O(N) model with Nienhuis action

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- action 'optimized' for simple loop formulation ($\leftrightarrow$ worm)
- investigate transfer matrix and Monte Carlo
- result: 'effectively' in the right universality class

O(N) model

partition function for standard lattice action:

$$Z = \int \left[\prod_z d\mu(s(z)) \right] e^{\beta \sum_{l=\langle x\, y \rangle} s(x) \cdot s(y)}, \quad s \in S_{N-1}$$

Nienhuis truncation (same universality class?!):

$$Z = \int \left[\prod_z d\mu(s(z)) \right] \prod_l \left[1 + \tilde{\beta} s(x) \cdot s(y) \right]$$

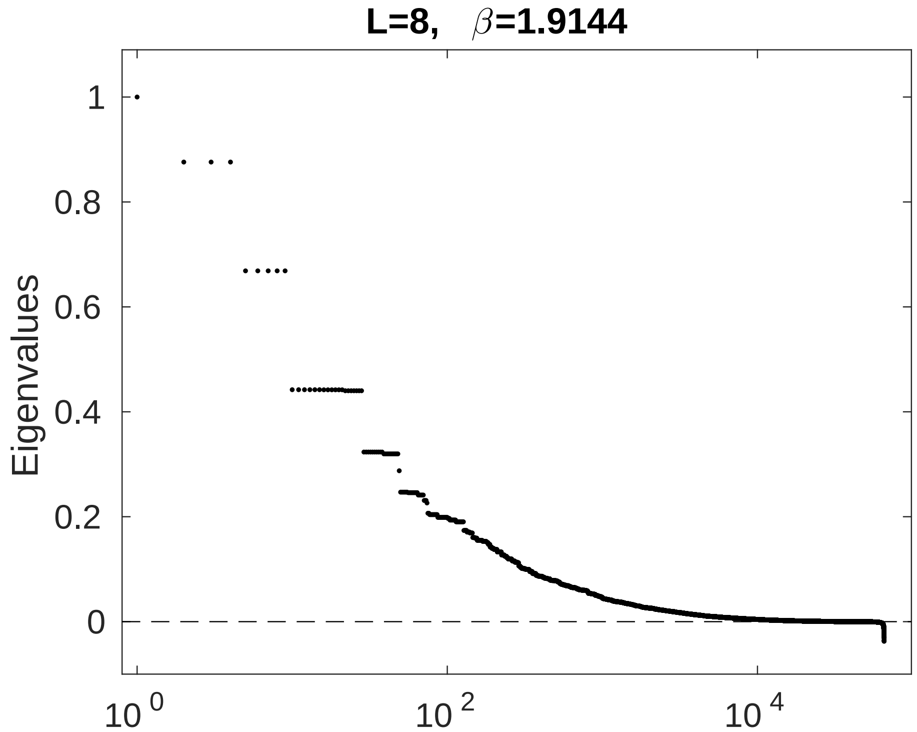
- exactly solved in $D = 2$ for $-2 \leqslant N \leqslant 2$ on honeycomb lattice
  - in these cases critical region covered for $\tilde{\beta} \leqslant 1$
- includes XY, Kosterlitz Thouless
- Ising, $N = 1$: equivalent for $\tilde{\beta} = \tanh(\beta)$
- $N = 3$ will require $\tilde{\beta} > 1 \Rightarrow$ serious sign problem for spins, not for worm
- truncation desirable: simpler worm simulation

Transfer matrix

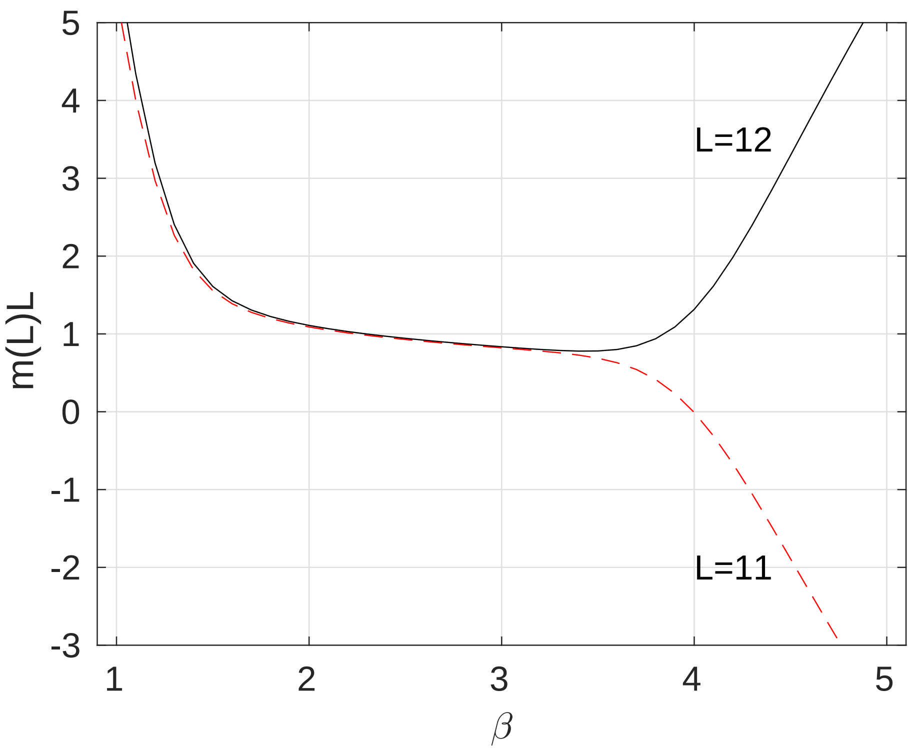
$$\mathbb{T}[s', s] = \mathbb{T}_0[s', s]\, \mathbb{T}_1[s]$$

$$\mathbb{T}_0[s', s] = \prod_{\vec{z}} \big(1 + \tilde{\beta} s'(\vec{z}\,) \cdot s(\vec{z}\,)\big), \qquad \mathbb{T}_1[s] = \prod_{\langle \vec{x}\, \vec{y}\,\rangle} \big(1 + \tilde{\beta} s(\vec{x}\,) \cdot s(\vec{y}\,)\big)$$

- $\mathbb{T}$ acts on $\psi[s]$ defined on a row of spins, L, periodic
- $\mathbb{T}_0$: finite dimensional projection, $\dim = (1+N)^L$
- $\Rightarrow$ build finite matrix, find spectrum [possible: $L \leqslant 14$]
- $\exists \mathbb{K}, \quad \mathbb{K}\,\mathbb{T}\,\mathbb{K}^{-1} =$ real symmetric
- eigenvalues real, $\lambda < 0$ possible
- two-step matrix $\mathbb{T}^2$: real, positive



spectrum for $\bar{g}^2 = m(L)L = 1.0595$



coupling vs. $\tilde{\beta}$ for $L = \text{odd}, \text{even}$

- $L = \text{odd}, m < 0$: triplet becomes groundstate!
- $L = \text{even}$: minimal $\bar{g}^2$ reached
- standard action: $\bar{g}^2 = 1/\beta + c(L)/\beta^2 + \ldots \searrow 0$ as $\beta \to \infty$ PT

Worm Simulation

start from:

$$\mathcal{Z} = \sum_{u,v,c} \rho^{-1}(u-v) \int Ds \prod_{l=\langle xy\rangle} \left[1 + \tilde{\beta}\, s(x)\cdot s(y)\right] s_c(u)\, s_c(v)$$

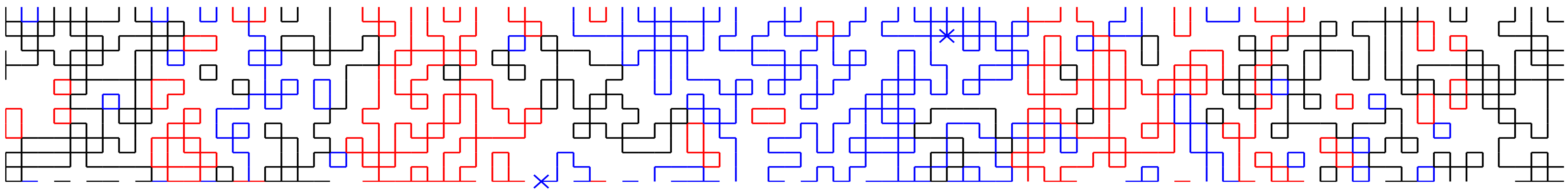
- $k_l = 0$ or $1 \ldots N$ on each link for term $1 (k=0)$ or $\tilde{\beta} s_k(x) s_k(y)$

$$\mathcal{Z} = \sum_{u,v,c,\{k\}} \rho^{-1}(u-v)\, \tilde{\beta}^{\sum_l (1-\delta_{k(l),0})} \prod_z C[q(z)]$$

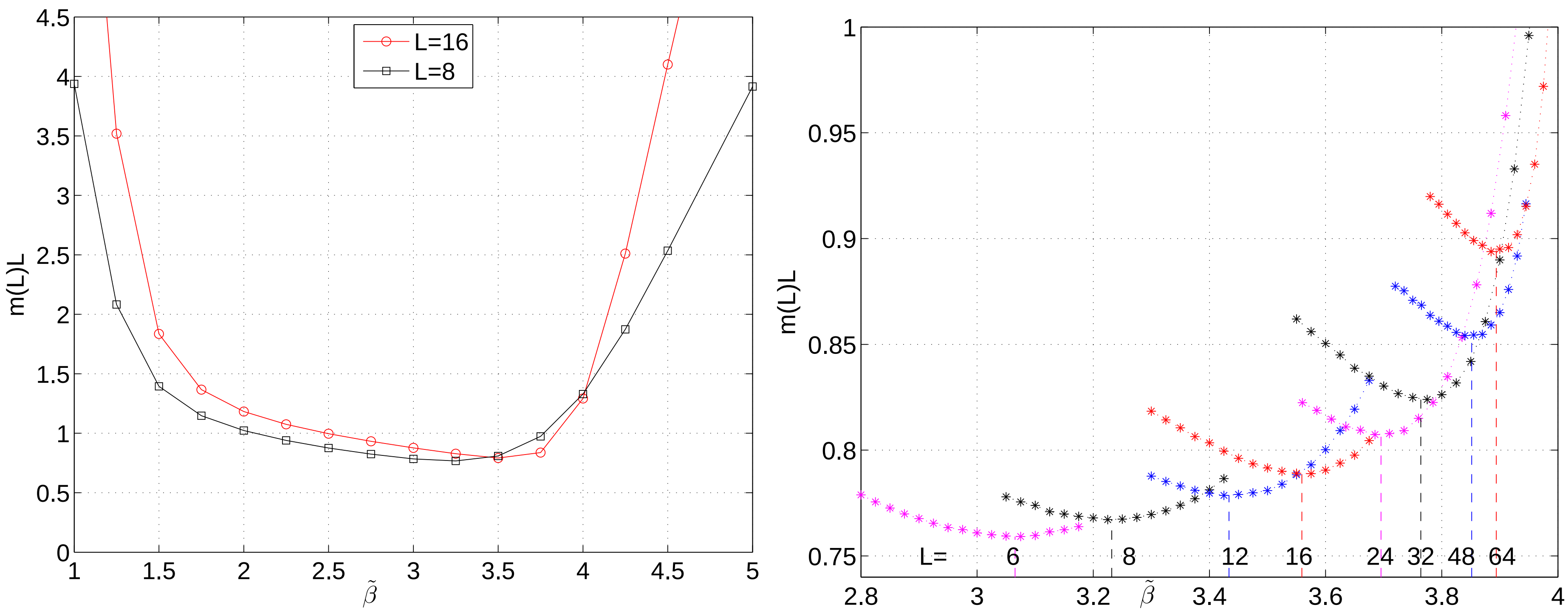
with

$$q_a(z) = \sum_{l,\partial l \ni z} k_a(l) + \delta_{a,c}(\delta_{z,u} + \delta_{z,v}), \qquad C[q] = \ldots.$$

$N = 3$(red,blue,black), $L = 12$, $T = 96$, $\tilde{\beta} = 2.6804$, $\bar{g}^2 = 0.9$

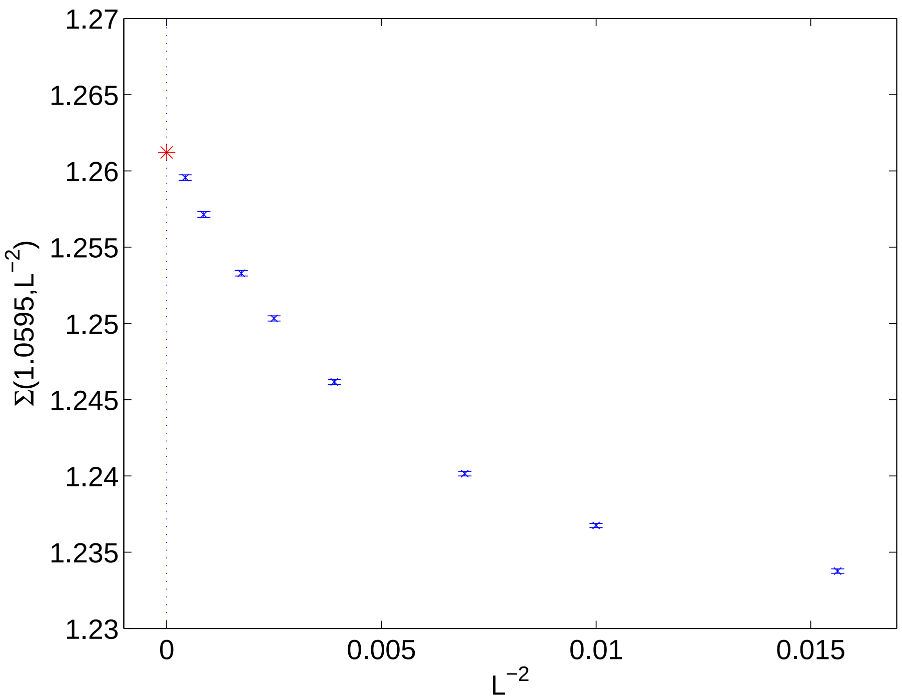


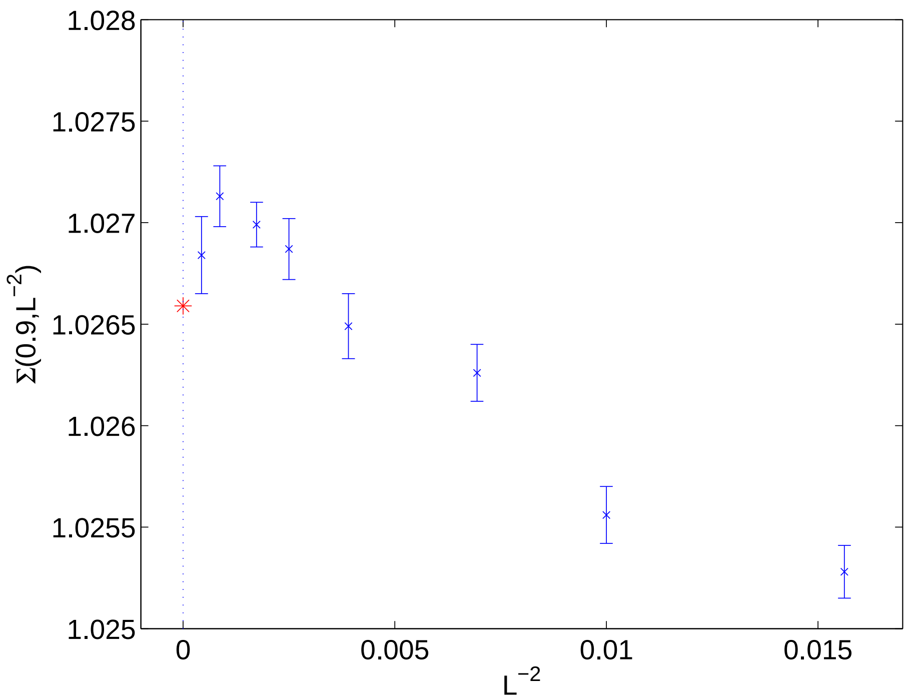
Monte Carlo Results



- $\tilde{\beta} > 1$ clearly required $\rightarrow$ sign problem in $s(x)$, not $k_a(l)$
- falling branches: $g^2(2L) > g^2(L)$, sign for as. freedom
- minima rising with L

Measuring SSFs $[\Sigma(u, L^{-2}) = \bar{g}^2(2L)_{\bar{g}^2(L)=u}]$





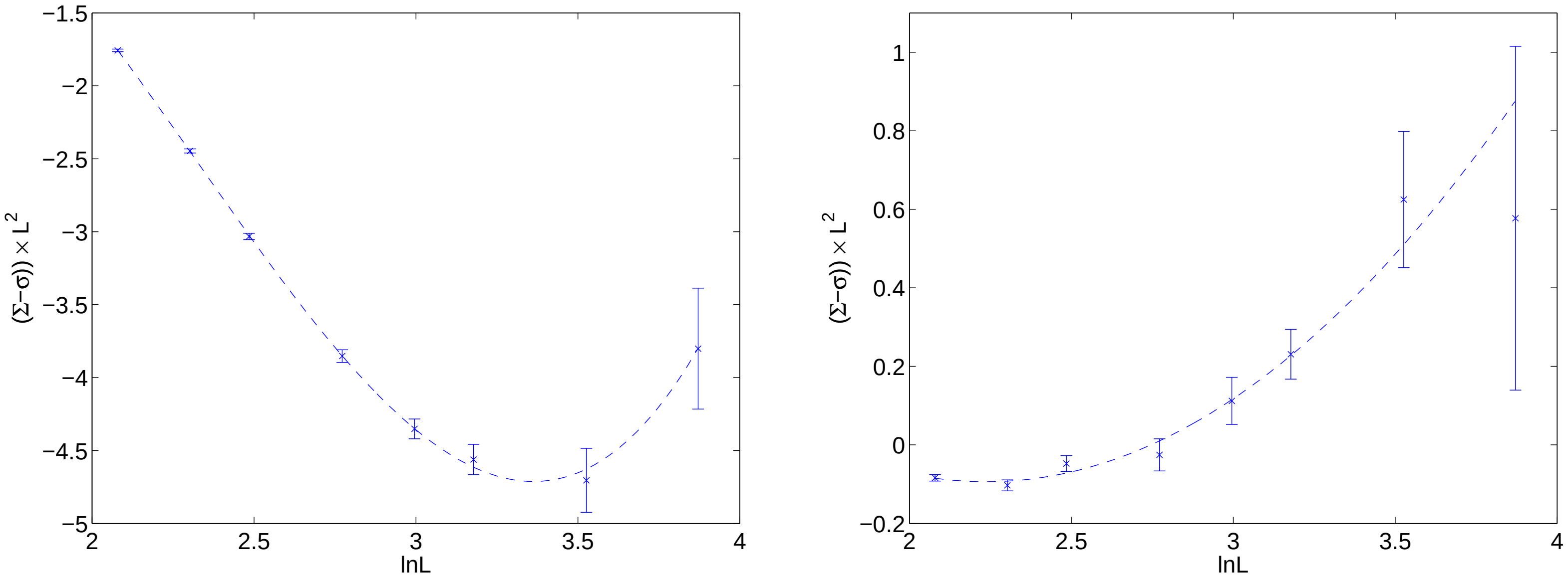
'traditional' $u = 1.0595$ $\qquad$ $u = 0.9$ close to min for $L = 48$

- run up to $L = 48 \rightarrow 2L = 96$, $*$ = exact
- data seem to 'know' $*$, deviations small
- cut-off effects look non-monotonic at our precision

Balog, Niedermayer, Weisz:

$$\Sigma - \sigma = \frac{1}{L^2}[A \ln^3 L + B \ln^2 L + C \ln L + D]$$

for a wide class of actions not including Nienhuis



$A = 0.97, B = -6.8, C = 13, D = -7.3$ $A = 0, B = 0.36, C = -1.6, D = 1.7$

- reasonable fits, competing logs

Conclusions

- for a given $\bar{g}^2$ only $(L/a) \leqslant c(\bar{g}^2)$ can be realized
  - $\rightarrow$ no complete continuum limit $a/L \rightarrow 0$
  - larger L/a possible for larger $\bar{g}^2$ $(c' > 0)$ (all $\bar{g}^2$??)
- perturbatively small $\bar{g}^2$ not reached; $mL \rightarrow \infty$ okay?
- BUT: for $\bar{g}^2$=1.0595, 0.9
  we got very close to exact answer up to tiny cutoff effects
- small $\bar{g}^2$ via ODD L: totally off...

- effective theory?
- analogue: triviality in 4d $\lambda\varphi^4$: for given $\lambda_R > 0$
  no complete limit $am_R \rightarrow 0$
- opposite to here: smaller λ_R allows smaller am_R