E.M.T. renormalization constants with the Wilson Flow

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renormalized E.M.T. on the lattice:why?

- Thermodynamic quantities

$$\langle \epsilon - 3p \rangle_T = -\left\langle \hat{\mathcal{T}}_{\mu\mu} \right\rangle_T \quad ; \quad \langle s \rangle_T = \left(-\left\langle \hat{\mathcal{T}}_{00} \right\rangle_T + \sum_{i=1}^{3} \left\langle \hat{\mathcal{T}}_{ii} \right\rangle_T \right) / T$$

- Transport coefficients

$$\eta = \pi \lim_{\omega \to 0} \mathrm{Im} \left\{ \left[i \int_0^\infty dt e^{i\omega t} \int d^3x \left\langle \hat{\mathcal{T}}_{12}(t,x) \hat{\mathcal{T}}^{12}(0,0) \right\rangle_T \right] \right\}$$

- Study of conformal field theories ($\langle \mathcal{T}_\mu^\mu(x) \rangle$ as order parameter)

E.M.T. IN YANG MILLS THEORY.

Two different strategies based on Wilson Flow

- T.W.I. for probe observables at positive flow time
  Del Debbio, Patella, Rago, arXiv:1306.1173 [hep-th]
- Small flow time expansion
  Suzuki, arXiv:1304.0533 [hep-lat]
  Asakawa, Hatsuda, Iritani, Itou, Kitazawa, Suzuki, arXiv:1412.4508 [hep-lat]

Other strategies

- Shifted Boundary Conditions
  Robaina, Meyer, arXiv:1310.6075 [hep-lat]
  Giusti, Pepe, arXiv:1503.07042 [hep-lat]
  Giusti, Meyer, arXiv:1310.7818 [hep-lat]

FULLY RENORMALIZED EMT

A possible way for probing T.W.I. using flowed observables have been proposed by Del Debbio, Patella and Rago.
Define $V(t, y)$ built with gauge fields evolved according to the Wilson Flow, then

$$\langle \partial_\mu T_{\mu\nu}(x) V(t, y) \rangle = -\langle \delta_{x,\nu} V(t, y) \rangle$$

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2. No contact terms ($x \to y$) arise
3. $[\delta_{x,\nu} V(t,y)]_R = Z_\delta \delta_{x,\nu} V(t,y)$, where Z_δ is finite and probe independent.

SETUP

1. SU(3) Yang Mills theory using OQCD
2. hypercubic lattice: spatial periodic B.C, O-SF along temporal direction (thanks to L.Giusti and M.Pepe for the values of β)

L/a	12	16	24	32
β	5.8506	6.0056	6.2670	6.4822
N_{meas}	96000	51048	2136	6700

MIXING

$$[\hat{T}_{\mu\nu}]_R = Z_1 [T^{[1]}_{\mu\nu} - \langle T^{[1]}_{\mu\nu} \rangle_0] + Z_3 T^{[3]}_{\mu\nu} + Z_6 T^{[6]}_{\mu\nu}$$

$$\hat{T}^{[1]}_{\mu\nu} = -\delta_{\mu\nu} \frac{1}{2g_0^2} \sum_{\sigma\tau} \mathrm{tr}\, \hat{F}_{\tau\sigma} \hat{F}_{\tau\sigma}$$

$$\hat{T}^{[3]}_{\mu\nu} = -\delta_{\mu\nu} \frac{2}{g_0^2} \sum_{\sigma} \mathrm{tr} \left\{ \hat{F}_{\sigma\mu} \hat{F}_{\sigma\mu} - \frac{1}{4} \sum_{\tau} \hat{F}_{\sigma\tau} \hat{F}_{\sigma\tau} \right\}$$

$$\hat{T}^{[6]}_{\mu\nu} = -(1 - \delta_{\mu\nu}) \frac{2}{g_0^2} \sum_{\sigma} \mathrm{tr}\, \hat{F}_{\mu\sigma} \hat{F}_{\nu\sigma}$$

Our method: Z_δ

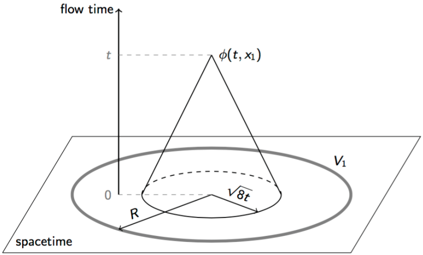
$\phi(t, x_1) = \frac{1}{L^3}\sum_{i=1}^{3} W_i(t, L_0/2)$:

exponentially suppressed corrections:

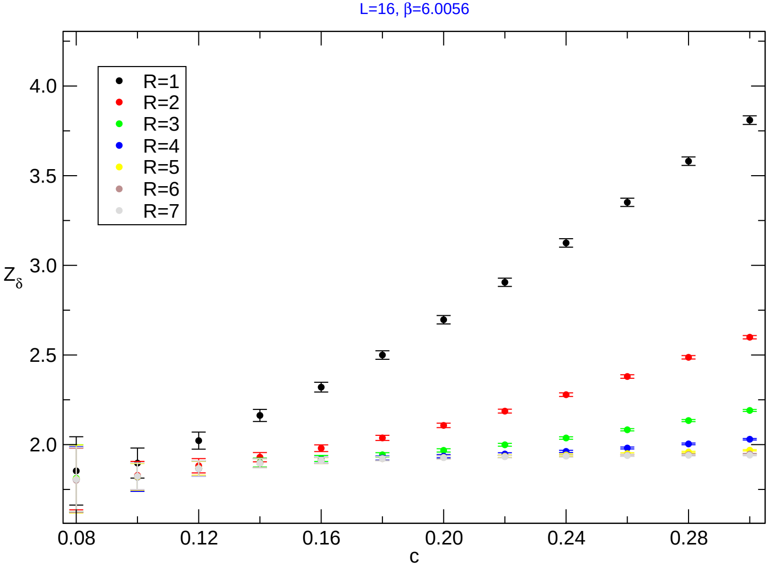
$$Z_\delta \left\langle \sum_{y\in V_1} \hat{\delta}_{y,0}\phi(t, x_1)\right\rangle = \left\langle \hat{\partial}_0\phi(t, x_1)\right\rangle + \mathcal{O}\left(\exp\left[-\frac{R^2}{16t}\right]\right)$$

sphere centered around the probe $\phi(t, x_1)$

$\neq 0$ only using proper boundary conditions : open-SF along $\mu = 0$



Results: Z_δ



Our method: Z_β / Z_δ

Use translational invariance

$$\left\langle [T_{\mu\nu}]_R(x)\hat{\partial}_\mu V_\nu^{[\alpha]}(t,y)\right\rangle = Z_\delta \left\langle \hat{\delta}_{x,\nu} V_\nu^{[\alpha]}(t,y)\right\rangle$$

Use translational invariance

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Choose $V_\mu(t,x)$

$$V_\mu^{[\alpha]}(t,x) = \partial_\nu T_{\nu\mu}^{[\alpha]}(t,x) \quad ; \quad \alpha = 1,3,6$$

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Next step: expand $[T_{\nu\mu}]_R$

$$\sum_{\beta=1,3,6} \left\langle \hat{T}_{\mu\nu}^{[\beta]}(y) \, \hat{\partial}_\mu V_\nu^{[\alpha]}(t,x) \right\rangle \frac{Z_\beta}{Z_\delta} = \left\langle \hat{\delta}_{y,\nu} V_\nu^{[\alpha]}(t,x)\right\rangle$$

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$$y = x = (\vec{x}, L_0/2)$$

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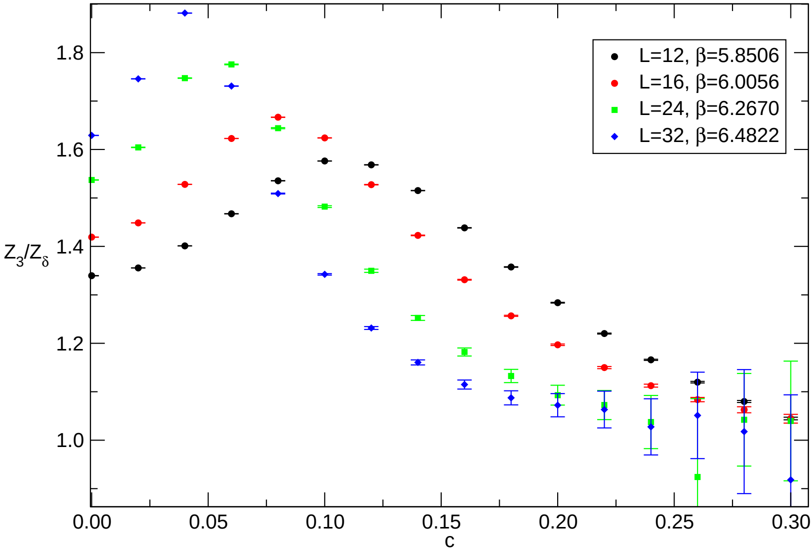
$$y = x = (\vec{x}, L_0/2)$$

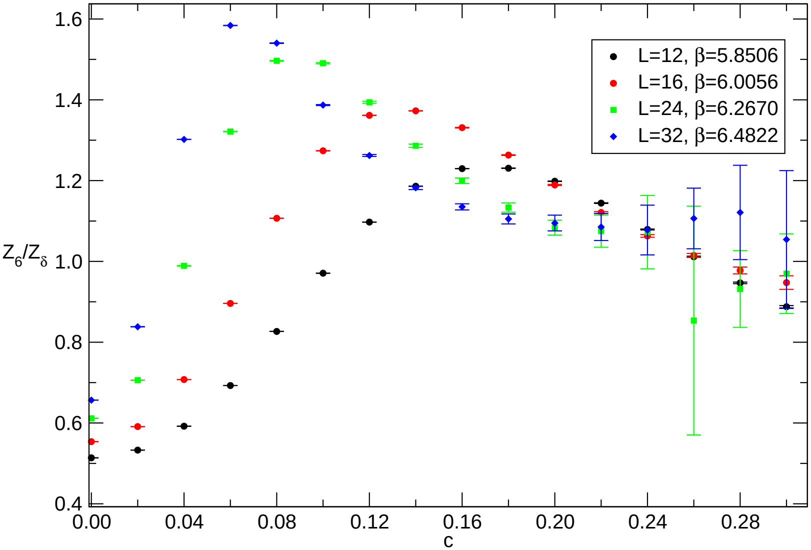
$$\sum_{\beta=1,3,6} M_{\alpha\beta} \frac{Z_\beta}{Z_\delta} = v^\alpha$$

3 equations for 3 coefficients.

Results: Z_3/Z_δ



Results: Z_6 / Z_δ

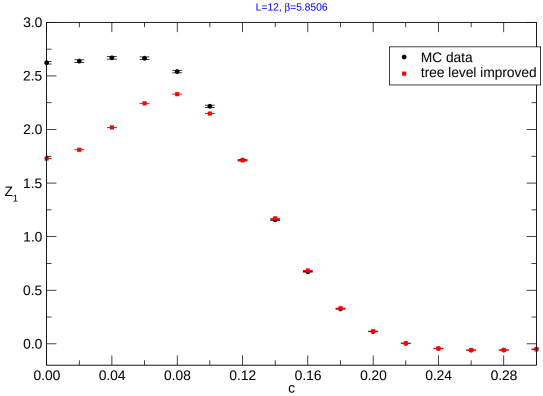


Our method

tree level improvement

$$\sum_{\beta=1,3,6}\left[\sum_{\rho\sigma}\langle \hat{T}^{[\beta]}_{\mu\rho}(x)\partial_\mu O^{[\alpha]}_{t,\rho}(x)\rangle\right] Z_\beta - Z_\delta \sum_{\rho}\langle \hat{\delta}_{x,\rho} O^{[\alpha]}_{t,\rho}(x)\rangle =$$

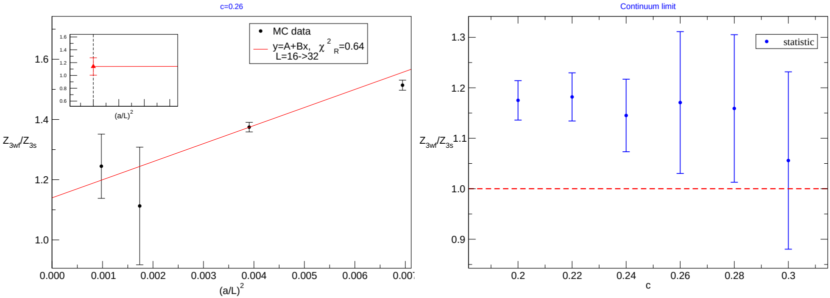
$$= \sum_{\beta=3,6}\left[\sum_{\rho\mu}\langle \hat{T}^{[\beta]}_{\mu\rho}(x)\partial_\mu O^{[\alpha]}_{t,\rho}(x)\rangle_{\text{tree}}\right] - \sum_{\rho}\langle \hat{\delta}_{x,\rho} O^{[\alpha]}_{t,\rho}(x)\rangle_{\text{tree}}$$



No improvements for $c > 0.1$

Preliminary check

Every *c* define a set of renormalization constants: each set can be compared with other available methods (shifted boundary conditions arXiv:1310.7818 [hep-lat])



CONCLUSION

- The renormalization constants of the E.M.T. can be measured probing TWI with observables at positive flow time.
- To obtain statistically reliable results, lots of measures are needed, especially for $0.2 < c < 0.3$: this require time.
- Probably a finer lattice would be helpful for making comparisons with other available methods, but it will be also computationally quite expensive.
- With the adopted setup, tree level improvements do not seem to reduce lattice artefacts in the region of interest of c.

Outlook

$$\left\langle \partial_\mu T_{\mu\nu}(x) V_\nu^{[\alpha]}(t,y) \right\rangle = -\left\langle \delta_{x,\nu} V_\nu^{[\alpha]}(t,y) \right\rangle$$

Use translational invariance and the following identity

$$\partial_\nu V_\nu^{[\alpha]}(t,y) = \int d^4z \left[\delta_{z,\nu} V_\nu^{[\alpha]}(t,y)\right]$$

to obtain

$$\left\langle \delta_{y,\nu} V_\nu^{[\alpha]}(t,x) \right\rangle = \left\langle T_{\mu\nu}(y) \int d^4z \left[\delta_{z,\mu} V_\nu^{[\alpha]}(t,x)\right] \right\rangle$$

Additional divergences arise: Z_δ cannot be factored out

Old approach

$$\left\langle \partial_\mu T_{\mu\nu}(x) V_\nu^{[\alpha]}(t,y) \right\rangle = - \left\langle \delta_{x,\nu} V_\nu^{[\alpha]}(t,y) \right\rangle$$

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