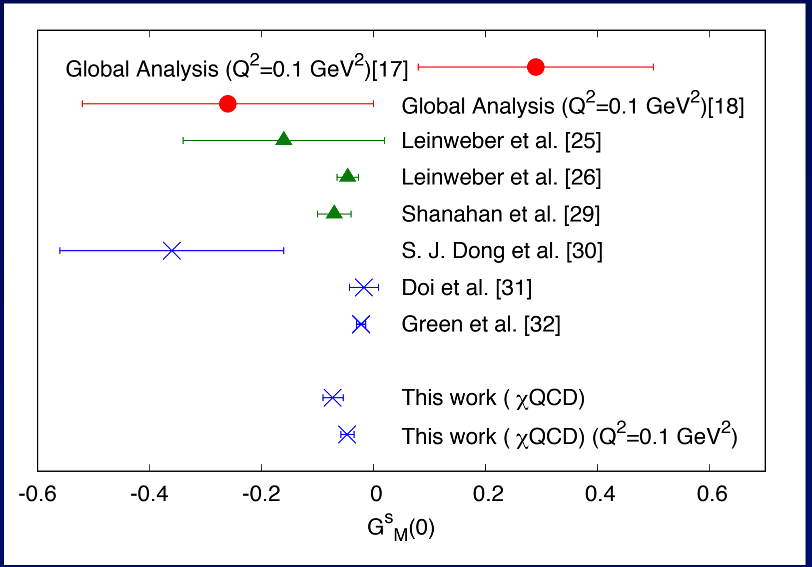
g_A(24I) = 1.153(6) → g_A(imp) = 1.188(7)
(3% increase)

g_A(32I) = 1.156(7) → g_A(imp) = 1.177(9)
(2% increase)

Strange quark magnetic Moment at the physical point (Sufian Poster)

R. Sufian et al., 1606.07075





$$G_M^s(0) = -\,0.073(17)(8)\ \mu_N$$

$$\langle r_s^2 \rangle_E = -\,0.0046(21)(10)\ \mathrm{fm}^2$$

Summary

- Except for the multi-mass with the same eigenmodes feature, which is unique to overlap, the LMS, LMA, Stochastic sandwich methods can be adopted by other fermion formulations.