Nucleon Matrix Elements at the Physical Point and Cost Comparison

- $Z_3$ smeared grid source and low mode substitution (LMS)
- Stochastic sandwich 3-pt correlators with LMS
- Comparison of cost of overlap fermions with twisted mass, clover, and domain-wall fermions.
- Isovector $g_A$

\[ \chi \text{ QCD Collaboration} \]
Overlap valence on 2+1 flavor DWF Configurations (RBC-UKQCD)

La ~ 4.6 fm  
\(m_\pi\) ~ 170 MeV  
32\(^3\) x 64, \(a =0.143\) fm

La ~ 2.7 fm  
\(m_\pi\) ~ 330 MeV  
24\(^3\) x 64, \(a =0.111\) fm

La ~ 2.7 fm  
\(m_\pi\) ~ 300 MeV  
32\(^3\) x 64, \(a =0.0828\) fm

(O(a\(^2\)) extrapolation)

La ~ 5.5 fm  
\(m_\pi\) ~ 140 MeV  
48\(^3\) x 96, \(a =0.114\) fm

La ~ 5.3 fm  
\(m_\pi\) ~ 140 MeV  
64\(^3\) x 128, \(a =0.083\) fm
Correlator improvement by exploiting special features of overlap fermion

• Eigenvectors of $D_{ov}$ (not $\gamma_5 D_{ov}$) are for all $m$ so that one do multi-mass deflation in calculating quark propagators.
• Storage: save only half of eigenvectors ($\Psi_2 = \gamma_5 \Psi_1$) and zero modes. This can be a big issue for large lattices.
• Smeared grid source with $Z_3$ noise ($\eta_i^3=1$)
• Low mode substitution to reverse the worse SNR

$$\frac{C_N(t)}{\sigma_N(t)} \approx \sqrt{n} e^{-(m_N - 3/2m_2)t}$$

• Low mode average for the quark loop is very efficient for scalar and pseudoscalar densities (> 90% saturated).
• Stochastic sandwich method for connected 3-point correlators with noise source and sink with LMS.
Nucleon with LLL and HLL substitution

\[ \langle \eta_i \eta_j \eta_k \rangle = \delta_{ij} \delta_{jk} \]

\[ \langle \eta_i^\dagger \eta_j \rangle = \delta_{ij} \]

\[ m_\pi \sim 300 \text{ MeV} \]

\[ \frac{C_N(t)}{\sigma_N(t)} \approx \sqrt{n} e^{-(m_\pi - 3/2m_\pi)t} \]

1005.5424
Nucleon Correlator

- Improvement of nucleon correlator with low-mode substitution

Point source: $m_N = 1.13(14)$ GeV;
Z$_3$ grid source: $m_N = 1.08(5)$ GeV;
Point smeared: $m_N = 1.11(5)$ GeV;
Z$_3$ grid smeared source: $m_N = 1.14(2)$ GeV;
Variation: $m_N = 1.16(1)$ GeV

$24^3 \times 64$ lattice with $m_\pi = 331$ MeV, $a = 1.73$ GeV$^{-1}$
47 configurations
Stochastic Sandwich Method for CI

- Grid smeared source with $Z_3$ noise ($4^3 \times 3$ for 48I lattice) with LMS.
- Grid noise sink for the high modes of the propagator connecting the sink and the current.
- Being a sandwich method, one sink inversion of the high modes is good for all flavor, polarization, and momentum transfer or different hadrons. This is independent of the source unlike sink sequential method.
- Has a choice of current sequential or sink sequential.
- Multi-mass is accommodated.
- Can become expensive if there are many currents and many low modes.
Simulation setup

On the $48^3 \times 96$

SL: The exact long distance part of the sequential propagator from the eigensystem of the Dirac operator.

SH: The stochastic estimate of the short distance part of the sequential propagator.

5 Z-3 noise grid sources: $4^3 \times 3$ points on $48^3 \times 96$ lattice, each point is a block smear source with size=5

- 4/8/12 noise grid sources (only for SH) at the six time slices separated to the **source** by 8a, 10a, 12a.
- No smear on the propagator while the block smear with size=5 is applied on the sink for the contraction.
$m_N(134) = 961(12) \text{ MeV}, \ m_N(\text{global}) = 950.7(7.1) \text{ MeV}$

**Cost comparison**

Clover on HISQ (1606.07049)

$$\frac{\text{Clover (a09m130)}}{\text{Overlap (48I)}} = \frac{(84768 + 7064 \times 3)}{(81 \times 32)} \times \frac{(\text{inver})}{1} \times \frac{(\text{time})}{44} \times \left[ \frac{0.011^2}{0.013} \right] \times \left[ \frac{0.011}{0.007} \right] \times \left( \frac{\text{(var)(unitary)}}{\text{(var)(multi-mass)}} \right) = 0.66$$

$$\frac{\text{DWF (48I)}}{\text{Overlap (48I)}} = \frac{(640 + 20 \times 3)}{(81 \times 32)} \times \frac{(\text{inver})}{1} \times \frac{(\text{time})}{5.7} \times \frac{(0.075)}{(0.013)} \times (\text{var}) = 1.7$$
Results:

Compare to the SNR of Twistedmass+Clover

\[48^3 \times 96\]

\[\text{The results of TM+C come from arXiv: 1507.04936}\]

<table>
<thead>
<tr>
<th>Method</th>
<th>(m_T)</th>
<th>(N_{cfg})</th>
<th>Inverstions</th>
<th>Measurements</th>
<th>(\text{Method})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overlap</td>
<td>133 MeV</td>
<td>81</td>
<td>5+4+8+12=29</td>
<td>0.4k(80k)</td>
<td>SSM+LMSS</td>
</tr>
<tr>
<td>TM+C</td>
<td>131 MeV</td>
<td>96</td>
<td>16*(1+3*8)=400</td>
<td>1.5k</td>
<td>Sequential</td>
</tr>
</tbody>
</table>

With the factor \(4^3 \times 3\) (points in the volume), the measurements would be 80k.

<table>
<thead>
<tr>
<th>(g_A^3)</th>
<th>OV, 0.91 fm</th>
<th>OV, 1.14 fm</th>
<th>OV, 1.37 fm</th>
<th>TMC, 0.9fm</th>
<th>TMC, 1.08 fm</th>
<th>TMC 1.26 fm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.133(15)</td>
<td>1.150(25)</td>
<td>1.233(66)</td>
<td>1.158(16)</td>
<td>1.162(30)</td>
<td>1.242(57)</td>
</tr>
<tr>
<td>(g_S^3)</td>
<td>0.72(8)</td>
<td>0.93(17)</td>
<td>0.782(41)</td>
<td>0.55(18)</td>
<td>1.18(34)</td>
<td>2.20(54)</td>
</tr>
<tr>
<td>(g_S^0(\text{CI}))</td>
<td>6.80(15)</td>
<td>7.23(33)</td>
<td>7.77(70)</td>
<td>6.46(27)</td>
<td>7.84(48)</td>
<td>8.93(86)</td>
</tr>
<tr>
<td>(&lt;X&gt;_{u-d})</td>
<td>0.214(9)</td>
<td>0.194(11)</td>
<td>0.208(27)</td>
<td>0.248(9)</td>
<td>0.218(15)</td>
<td>0.208(24)</td>
</tr>
<tr>
<td>(&lt;X&gt;_{u+d(\text{CI})})</td>
<td>0.519(11)</td>
<td>0.456(15)</td>
<td>0.400(36)</td>
<td>0.645(13)</td>
<td>0.587(18)</td>
<td>0.555(63)</td>
</tr>
</tbody>
</table>
## Cost Comparison with TM (1601.01624)

<table>
<thead>
<tr>
<th>Fermion</th>
<th>a (fm)</th>
<th>Conf.</th>
<th>Source</th>
<th>Inversion</th>
<th>time</th>
<th>LMS</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM (CI)</td>
<td>0.093</td>
<td>96</td>
<td>16</td>
<td>16*(8*3)</td>
<td>1</td>
<td>0</td>
<td>x%</td>
</tr>
<tr>
<td>Overlap (CI)</td>
<td>0.112</td>
<td>81</td>
<td>5</td>
<td>5+4+8+12</td>
<td>~10</td>
<td>25%</td>
<td>x%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fermion</th>
<th>a (fm)</th>
<th>Conf.</th>
<th>Source</th>
<th>Loop</th>
<th>time</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM (strange)</td>
<td>0.093</td>
<td>1800</td>
<td>100</td>
<td>300</td>
<td>1</td>
<td>12%</td>
</tr>
<tr>
<td>Overlap (strange)</td>
<td>0.112</td>
<td>81</td>
<td>32</td>
<td>32</td>
<td>~10</td>
<td>40%/17% (global)</td>
</tr>
</tbody>
</table>

\[
\text{CI} = \frac{\text{TM (48)} \times \text{Conf.} \times \text{inver} \times \frac{1}{\text{time}} \times \frac{1}{\text{LMS}} \times \frac{x}{x}}{\text{Overlap (48I)}} = \frac{96 \times 384}{81 \times 29} \times \frac{1}{10} \times \frac{1}{1.25} \times \frac{x}{x} = 1.3
\]

\[
\text{DI (scalar)} = \frac{\text{TM (48)} \times \text{conf} \times \text{inver} \times \frac{1}{\text{time}}}{\text{Overlap (48I)}} = \frac{1800 \times 100 + 300}{81 \times 32 + 32} \times \frac{1}{10} \times \left\{ \frac{0.12}{0.40} \right\}^2 (\text{var}(\text{unitary})) = 1.3
\]

\[
\text{DI (scalar)} = \frac{\text{TM (48)} \times \text{conf} \times \text{inver} \times \frac{1}{\text{time}}}{\text{Overlap (48I)}} = \left\{ \frac{0.12}{0.17} \right\}^2 (\text{var}(\text{global})) = 6.9
\]
## Cost Comparison with Clover for $g_s^3$ (1602.07737)

<table>
<thead>
<tr>
<th>Fermion</th>
<th>a (fm)</th>
<th>Conf.</th>
<th>Source</th>
<th>Sink</th>
<th>tsep</th>
<th>time</th>
<th>LMS</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clover (CI)</td>
<td>0.081</td>
<td>400</td>
<td>100</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>10%</td>
</tr>
<tr>
<td>Overlap (CI)</td>
<td>0.083</td>
<td>300</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>~ 44</td>
<td>8%</td>
<td>6%</td>
</tr>
</tbody>
</table>

\[
\frac{\text{Clover (32)}}{\text{Overlap (32I)}} = \frac{400}{300} \times \frac{49(L) + 4.5(H)}{1 \times 16 + (3 \times 3) \times 13} \times \frac{1}{1.08} \times \frac{0.10}{0.06} \times 1.4 = 1.4
\]

## Cost Comparison with DWF for $g_A$ (C14-08-11.3)

<table>
<thead>
<tr>
<th>Fermion</th>
<th>a (fm)</th>
<th>Conf.</th>
<th>Source</th>
<th>Sink</th>
<th>tsep</th>
<th>time</th>
<th>LMS</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>DWF (CI)</td>
<td>0.114</td>
<td>20</td>
<td>32+1</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>9.7%</td>
</tr>
<tr>
<td>Overlap (CI)</td>
<td>0.114</td>
<td>81</td>
<td>5</td>
<td>4/8/12</td>
<td>3</td>
<td>~ 5.7</td>
<td>25%</td>
<td>2.2%</td>
</tr>
</tbody>
</table>

\[
\frac{\text{DWF (48I)}}{\text{Overlap (48I)}} = \frac{20}{81} \times \frac{(32 + 1 \times 3) \times (2 \times 4)}{5 \times (4+8+12)} \times \frac{1}{5.7} \times \frac{1}{1.25} \times \frac{0.097}{0.022} \times 6.5 = 6.5
\]
An issue with isovector $g_A$
Operator improvement for $g_A$ (J. Liang poster)

Improvement with dimension 4 op.

\[ A_i + f \bar{\psi} \gamma_5 \sigma_{i \mu} \bar{D}_\mu \psi \]

\[ A_4 + f \bar{\psi} \sigma_{4j} \bar{D}_j \psi \]

\[ g_A(24I) = 1.153(6) \rightarrow g_A(\text{imp}) = 1.188(7) \]

(3% increase)

\[ g_A(32I) = 1.156(7) \rightarrow g_A(\text{imp}) = 1.177(9) \]

(2% increase)
Strange quark magnetic Moment at the physical point (Sufian Poster)

R. Sufian et al., 1606.07075

\[ G_M^s(0) = -0.073(17)(8) \mu_N \]

\[ \langle r_s^2 \rangle_E = -0.0046(21)(10) \text{ fm}^2 \]
Summary

• Except for the multi-mass with the same eigenmodes feature, which is unique to overlap, the LMS, LMA, Stochastic sandwich methods can be adopted by other fermion formulations.